11.1 Fundamental Theorem of Algebra

Answers

1. $f(x) = (x-2)^2(x-4)^3(x-1)\left(x-\sqrt{2}i\right)\left(x+\sqrt{2}i\right) = x^8 - 17x^7 + 118x^6 - 438x^5 + 984x^4 - 1512x^3 + 1760x^2 - 1408x + 512$

2. $f(x) = (x-1)(x+3)^3(x+1)\left(x-\sqrt{3}i\right)\left(x+\sqrt{3}i\right) = x^7 + 9x^6 + 29x^5 + 45x^4 + 51x^3 + 27x^2 - 81x - 81$

3. $f(x) = (x-5)^2(x+1)^2(x-2i)(x+2i) = x^6 - 8x^5 + 10x^4 + 8x^3 + 49x^2 + 160x + 100$

4. $f(x) = (x-i)(x+i)\left(x-\sqrt{2}i\right)\left(x+\sqrt{2}i\right) = x^4 + 3x^2 + 2$

5. $f(x) = (x+3)^2(x-2)(x-i)(x+i)$; Roots are $-3\ (multiplicity\ 2), 2, i, -i$

6. $g(x) = (x-1)(x+1)(x-i)(x+i)$; Roots are $1, -1, i, -i$

7. $h(x) = (x-4)^2(x-2)^2(x+3i)(x-3i)$; Roots are 4 (multiplicity 2), 2 (multiplicity 2), $-3i, 3i$

8. $j(x) = (x-1)^2(x-3)^3\left(x+\sqrt{3}i\right)\left(x-\sqrt{3}i\right)$; Roots are 1 (multiplicity 2), 3 (multiplicity 3), $-\sqrt{3}i$, $\sqrt{3}i$.

9. $k(x) = (x-2)(x+3)(x+4)(x-1)^2$; Roots are $2, -3, -4, 1(multiplicity\ 2)$

10. $m(x) = (x-2)(x+2)(x-3)(x+3)(x-i)(x+i)$; Roots are $2, -2, 3, -3, i, -i$

11. $n(x) = (x-6)(x+1)^3(x-\sqrt{5}i)(x+\sqrt{5}i)$; Roots are 6, -1 (multiplicity 3), $\sqrt{5}i$, $-\sqrt{5}i$

12. $p(x) = (x-2)(x+2)^3\left(x-\sqrt{7}i\right)(x+\sqrt{7}i)$; Roots are 2, -2 (multiplicity 3), $\sqrt{7}i, -\sqrt{7}i$

13. The degree of the polynomial is the number of roots with multiplicity.

14. Multiplicity refers to a root that counts more than once because when the polynomial is in factored form, the degree of its corresponding binomial is greater than 1.

15. $-\sqrt{3}i$

11.2 Arithmetic with Complex Numbers

Answers

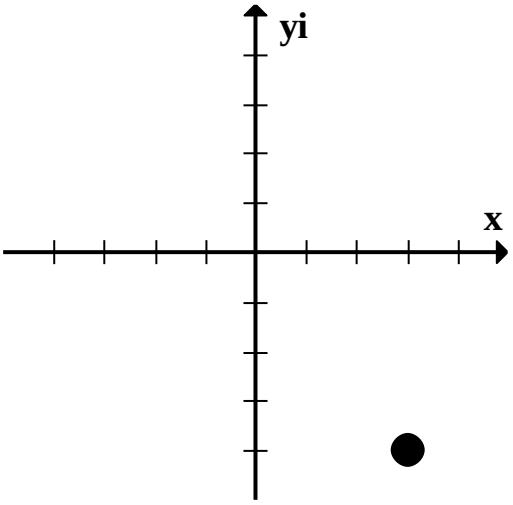
1. 1

2. 1

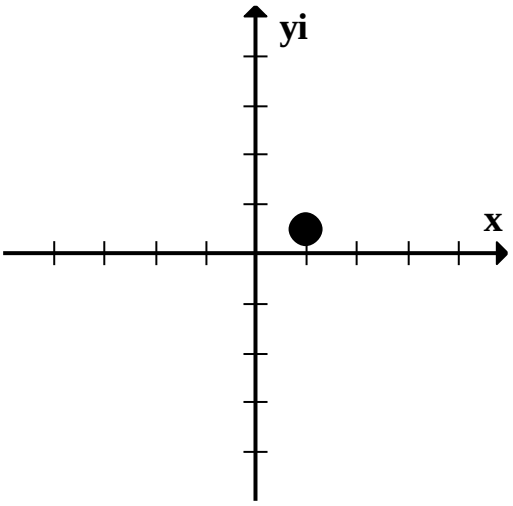
3. $-i$

4. i

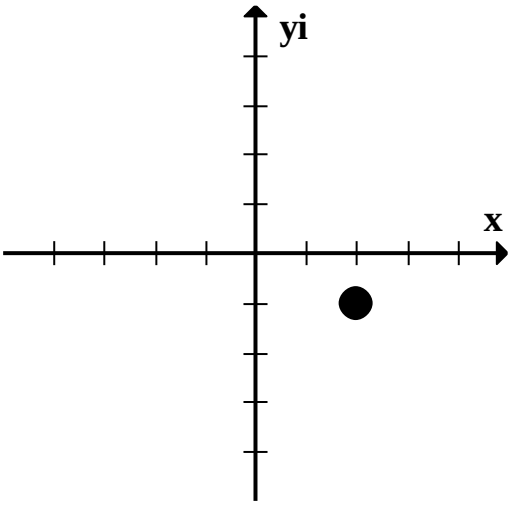
5. $|6-8i|=10$



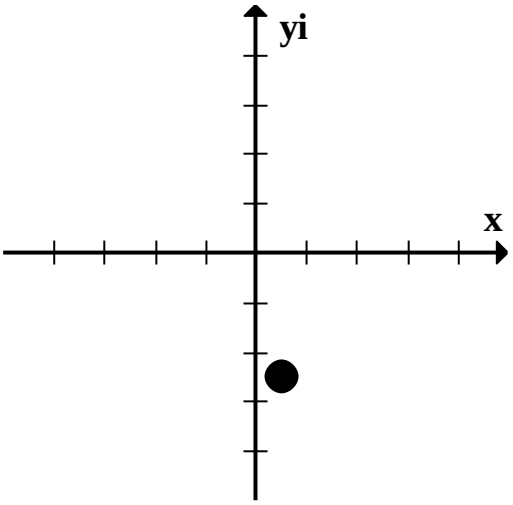
6. $|2+i|=\sqrt{5}$



7. $|4 - 2i| = \sqrt{20} = 2\sqrt{5}$



8. $|-5i + 1| = \sqrt{26}$



9. $5 - 2i$

10. $-1 + 12i$

11. $41 + 11i$

12. $-8 + 34i$

13. $328 + 88i$

14. $-\frac{29}{34} + \frac{31}{34}i$

15. $-29 + 58i$

11.3 Trigonometric Polar Form of Complex Numbers

Answers

1. $-5i$
2. $\sqrt{3}+i$
3. $-2\sqrt{2}-2\sqrt{2}i$
4. $3+3\sqrt{3}i$
5. $2i$
6. $\sqrt{5}\text{ cis } 333.43°$
7. $13\text{ cis } 67.38°$
8. $10\text{ cis } 36.87°$
9. $1\text{ cis}\frac{\pi}{2}$
10. $\frac{6}{5}\text{cis } 132°$
11. $3\text{ cis } 86°$
12. $30\text{ cis}\frac{5\pi}{12}$
13. $-\frac{12}{15}\text{ cis}\frac{3\pi}{2}$
14. $r_1 \cdot \text{cis}\,\theta_1 \cdot\ r_2 \cdot \text{cis}\,\theta_2 = r_1 \cdot r_2 \cdot (\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)$

    $= r_1 \cdot r_2((\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) + i(cos\theta_2 sin\theta_1 + cos\theta_1 sin\theta_2))$

    $= r_1 \cdot r_2[(\cos(\theta_1 + \theta_2) + i(\sin\theta_1 + \theta_2)]$

    $= r_1 \cdot r_2 \cdot \text{cis}\,(\theta_1 + \theta_2)$
15. $(r_1 \cdot \text{cis}\,\theta_1) \div (r_2 \cdot \text{cis}\,\theta_2) = \frac{r_1}{r_2} \cdot \frac{\cos\theta_1 + i\sin\theta_1}{\cos\theta_2 + i\sin\theta_2}$

    $= \frac{r_1}{r_2} \cdot \frac{\cos\theta_1 + i\sin\theta_1}{\cos\theta_2 + i\sin\theta_2} \cdot \frac{\cos\theta_2 - i\sin\theta_2}{\cos\theta_2 - i\sin\theta_2}$

    $= \frac{r_1}{r_2} \cdot \frac{(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 - i\sin\theta_2)}{1}$

    $= \frac{r_1}{r_2} \cdot ((\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2) + i(cos\theta_2 sin\theta_1 - cos\theta_1 sin\theta_2))$

    $= \frac{r_1}{r_2}[(\cos(\theta_1 - \theta_2) + i(\sin\theta_1 - \theta_2)]$

    $= \frac{r_1}{r_2} \cdot \text{cis}\,(\theta_1 - \theta_2)$

11.4 De Moivre's Theorem and nth Roots

Answers

1. $-4-4i$

2. -8

3. $117+44i$

4. $-16\sqrt{3}-16i$

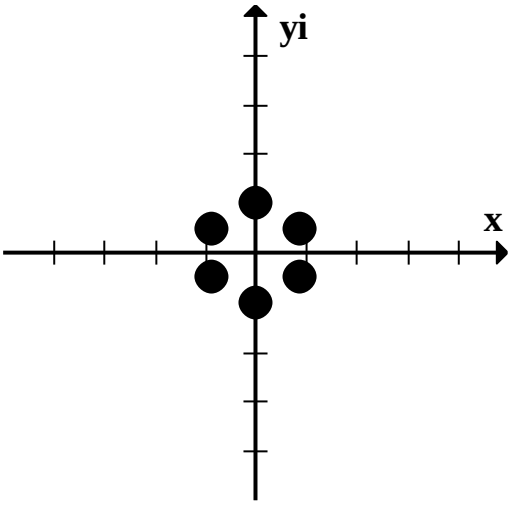
5. $-\frac{1}{2}-\frac{\sqrt{3}}{2}i$

6. $5^{\frac{1}{3}}\text{ cis } 17.71° = 1.63+0.52i$, $5^{\frac{1}{3}}\text{cis } 137.71 = -1.27+1.15i$, $5^{\frac{1}{3}}\text{cis } 257.71 = -0.36-1.67i$

7. $2\ cis\ 18 = 1.9+0.62i, 2cis90 = 2i, 2cis162 = -1.9+0.62i, 2cis252 = -1.18-1.6i, 2cis342 = 1.18-1.62i$

8. $\sqrt{6}cis\ 13.18 = 1.16+0.27i, \sqrt{6}cis\ 85.18 = 0.1+1.19i, \sqrt{6}cis\ 157.18 = -1.1+0.46i,$ $\sqrt{6}cis\ 247.18 = -0.78-0.91i, \sqrt{6}cis\ 337.18 = 0.62-1.02i$

9. $-2i, 2i, -1.732+i, 1.732-i, -1.732-i, 1.732+i$

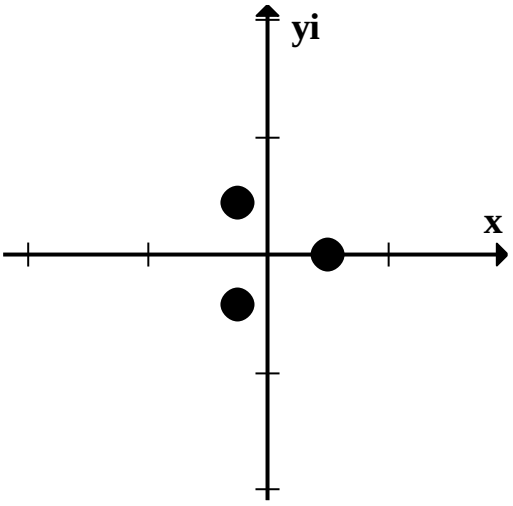


10. $x = -2i, 2i, -1.732+i, 1.732-i, -1.732-i, 1.732+i$

11. a) 3 roots

b) $x = 1, x = -0.5 - \frac{\sqrt{3}}{2}i, x = -0.5 + \frac{\sqrt{3}}{2}i$

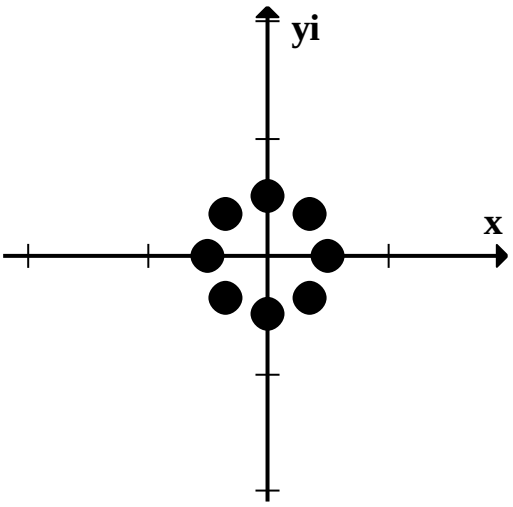
c)



12. a) 8 roots

b) $x = -1, x = 1, x = -i, x = i, x = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i, x = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i, x = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i, x = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

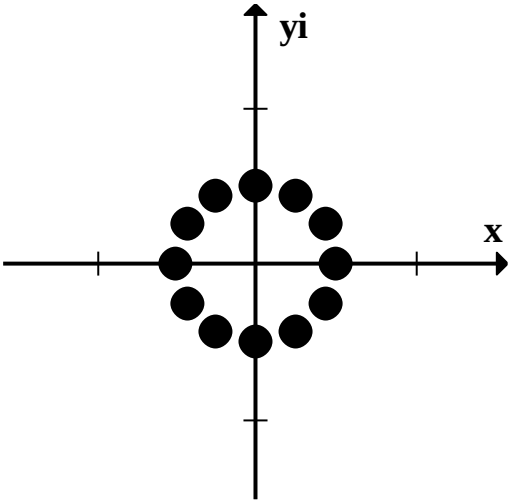
c)



13. a) 12 roots

b) $x = -1, x = 1, x = -i, x = i, x = -\frac{\sqrt{3}}{2} - 0.5i, x = \frac{\sqrt{3}}{2} + 0.5i, x = -0.5 - \frac{\sqrt{3}}{2}i, x = 0.5 + \frac{\sqrt{3}}{2}i, x = 0.5 - \frac{\sqrt{3}}{2}i, x = -0.5 + \frac{\sqrt{3}}{2}i, x = \frac{\sqrt{3}}{2} - 0.5i, x = -\frac{\sqrt{3}}{2} + 0.5i$

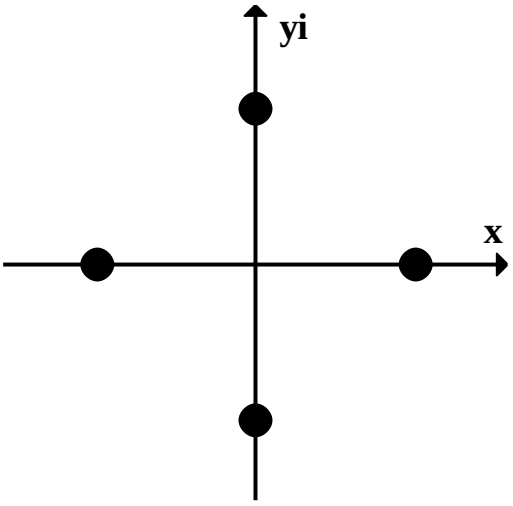
c)



14. a) 4 roots

b) $x = -2, x = 2, x = 2i, x = -2i$

c)



15.) 3 roots

b) $x = 3, x = \left(\frac{3}{2}\right)(-1 - i\sqrt{3}), x = \left(\frac{3}{2}\right)(-1 + i\sqrt{3})$

c)