5.1 Midsegment Theorem

Answers

1. True

2. True

3. False; there are *four* congruent triangles formed by the midsegments and the sides of a triangle.

4. True

5. $RS = TU = 6$

6. $TU = 8$

7. $x = 5$, $TU = 10$

8. $x = 2$

9. $y = 18$

10. $x = 5.5$

11. $x = 6$

12. $x = 14$, $y = 24$

13. $x = 6$, $z = 26$

14. $x = 5$, $y = 3$

15. $x = 1$, $z = 11$

16. a) 13, 19, and 21

    b) 53

    c) 106

    d) The perimeter of the larger triangle is double the perimeter of the midsegment triangle. Or, the perimeter of the midsegment triangle is half the perimeter of the larger triangle.

17. (7, 1), (3, 6), and (1, 3)

18. (3, 6), (2, 2), and (−5, −3)

19. (2, 10), (7, 1), and (4, −1)

20. (−1, −8), (−9, 5), and (−12, −2)

5.2 Perpendicular Bisectors

Answers

1. $x = 5$

2. $x = \frac{1}{2}$

3. $x = 31°$

4. $x = 34$

5. $AE = EB$, $AD = DB$

6. No, because $AC \neq CB$

7. Yes, because $AD = DB$

8. No, we don't know if T is the midpoint of $\overline{XY}$ or if $\overleftrightarrow{ST}$ passes though the vertex at the top of the triangle.

9. Equilateral Triangle

10.

Statement	***Reason***
1. $\overleftrightarrow{CD}$ is the perpendicular bisector of $\overline{AB}$	Given
2. D is the midpoint of $\overline{AB}$	Definition of a perpendicular bisector
3. $\overline{AD} \cong \overline{DB}$	Definition of a midpoint
4. $\angle CDA$ and $\angle CDB$ are right angles	Definition of a perpendicular bisector
5. $\angle CDA \cong \angle CDB$	Definition of right angles
6. $\overline{CD} \cong \overline{CD}$	Reflexive PoC
7. $\Delta CDA \cong \Delta CDB$	SAS
8. $\overline{AC} \cong \overline{CB}$	CPCTC

5.3 Angle Bisectors in Triangles

Answers

1. $x = 8°$
2. $x = 7°$
3. $x = 9$
4. $x = 9$
5. No, the line segment must also be perpendicular to the sides of the angle.
6. Yes, the angles are marked congruent.
7. Every type of triangle.
8. A diagonal
9. 4 isosceles right triangles with half of each diagonal as the legs and 4 isosceles right triangles with the sides of the squares as the legs.

10.

Statement	***Reason***
1. $\overline{AD} \cong \overline{DC}$	Given
2. $\overrightarrow{BA} \perp \overline{AD}$ and $\overrightarrow{BC} \perp \overline{DC}$	The shortest distance from a point to a line is perpendicular.
3. $\angle DAB$ and $\angle DCB$ are right angles	Definition of perpendicular lines
4. $\angle DAB \cong \angle DCB$	All right angles are congruent
5. $\overline{BD} \cong \overline{BD}$	Reflexive PoC
6. $\Delta ABD \cong \Delta CBD$	HL
7. $\angle ABD \cong \angle DBC$	CPCTC
8. $\overrightarrow{BD}$ bisects $\angle ABC$	Definition of an angle bisector

5.4 Medians

Answers

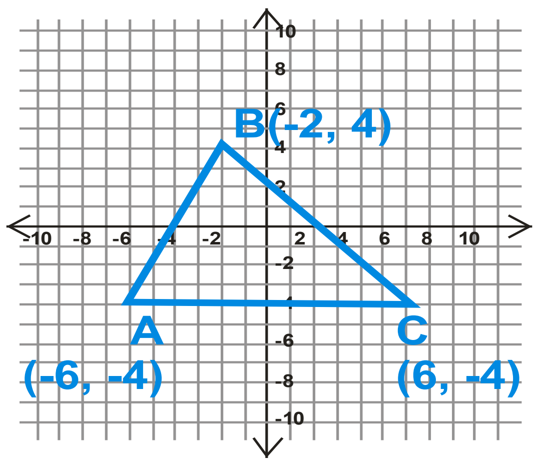
1. $GF = 8$, $CF = 24$

2. $AG = 20$, $GD = 10$

3. $GC = 2x$, $CF = 3x$

4. $x = 2$, $AD = 27$

5.

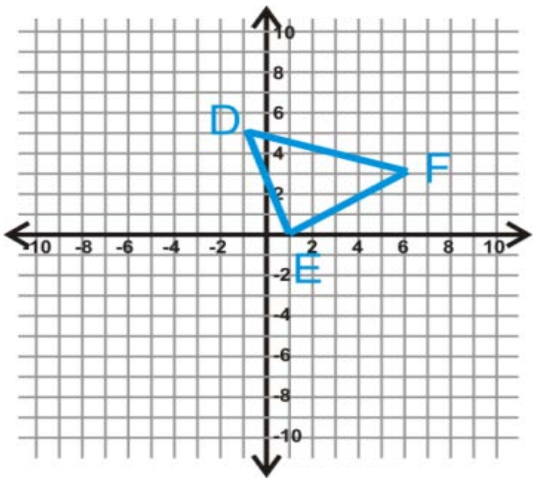


6. $D(0, -4)$

7. $m_{BD} = -4$

8. $y = -4x - 4$

9.



10. $G(3, 1)$

11. $m_{DG} = -1$

12. $y = -x + 4$

13. True

5.5 Altitudes

Answers

1. Outside
2. Inside
3. A leg
4. Inside
5. Outside
6. Inside
7. A leg
8. Outside
9. A leg
10. $\overline{AB}$ or $\overline{BC}$
11. Drawing is confusing here. Assuming the triangle is ΔEHG, $\overline{HG}$ would be the altitude.
12. $\overline{JL}$
13. $\overline{NP}$
14. $\overline{TS}$
15. $\overline{UV}$ or $\overline{UW}$

5.6 Comparing Angles and Sides in Triangles

Answers

1. *AB*, *BC*, *AC*

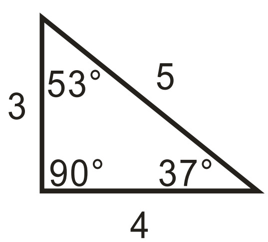
2. *BC*, *AB*, *AC*

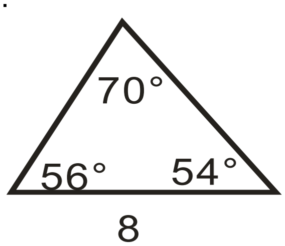
3. *AC*, *BC*, *AB*

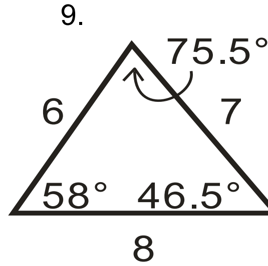
4. $\angle B$, $\angle A$, $\angle C$

5. $\angle B$, $\angle C$, $\angle A$

6. $\angle C$, $\angle B$, $\angle A$

7.


8.


9.


10. In the isosceles triangle on the left, $x + 3$ is the shortest side. In the isosceles triangle on the right, $3x + 1$ is the largest side. $0 < x < 10.\overline{3}$ (By the Triangle Inequality Theorem, which is in the next concept.)

11. By the SAS Inequality Theorem, $m\angle 1 > m\angle 2$ because $7 > 6$ and the other two sides of the triangles are equal.

12. In ΔGIJ, the sides from smallest to largest: IJ, IG, GJ

    In ΔGHJ, the sides from smallest to largest: GJ, GH, JH

    Because GJ is the longest side in ΔGIJ and the shortest side in ΔGHJ, we can put the five lengths in one list: IJ, IG, GJ, GH, JH.

13. $m\angle 1 < m\angle 2$ because the sides they are between are congruent and $\angle 1$ is opposite the smaller side (SAS Inequality Theorem). If $m\angle 1 < m\angle 2$ and the other two angles in the triangles are congruent, then $m\angle 3 > m\angle 4$.

5.7 Triangle Inequality Theorem

Answers

1. No, 6 + 6 < 13; the two smaller sides must add up to be larger than the third side.
2. No, 1 + 2 = 3; the two smaller sides must add up to be larger than the third side.
3. Yes; 7 + 8 > 10
4. Yes; 3 + 4 > 5
5. No, 23 + 56 < 85; the two smaller sides must add up to be larger than the third side.
6. Yes; 30 + 40 > 50
7. Yes; 7 + 8 > 14
8. No, 7 + 8 = 15; the two smaller sides must add up to be larger than the third side.
9. Yes; 7 + 8 > 14.99
10. 1 < 3^{rd} side < 17
11. 11 < 3^{rd} side < 19
12. 12 < 3^{rd} side < 52
13. 3 < 3^{rd} side < 7
14. 2 < 3^{rd} side < 18
15. x < 3^{rd} side < $3x$
16. 0 < base < 24

5.8 Indirect Proof in Algebra and Geometry

Answers

1. Assume the opposite.

   n is odd. Therefore *n* = 2*a* + 1, where *a* is any whole number. Now we need to show that n^2 is even.

   Find n^2.

   $n^2 = (2a + 1)^2 = (2a + 1)(2a + 1) = 4a^2 + 2a + 2a + 1 = 4a^2 + 4a + 1$

   Contradiction: $4a^2 + 4a + 1$ can never be even. $4a^2$ will always be even because 4 is a factor and is even, and 4a will always be even for the same reason. Therefore, $4a^2 + 4a$ is an even number, making $4a^2 + 4a + 1$ odd. This contradicts our assumption, making the original statement true: If *n* is an integer and n^2 is even, then *n* is even.

2. Assume the opposite: $\triangle ABC$ is equilateral.

   If $\triangle ABC$ is equilateral, then all the angles are equal and 60°.

   Contradiction: This statement contradicts $m\angle A \neq m\angle B$. Therefore, our original statement is true: If $m\angle A \neq m\angle B$ in $\triangle ABC$, then $\triangle ABC$ is not equilateral.

3. Assume the opposite: $x^2 < 9$

   Solving for *x*, we get $x < 3$ or $x > -3$.

   Contradiction: Our original statement says $x > 3$, which contradicts $x < 3$ from above.

4. <u>Assumption</u>: The base angles of an isosceles triangle are not congruent.

If the base angles are not congruent, then this is a scalene triangle. Recall that an isosceles triangle has two congruent sides. If all the sides are different, then it cannot be isosceles.

<u>Contradiction</u>: This statement contradicts our original statement. Therefore, the base angles of an isosceles triangle are congruent.

5. <u>Assumption</u>: $x + y$ is even. Let's say $x + y = 2n$, where n is any integer.

Solving for n, we have $n = \frac{x+y}{2} = \frac{x}{2} + \frac{y}{2}$. This implies that x and y are both even, so they can be divided by 2 evenly.

<u>Contradiction</u>: This statement contradicts the original statement that y is odd. Therefore, if x is even and y is odd, then $x + y$ is odd.

6. <u>Assumption</u>: In ΔABE, $\angle A$ is a right angle and $\angle B$ is obtuse.

If $\angle B$ is obtuse, then $m\angle B > 90^\circ$. Let's say $m\angle B = 91^\circ$. Using the Triangle Sum Theorem, we have:

$m\angle \text{A} + m\angle \text{B} + m\angle \text{E} = 180^\circ$

$90^\circ + 91^\circ\ m\angle \text{E} = 180^\circ$

$m\angle \text{E} = -1^\circ$

<u>Contradiction</u>: We know that an angle in a triangle cannot be negative, so our assumption is false. Therefore, in ΔABE, if $\angle A$ is a right angle, then $\angle B$ cannot be obtuse.

7. Assumption: $AB + BC \neq AC$.

   If $AB + BC \neq AC$, then A, B, and C could be the vertices of a triangle, making the three points non-collinear.

   Contradiction: This contradicts our original statement, so if A, B, and C are collinear, then $AB + BC = AC$ (Segment Addition Postulate).

8. Assumption: ΔABC is not equilateral.

   If ΔABC is not equilateral, then the angles are not all 60°. Two of the three angles could be 72°, making the third angle 36°.

   Contradiction: This contradicts our original statement, where the base angles cannot be 72°. Therefore, if a triangle is equilateral, the base angles cannot be 72°.

9. Assumption: $x \neq 11$

   If we solve for x in the equation, then $x = 9$ to make the equation true.

   Contradiction: The equation is not true, so this is our contradiction, meaning that $x = 11$.

10. Assumption: ΔABC is not a right triangle.

    If ΔABC is not a right triangle, then it can have side lengths 3, 4, and 6.

    Contradiction: The original statement says that the side lengths cannot be 3, 4, and 6. Therefore, this is our contradiction and ΔABC is a right triangle.