Conjectures and Counterexamples

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Here you’ll learn how to make educated guesses, or conjectures, based on patterns. You’ll also learn how to disprove conjectures with counterexamples.

Suppose you were given a mathematical pattern like \( h = \frac{-16}{t^2} \). What if you wanted to make an educated guess, or conjecture, about \( h \)? After completing this Concept, you’ll be able to make such a guess and provide counterexamples that disprove incorrect guesses.

**Watch This**

**CK-12 Conjectures and Counterexamples**

Watch the final part of this video.

**James Sousa: Counterexamples**

**Guidance**

A **conjecture** is an “educated guess” that is based on examples in a pattern. A **counterexample** is an example that disproves a conjecture.

**Example A**

Here’s an algebraic equation and a table of values for \( n \) and \( t \).

\[
t = (n - 1)(n - 2)(n - 3)
\]
After looking at the table, Pablo makes this conjecture:

The value of \((n-1)(n-2)(n-3)\) is 0 for any number \(n\).

Is this a true conjecture?

This is not a valid conjecture. If Pablo were to continue the table to \(n = 4\), he would have see that \((n-1)(n-2)(n-3) = (4-1)(4-2)(4-3) = (3)(2)(1) = 6\)

In this example \(n = 4\) is the counterexample.

**Example B**

Arthur is making figures for an art project. He drew polygons and some of their diagonals.

![Diagonals of polygons](image)

From these examples, Arthur made this conjecture:

If a convex polygon has \(n\) sides, then there are \(n - 2\) triangles formed when diagonals are drawn from any vertex of the polygon.

Is Arthur’s conjecture correct? Or, can you find a counterexample?

The conjecture appears to be correct. If Arthur draws other polygons, in every case he will be able to draw \(n - 2\) triangles if the polygon has \(n\) sides.

*Notice that we have not proved Arthur’s conjecture, but only found several examples that hold true. So, at this point, we say that the conjecture is true.*

**Example C**

Give a counterexample to this statement: Every prime number is an odd number.

The only counterexample is the number 2: an even number (not odd) that is prime.

CK-12 Conjectures and Counterexamples
**Guided Practice**

A car salesman sold 5 used cars to five different couples. He noticed that each couple was under 30 years old. The following day, he sold a new, luxury car to a couple in their 60’s. The salesman determined that only younger couples by used cars.

1. Is the salesman’s conjecture logical? Why or why not?
2. Can you think of a counterexample?

**Answers:**

1. It is logical based on his experiences, but is not true.
2. A counterexample would be a couple that is 30 years old or older buying a used car.

**Practice**

Give a counterexample for each of the following statements.

1. If \( n \) is a whole number, then \( n^2 > n \).
2. All numbers that end in 1 are prime numbers.
3. All positive fractions are between 0 and 1.
4. Any three points that are coplanar are also collinear.
5. All girls like ice cream.
6. All high school students are in choir.
7. For any angle there exists a complementary angle.
8. All teenagers can drive.
9. If \( n \) is an integer, then \( n > 0 \).
10. All equations have integer solutions.