Systems of Equations and Inequalities; Counting Methods

Andrew Gloag, (AndrewG)
Anne Gloag, (AnneG)
Melissa Kramer, (MelissaK)
CK12 Editor

Say Thanks to the Authors
Click http://www.ck12.org/saythanks
(No sign in required)
James is trying to expand his pastry business to include cupcakes and personal cakes. He has a limited amount of manpower to decorate the new items and a limited amount of material to make the new cakes. In this chapter, you will learn how to solve this type of situation.

Every equation and inequality you have studied thus far is an example of a system. A system is a set of equations or inequalities with the same variables. This chapter focuses on the methods used to solve a system such as graphing, substitution and elimination. You will combine your knowledge of graphing inequalities to solve a system of inequalities.
In a previous chapter, you learned that the intersection of two sets is joined by the word “and.” This word also joins two or more equations or inequalities. A set of algebraic sentences joined by the word “and” is called a system.

The solution(s) to a system is the set of ordered pairs that is in common to each algebraic sentence.

**Example 1:** Determine which of the points (1, 3), (0, 2), or (2, 7) is a solution to the following system of equations.

\[
\begin{align*}
  y &= 4x - 1 \\
  y &= 2x + 3
\end{align*}
\]

**Solution:** A solution to a system is an ordered pair that is in common to all the algebraic sentences. To determine if a particular ordered pair is a solution, substitute the coordinates for the variables \(x\) and \(y\) in each sentence and check.

Check (1, 3) :
\[
\begin{align*}
  3 &= 4(1) - 1; \quad 3 = 3. \text{Yes, this ordered pairs checks.} \\
  3 &= 2(1) + 3; \quad 3 = 5. \text{No, this ordered pair does not check.}
\end{align*}
\]

Check (0, 2) :
\[
\begin{align*}
  2 &= 4(0) - 1; \quad 2 = -1. \text{No, this ordered pair does not check.} \\
  2 &= 2(0) + 3; \quad 2 = 3. \text{No, this ordered pair does not check.}
\end{align*}
\]

Check (2, 7) :
\[
\begin{align*}
  7 &= 4(2) - 1; \quad 7 = 7. \text{Yes, this ordered pairs checks.} \\
  7 &= 2(2) + 3; \quad 7 = 7. \text{Yes, this ordered pairs checks.}
\end{align*}
\]

Because the coordinate (2, 7) works in both equations simultaneously, it is a solution to the system.

To determine the coordinate that is in common to each sentence in the system, each equation can be graphed. The point at which the lines intersect represents the solution to the system. The solution can be written two ways:

- As an ordered pair, such as (2, 7)
- By writing the value of each variable, such as \(x = 2, y = 7\)

**Example:** Find the solution to the system \[\begin{align*}
  y &= 3x - 5 \\
  y &= -2x + 5
\end{align*}\].

**Solution:** By graphing each equation and finding the point of intersection, you find the solution to the system. Each equation is written in slope-intercept form and can be graphed using the methods learned in Chapter 4. The lines appear to intersect at the ordered pair (2, 1). Is this the solution to the system?
The coordinate checks in both sentences. Therefore, (2, 1) is a solution to the system \[
\begin{align*}
y &= 3x - 5 \\
y &= -2x + 5
\end{align*}
\].

**Example 2:** Solve the system \[
\begin{align*}
x + y &= 2 \\
y &= 3
\end{align*}
\].

**Solution:** The first equation is written in standard form. Using its intercepts will be the easiest way to graph this line.

The second equation is a horizontal line three units up from the origin.

The lines appear to intersect at (–1, 3).

\[
\begin{align*}
-1 + 3 &= 2; 
2 &= 2 \\
3 &= 3
\end{align*}
\]

The coordinate is in common to each sentence and is a solution to the system.

The greatest strength of the graphing method is that it offers a very visual representation of a system of equations and its solution. You can see, however, that determining a solution from a graph would require very careful graphing of the lines and is really practical only when you are certain that the solution gives integer values for \(x\) and \(y\). In most cases, this method can offer only approximate solutions to systems of equations. For exact solutions, other methods are necessary.

---

**Solving Systems Using a Graphing Calculator**

A graphing calculator can be used to find or check solutions to a system of equations. To solve a system graphically, you must graph the two lines on the same coordinate axes and find the point of intersection. You can use a graphing calculator to graph the lines as an alternative to graphing the equations by hand.

Using the system from the above example, \[
\begin{align*}
y &= 3x - 5 \\
y &= -2x + 5
\end{align*}
\], we will use the graphing calculator to find the approximate solutions to the system.
1.1. Linear Systems by Graphing

Begin by entering the equations into the $Y =$ menu of the calculator.

You already know the solution to the system is (2, 1). The window needs to be adjusted so an accurate picture is seen. Change your window to the default window.

```
WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
Xres=1
```

See the graphs by pressing the GRAPH button.

The solution to a system is the intersection of the equations. To find the intersection using a graphing calculator, locate the Calculate menu by pressing 2nd and TRACE. Choose option #5 – INTERSECTION.

```
CALCULATE
1:value
2:zero
3:minimum
4:maximum
5:intersect
6:dy/dx
7:int f(x)dx
```

The calculator will ask you “First Curve?” Hit ENTER. The calculator will automatically jump to the other curve and ask you “Second Curve?” Hit ENTER. The calculator will ask, “Guess?” Hit ENTER. The intersection will appear at the bottom of the screen.

Example: Peter and Nadia like to race each other. Peter can run at a speed of 5 feet per second and Nadia can run at a speed of 6 feet per second. To be a good sport, Nadia likes to give Peter a head start of 20 feet. How long does Nadia take to catch up with Peter? At what distance from the start does Nadia catch up with Peter?
Solution: Begin by translating each runner’s situation into an algebraic sentence using \( distance = rate \times time \).

Peter: \( d = 5t + 20 \)

Nadia: \( d = 6t \)

The question asks when Nadia catches Peter. The solution is the point of intersection of the two lines. Graph each equation and find the intersection.

![Graph showing the intersection of two lines]

The two lines cross at the coordinate \( t = 20, \ d = 120 \). This means after 20 seconds Nadia will catch Peter. At this time, they will be at a distance of 120 feet. Any time after 20 seconds Nadia will be farther from the starting line than Peter.

**Practice Set**

Sample explanations for some of the practice exercises below are available by viewing the following video. Note that there is not always a match between the number of the practice exercise in the video and the number of the practice exercise listed in the following exercise set. However, the practice exercise is the same in both.

**CK-12 Basic Algebra: Solving Linear Systems by Graphing (8:30)**

1. Define a system.
2. What is the solution to a system?
3. Explain the process of solving a system by graphing.
4. What is one problem with using a graph to solve a system?
5. What are the two main ways to write the solution to a system of equations?
6. Suppose Horatio says the solution to a system is \( (4, -6) \). What does this mean visually?
7. Where is the “Intersection” command located in your graphing calculator? What does it do?
8. In the race example, who is farther from the starting line at 19.99 seconds? At 20.002 seconds?
Determine which ordered pair satisfies the system of linear equations.

9. \[
\begin{align*}
y & = 3x - 2 \\
y & = -x
\end{align*}
\]
\{(1, 4), (2, 9), \left(\frac{1}{2}, -\frac{1}{2}\right)\}

10. \[
\begin{align*}
y & = 2x - 3 \\
y & = x + 5
\end{align*}
\]
\{(8, 13), (-7, 6), (0, 4)\}

11. \[
\begin{align*}
2x + y & = 8 \\
5x + 2y & = 10
\end{align*}
\]
\{(-9, 1), (-6, 20), (14, 2)\}

12. \[
\begin{align*}
3x + 2y & = 6 \\
y & = \frac{x}{2} - 3
\end{align*}
\]
\{(3, -\frac{3}{2}), (-4, 3), \left(\frac{1}{2}, 4\right)\}

In 13 – 22, solve the following systems by graphing.

13. \[
\begin{align*}
y & = x + 3 \\
y & = -x + 3
\end{align*}
\]

14. \[
\begin{align*}
y & = 3x - 6 \\
y & = -x + 6
\end{align*}
\]

15. \[
\begin{align*}
2x & = 4 \\
y & = -3
\end{align*}
\]

16. \[
\begin{align*}
y & = -x + 5 \\
-x + y & = 1
\end{align*}
\]

17. \[
\begin{align*}
x + 2y & = 8 \\
5x + 2y & = 0
\end{align*}
\]

18. \[
\begin{align*}
3x + 2y & = 12 \\
4x - y & = 5
\end{align*}
\]

19. \[
\begin{align*}
5x + 2y & = -4 \\
x - y & = 2
\end{align*}
\]

20. \[
\begin{align*}
2x + 4 & = 3y \\
x - 2y + 4 & = 0
\end{align*}
\]

21. \[
\begin{align*}
y & = \frac{x}{2} - 3 \\
2x - 5y & = 5
\end{align*}
\]

22. \[
\begin{align*}
y & = 4 \\
x & = 8 - 3y
\end{align*}
\]

23. Mary’s car is 10 years old and has a problem. The repair man indicates that it will cost her $1200.00 to repair her car. She can purchase a different, more efficient car for $4,500.00. Her present car averages about $2,000.00 per year for gas while the new car would average about $1,500.00 per year. Find the number of years for when the total cost of repair would equal the total cost of replacement.

24. Juan is considering two cell phone plans. The first company charges $120.00 for the phone and $30 per month for the calling plan that Juan wants. The second company charges $40.00 for the same phone, but charges $45 per month for the calling plan that Juan wants. After how many months would the total cost of the two plans be the same?

25. A tortoise and hare decide to race 30 feet. The hare, being much faster, decided to give the tortoise a head start of 20 feet. The tortoise runs at 0.5 feet/sec and the hare runs at 5.5 feet per second. How long will it be until the hare catches the tortoise?

**Mixed Review**

26. Solve for \(h\): \(25 \geq |2h + 5|\).

27. Subtract \(\frac{4}{3} - \frac{1}{2}\).

28. You write the letters to ILLINOIS on separate pieces of paper and place them into a hat.
a. Find \( P(\text{drawing an } I) \).

b. Find the odds for drawing an \( L \).

29. Graph \( x < 2 \) on a number line and on a Cartesian plane.

30. Give an example of an ordered pair in quadrant II.

31. The data below show the average life expectancy in the United States for various years.

   a. Use the method of interpolation to find the average life expectancy in 1943.
   
   b. Use the method of extrapolation to find the average life expectancy in 2000.
   
   c. Find an equation for the line of best fit. How do the predictions of this model compare to your answers in questions a) and b)?

**Table 1.1: U. S. Life Expectancy at Birth**

<table>
<thead>
<tr>
<th>Birth Year</th>
<th>Female</th>
<th>Male</th>
<th>Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>1940</td>
<td>65.2</td>
<td>60.8</td>
<td>62.9</td>
</tr>
<tr>
<td>1950</td>
<td>71.1</td>
<td>65.6</td>
<td>68.2</td>
</tr>
<tr>
<td>1960</td>
<td>73.1</td>
<td>66.6</td>
<td>69.7</td>
</tr>
<tr>
<td>1970</td>
<td>74.7</td>
<td>67.1</td>
<td>70.8</td>
</tr>
<tr>
<td>1975</td>
<td>76.6</td>
<td>68.6</td>
<td>72.6</td>
</tr>
<tr>
<td>1980</td>
<td>77.5</td>
<td>70.0</td>
<td>73.7</td>
</tr>
<tr>
<td>1985</td>
<td>78.2</td>
<td>71.2</td>
<td>74.7</td>
</tr>
<tr>
<td>1990</td>
<td>78.8</td>
<td>71.8</td>
<td>75.4</td>
</tr>
<tr>
<td>1995</td>
<td>78.9</td>
<td>72.5</td>
<td>75.8</td>
</tr>
<tr>
<td>1998</td>
<td>79.4</td>
<td>73.9</td>
<td>76.7</td>
</tr>
</tbody>
</table>
1.2 Solving Systems by Substitution

While the graphical approach to solving systems is helpful, it may not always provide exact answers. Therefore, we will learn a second method to solving systems. This method uses the **Substitution Property of Equality**.

**Substitution Property of Equality**: If \( y = \text{an algebraic expression} \), then the algebraic expression can be substituted for any \( y \) in an equation or an inequality.

Consider the racing example from the previous lesson.

Peter and Nadia like to race each other. Peter can run at a speed of 5 feet per second and Nadia can run at a speed of 6 feet per second. To be a good sport, Nadia likes to give Peter a head start of 20 feet. How long does Nadia take to catch up with Peter? At what distance from the start does Nadia catch up with Peter?

The two racers' information was translated into two equations.

Peter: \( d = 5t + 20 \)

Nadia: \( d = 6t \)

We want to know when the two racers will be the same distance from the start. This means we can set the two equations equal to each other.

\[
5t + 20 = 6t
\]

Now solve for \( t \).

\[
5t - 5t + 20 = 6t - 5t
\]
\[
20 = t
\]

After 20 seconds, Nadia will catch Peter.

Now we need to determine how far from the distance the two runners are. You already know \( 20 = t \), so we will substitute to determine the distance. Using either equation, substitute the known value for \( t \) and find \( d \).

\[
d = 5(20) + 20 \rightarrow 120
\]

When Nadia catches Peter, the runners are 120 feet from the starting line.

The **Substitution Method** is useful when one equation of the system is of the form \( y = \text{algebraic expression} \) or \( x = \text{algebraic expression} \).
Example 1: Solve the system \[
\begin{align*}
  x + y &= 2 \\
  y &= 3
\end{align*}
\]

Solution: The second equation is solved for the variable \(x\). Therefore, we can substitute the value “3” for any \(y\) in the system.

\[
x + y = 2 \rightarrow x + 3 = 2
\]

Now solve the equation for \(x\):

\[
x + 3 - 3 = 2 - 3 \\
x = -1
\]

The \(x\)–coordinate of the intersection of these two equations is \(-1\). Now we must find the \(y\)–coordinate using substitution.

\[
x + y = 2 \rightarrow (-1) + y = 2 \\
-1 + 1 + y = 2 + 1 \\
y = 3
\]

As seen in the previous lesson, the solution to the system is \((-1, 3)\).

Example: Find the solution to the system \[
\begin{align*}
  y &= 3x - 5 \\
  y &= -2x + 5
\end{align*}
\] using substitution.

Solution: Each equation is equal to the variable \(y\), therefore the two algebraic expressions must equal each other.

\[
3x - 5 = -2x + 5
\]

Solve for \(x\).

\[
3x - 5 + 5 = -2x + 5 + 5 \\
3x + 2x = -2x + 2x + 10 \\
5x = 10 \\
x = 2
\]

The \(x\)–coordinate of the intersection of the two lines is 2. Now you must find the \(y\)–coordinate using either of the two equations.

\[
y = -2(2) + 5 = 1
\]

The solution to the system is \(x = 2, \ y = 1\) or \((2, 1)\).
Solving Real-World Systems by Substitution

Example: Anne is trying to choose between two phone plans. Vendafone’s plan costs $20 per month, with calls costing an additional 25 cents per minute. Sellnet’s plan charges $40 per month, but calls cost only 8 cents per minute. Which should she choose?

Solution: Anne’s choice will depend upon how many minutes of calls she expects to use each month. We start by writing two equations for the cost in dollars in terms of the minutes used. Since the number of minutes is the independent variable, it will be our $x$. Cost is dependent on minutes. The cost per month is the dependent variable and will be assigned $y$.

For Vendafone: $y = 0.25x + 20$

For Sellnet: $y = 0.08x + 40$

By graphing two equations, we can see that at some point the two plans will charge the same amount, represented by the intersection of the two lines. Before this point, Sellnet’s plan is more expensive. After the intersection, Sellnet’s plan is cheaper.

Use substitution to find the point that the two plans are the same. Each algebraic expression is equal to $y$, so they must equal each other.

\[
0.25x + 20 = 0.08x + 40
\]

Subtract 20 from both sides.

\[
0.25x = 0.08x + 20
\]

Subtract 0.08x from both sides.

\[
0.17x = 20
\]

Divide both sides by 0.17.

\[
x = 117.65\text{ minutes}
\]

Rounded to two decimal places.

We can now use our sketch, plus this information, to provide an answer. If Anne will use 117 minutes or fewer every month, she should choose Vendafone. If she plans on using 118 or more minutes, she should choose Sellnet.

Mixture Problems

Systems of equations arise in chemistry when mixing chemicals in solutions and can even be seen in things like mixing nuts and raisins or examining the change in your pocket!

By rearranging one sentence in an equation into $y = \text{algebraic expression}$ or $x = \text{algebraic expression}$, you can use the Substitution Method to solve the system.
Example: *Nadia empties her purse and finds that it contains only nickels and dimes. If she has a total of 7 coins and they have a combined value of 55 cents, how many of each coin does she have?*

Solution: Begin by choosing appropriate variables for the unknown quantities. Let \( n = \text{the number of nickels} \) and \( d = \text{the number of dimes} \).

There are seven coins in Nadia’s purse: \( n + d = 7 \).

The total is 55 cents: \( 0.05n + 0.10d = 0.55 \).

The system is:

\[
\begin{align*}
    n + d &= 7 \\
    0.05n + 0.10d &= 0.55
\end{align*}
\]

We can quickly rearrange the first equation to isolate \( d \), the number of dimes: \( d = 7 - n \).

Using the Substitution Property, every \( d \) can be replaced with the expression \( 7 - n \).

\[
0.05n + 0.10(7 - n) = 0.55
\]

Now solve for \( n \):

\[
\begin{align*}
    0.05n + 0.70 - 0.10n &= 0.55 & \text{Distributive Property} \\
    -0.05n + 0.70 &= 0.55 & \text{Add like terms.} \\
    -0.05n &= -0.15 & \text{Subtract 0.70.} \\
    n &= 3 & \text{Divide by } -0.05.
\end{align*}
\]

Nadia has 3 nickels. There are seven coins in the purse; three are nickels so four must be dimes.

Check to make sure this combination is 55 cents: \( 0.05(3) + 0.10(4) = 0.15 + 0.40 = 0.55 \).

---

**Chemical Mixtures**

Example: *A chemist has two containers, Mixture A and Mixture B. Mixture A has a 60% copper sulfate concentration. Mixture B has a 5% copper sulfate concentration. The chemist needs to have a mixture equaling 500 mL with a 15% concentration. How much of each mixture does the chemist need?*
1.2. Solving Systems by Substitution

Solution: Although not explicitly stated, there are two equations involved in this situation.

- Begin by stating the variables. Let \( A = \text{mixture A and} \ B = \text{mixture B} \).
- The total mixture needs to have 500 mL of liquid.

Equation 1 (how much total liquid): \( A + B = 500 \).

- The total amount of copper sulfate needs to be 15\% of the total amount of solution (500 mL). \( 0.15 \cdot 500 = 75 \text{ ounces} \)

Equation 2 (how much copper sulfate the chemist needs): \( 0.60A + 0.05B = 75 \)

\[
\begin{align*}
A + B &= 500 \\
0.60A + 0.05B &= 75 
\end{align*}
\]

By rewriting equation 1, the Substitution Property can be used: \( A = 500 - B \).
Substitute the expression \( 500 - B \) for the variable \( A \) in the second equation.

\[
0.60(500 - B) + 0.05B = 75
\]

Solve for \( B \).

\[
\begin{align*}
300 - 0.60B + 0.05B &= 75 \\
300 - 0.55B &= 75 \\
-0.55B &= -225 \\
B &\approx 409 \text{ mL}
\end{align*}
\]

The chemist needs approximately 409 mL of mixture \( B \). To find the amount of mixture \( A \), use the first equation: \( A + 409 = 500 \)

\[
A = 91 \text{ mL}
\]

The chemist needs 91 milliliters of mixture \( A \) and 409 milliliters of mixture \( B \) to get a 500 mL solution with a 15\% copper sulfate concentration.
Practice Set

Sample explanations for some of the practice exercises below are available by viewing the following video. Note that there is not always a match between the number of the practice exercise in the video and the number of the practice exercise listed in the following exercise set. However, the practice exercise is the same in both.

CK-12 Basic Algebra: Solving Linear Systems by Substitution (9:21)

1. Explain the process of solving a system using the Substitution Property.
2. Which systems are easier to solve using substitution?

Solve the following systems. Remember to find the value for both variables!

3. \[
\begin{align*}
  y &= -3 \\
  6x - 2y &= 0
\end{align*}
\]

4. \[
\begin{align*}
  -3 - 3y &= 6 \\
  y &= -3x + 4
\end{align*}
\]

5. \[
\begin{align*}
  y &= 3x + 16 \\
  y &= x + 8
\end{align*}
\]

6. \[
\begin{align*}
  y &= -6x - 3 \\
  y &= 3
\end{align*}
\]

7. \[
\begin{align*}
  y &= -2x + 5 \\
  y &= -1 - 8x
\end{align*}
\]

8. \[
\begin{align*}
  y &= 6 + x \\
  y &= -2x - 15
\end{align*}
\]

9. \[
\begin{align*}
  y &= -2 \\
  y &= 5x - 17
\end{align*}
\]

10. \[
\begin{align*}
  x + y &= 5 \\
  3x + y &= 15
\end{align*}
\]

11. \[
\begin{align*}
  12y - 3x &= -1 \\
  x - 4y &= 1
\end{align*}
\]

12. \[
\begin{align*}
  x + 2y &= 9 \\
  3x + 5y &= 20
\end{align*}
\]

13. \[
\begin{align*}
  x - 3y &= 10 \\
  2x + y &= 13
\end{align*}
\]

14. Solve the system \[
\begin{align*}
  y &= \frac{1}{3}x - 14 \\
  y &= \frac{19}{3}x + 7
\end{align*}
\] by graphing and substitution. Which method do you prefer? Why?

15. Of the two non-right angles in a right angled triangle, one measures twice that of the other. What are the angles?
16. The sum of two numbers is 70. They differ by 11. What are the numbers?

17. A rectangular field is enclosed by a fence on three sides and a wall on the fourth side. The total length of the fence is 320 yards. If the field has a total perimeter of 400 yards, what are the dimensions of the field?

18. A ray cuts a line forming two angles. The difference between the two angles is 18°. What does each angle measure?

19. I have $15.00 and wish to buy five pounds of mixed nuts for a party. Peanuts cost $2.20 per pound. Cashews cost $4.70 per pound. How many pounds of each should I buy?

20. A chemistry experiment calls for one liter of sulfuric acid at a 15% concentration, but the supply room only stocks sulfuric acid in concentrations of 10% and in 35%. How many liters of each should be mixed to give the acid needed for the experiment?

21. Bachelle wants to know the density of her bracelet, which is a mix of gold and silver. Density is total mass divided by total volume. The density of gold is 19.3 g/cc and the density of silver is 10.5 g/cc. The jeweler told her that the volume of silver used was 10 cc and the volume of gold used was 20 cc. Find the combined density of her bracelet.

22. Jeffrey wants to make jam. He needs a combination of raspberries and blackberries totaling six pounds. He can afford $11.60. How many pounds of each berry should he buy?

Mixed Review

23. The area of a square is 96 inches². Find the length of a square exactly.

24. The volume of a sphere is $V = \frac{4}{3}\pi r^3$, where $r = radius$. Find the volume of a sphere with a diameter of 11 centimeters.

25. Find:
   a. the additive inverse and
   b. the multiplicative inverse of 7.6.

26. Solve for $x$: $\frac{12}{x} = 6$.

27. The temperature in Fahrenheit can be approximated by crickets using the rule “Count the number of cricket chirps in 15 seconds and add 40.”
   a. What is the domain of this function?
   b. What is the range?
c. Would you expect to hear any crickets at 32°F? Explain your answer.

d. How many chirps would you hear if the temperature were 57°C?

28. Is 4.5 a solution to 45 - 6x ≤ 18?
1.3 Solving Linear Systems by Addition or Subtraction

As you noticed in the previous lesson, solving a system algebraically will give you the most accurate answer and in some cases, it is easier than graphing. However, you also noticed that it took some work in several cases to rewrite one equation before you could use the Substitution Property. There is another method used to solve systems algebraically: the elimination method.

The purpose of the elimination method to solve a system is to cancel, or eliminate, a variable by either adding or subtracting the two equations. This method works well if both equations are in standard form.

**Example 1:** If one apple plus one banana costs $1.25 and one apple plus two bananas costs $2.00, how much does it cost for one banana? One apple?

**Solution:** Begin by defining the variables of the situation. Let \(a\) = the number of apples and \(b\) = the number of bananas. By translating each purchase into an equation, you get the following system:

\[
\begin{align*}
    a + b &= 1.25 \\
    a + 2b &= 2.00
\end{align*}
\]

You could rewrite the first equation and use the Substitution Property here, but because both equations are in standard form, you can also use the elimination method.

Notice that each equation has the value 1a. If you were to subtract these equations, what would happen?

\[
\begin{align*}
    a + b &= 1.25 \\
    -(a + 2b &= 2.00) \\
    -b &= -0.75 \\
    b &= 0.75
\end{align*}
\]

Therefore, one banana costs $0.75, or 75 cents. By subtracting the two equations, we were able to eliminate a variable and solve for the one remaining.

How much is one apple? Use the first equation and the Substitution Property.

\[
\begin{align*}
    a + 0.75 &= 1.25 \\
    a &= 0.50 \rightarrow \text{one apple costs 50 cents}
\end{align*}
\]

**Example:** Solve the system \[
\begin{align*}
    3x + 2y &= 11 \\
    5x - 2y &= 13
\end{align*}
\]

**Solution:** These equations would take much more work to rewrite in slope-intercept form to graph or to use the Substitution Property. This tells us to try to eliminate a variable. The coefficients of the \(x\)–variables have nothing in common, so adding will not cancel the \(x\)–variable.

Looking at the \(y\)–variable, you can see the coefficients are 2 and –2. By adding these together, you get zero. Add these two equations and see what happens.
The resulting equation is $8x = 24$. Solving for $x$, you get $x = 3$. To find the $y$-coordinate, choose either equation, and substitute the number 3 for the variable $x$.

$$3(3) + 2y = 11$$
$$9 + 2y = 11$$
$$2y = 2$$
$$y = 1$$

The point of intersection of these two equations is $(3, 1)$.

Example: Andrew is paddling his canoe down a fast-moving river. Paddling downstream he travels at 7 miles per hour, relative to the river bank. Paddling upstream, he moves slower, traveling at 1.5 miles per hour. If he paddles equally hard in both directions, calculate, in miles per hour, the speed of the river and the speed Andrew would travel in calm water.

Solution: We have two unknowns to solve for, so we will call the speed that Andrew paddles at $x$, and the speed of the river $y$. When traveling downstream, Andrew’s speed is boosted by the river current, so his total speed is the canoe speed plus the speed of the river ($x + y$). Upstream, his speed is hindered by the speed of the river. His speed upstream is $(x - y)$.

Downstream Equation  \[ x + y = 7 \]
Upstream Equation  \[ x - y = 1.5 \]

Notice $y$ and $-y$ are additive inverses. If you add them together, their sum equals zero. Therefore, by adding the two equations together, the variable $y$ will cancel, leaving you to solve for the remaining variable, $x$.

\[
\begin{align*}
  x + y &= 7 \\
  + (x - y) &= 1.5 \\
  2x &= 8.5 \\
  x &= 8.5
\end{align*}
\]
Therefore, \(x = 4.25\); Andrew is paddling 4.25\(\text{miles/hour}\). To find the speed of the river, substitute your known value into either equation and solve.

\[
4.25 - y = 1.5 \\
-y = -2.75 \\
y = 2.75
\]

The stream’s current is moving at a rate of 2.75\(\text{miles/hour}\).


### Practice Set

Sample explanations for some of the practice exercises below are available by viewing the following video. Note that there is not always a match between the number of the practice exercise in the video and the number of the practice exercise listed in the following exercise set. However, the practice exercise is the same in both.

**CK-12 Basic Algebra:SolvingLinear Systems by Elimination (12:44)**

1. What is the purpose of the elimination method to solve a system? When is this method appropriate?

In 2 – 10, solve each system using elimination.

2. \[
\begin{align*}
2x + y &= -17 \\
8x - 3y &= -19
\end{align*}
\]
3. \[
\begin{align*}
x + 4y &= -9 \\
-2x - 5y &= 12
\end{align*}
\]
4. \[
\begin{align*}
-2x - 5y &= -10 \\
x + 4y &= 8
\end{align*}
\]
5. \[
\begin{align*}
x - 3y &= -10 \\
-8x + 5y &= -15
\end{align*}
\]
6. \[
\begin{align*}
x - 6y &= -18 \\
x - 6y &= -6
\end{align*}
\]
7. \[
\begin{align*}
5x - 3y &= -14 \\
x - 3y &= 2
\end{align*}
\]
8. \[
\begin{align*}
3x + 4y &= 2.5 \\
5x - 4y &= 25.5
\end{align*}
\]
9. \[5x + 7y = -31\]
\[5x - 9y = 17\]

10. \[3y - 4x = -33\]
\[5x - 3y = 40.5\]

11. Nadia and Peter visit the candy store. Nadia buys three candy bars and four fruit roll-ups for $2.84. Peter also buys three candy bars, but he can afford only one fruit roll-up. His purchase costs $1.79. What is the cost of each candy bar and each fruit roll-up?

12. A small plane flies from Los Angeles to Denver with a tail wind (the wind blows in the same direction as the plane), and an air-traffic controller reads its ground-speed (speed measured relative to the ground) at 275 miles per hour. Another identical plane moving in the opposite direction has a ground-speed of 227 miles per hour. Assuming both planes are flying with identical air-speeds, calculate the speed of the wind.

13. An airport taxi firm charges a pick-up fee, plus an additional per-mile fee for any rides taken. If a 12-mile journey costs $14.29 and a 17-mile journey costs $19.91, calculate:
   a. the pick-up fee
   b. the per-mile rate
   c. the cost of a seven-mile trip

14. Calls from a call-box are charged per minute at one rate for the first five minutes, then a different rate for each additional minute. If a seven-minute call costs $4.25 and a 12-minute call costs $5.50, find each rate.

15. A plumber and a builder were employed to fit a new bath, each working a different number of hours. The plumber earns $35 per hour, and the builder earns $28 per hour. Together they were paid $330.75, but the plumber earned $106.75 more than the builder. How many hours did each work?

16. Paul has a part-time job selling computers at a local electronics store. He earns a fixed hourly wage, but he can earn a bonus by selling warranties for the computers he sells. He works 20 hours per week. In his first week, he sold eight warranties and earned $220. In his second week, he managed to sell 13 warranties and earned $280. What is Paul’s hourly rate, and how much extra does he get for selling each warranty?

Mixed Review

17. Graph \(y = |x| - 5\).

18. Solve. Write the solution using interval notation and graph the solution on a number line: \(-9 \geq \frac{x}{3}\).

19. The area of a rectangle is 1,440 square centimeters. Its length is ten times more than its width. What are the dimensions of the rectangle?

20. Suppose \(f(x) = 8x^2 - 10\). Find \(f(-6)\).

21. Torrey is making candles from beeswax. Each taper candle needs 86 square inches and each pillar candle needs 264 square inches. Torrey has a total of 16 square feet of beeswax. Graph all the possible combinations of taper and pillar candles Torrey could make (Hint: one square foot = 144 square inches).
This chapter has provided three methods to solve systems: graphing, substitution, and elimination through addition and subtraction. As stated in each lesson, these methods have strengths and weaknesses. Below is a summary.

**Graphing**
✓ A good technique to visualize the equations and when both equations are in slope-intercept form.

- Solving a system by graphing is often imprecise and will not provide exact solutions.

**Substitution**
✓ Works particularly well when one equation is in standard form and the second equation is in slope-intercept form.
✓ Gives exact answers.

- Can be difficult to use substitution when both equations are in standard form.

**Elimination by Addition or Subtraction**
✓ Works well when both equations are in standard form and the coefficients of one variable are additive inverses.
✓ Answers will be exact.

- Can be difficult to use if one equation is in standard form and the other is in slope-intercept form.
- Addition or subtraction does not work if the coefficients of one variable are not additive inverses.

Although elimination by only addition and subtraction does not work without additive inverses, you can use the Multiplication Property of Equality and the Distributive Property to create additive inverses.

**Multiplication Property and Distributive Property:**

If \( ax + by = c \), then \( m(ax + by) = m(c) \) and \( m(ax + by) = m(c) \rightarrow (am)x + (bm)y = mc \)

While this definition box may seem complicated, it really states you can multiply the entire equation by a particular value and then use the Distributive Property to simplify. The value you are multiplying is called a **scalar**.

Example: Solve the system \[
\begin{align*}
7x + 4y &= 12 \\
5x - 2y &= 11
\end{align*}
\]

Solution: Neither variable has additive inverse coefficients. Therefore, simply adding or subtracting the two equations will not cancel either variable. However, there is a relationship between the coefficients of the \( y \)-variable.

\[4 \text{ is the additive inverse of } -2 \times (2)\]

By multiplying the second equation by the scalar 2, you will create additive inverses of \( y \). You can then add the equations.

\[
\begin{align*}
7x + 4y &= 12 \\
2(5x - 2y &= 11)
\end{align*} \quad \rightarrow \quad \begin{align*}
7x + 4y &= 12 \\
10x - 4y &= 22
\end{align*}
\]
Add the two equations. 

\[ 17x = 34 \]
\[ x = 2 \]

Divide by 17.

To find the \( y \)-value, use the Substitution Property in either equation.

\[ 7(2) + 4y = 12 \]
\[ 14 + 4y = 12 \]
\[ 4y = -2 \]
\[ y = -\frac{1}{2} \]

The solution to this system is \((2, -\frac{1}{2})\).

Example: Andrew and Anne both use the I-Haul truck rental company to move their belongings from home to the dorm rooms on the University of Chicago campus. I-Haul has a charge per day and an additional charge per mile. Andrew travels from San Diego, California, a distance of 2,060 miles in five days. Anne travels 880 miles from Norfolk, Virginia, and it takes her three days. If Anne pays $840 and Andrew pays $1,845.00, what does I-Haul charge:

a) per day?

b) per mile traveled?

Solution: Begin by writing a system of linear equations: one to represent Anne and the second to represent Andrew. Let \( x = \text{amount charged per day} \) and \( y = \text{amount charged per mile} \).

\[
\begin{align*}
3x + 880y &= 840 \\
5x + 2060y &= 1845
\end{align*}
\]

There are no relationships seen between the coefficients of the variables. Instead of multiplying one equation by a scalar, we must multiply both equations by the least common multiple.

The least common multiple is the smallest value that is divisible by two or more quantities without a remainder. Suppose we wanted to eliminate the variable \( x \) because the numbers are smaller to work with. The coefficients of \( x \) must be additive inverses of the least common multiple.

\[
\text{LCM of 3 and 5} = 15
\]

\[
\begin{align*}
-5(3x + 880y &= 840) \\
3(5x + 2060y &= 1845)
\end{align*}
\rightarrow
\begin{align*}
-15x - 4400y &= -4200 \\
15x + 6180y &= 5535
\end{align*}
\]

Adding the two equations yields:

\[ 1780y = 1335 \]
\[ y = 0.75 \]
1.4. Solving Linear Systems by Multiplication

The company charges $0.75 per mile.
To find the amount charged per day, use the Substitution Property and either equation.

\[
5x + 2060(0.75) = 1845 \\
5x + 1545 = 1845 \\
5x + 1545 - 1545 = 1845 - 1545 \\
5x = 300 \\
x = 60
\]

I-Haul charges $60.00 per day and $0.75 per mile.

**Multimedia Link:** For even more practice, we have this video. One common type of problem involving systems of equations (especially on standardized tests) is “age problems.” In the following video, the narrator shows two examples of age problems, one involving a single person and one involving two people. [KhanAcademy Age Problems](7:13)


**Practice Set**

Sample explanations for some of the practice exercises below are available by viewing the following video. Note that there is not always a match between the number of the practice exercise in the video and the number of the practice exercise listed in the following exercise set. However, the practice exercise is the same in both.

[CK-12 Basic Algebra: Solving Linear Systems by Multiplication](12:00)

Find the least common multiple of the values given.

1. 5 and 7
2. –11 and 6
3. 15 and 8
4. 7 and 12
5. 2 and 17
6. –3 and 6
7. 6 and \( \frac{1}{3} \)
8. 3 and 111
9. 9 and 14
10. 5 and –5

List the scalar needed to create the additive inverse.

11. ____multiply by 6 to create additive inverse of 12.
12. ____multiply by 5 to create additive inverse of 35.
13. ____multiply by –10 to create additive inverse of 80.
14. ____multiply by –7 to create additive inverse of 63.
15. What could you multiply 11 by to create the additive inverse of 121?
16. What scalar could you multiply 4 by to create the additive inverse of –16?

Solve the following systems using multiplication.

17. \( \begin{align*} 5x - 10y &= 15 \\ 3x - 2y &= 3 \end{align*} \)
18. \( \begin{align*} 5x - y &= 10 \\ 3x - 2y &= -1 \end{align*} \)
19. \( \begin{align*} 5x + 7y &= 15 \\ 7x - 3y &= 5 \end{align*} \)
20. \( \begin{align*} 9x + 5y &= 9 \\ 12x + 8y &= 12.8 \end{align*} \)
21. \( \begin{align*} 4x - 3y &= 1 \\ 3x - 4y &= 4 \end{align*} \)
22. \( \begin{align*} 7x - 3y &= -3 \\ 6x + 4y &= 3 \end{align*} \)

In 23 – 28, solve the systems using any method.

23. \( \begin{align*} x &= 3y \\ x - 2y &= -3 \end{align*} \)
24. \( \begin{align*} y &= 3x + 2 \\ y &= -2x + 7 \end{align*} \)
25. \( \begin{align*} 5x - 5y &= 5 \\ 5x + 5y &= 35 \end{align*} \)
26. \( \begin{align*} y &= -3x - 3 \\ 3x - 2y + 12 &= 0 \end{align*} \)
27. \( \begin{align*} 3x - 4y &= 3 \\ 4y + 5x &= 10 \end{align*} \)
28. \( \begin{align*} 9x - 2y &= -4 \\ 2x - 6y &= 1 \end{align*} \)

29. Supplementary angles are two angles whose sum is 180°. Angles A and B are supplementary angles. The measure of Angle A is 18° less than twice the measure of Angle B. Find the measure of each angle.
30. A farmer has fertilizer in 5% and 15% solutions. How much of each type should he mix to obtain 100 liters of fertilizer in a 12% solution?
31. A 150-yard pipe is cut to provide drainage for two fields. If the length of one piece is three yards less than twice the length of the second piece, what are the lengths of the two pieces?
32. Mr. Stein invested a total of $100,000 in two companies for a year. Company A’s stock showed a 13% annual gain, while Company B showed a 3% loss for the year. Mr. Stein made an overall 8% return on his investment over the year. How much money did he invest in each company?

33. A baker sells plain cakes for $7 or decorated cakes for $11. On a busy Saturday, the baker started with 120 cakes, and sold all but three. His takings for the day were $991. How many plain cakes did he sell that day, and how many were decorated before they were sold?

34. Twice John’s age plus five times Claire’s age is 204. Nine times John’s age minus three times Claire’s age is also 204. How old are John and Claire?

Mixed Review

35. Baxter the golden retriever is lying in the sun. He casts a shadow of 3 feet. The doghouse he is next to is 3 feet tall and casts an 8-foot shadow. What is Baxter’s height?

36. A botanist watched the growth of a lily. At 3 weeks, the lily was 4 inches tall. Four weeks later, the lily was 21 inches tall. Assuming this relationship is linear:
   a. Write an equation to show the growth pattern of this plant.
   b. How tall was the lily at the 5.5-week mark?
   c. Is there a restriction on how high the plant will grow? Does your equation show this?

37. The “Wave” is an exciting pastime at football games. To prepare, students in a math class took the data in the table below.

   a. Find a linear regression equation for this data. Use this model to estimate the number of seconds it will take for 18 students to complete a round of the wave.
   b. Use the method on interpolation to determine the amount of time it would take 18 students to complete the wave.

   | s (number of students in wave) | 4  | 8  | 12 | 16 | 20 | 24 | 28 | 30 |
   | t (time in seconds to complete on full round) | 2  | 3.2 | 4  | 5.6 | 7  | 7.9 | 8.6 | 9.1 |

Quick Quiz

1. Is (–3, –5) a solution to the system \(\begin{cases} -3y = 3x + 6 \\ y = -3x + 4 \end{cases}\) ?

2. Solve the system: \(\begin{cases} y = 6x + 17 \\ y = 7x + 20 \end{cases}\).

3. Joann and Phyllis each improved their flower gardens by planting daisies and carnations. Joann bought 10 daisies and 4 carnations and paid $52.66. Phyllis bought 3 daisies and 6 carnations and paid $43.11. How much is each daisy? How much is each carnation?

4. Terry’s Rental charges $49 per day and $0.15 per mile to rent a car. Hurry-It-Up charges a flat fee of $84 per day to rent a car. Write these two companies’ charges in equation form and use the system to determine at what mileage the two companies will charge the same for a one-day rental.
1.5 Special Types of Linear Systems

Solutions to a system can have several forms:

- One intersection
- Two or more solutions
- No solutions
- An infinite amount of solutions

Inconsistent Systems

This lesson will focus on the last two situations: systems with no solutions or systems with an infinite amount of solutions.

A system with parallel lines will have no solutions.

Remember from chapter 5 that parallel lines have the same slope. When graphed, the lines will have the same steepness with different \( y \)-intercepts. Therefore, parallel lines will never intersect, thus they have no solution.

\[
\begin{align*}
4y &= 5 - 3x \\
6x + 8y &= 7
\end{align*}
\]

Algebraically, a system with no solutions looks like this when solved.

Using the Substitution Property, replace the \( y \)-variable in the second equation with its algebraic expression in equation #1.
1.5. Special Types of Linear Systems

Apply the Distributive Property.

\[6x + 8 \left(\frac{5}{4} - \frac{3}{4}x\right) = 7\]

Add like terms.

\[6x + 10 - 6x = 7\]

\[10 = 7\]

You have solved the equation correctly, yet the answer does not make sense.

When solving a system of parallel lines, the final equation will be untrue.

Because \(10 \neq 7\) and you have done your math correctly, you can say this system has “no solutions.”

A system with no solutions is called an **inconsistent system**.

---

**Consistent Systems**

Consistent systems, on the contrary, have at least one solution. This means there is at least one intersection of the lines. There are three cases for consistent systems:

- One intersection, as you have practiced the majority of this chapter
- Two or more intersections, as you will see when a quadratic equation intersects a linear equation
- Infinitely many intersections, as with **coincident lines**

![Graph of coincident lines]

**Coincident lines** are lines with the same slope and \(y\)–intercept. The lines completely overlap.

When solving a consistent system involving coincident lines, the solution has the following result.

\[
\begin{align*}
x + y &= 3 \\
3x + 3y &= 9
\end{align*}
\]
Multiply the first equation by $-3$:

\[
\begin{align*}
-3(x + y) &= 3 \\
3x + 3y &= 9
\end{align*}
\]

\[
\begin{align*}
-3x - 3y &= -9 \\
3x + 3y &= 9
\end{align*}
\]

Add the equations together.

\[0 = 0\]

There are no variables left and you KNOW you did the math correctly. However, this is a true statement.

When solving a system of coincident lines, the resulting equation will be without variables and the statement will be true. You can conclude the system has an infinite number of solutions. This is called a consistent-dependent system.

**Example 1:** Identify the system as consistent, inconsistent, or consistent-dependent.

\[
\begin{align*}
3x - 2y &= 4 \\
9x - 6y &= 1
\end{align*}
\]

**Solution:** Because both equations are in standard form, elimination is the best method to solve this system.

Multiply the first equation by 3.

\[
\begin{align*}
3(3x - 2y) &= 4 \\
9x - 6y &= 12
\end{align*}
\]

Subtract the two equations.

\[
\begin{align*}
9x - 6y &= 12 \\
9x - 6y &= 1
\end{align*}
\]

This Statement is not true.

This is an untrue statement; therefore, you can conclude:

a. These lines are parallel.

b. The system has no solution.

c. The system is inconsistent.

**Example 2:** Two movie rental stores are in competition. Movie House charges an annual membership of $30 and charges $3 per movie rental. Flicks for Cheap charges an annual membership of $15 and charges $3 per movie rental. After how many movie rentals would Movie House become the better option?

Solution: It should already be clear to see that Movie House will never become the better option, since its membership is more expensive and it charges the same amount per move as Flicks for Cheap.

The lines that describe each option have different $y$--intercepts, namely 30 for Movie House and 15 for Flicks for Cheap. They have the same slope, three dollars per movie. This means that the lines are parallel and the system is inconsistent.
Let’s see how this works algebraically.

Define the variables: Let $x =$number of movies rented and $y =$total rental cost

$$\begin{align*}
y &= 30 + 3x \\
y &= 15 + 3x
\end{align*}$$

Because both equations are in slope-intercept form, solve this system by substituting the second equation into the first equation.

$$15 + 3x = 30 + 3x \Rightarrow 15 = 30$$

This statement is always false. Therefore, the system is inconsistent with no solutions.

**Practice Set**

Sample explanations for some of the practice exercises below are available by viewing the following video. Note that there is not always a match between the number of the practice exercise in the video and the number of the practice exercise listed in the following exercise set. However, the practice exercise is the same in both.

**CK-12 Basic Algebra: Special Types of Linear Systems (15:18)**

1. Define an inconsistent system. What is true about these systems?
2. What are the three types of consistent systems?
3. You graph a system and see only one line. What can you conclude?
4. You graph a system and see the lines have an intersection point. What can you conclude?
5. The lines you graphed appear parallel. How can you verify the system will have no solution?
6. You graph a system and obtain the following graph. Is the system consistent or inconsistent? How many solutions does the system have?
In 7 – 24, find the solution of each system of equations using the method of your choice. Please state whether the system is inconsistent, consistent, or consistent-dependent.

7. \(3x - 4y = 13\)
   \(y = -3x - 7\)
8. \(4x + y = 3\)
   \(12x + 3y = 9\)
9. \(10x - 3y = 3\)
   \(2x + y = 9\)
10. \(2x - 5y = 2\)
    \(4x + y = 5\)
11. \(\frac{3x}{2} + y = 3\)
    \(1.2x + 2y = 6\)
12. \(3x - 4y = 13\)
    \(y = -3x - 7\)
13. \(3x - 3y = 3\)
    \(x - y = 1\)
14. \(0.5x - y = 30\)
    \(0.5x - y = -30\)
15. \(4x - 2y = -2\)
    \(3x + 2y = -12\)
16. \(3x + 2y = 4\)
    \(-2x + 2y = 24\)
17. \(5x - 2y = 3\)
    \(2x - 3y = 10\)
18. \(3x - 4y = 13\)
    \(y = -3x - y\)
19. \(5x - 4y = 1\)
    \(-10x + 8y = -30\)
20. \(4x + 5y = 0\)
    \(3x = 6y + 4.5\)
21. \(-2y + 4x = 8\)
    \(y - 2x = -4\)
22. \(x - \frac{y}{2} = \frac{3}{2}\)
    \(3x + y = 6\)
23. \(0.05x + 0.25y = 6\)
   \(x + y = 24\)

24. \(x + \frac{2y}{3} = 6\)
   \(3x + 2y = 2\)

25. Peter buys two apples and three bananas for $4. Nadia buys four apples and six bananas for $8 from the same store. How much does one banana and one apple costs?

26. A movie rental store, CineStar, offers customers two choices. Customers can pay a yearly membership of $45 and then rent each movie for $2, or they can choose not to pay the membership fee and rent each movie for $3.50. How many movies would you have to rent before membership becomes the cheaper option?

27. A movie house charges $4.50 for children and $8.00 for adults. On a certain day, 1200 people enter the movie house and $8,375 is collected. How many children and how many adults attended?

28. Andrew placed two orders with an internet clothing store. The first order was for 13 ties and four pairs of suspenders, and it totaled $487. The second order was for six ties and two pairs of suspenders, and it totaled $232. The bill does not list the per-item price but all ties have the same price and all suspenders have the same price. What is the cost of one tie and of one pair of suspenders?

29. An airplane took four hours to fly 2400 miles in the direction of the jet-stream. The return trip against the jet-stream took five hours. What were the airplane’s speed in still air and the jet-stream’s speed?

30. Nadia told Peter that she went to the farmer’s market, that she bought two apples and one banana, and that it cost her $2.50. She thought that Peter might like some fruit so she went back to the seller and bought four more apples and two more bananas. Peter thanked Nadia, but he told her that he did not like bananas, so he would pay her for only four apples. Nadia told him that the second time she paid $6.00 for the fruit. Please help Peter figure out how much to pay Nadia for four apples.

Mixed Review

31. A football stadium sells regular and box seating. There are twelve times as many regular seats as there are box seats. The total capacity of the stadium is 10,413. How many box seats are in the stadium? How many regular seats?

32. Find an equation for the line perpendicular to \(y = -\frac{3}{5}x - 8.5\) containing the point (2, 7).

33. Rewrite in standard form: \(y = \frac{1}{6}x - 4\).

34. Find the sum: \(7\frac{2}{3} + 4\frac{1}{2}\).

35. Divide \(\frac{7}{8} \div -\frac{3}{4}\).

36. Is the product of two rational numbers always a rational number? Explain your answer.
The chapter moves on to the concept of systems of linear inequalities. In the last chapter, you learned how to graph a linear inequality in two variables.

**Step 1**: Graph the equation using the most appropriate method.

- Slope-intercept form uses the $y$–intercept and slope to find the line.
- Standard form uses the intercepts to graph the line.
- Point-slope uses a point and the slope to graph the line.

**Step 2**: If the equal sign is not included draw a dashed line. Draw a solid line if the equal sign is included.

**Step 3**: Shade the half plane above the line if the inequality is “greater than.” Shade the half plane under the line if the inequality is “less than.”

In this section, we will learn how to graph two or more linear inequalities on the same coordinate plane. The inequalities are graphed separately on the same graph and the **solution for the system of inequalities** is the common shaded region between all the inequalities in the system.

The common shaded region of the system of inequalities is called the **feasible region**.

Example: **Solve the system of inequalities**

$$\begin{cases}
2x + 3y \leq 18 \\
x - 4y \leq 12
\end{cases}$$

Solution: The first equation is written in standard form and can be graphed using its intercepts. The line is solid because the equal sign is included in the inequality. Since the inequality is less than or equal to, shade the half plane below the line.

The second equation is a little tricky. Rewrite this in slope-intercept form to graph.

$$\Rightarrow 
-4y \leq -3x + 12 \quad \quad \quad \quad \quad y \geq \frac{x}{4} - 3$$
The division by $-4$ causes the inequality to reverse. The line is solid again because the equal sign is included in the inequality. Shade the half plane above the boundary line because $y$ is greater than or equal.

When we combine the graphs, we see that the blue and red shaded regions overlap. This overlap is where both inequalities work. Thus the purple region denotes the solution of the system, the feasible region.

The kind of solution displayed in this example is called unbounded, because it continues forever in at least one direction (in this case, forever upward and to the left).

**Bounded** regions occur when more than two inequalities are graphed on the same coordinate plane, as in the next example.

**Example:** Find the solution set to the following system.

\[
\begin{align*}
  y &> 3x - 4 \\
  y &< -\frac{9}{4}x + 2 \\
  x &\geq 0 \\
  y &\geq 0
\end{align*}
\]
Solution: Graph each line and shade appropriately.

\[ y > 3x - 4 \]

Finally we graph and \( x \geq 0 \) and \( y \geq 0 \), and the intersecting region is shown in the following figure.

\[ y < -\frac{9}{4}x + 2 \]
1.6. Systems of Linear Inequalities

Writing Systems of Linear Inequalities

In some cases, you are given the feasible region and asked to write the system of inequalities. To do this, you work in reverse order of graphing.

- Write the equation for the boundary line.
- Determine whether the sign should include “or equal to.”
- Determine which half plane is shaded.
- Repeat for each boundary line in the feasible region.

Example: Write the system of inequalities shown below.

There are two boundary lines, so there are two inequalities. Write each one in slope-intercept form.

\[ y \leq \frac{1}{4}x + 7 \]
\[ y \geq -\frac{5}{2}x - 5 \]
Linear Programming – Real-World Systems of Linear Inequalities

Entire careers are devoted to using systems of inequalities to ensure a company is making the most profit by producing the right combination of items or is spending the least amount of money to make certain items. Linear programming is the mathematical process of analyzing a system of inequalities to make the best decisions given the constraints of the situation.

Constraints are the particular restrictions of a situation due to time, money, or materials.

The goal is to locate the feasible region of the system and use it to answer a profitability, or optimization, question.

Theorem: The maximum or minimum values of an optimization equation occur at the vertices of the feasible region – at the points where the boundary lines intersect.

This theorem provides an important piece of information. While the individual colors of the inequalities will overlap, providing an infinite number of possible combinations, only the vertices will provide the maximum (or minimum) solutions to the optimization equation.

Let’s go back to the situation presented in the chapter opener.

James is trying to expand his pastry business to include cupcakes and personal cakes. He has 40 hours available to decorate the new items and can use no more than 22 pounds of cake mix. Each personal cake requires 2 pounds of cake mix and 2 hours to decorate. Each cupcake order requires one pound of cake mix and 4 hours to decorate. If he can sell each personal cake for $14.99 and each cupcake order for $16.99, how many personal cakes and cupcake orders should James make to make the most revenue?

There are four inequalities in this situation. First, state the variables. Let \( p = \text{the number of personal cakes} \) and \( c = \text{the number of cupcake orders} \).

Translate this into a system of inequalities.

\[ 2p + 1c \leq 22 \] – This is the amount of available cake mix.
2p + 4c ≤ 40 – This is the available time to decorate.
p ≥ 0 – You cannot make negative personal cakes.
c ≥ 0 – You cannot make negative cupcake orders.

Now graph each inequality and determine the feasible region.

The feasible region has four vertices (0, 0),(0, 10),(11, 0),(8, 6). According to our theorem, the optimization answer will only occur at one of these vertices.

Write the optimization equation: How much of each type of order should James make to bring in the most revenue?

\[ 14.99p + 16.99c = \text{maximum revenue} \]

Substitute each ordered pair to determine which makes the most money

- (0, 0) → $0.00
- (0, 10) → 14.99(0) + 16.99(10) = $169.90
- (11, 0) → 14.99(11) + 16.99(0) = $164.89
- (8, 6) → 14.99(8) + 16.99(6) = $221.86

To make the most revenue, James should make 8 personal cakes and 6 cupcake orders.


**Practice Set**

Sample explanations for some of the practice exercises below are available by viewing the following video. Note that there is not always a match between the number of the practice exercise in the video and the number of the
1. What is linear programming?
2. What is the feasible region of a system of inequalities?
3. How do constraints affect the feasible region?
4. What is an optimization equation? What is its purpose?
5. You have graphed a feasible region. Where are the maximum (or minimum) points of the optimization equation located?

Find the solution region of the following systems of inequalities.

6. \(4y - 5x < 8\)
   \(-5x \geq 16 - 8y\)
7. \(5x - y \geq 5\)
   \(2y - x \geq -10\)
8. \(2x - 3y \leq 21\)
   \(x + 4y \leq 6\)
   \(3x + y \geq -4\)
9. \(\begin{cases} y \geq \frac{1}{4}x - 3 \\ y < \frac{13}{8}x + 8 \end{cases}\)
10. \(\begin{cases} y \leq \frac{3}{4}x - 5 \\ y \geq -2x + 2 \end{cases}\)
11. \(\begin{cases} y > -x + 1 \\ y > \frac{1}{4}x + 6 \end{cases}\)
12. \(\begin{cases} y > -\frac{1}{2}x + 4 \\ x < -4 \end{cases}\)
13. \(\begin{cases} y \leq 6 \\ y > \frac{1}{4}x + 6 \end{cases}\)

Write the system of inequalities for each feasible region pictured below.
Given the following constraints find the maximum and minimum values for:

17. \( z = -x + 5y \)
\( x + 3y \leq 0 \)
\( x - y \geq 0 \)
\( 3x - 7y \leq 16 \)

18. Find the maximum and minimum value of \( z = 2x + 5y \) given the constraints.

\[
\begin{align*}
2x - y & \leq 12 \\
4x + 3y & \geq 0 \\
x - y & \leq 6
\end{align*}
\]
19. In Andrew’s Furniture Shop, he assembles both bookcases and TV cabinets. Each type of furniture takes him about the same time to assemble. He figures he has time to make at most 18 pieces of furniture by this Saturday. The materials for each bookcase cost him $20.00 and the materials for each TV stand cost him $45.00. He has $600.00 to spend on materials. Andrew makes a profit of $60.00 on each bookcase and a profit of $100.00 for each TV stand. Find how many of each piece of furniture Andrew should make so that he maximizes his profit.

20. You have $10,000 to invest, and three different funds from which to choose. The municipal bond fund has a 5% return, the local bank’s CDs have a 7% return, and a high-risk account has an expected 10% return. To minimize risk, you decide not to invest any more than $1,000 in the high-risk account. For tax reasons, you need to invest at least three times as much in the municipal bonds as in the bank CDs. Assuming the year-end yields are as expected, what are the optimal investment amounts?

Mixed Review

21. Solve by elimination \[
\begin{align*}
12x + 8y &= 24 \\
-6x + 3y &= 9
\end{align*}
\].

22. Solve \(36 = |5t - 6|\).

23. Determine the intercepts of \(y = -\frac{5}{6}x - 3\).

24. Jerry’s aunt repairs upholstery. For three hours’ worth of work, she charges $145. For nine hours of work, she charges $355. Assuming this relationship is linear, write the equation for this line in point-slope form. How much would Jerry’s aunt charge for 1.25 hours worth of work?

25. Translate into an algebraic sentence: “Yoder is four years younger than Kate. Kate is six years younger than Dylan. Dylan is 20.” How old is each person?
Congratulations! You have won a free trip to Europe. On your trip you have the opportunity to visit 6 different cities. You are responsible for planning your European vacation. How many different ways can you schedule your trip? The answer may surprise you!

This is an example of a permutation. A permutation is an arrangement of objects in a specific order. It is the product of the counting numbers 1 through $n$.

$$n! = n(n-1)(n-2) \cdots 1$$

How many ways can you visit the European cities? There are 6 choices for the first stop. Once you have visited this city, you cannot return so there are 5 choices for the second stop, and so on.

$$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$$

There are 720 different ways to plan your European vacation!

A permutation of $n$ objects arranged \textit{kat a time} is expressed as $nP_k$.

$$nP_k = \frac{n!}{(n-k)!}$$

Example 1: Evaluate $6P_3$.

Solution: This equation asks, “How many ways can 6 objects be chosen 3 at a time?”

There are 6 ways to choose the first object, 5 ways to choose the second object, and 4 ways to choose the third object.
There are 120 different ways 6 objects can be chosen 3 at a time.

Example 1 can also be written using the formula for permutation: \( \binom{6}{3} = \frac{6!}{(6-3)!} = \frac{6!}{3!} = 6 \cdot 5 \cdot 4 = 120. \)

### Permutations and Graphing Calculators

Most graphing calculators have the ability to calculate a permutation.

*Evaluate \( \binom{6}{3} \) using a graphing calculator.*

Type the first value of the permutation, the \( n \). Choose the [MATH] button, directly below the [ALPHA] key. Move the cursor once to the left to see this screen:

Option #2 is the permutation option. Press [ENTER] and then the second value of the permutation, the value of \( k \). Press [ENTER] to evaluate.

\[
6 \ \text{nPr} \ 3 = 120
\]

### Permutations and Probability

The letters of the word HOSPITAL are arranged at random. How many different arrangements can be made? What is the probability that the last letter is a vowel?

There are eight ways to choose the first letter, seven ways to choose the second, and so on. The total number of arrangements is \( 8! = 40,320. \)

There are three vowels in HOSPITAL; therefore, there are three possibilities for the last letter. Once this letter is chosen, there are seven choices for the first letter, six for the second, and so on.

\[
7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 = 15,120
\]

Probability, as you learned in a previous chapter, has the formula:

\[
\text{Probability (success)} = \frac{\text{number of ways to get success}}{\text{total number of possible outcomes}}
\]
There are 15,120 ways to get a vowel as the last letter; there are 40,320 total combinations.

\[
P(\text{last letter is a vowel}) = \frac{15,120}{40,320} = \frac{3}{8}
\]

**Multimedia Link:** For more help with permutations, visit the [http://regentsprep.org/REgents/math/ALGEBRA/APR2/L.permProb.htm](http://regentsprep.org/REgents/math/ALGEBRA/APR2/L.permProb.htm) - Algebra Lesson Page by Regents Prep.

---

### Practice Set

1. Define **permutation**.

In 2 – 19, evaluate each permutation.

2. 7!
3. 10!
4. 1!
5. 5!
6. 9!
7. 3!
8. 4! + 4!
9. 16! − 5!
10. \(\frac{98!}{96!}\)
11. \(\frac{11!}{7!}\)
12. \(\frac{300!}{30!}\)
13. \(\frac{8!}{7!}\)
14. 2! + 9!
15. \(11P_2\)
16. \(5P_3\)
17. \(5P_3\)
18. \(15P_{10}\)
19. \(60P_{59}\)
20. How many ways can 14 books be organized on a shelf?
21. How many ways are there to choose 10 objects, four at a time?
22. How many ways are there to choose 21 objects, 13 at a time?
23. A running track has eight lanes. In how many ways can 8 athletes be arranged to start a race?
24. Twelve horses run a race.
   a. How many ways can first and second places be won?
   b. How many ways will all the horses finish the race?
25. Six actors are waiting to audition. How many ways can the director choose the audition schedule?
26. Jerry, Kerry, Larry, and Mary are waiting at a bus stop. What is the probability that Mary will get on the bus first?
27. How many permutations are there of the letters in the word “HEART”?
28. How many permutations are there of the letters in the word “AMAZING”?
29. Suppose I am planning to get a three-scoop ice cream cone with chocolate, vanilla, and Superman. How many ice cream cones are possible? If I ask the server to “surprise me,” what is the probability that the Superman scoop will be on top?
30. What is the probability you choose two cards (without replacement) from a standard 52-card deck and both cards are jacks?

31. The Super Bowl Committee has applications from 9 towns to host the next two Super Bowls. How many ways can they select the host if:
   a. The town cannot host a Super Bowl two consecutive years?
   b. The town can host a Super Bowl two consecutive years?

Mixed Review

32. Graph the solution to the following system:

\[ 2x - 3y > -9 \]
\[ y < 1 \]

33. Convert 24 meters/minute to feet/second.

34. Solve for \( t \) : \( |t - 6| \leq -14 \).

35. Find the distance between 6.15 and -9.86.

36. Which of the following vertices provides the minimum cost according to the equation \( 12x + 20y = \text{cost} \):
   (3,6), (9,0), (6,2), (0,11)?

37. Write the system of inequalities pictured below.
1.8 Probability and Combinations

When the order of objects is not important and/or the objects are replaced, **combinations** are formed.

A **combination** is an arrangement of objects in no particular order.

Consider a sandwich with salami, ham, and turkey. It does not matter the order in which we place the deli meat, as long as it’s on the sandwich.

There is only one way to stack the meat on the sandwich if the order does not matter. However, if the order mattered, there are 3 choices for the first meat, 2 for the second, and one for the last choice: \(3 \cdot 2 \cdot 1 = 6\).

**Combination ≠ Permutation**

A combination of \(n\) objects chosen \(k\) at a time is expressed as \(nC_k\).

\[
\begin{align*}
\binom{n}{k} & = \frac{n!}{k!(n-k)!} \\
& = \binom{n}{k}
\end{align*}
\]

This is read “\(n\) choose \(k\).”

**Example 1:** How many ways can 8 students be chosen from a class of 21?

**Solution:** It does not matter how the eight students are chosen. Use the formula for combination rather than permutation.

\[
\binom{21}{8} = \frac{21!}{8!(21-8)!} = 203,490
\]

There are 203,490 different ways to choose eight students from 21.

Example: The Senate is made of 100 people, two per state. How many different four-person committees are possible?

**Solution:** This question does not care how the committee members are chosen; we will use the formula for combination.

\[
\binom{100}{4} = \frac{100!}{4!(100-4)!} = 3,921,225 \text{ ways}
\]

That is a lot of possibilities!
Combinations on the Graphing Calculator

Just like permutations, most graphing calculators have the capability to calculate combinations. On the TI calculators, use these directions.

• Enter the \( n \), or the total to choose from.
• Choose the [MATH] button, directly below the [ALPHA] key. Move the cursor once to the left to see this screen:

```
MATH NUM CPX PRT
1:rand
2:Inr
3:nCr
4:!
5:randInt()
6:randNorm()
7:randBin()
```

• Choose option #3, \( nC_r \). Type in the \( k \) value, the amount you want to choose.

\[
100 \text{ nCr } 4 = 3921225
\]

Probability and Combinations

Combinations are used in probability when there is a replacement of objects or the order does not matter. Suppose you have ten marbles: four blue and six red. You choose three marbles without looking. What is the probability that all three marbles are blue?

\[
\text{Probability (success)} = \frac{\text{number of ways to get success}}{\text{total number of possible outcomes}}
\]

There are \( 4C_3 \) ways to choose the blue marbles. There are \( 10C_3 \) total combinations.

\[
P(\text{all 3 marbles are blue}) = \frac{\binom{4}{3}}{\binom{10}{3}} = \frac{4}{120} = \frac{1}{30}
\]

There is approximately 3.33% chance of all three marbles being drawn are blue.

Practice Set

1. What is a combination? How is it different from a permutation?
2. How many ways can you choose \( k \) objects from \( n \) possibilities?
3. Why is \( 3C_9 \) impossible to evaluate?
In 4 – 19, evaluate the combination.

4. \( \binom{12}{2} \)
5. \( \binom{5}{3} \)
6. \( \binom{7}{2} \)
7. \( \binom{3}{0} \)
8. \( \binom{6}{4} \)
9. \( \binom{9}{1} \)
10. \( \binom{10}{6} \)
11. \( \binom{19}{18} \)
12. \( \binom{20}{14} \)
13. \( \binom{13}{9} \)
14. \( 7C_3 \)
15. \( 11C_5 \)
16. \( 5C_4 \)
17. \( 13C_9 \)
18. \( 20C_5 \)
19. \( 15C_{15} \)

20. Your backpack contains 6 books. You select two at random. How many different pairs of books could you select?

21. Seven people go out for dinner. In how many ways can 4 order steak, 2 order vegan, and 1 order seafood?

22. A pizza parlor has 10 toppings to choose from. How many four-topping pizzas can be created?

23. Gooies Ice Cream Parlor offers 28 different ice creams. How many two-scooped cones are possible, given the order does not matter?

24. A college football team plays 14 games. In how many ways can the season end with 8 wins, 4 losses, and 2 ties?

25. Using the marble situation from the lesson, determine the probability that the three marbles chosen are all red?

26. Using the marble situation from the lesson, determine the probability that two marbles are red and the third is blue.

27. Using the Senate situation from the lesson, how many two-person committees can be made using Senators?

28. Your English exam has seven essays and you must answer four. How many combinations can be made?

29. The sociology test has 15 true/false questions. In how many ways can you answer 11 correctly?

30. Seven people are applying for two vacant school board positions; four are women, three are men. In how many ways can these vacancies be filled ...
   a. With any two applicants?
   b. With only women?
   c. With one man and one woman?

**Mixed Review**

31. How many ways can 15 paintings be lined along a wall?

32. Your calculator gives an “Overload” error when trying to simplify \( \frac{300!}{295!} \). What can you do to help evaluate this fraction?

33. Consider a standard six-sided die. What is the probability that the number rolled will be a multiple of 2?

34. Solve the following system: The sum of two numbers is 70.6 and their product is 1,055.65. Find the two numbers.
1. Match the following terms to their definitions.
   a. System – Restrictions imposed by time, materials, or money
   b. Feasible Region – A system with an infinite amount of solutions
   c. Inconsistent System – An arrangement of objects when order matters
   d. Constraints – A method used by businesses to determine the most profitable or least cost given constraints
   e. Consistent-dependent System – An arrangement of objects in which order does not matter
   f. Permutation – Two or more algebraic sentences joined by the word and
   g. Combination – A system with no solutions
   h. Linear programming - A solution set to a system of inequalities

2. Where are the solutions to a system located?

3. Suppose one equation of a system is in slope-intercept form and the other is in standard form. Which method presented in this chapter would be the most effective to solve this system? Why?

4. Is (–3, –8) a solution to \( \begin{cases} 7x - 4y = 11 \\ x + 2y = -19 \end{cases} \) ?

5. Is (–1, 0) a solution to \( \begin{cases} y = 0 \\ 8x + 7y = 8 \end{cases} \) ?

Solve the following systems by graphing.

6. \( \begin{cases} y = -2 \\ y = -6x + 1 \end{cases} \)

7. \( \begin{cases} y = 3 - \frac{1}{3}x \\ x + 3y = 4 \end{cases} \)

8. \( \begin{cases} y = \frac{1}{2}x - 6 \\ 4y = 2x - 24 \end{cases} \)

9. \( \begin{cases} y = -\frac{3}{2}x + 7 \\ y = \frac{2}{5}x + 1 \\ x = 2 \end{cases} \)

10. \( \begin{cases} y = 4 \\ y = \frac{1}{2}x + 3 \end{cases} \)

Solve the following system by substitution.

11. \( \begin{cases} y = 2x - 7 \\ y + 7 = 4x \end{cases} \)

12. \( \begin{cases} y = -3x + 22 \\ y = -2x + 16 \end{cases} \)

13. \( \begin{cases} y = 3 - \frac{1}{3}x \\ x + 3y = 4 \end{cases} \)
14. \[ \begin{align*}
2x + y &= -10 \\
y &= x + 14
\end{align*} \]
15. \[ \begin{align*}
y + 19 &= -7x \\
y &= -2x - 9
\end{align*} \]
16. \[ \begin{align*}
y &= 0 \\
5x &= 15
\end{align*} \]
17. \[ \begin{align*}
y &= 3 - \frac{1}{3}x \\
x + 3y &= 4
\end{align*} \]
18. \[ \begin{align*}
7x + 3y &= 3 \\
y &= 8
\end{align*} \]

Solve the following systems using elimination.

19. \[ \begin{align*}
2x + 4y &= -14 \\
-2x + 4y &= 8
\end{align*} \]
20. \[ \begin{align*}
6x - 9y &= 27 \\
6x - 8y &= 24
\end{align*} \]
21. \[ \begin{align*}
3x - 2y &= 0 \\
2y - 3x &= 0
\end{align*} \]
22. \[ \begin{align*}
4x + 3y &= 2 \\
-8x + 3y &= 14
\end{align*} \]
23. \[ \begin{align*}
-8x + 8y &= 8 \\
6x + y &= 1
\end{align*} \]
24. \[ \begin{align*}
7x - 4y &= 11 \\
x + 2y &= -19
\end{align*} \]
25. \[ \begin{align*}
y &= -2x - 1 \\
4x + 6y &= 10
\end{align*} \]
26. \[ \begin{align*}
x - 6y &= 20 \\
2y - 3x &= -12
\end{align*} \]
27. \[ \begin{align*}
-4x + 4y &= 0 \\
8x - 8y &= 0
\end{align*} \]
28. \[ \begin{align*}
-9x + 6y &= -27 \\
-3x + 2y &= -9
\end{align*} \]

Graph the solution set to each system of inequalities.

29. \[ \begin{align*}
y &> -\frac{3}{2}x - 5 \\
y &\geq -2x + 2
\end{align*} \]
30. \[ \begin{align*}
y &> \frac{13}{8}x + 8 \\
y &\geq \frac{1}{2}x - 3
\end{align*} \]
31. \[ \begin{align*}
y &\leq \frac{3}{2}x - 5 \\
y &\geq -2x + 8
\end{align*} \]
32. \[ \begin{align*}
y &\leq -\frac{7}{2}x - 3 \\
y &\geq \frac{4}{5}x + 4
\end{align*} \]
33. \[
\begin{align*}
x < 5 \\
y &\geq \frac{9}{x} - 2
\end{align*}
\]

Write a system of inequalities for the regions below.

36. Yolanda is looking for a new cell phone plan. Plan A charges $39.99 monthly for talking and $0.08 per text. Plan B charges $69.99 per month for an “everything” plan.
   a. At how many texts will these two plans charge the same?
   b. What advice would you give Yolanda?

37. The difference of two numbers is –21.3. Their product is –72.9. What are the numbers?

38. Yummy Pie Company sells two kinds of pies: apple and blueberry. Nine apples pies and 6 blueberry pies cost $126.00. 12 apples pies and 12 blueberry pies cost $204.00. What is the cost of one apple pie and 2 blueberry pies?

39. A jet traveled 784 miles. The trip took seven hours, traveling with the wind. The trip back took 14 hours, against the wind. Find the speed of the jet and the wind speed.

40. A canoe traveling downstream takes one hour to travel 7 miles. On the return trip, traveling against current, the same trip took 10.5 hours. What is the speed of the canoe? What is the speed of the river?

41. The yearly musical production is selling two types of tickets: adult and student. On Saturday, 120 student tickets and 45 adult tickets were sold, bringing in $1,102.50 in revenue. On Sunday, 35 student tickets and 80 adult tickets were sold, bringing in $890.00 in revenue. How much was each type of ticket?

42. Rihanna makes two types of jewelry: bracelets and necklaces. Each bracelet requires 36 beads and takes 1 hour to make. Each necklace requires 80 beads and takes 3 hours to make. Rihanna only has 600 beads and 20 hours of time. 1. Write the constraints of this situation as a system of inequalities. 2. Graph the feasible region and locate its vertices. 3. Rihanna makes $8.00 profit per bracelet and $7.00 profit per necklace. How
many of each should she make to maximize her profit?

43. A farmer plans to plant two types of crops: soybeans and wheat. He has 65 acres of available land. He wants to plant twice as much soybeans as wheat. Wheat costs $30 per acre and soybeans cost $30 per acre. 1. Write the constraints as a system of inequalities. 2. Graph the feasible region and locate its vertices. 3. How many acres of each crop should the farmer plant in order to minimize cost?

44. How many ways can you organize 10 items on a shelf?

45. Evaluate 5!

46. Simplify \( \frac{100!}{97!} \)

47. How many ways can a football team of 9 be arranged if the kicker must be in the middle?

48. How many one-person committees can be formed from a total team of 15?

49. How many three-person committees can be formed from a total team of 15?

50. There are six relay teams running a race. How many different combinations of first and second place are there?

51. How many ways can all six relay teams finish the race?

52. Evaluate \( \binom{14}{12} \).

53. Evaluate \( \binom{8}{8} \) and explain its meaning.

54. A baked potato bar has 9 different choices. How many potatoes can be made with four toppings?

55. A bag contains six green marbles and five white marbles. Suppose you choose two without looking. What is the probability that both marbles will be green?

56. A principal wants to make a committee of four teachers and six students. If there are 22 teachers and 200 students, how many different committees can be formed?
1. **True or false?** A shorter way to write a permutation is \( P(n, k) \).

2. Is \((-17, 17)\) a solution to \( \begin{cases} \frac{1}{17}x + 18 \\ y = -\frac{2}{17}x - 4 \end{cases} \)?

3. What is the primary difference between a combination and a permutation?

4. An airplane is traveling a distance of 1,150 miles. Traveling against the wind, the trip takes 12.5 hours. Traveling with the wind, the same trip takes 11 hours. What is the speed of the plane? What is the speed of the wind?

5. A solution set to a system of inequalities has two dashed boundary lines. What can you conclude about the coordinates on the boundaries?

6. What does \( k \) have to be to create a dependent-consistent system? \( \begin{cases} 5x + 2y = 20 \\ 15x + ky = 60 \end{cases} \)

7. Joy Lynn makes two different types of spring flower arrangements. The Mother’s Day arrangement has 8 roses and 6 lilies. The Graduation arrangement has 4 roses and 12 lilies. Joy Lynn can use no more than 120 roses and 162 lilies. If each Mother’s Day arrangement costs $32.99 and each Graduation arrangement costs $27.99, how many of each type of arrangement should Joy Lynn make to make the most revenue?

8. Solve the system \( \begin{cases} -6x + y = -1 \\ -7x - 2y = 2 \end{cases} \).

9. Solve the system \( \begin{cases} y = 0 \\ 8x + 7y = 8 \end{cases} \).

10. Solve \( \begin{cases} y = x + 8 \\ y = 3x + 16 \end{cases} \).

11. How many solutions does the following system have? \( \begin{cases} y = -2x - 2 \\ y = -2x + 17 \end{cases} \)

12. The letters to the word VIOLENT are placed into a bag.
   a. How many different ways can all the letters be pulled from the bag?
   b. What is the probability that the last letter will be a consonant?

13. Suppose an ice cream shop has 12 different topping choices for an ice cream sundae. How many ways can you choose 5 of the 12 toppings?

14. A saleswoman must visit 13 cities exactly once without repeating. In how many ways can this be done?

---

**Texas Instruments Resources**

*In the CK-12 Texas Instruments Algebra I FlexBook, there are graphing calculator activities designed to supplement the objectives for some of the lessons in this chapter. See [http://www.ck12.org/flexr/chapter/9617](http://www.ck12.org/flexr/chapter/9617).*