

## Chapter 2

### Lesson 2.1

1. Absolute value represents the distance from zero when graphed on a number line.
2. proper fractions, improper fractions, equivalent
3. There are many possible answers. Some examples are 1.3, 0.55, and  $\frac{12}{37}$ .
4. Rational numbers can be written as a ratio of two integers, where irrational numbers cannot.
5. Since the tic marks are evenly spaced, we can determine that each mark represents 3 units because there are 7 marks from 0 to 21 and  $21 \div 7 = 3$ . Therefore,  $a: -3, b: 3, c: 9, d: 12, e: 15$
6. There are 3 sections and one is shaded:  $\frac{1}{3}$ .
7. There are 12 sections and 7 are shaded:  $\frac{7}{9}$ .
8. There are 70 squares and 44 squares shaded:  $\frac{44}{70} = \frac{22}{35}$ .
9. If the numerators are the same, the larger the denominator, the smaller the fraction. So,  $\frac{1}{4}, \frac{1}{3}, \frac{1}{2}$ .

10. The fractions are in reverse order. Since they all have different denominators and numerators, we must get a common denominator:

$$10(11)(12) = 1320$$

$$\frac{13 \times 132}{10 \times 132} = \frac{1,716}{1,320}$$

$$\frac{12 \times 120}{11 \times 120} = \frac{1,440}{1,320}$$

$$\frac{11 \times 110}{12 \times 110} = \frac{1,210}{1,320}$$

Now we know the correct order by comparing the numerators:  $\frac{11}{12}, \frac{12}{11}, \frac{13}{10}$ .

11. We need to find a common denominator between 60, 80, and 100.

$$\frac{39}{60} = \frac{39(80)}{4800} = \frac{3120}{4800}$$

$$\frac{49}{80} = \frac{49(60)}{4800} = \frac{2940}{4800}$$

$$\frac{59}{100} = \frac{59(48)}{4800} = \frac{2832}{4800}$$

By comparing the denominators, we find the order should be  $\frac{59}{100}, \frac{49}{80}, \frac{39}{60}$ .

12. We must first get a common denominator.

$$11(13)(19) = 2717$$

$$\frac{7 \times 247}{11 \times 247} = \frac{1,729}{2,717}$$

$$\frac{8 \times 209}{13 \times 209} = \frac{1,672}{2,717}$$

$$\frac{12 \times 143}{19 \times 143} = \frac{1,716}{2,717}$$

We can now determine the order of  $\frac{8}{13}, \frac{12}{19}, \frac{7}{11}$ .

13.  $\frac{22 \div 22}{44 \div 22} = \frac{1}{2}$

14.  $\frac{9 \div 9}{27 \div 9} = \frac{1}{3}$

15.  $\frac{12 \div 6}{18 \div 6} = \frac{2}{3}$

16.  $\frac{315 \div 5}{420 \div 5} = \frac{63 \div 21}{84 \div 21} = \frac{3}{4}$

17. Since 19 is a prime number, this fraction can not be reduced.

18.  $\frac{99 \div 11}{11 \div 11} = \frac{9}{1} = 9$

19. The opposite of 1.001 is  $-1.001$ .

20. The opposite of  $-9.345$  is  $9.345$ .

21. The opposite of  $(16 - 45) = -(16 - 45)$ . Simplified, the answer is  $-(-29) = 29$ .

22. The opposite of  $(5 - 11) = -(5 - 11)$ . Simplified, the answer is  $-(-6) = 6$ .

23. The opposite of  $(x + y) = -(x + y)$  or  $-x - y$ .
24. The opposite of  $(x - y) = -(x - y) = -x + y$ .
25.  $|-98.4| = 98.4$
26.  $|123.567| = 123.567$
27.  $-|16 - 98| = -|-82| = -(82) = -82$
28.  $11 - |-4| = 11 - 4 = 7$
29.  $|4 - 9| - |-5| = |-5| - 5 = 5 - 5 = 0$
30.  $|-5 - 11| = |-16| = 16$
31.  $7 - |22 - 15 - 19| = 7 - |-12| = 7 - 12 = -5$
32.  $-|-7| = -7$
33.  $|-2 - 88| - |88 + 2| = |-90| - |90| = 90 - 90 = 0$
34.  $|-5 - 99| + |-16 - 7| = |-104| - |9| = 104 - 9 = 95$
35. To compare the numbers, compare the first two digits. Since  $8 > 7$ , then  $8 > 7.99999$ .
36. First, convert  $\frac{-17}{4}$  to a decimal.  $\frac{-17}{4} = -4.25$ .  
Then we can see that  $-4.5 = \frac{-17}{4}$ .
37.  $65 > -1$
38. 10 units left of zero is  $-10$  and 9 units right of zero is 9. So,  $-10 < 9$ .
39. After one jump the frog will land on either  $7 + 2 = 9$ , or  $7 - 2 = 5$ . After 5 jumps, the frog could end up at  $7 - 5(2) = 7 - 10 = -3$ ,  $7 + 5(2) = 7 + 10 = 17$ , or any integer in between. Written differently,  $-3 < x < 17$  where  $x$  = the location of the frog.
40. All real numbers will have an additive identity except 0 because it does not have an opposite.
41.  $\frac{5}{6}d + 7a^2 = \frac{5}{6}(24) + 7(-1)^2 = 20 + 7 = 27$

42. Let  $l$  = the length of the rectangle and  $w$  = the width of the rectangle. Then we have  $2l + 2w = 22$ . Also we know that  $w = l - 1$ . By substitution,

$$2l + 2(l - 1) = 22$$

$$2l + 2l - 2 = 22$$

$$4l - 2 = 22$$

$$\frac{4l}{4} = \frac{24}{4}$$

$$l = 6$$

$$l = 6$$

That means that  $w = 6 - 1 = 5$ . The length is 6 inches, and the width is 5 inches.

43.  $4(2) + 7 \leq 15$

$$8 + 7 \leq 15$$

$$15 \leq 15 \checkmark$$

44.  $\frac{(7+3) \div 2 \times 3^2 - 5}{(58-8)} = \frac{10 \div 2 \times 9 - 5}{50} = \frac{45 - 5}{50} = \frac{40}{50} = \frac{4}{5}$

## Lesson 2.2

1.  $3 + 3 + 1 - (-1) = 7 + 1 = 8$

2.  $0 - 2 + 7 + 1 = 6$

3.  $\frac{3}{7} + \frac{2}{7} = \frac{3+2}{7} = \frac{5}{7}$

4.  $\frac{3}{10} + \frac{1}{5} = \frac{3+1(2)}{10} = \frac{5}{10} = \frac{1}{2}$

5.  $\frac{5}{16} + \frac{5}{12} = \frac{5(3)+5(4)}{48} = \frac{15+20}{48} = \frac{35}{48}$

6.  $\frac{3}{8} + \frac{9}{16} = \frac{3(2)+9}{16} = \frac{15}{16}$

7.  $\frac{8}{25} + \frac{7}{10} = \frac{8(2)+7}{50} = \frac{23}{50}$

8.  $\frac{1}{6} + \frac{1}{4} = \frac{1(2)+1(3)}{12} = \frac{5}{12}$

9.  $\frac{7}{15} + \frac{2}{9} = \frac{7(3)+2(5)}{45} = \frac{21+10}{45} = \frac{31}{45}$

10.  $\frac{5}{19} + \frac{2}{27} = \frac{5(27)+2(19)}{513} = \frac{135+38}{513} = \frac{173}{513}$

11.  $-2.6 + 11.19 = 8.59$

12.  $-8 + 13 = 5$

13.  $-7.1 + (-5.63) = -7.1 - 5.63 = -12.73$

14.  $9.99 + (-0.01) = 9.99 - 0.01 = 9.98$

15.  $4\frac{7}{8} + 1\frac{1}{2} = \frac{39}{8} + \frac{3}{2} = \frac{39+3(4)}{8} = \frac{39+12}{8} = \frac{51}{8}$

16.  $-3\frac{1}{3} + \left(-2\frac{3}{4}\right) = \frac{-10}{3} + \frac{-11}{4} = \frac{-10(4)-11(3)}{12} = \frac{-40-33}{12} = \frac{-73}{12}$

17. Order refers to the commutative property, so the commutative property of addition.
18. This is the associative property of addition.
19. This is a variation of the additive identity property.
20. This is the additive identity property.
21. Let  $c$  = the total cost. Nadia contributes is  $\frac{1}{2}c$ , Ian contributes  $\frac{1}{3}c$ , and Peter contributes  $\frac{1}{4}c$ . In total, they contribute
- $$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{1(6) + 1(4) + 1(3)}{12} = \frac{6 + 4 + 3}{12} = \frac{13}{12}$$
- Therefore, the amount of tax is  $\frac{13}{12} - 1 = \frac{13}{12} - \frac{12}{12} = \frac{1}{12}$  the total cost.
22.  $-160 + 8 = -152$  feet
23.  $-8 + 25 = 17$  degrees
24.  $9.1 + 5.8 = 14.9$
25.  $5.8 + (-5.8) = 0$
26.  $-5.8 + 4.12 = -1.68$
27.  $-23.14 + (-5.8) = -28.94$
28.  $7.86 + (-5.8) = 2.06$
29.  $-5.8 + 3.5 = -2.3$
30.  $-5.8 + 5.8 = 0$
31. The opposite of  $-72$  is  $72$ .
32.  $|16 - 29 + 78 - 114| = |-49| = 49$
33. domain =  $\{1, 2, 3, 4\}$  and range =  $\{-3, -1, 0, 6\}$
34. Mass =  $20.1(\text{Volume})$

### Lesson 2.3

1.  $9 - 14 = -5$

2.  $2 - 7 = -5$

3.  $21 - 8 = 13$

4.  $8 - (-14) = 8 + 14 = 22$

5.  $-11 - (-50) = -11 + 50 = 39$

6.  $\frac{5}{12} - \frac{9}{18} = \frac{5}{12} - \frac{1}{2} = \frac{5-1(6)}{12} = \frac{5-6}{12} = \frac{-1}{12}$

7.  $5.4 - 1.01 = 4.39$

8.  $\frac{2}{3} - \frac{1}{4} = \frac{2(4)-1(3)}{12} = \frac{8-3}{12} = \frac{5}{12}$

9.  $\frac{3}{4} - \frac{1}{3} = \frac{3(3)-1(4)}{12} = \frac{9-4}{12} = \frac{5}{12}$

10.  $\frac{1}{4} - \left(-\frac{2}{3}\right) = \frac{1}{4} + \frac{2}{3} = \frac{1(3)+2(4)}{12} = \frac{3+8}{12} = \frac{11}{12}$

11.  $\frac{15}{11} - \frac{9}{7} = \frac{15(7)-9(11)}{77} = \frac{105-99}{77} = \frac{6}{77}$

12.  $\frac{2}{13} - \frac{1}{11} = \frac{2(11)-1(13)}{143} = \frac{22-13}{143} = \frac{9}{143}$

13.  $-\frac{7}{8} - \left(-\frac{8}{3}\right) = -\frac{7}{8} + \frac{8}{3} = \frac{-7(3)+8(8)}{24} = \frac{-21+64}{24} = \frac{43}{24}$

14.  $\frac{7}{27} - \frac{9}{39} = \frac{7(39)-9(27)}{1053} = \frac{273-243}{1053} = \frac{30}{1053} = \frac{10}{351}$

15.  $\frac{6}{11} - \frac{3}{22} = \frac{6(2)-3}{22} = \frac{12-3}{22} = \frac{9}{22}$

16.  $-3.1 - 21.49 = -24.59$

17.  $\frac{13}{64} - \frac{7}{40} = \frac{13(5)-7(8)}{320} = \frac{80-56}{320} = \frac{24}{320} = \frac{3}{40}$

$$18. \frac{11}{70} - \frac{11}{30} = \frac{11(3) - 11(7)}{210} = \frac{33 - 77}{210} = \frac{-44}{210} = -\frac{22}{105}$$

$$19. -68 - (-22) = -68 + 22 = -46$$

$$20. \frac{1}{3} - \frac{1}{2} = \frac{1(2) - 1(3)}{6} = \frac{2 - 3}{6} = -\frac{1}{6}$$

$$21. 9 - (-14) = 9 + 14 = 23$$

22. When  $x = 3$ ,  $y = 3(3) + 2 = 6 + 2 = 8$ . When  $x = 7$ ,  $y = 3(7) + 2 = 21 + 2 = 23$ . So, the difference is  $23 - 8 = 15$ .

$$23. \text{ When } x = 1, y = \frac{2}{3}(1) + \frac{1}{2} = \frac{2}{3} + \frac{1}{2} = \frac{2(2) + 1(3)}{6} = \frac{4 + 3}{6} = \frac{7}{6}. \text{ When } x = 7,$$

$$y = \frac{2}{3}(7) + \frac{1}{2} = \frac{14}{3} + \frac{1}{2} = \frac{4(7) + 1(3)}{6} = \frac{28 + 3}{6} = \frac{31}{6}. \text{ So, the difference is}$$

$$\frac{31}{6} - \frac{7}{6} = \frac{24}{6} = 4.$$

24. This statement is false. Consider the example  $3 - (-4) = 12$ . Twelve is greater than both 3 and  $-4$ .

25. This statement is true. For any number,  $x$ ,  $x - (-x) = x + x = 2x$ .

26. The closing price will be  $\$4.83 - \$0.97 = \$3.86/\text{share}$ .

$$27. (a - b) + c = 2 - (-3) + (-1.5) = 5 - 1.5 = 3.5$$

$$28. |b + c| - a = |-3 + (-1.5)| - 2 = |-4.5| - 2 = 4.5 - 2 = 2.5$$

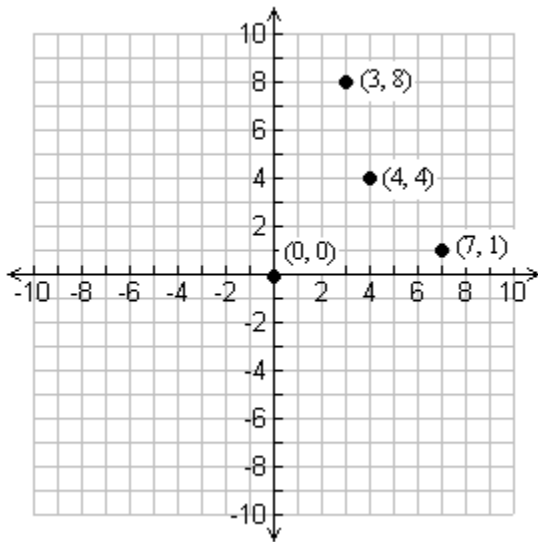
$$29. a - (b + c) = 2 - (-3 + (-1.5)) = 2 - (-4.5) = 2 + 4.5 = 6.5$$

$$30. |b| + |c| + a = |-3| + |-4.5| + 2 = 3 + 4.5 + 2 = 9.5$$

$$31. 7b + 4a = 7(-3) + 4(2) = -21 + 8 = -13$$

$$32. (c - a) - b = (-1.5 - 2) - (-3) = -3.5 + 3 = -0.5$$

33.



34. When  $m = -\frac{2}{3}$ , the expression is  $\frac{2^3 + -\frac{2}{3}}{4} = \frac{8 - \frac{2}{3}}{4} = \frac{\frac{24-2}{3}}{4} = \frac{\frac{22}{3}}{4} = \frac{22}{3} \left( \frac{4}{1} \right) = \frac{88}{3}$ .

35. Let  $r$  = the amount of money Ricky has,  $s$  = the amount of money Stacy has, and  $a$  = the amount of money Aaron has. Then we know that  $r + s + a = 62$ . Since we know Ricky,  $r$ , has \$12 more than Stacy,  $s$ , then we can write  $r = s + 12$ . We also know that Stacy,  $s$ , has \$5 fewer than Aaron,  $a$ , so we can write  $a = s + 5$ . By substitution,  $r + s + a = 62$  becomes  $(s + 12) + s + (s + 5) = 62$ . Simplifying gives  $3s + 17 = 62$ .

36.  $\frac{1}{3} + \frac{7}{5} = \frac{1(5) + 7(3)}{15} = \frac{5 + 21}{15} = \frac{26}{15}$

37.  $\frac{21}{4} - \frac{2}{3} = \frac{21(3) - 2(4)}{12} = \frac{63 - 8}{12} = \frac{55}{12}$

### Lesson 2.4

$$1. \frac{1}{2} \cdot \frac{3}{4} = \frac{1(3)}{2(4)} = \frac{3}{8}$$

$$2. -7.85 - 2.3 = -10.15$$

$$3. \frac{2}{5} \cdot \frac{5}{9} = \frac{2(5)}{5(9)} = \frac{2}{9}$$

$$4. \frac{1}{3} \cdot \frac{2}{7} \cdot \frac{2}{5} = \frac{1(2)(2)}{3(7)(5)} = \frac{4}{105}$$

$$5. 4.5 \cdot -3 = -13.5$$

$$6. \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} = \frac{1(2)(3)(4)}{2(3)(4)(5)} = \frac{1}{5}$$

$$7. \frac{5}{12} \times \frac{9}{10} = \frac{5(9)}{12(10)} = \frac{45}{120} = \frac{3}{8}$$

$$8. \text{Since anything times 0 is 0, } \frac{27}{5} \cdot 0 = 0$$

$$9. \frac{2}{3} \times \frac{1}{4} = \frac{2(1)}{3(4)} = \frac{2}{12} = \frac{1}{6}$$

$$10. -11.1(4.1) = -45.51$$

$$11. \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$$

$$12. \frac{15}{11} \times \frac{9}{7} = \frac{15(9)}{11(7)} = \frac{135}{77}$$

$$13. \frac{2}{7} \cdot -3.5 = \frac{2(-3.5)}{7} = \frac{-7}{7} = -1$$

$$14. \frac{1}{13} \times \frac{1}{11} = \frac{1}{143}$$

$$15. \frac{7}{27} \times \frac{9}{14} = \frac{7(9)}{27(14)} = \frac{1}{6}$$

$$16. \left(\frac{3}{5}\right)^2 = \frac{3^2}{5^2} = \frac{9}{25}$$

$$17. \frac{1}{11} \times \frac{22}{21} \times \frac{17}{10} = \frac{1(22)(17)}{11(21)(10)} = \frac{17}{1155}$$

$$18. 5.75 \cdot 0 = 0$$

$$19. 79.5(-1) = -79.5$$

$$20. \pi(-1) = -\pi$$

$$21. (x + 1)(-1) = -(x + 1)$$

$$22. |x|(-1) = -|x|$$

$$23. 25(-1) = -25$$

$$24. -105(-1) = 105$$

$$25. x^2(-1) = -x^2$$

$$26. (-1)(3 + x) = -1(3 + x) = -3 - x$$

$$27. (-1)(3 - x) = -1(3 - x) = -3 + x$$

28. Since  $3 + 5 = 8$ , this is the distributive property because  $8(7) = 3(7) + 5(7)$ .

29. Since there is only one pile, this is the multiplicative identity, or identity of multiplication because  $10(1) = 10$ .

30. This is the commutative property of addition since  $3(5) = 5(3)$ .

31. To compare these numbers, we first need to get a common denominator. The common denominator is 27, so the first fraction,  $\frac{16}{27}$ , will stay the same. We need to convert the second fraction.

$$\frac{2 \times 9}{3 \times 9} = \frac{18}{27}. \text{ Since } 16 < 18, \frac{16}{27} < \frac{18}{27} \text{ therefore } \frac{16}{27} < \frac{2}{3}.$$

32. rational numbers – numbers that can be written as a ratio of two integers.

33. A proper fraction is a fraction where the absolute value of the numerator is smaller than the absolute value of the denominator, such as  $\frac{1}{2}$ ,  $\frac{112}{245}$ , or  $-\frac{18}{93}$ . An improper fraction is where the absolute value of the numerator is larger than the absolute value of the denominator, like  $\frac{-8}{3}$ ,  $\frac{11}{2}$ , or  $\frac{213}{5}$ .

35. This property is the additive inverse property.

$$36. 11\frac{2}{9} + \frac{7}{8} = \frac{101}{9} + \frac{7}{8} = \frac{101(8) + 7(9)}{72} = \frac{808 + 63}{72} = \frac{871}{72}$$

### Quick Quiz

1. To order these, we could find a common denominator for all of the fractions, but an easier method would be to convert all of the fractions to decimals and compare them.

$$\frac{5}{6} = 0.83333, \frac{23}{26} = 0.8846, \frac{31}{32} = 0.9393, \frac{3}{14} = 0.2143$$

Now, the order, from least to greatest is  $\frac{3}{14}, \frac{5}{6}, \frac{23}{26}, \frac{31}{32}$ .

$$2. \frac{5}{9} \times \frac{27}{4} = \frac{15}{4}$$

$$3. |-5 + 11| - |9 - 37| = |6| - |-28| = 6 - 28 = -22$$

$$4. \frac{21}{5} + \frac{7}{8} = \frac{21(8) + 7(5)}{40} = \frac{168 + 35}{40} = \frac{203}{40}$$

## Lesson 2.5

1.  $(x + 4) - 2(x + 5) = x + 4 - 2x - 10 = -x - 6$
2.  $\frac{1}{2}(4z + 6) = 2z + 3$
3.  $(4 + 5) - (5 + 2) = 9 - 5 - 2 = 2$
4.  $(x + 2 + 7) = x + 9$
5.  $0.25(6q + 32) = 1.5q + 8$
6.  $y(x + 7) = xy + 7y$
7.  $-4.2(h - 11) = -4.2h + 46.2$
8.  $13x(3y + z) = 39xy + 13xz$
9.  $\frac{1}{2}(x - y) - 4 = \frac{1}{2}x - \frac{1}{2}y - 4$
10.  $0.6(0.2x + 0.7) = 0.12x + 0.42$
11.  $(2 - j)(-6) = -12 + 6j$
12.  $(r + 3)(-5) = -5r - 15$
13.  $6 + (x - 5) + 7 = 6 + x - 5 + 7 = x + 8$
14.  $6 - (x - 5) + 7 = 6 - x + 5 + 7 = -x + 18$
15.  $4(m + 7) - 6(4 - m) = 4m + 28 - 24 + 6m = 10m + 4$
16.  $-5(y - 11) + 2y = -5y + 55 + 2y = -3y + 55$
17.  $\frac{8x+12}{4}$  is the same as  $\frac{1}{4}(8x+12) = 2x+3$
18.  $\frac{9x+12}{3}$  is the same as  $\frac{1}{3}(9x+12) = 3x+4$
19.  $\frac{11x+12}{2} = \frac{1}{2}(11x+12) = \frac{11}{2}x+6$

20.  $\frac{3y+2}{6} = \frac{1}{6}(3y+2) = \frac{1}{2}y + \frac{1}{3}$

21.  $-\frac{6z-2}{3} = -\frac{1}{3}(6z-2) = -2z + \frac{2}{3}$

22.  $\frac{7-6p}{3} = \frac{7}{3} - 2p$

23.  $\frac{2}{3}n + 16$

24.  $2m - 3$

25.  $-4x(x) + 2$

26. Let  $p$  = the number of poetry books and  $n$  = the number of novels. Then the expression is  $5(7p + 11n)$  OR  $35p + 55n$

27. One could estimate 19.99 to 20. Then  $20 - 19.99 = 0.01$ . So,  $6(19.99) = 6(20) - 6(0.01) = 120 - 0.06 = 119.94$ .

28. The student did not distribute the 4 to the second value (10) in the parentheses. The correct answer would be  $4(9x + 10) = 36x + 40$ .

29. Like item 27, we can round 5998 up to 6000. Then  $9(5998) = 9(6000) - 9(2) = 54,000 - 18 = 53,982$

30. a. First,  $5 \text{ lbs} = 5(16) = 80 \text{ oz}$ . Then he can make  $\frac{80}{6} = 13.\bar{3}$ , or 13 cookies.

b. Second, using the number of cookies from a., he can put  $\frac{60}{13} \approx 4.62$ , or about 5 nuts on each cookie.

31. Let  $x$  = how much more Jacob needs. Then we have  $104 + x \geq 244$ .

32. The product of 6 and a number is 4 less than 16.

33. To represent the mixed number, fill in 3 whole rows for the whole number and 3 out of 4 parts of the row for  $\frac{3}{4}$ .


34. When  $x = 3$ ,  $y = \frac{1}{6}(3) - 4 = \frac{1}{2} - 4 = \frac{1 - 4(2)}{2} = -\frac{7}{2}$ .

When  $x = 9$ ,  $y = \frac{1}{6}(9) - 4 = \frac{3}{2} - 4 = \frac{3 - 4(2)}{2} = -\frac{5}{2}$ .

So, the change in  $y = -\frac{5}{2} - \left(-\frac{7}{2}\right) = -\frac{5}{2} + \frac{7}{2} = \frac{2}{2} = 1$ .

## Lesson 2.6

1. inverse – the opposite of something. In relation to operations, it is the operation that undoes another operation.
2. The multiplicative inverse for any number  $x$  is  $\frac{1}{x}$ . Their product is 1. The additive inverse for any number  $x$  is  $-x$ . Their sum is 0.
3. The multiplicative inverse of 100 is  $\frac{1}{100}$ .
4. The multiplicative inverse of  $\frac{2}{8}$  is  $\frac{8}{2}$ .
5. The multiplicative inverse of  $-\frac{19}{21}$  is  $-\frac{21}{19}$ .
6. The multiplicative inverse of 7 is  $\frac{1}{7}$ .
7. The multiplicative inverse of  $-\frac{z^3}{2xy^2}$  is  $-\frac{2xy^2}{z^3}$ .
8. The multiplicative inverse of 0 is 0.
9. The multiplicative inverse of  $\frac{1}{3}$  is 3.
10. The multiplicative inverse of  $\frac{-19}{18}$  is  $\frac{-18}{19}$ .
11. The multiplicative inverse of  $\frac{3xy}{8z}$  is  $\frac{8z}{3xy}$ .
12.  $\frac{5}{2} \div \frac{1}{4} = \frac{5}{2} \left( \frac{4}{1} \right) = \frac{20}{2} = 10$
13.  $\frac{1}{2} \div \frac{7}{9} = \frac{1}{2} \left( \frac{9}{7} \right) = \frac{9}{14}$
14.  $\frac{5}{11} \div \frac{6}{7} = \frac{5}{11} \left( \frac{7}{6} \right) = \frac{35}{66}$

$$15. \frac{1}{2} \div \frac{1}{2} = \frac{1}{2} \left( \frac{2}{1} \right) = \frac{2}{2} = 1 \text{ (multiplicative inverse)}$$

$$16. -\frac{x}{2} \div \frac{5}{7} = -\frac{x}{2} \left( \frac{7}{5} \right) = -\frac{7x}{10}$$

$$17. \frac{1}{2} \div \frac{x}{4y} = \frac{1}{2} \left( \frac{4y}{x} \right) = \frac{4y}{2x} = \frac{2y}{x}$$

$$18. -\frac{1}{3} \div \left( -\frac{3}{5} \right) = -\frac{1}{3} \left( -\frac{5}{3} \right) = \frac{5}{9}$$

$$19. \frac{7}{2} \div \frac{7}{4} = \frac{7}{2} \left( \frac{4}{7} \right) = \frac{4}{2} = 2$$

$$20. 11 \div \left( -\frac{x}{4} \right) = 11 \left( -\frac{4}{x} \right) = -\frac{44}{x}$$

$$21. \frac{x}{y} = \frac{\frac{3}{8}}{\frac{4}{3}} = \frac{3}{8} \left( \frac{3}{4} \right) = \frac{9}{32}$$

$$22. 4z \div u = 4(10) \div 0.5 = 40 \div 0.5 = 80$$

$$23. \frac{-6}{m} = \frac{-6}{\frac{2}{5}} = -6 \left( \frac{5}{2} \right) = -15$$

24. We can determine if  $\frac{\text{painted coverage (sample)}}{\text{amount of paint (sample)}}$  is  $<$ ,  $>$  or  $=$  50 sq ft/pint. So with

the values given:  $\frac{2(3)}{\frac{1}{8}} = \frac{6}{\frac{1}{8}} = 6(8) = 48$  sq ft. Since  $50 > 48$ , the coverage in the

sample is less than the stated coverage.

25. To find the amount of time, we can use  $d = rt$ , or distance = rate (time). Since we need to know how much time, we can solve for time in the equation to get

$$\text{time} = \frac{\text{distance}}{\text{rate}} = \frac{\frac{2}{3}}{\frac{8}{3}} = \frac{2}{3} \left( \frac{3}{8} \right) = \frac{16}{9} \approx 1.78 \text{ hours OR 1 hour and 47 minutes.}$$

26. Newton's 2<sup>nd</sup> law gives the equation  $F = ma$  (force = mass x acceleration). We need to know the mass of the object, so  $m = \frac{F}{a}$ . Using our values yields

$$m = \frac{\frac{2}{7}}{\frac{3}{10}} = \frac{2}{7} \left( \frac{10}{3} \right) = \frac{20}{21} \text{ kg.}$$

27. The reciprocal must be a number when multiplied by the original gives a product of 1, whereas the opposite of a number will add to the original to give a sum of 0.

28. Zero does not have a reciprocal because dividing by zero is undefined.

29.  $199 - (-11) = 199 + 11 = 210$

30.  $-2.3 - (-3.1) = -2.3 + 3.1 = 0.8$

31.  $|16 - 84| = |-68| = 68$

32.  $|\frac{-11}{4}| = \frac{11}{4}$

33.  $(4 \div 2 \times 6 + 10 - 5)^2 = (12 + 5)^2 = (17)^2 = 289$

34.  $f(21) = \frac{1}{9}(21 - 3) = \frac{1}{9}(18) = \frac{18}{9} = 2$

35. range = all the values represented by the dependent variable (resulting from substituting the values in from the domain.)

**Lesson 2.7**

1.  $\sqrt{25} = 5$

2.  $\sqrt{24} = \sqrt{4(6)} = 2\sqrt{6}$

3.  $\sqrt{20} = \sqrt{4(5)} = 2\sqrt{5}$

4.  $\sqrt{200} = \sqrt{100(2)} = 10\sqrt{2}$

5.  $\sqrt{2000} = \sqrt{400(5)} = 20\sqrt{5}$

6.  $\sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}} = \frac{1}{2}$

7.  $\sqrt{\frac{9}{4}} = \frac{\sqrt{9}}{\sqrt{4}} = \frac{3}{2}$

8.  $\sqrt{0.16} = \sqrt{0.4(0.4)} = 0.4$

9.  $\sqrt{0.1} = \sqrt{0.1}$  (cannot reduce further to an exact value)

10.  $\sqrt{0.01} = 0.1$

11.  $\sqrt{13} \approx 3.61$

12.  $\sqrt{99} \approx 9.95$

13.  $\sqrt{123} \approx 11.09$

14.  $\sqrt{2} \approx 1.41$

15.  $\sqrt{2000} \approx 44.72$

16.  $\sqrt{0.25} = 0.50$

17.  $\sqrt{1.35} \approx 1.16$

18.  $\sqrt{0.37} \approx 0.61$

19.  $\sqrt{0.7} \approx 0.84$

20.  $\sqrt{0.01} = 0.1$

21. real number, rational number

22. real number, irrational number

23. real number, irrational number

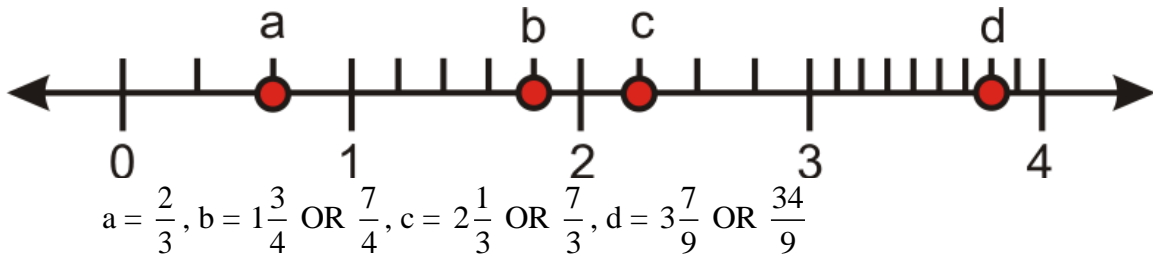
24.  $\sqrt{25} = 5$  real number, rational number, whole number, and integer

25.  $\sqrt{100} = 10$  real number, rational number, whole number, and integer

26.  $\frac{\sqrt{6}}{2} \approx 1.224744871\dots$ ,  $\frac{61}{50} = 1.22$ ,  $\sqrt{1.5} \approx 1.224744871\dots$ ,  $\frac{16}{13} \approx 1.23$

Therefore, the order is  $\frac{61}{50}, \frac{\sqrt{6}}{2}, \sqrt{1.5}, \frac{16}{13}$  OR  $\frac{61}{50}, \sqrt{1.5}, \frac{\sqrt{6}}{2}, \frac{16}{13}$

27.



28.  $\frac{9}{4} \div 6 = \frac{9}{4} \left( \frac{1}{6} \right) = \frac{9}{24} = \frac{3}{8}$

29.  $A = \frac{3(7)}{2} = \frac{21}{2} = 10.5$  sq. ft.

30.  $\frac{144 \div 6}{6 \div 6} = \frac{24}{1} = 24$

31.

$x$	$y$	$y = 60x$
0	0	$60(0) = 0$
1	60	$60(1) = 60$
2	120	$60(2) = 120$
3	180	$60(3) = 180$
4	240	$60(4) = 240$
5	300	$60(5) = 300$
6	360	$60(6) = 360$

## Lesson 2.8

1. *Nadia's father is 36. He is 16 years older than four times Nadia's age. How old is Nadia?*

Using the equation defined in Example 3, we have  $4 \times (\text{Nadia's age}) + 16 = 36$ .

So  $4 \times (\text{Nadia's age}) = 20$

Nadia's age = 5

2. Let  $N$  = the number of miles Nadia walks and  $P$  = the number of miles Peter walks. Then  $N + P = 6$ . We can then set up this table with our information.

	$d$	$r$	$t$
Nadia	$d$	3.5	$t$
Peter	$6 - d$	6	$t$
total	6	---	---

The total distance is 6 miles. Using the formula  $d = rt$ , we can write  $6 = 3.5t + 6t$  (since they are traveling towards each other). We can solve for  $t$  to get how long it will take for them to meet.

$$6 = 3.5t + 6t$$

$$\underline{6 = 9.5t}$$

$$9.5 \quad 9.5$$

$$t \approx 0.6316 \text{ hours, or } 0.6316(60) \approx 37.9 \text{ minutes}$$

To find the distance from Nadia's home, use  $d = rt$  where  $r = 3.5$  and  $t = 0.6316$  hours.

$$d = 3.5(0.6316) \approx 2.2 \text{ miles}$$

They will meet approximately 2.2 miles from Nadia's home.

3. Let  $R$  = the number of notebooks bought at Rite-Aid, and let  $S$  = the number of notebooks bought at Staples. Then we can write  $R + S = 17$

We also know that  $2.25S = 2R$ .

If we solve the first equation for  $R$ , we get  $R = 17 - S$ .

Then,

$$2.25S = 2(17 - S)$$

$$2.25S = 34 - 2S$$

$$\underline{4.25S = 34}$$

$$4.25 \quad 4.25$$

$$S = 8$$

That means that  $R = 17 - 8 = 11$ .

Peter bought 11 notebooks at Rite-Aid and 8 notebooks at Staples.

4. Let  $d$  = the number of dimes and  $q$  = the number of quarters. We know that  $d + q = 22$ . We also know that  $0.25q + 0.10d = 4$ . We can take the first equation and solve for  $d$ .

$$d + q = 22$$

$$q = 22 - d$$

Now we can substitute that value into the other equation to get

$$0.25(22 - d) + 0.10d = 4$$

Now we can find the number of dimes.

$$0.25(22 - d) + 0.10d = 4$$

$$5.5 - 0.25d + 0.10d = 4$$

$$5.5 - 0.15d = 4$$

$$\underline{-0.15d = -1.5}$$

$$\begin{array}{r} -0.15 \\ -0.15 \end{array}$$

$$d = 10$$

We now know there are 10 dimes. We can find the number of quarters by using the equation  $q = 22 - d = 22 - 10 = 12$ . There are 12 quarters.

5. Let  $l$  = the length and  $w$  = the width. From the given information, we know that  $l = 1.5w$ . We also know that the formula for the perimeter of this rectangle is  $2l + 2w = 50$ . Using substitution, we get

$$2(1.5w) + 2w = 50$$

$$3w + 2w = 50$$

$$\underline{5w = 50}$$

$$\begin{array}{r} 5 \\ 5 \end{array}$$

$$w = 10 \text{ feet}$$

Then the length,  $l = 1.5w = 1.5(10) = 15$  feet

The dimensions of the garden are 10 x 15 feet.

6. We know that every animal has a head, so  $p + c = 13$  if  $p$  = pigs and  $c$  = chickens. Also, we know that each pig has 4 feet and each chicken has 2 feet, so  $4p + 2c = 36$ . We can solve the first equation for  $p$  to get  $p = 13 - c$ . Now substitute that into the second equation.

$$4(13 - c) + 2c = 36$$

$$52 - 4c + 2c = 36$$

$$\underline{-2c = -16}$$

$$\begin{array}{r} -2 \\ -2 \end{array}$$

$$c = 8$$

Substitute that into the first equation to get  $p = 13 - 8 = 5$ .

There were 8 chickens and 5 pigs.

7. The simple interest formula is  $I = prt$ , where  $I$  is the interest earned,  $p$  = the principle (original amount invested),  $r$  = the interest rate (as a decimal) and  $t$  = time (in years).

We know that  $p_1 + p_2 = 8000$ . From that we can write  $p_2 = 8000 - p_1$ .

We also know that  $I = p_1r_1t + p_2r_2t$ . Using substitution, we can write

$$I = p_1r_1t + (8000 - p_1)r_2t$$

$$450 = p_1(0.0525)(1) + (8000 - p_1)(0.09)(1)$$

$$450 = 0.0525p_1 + 720 - 0.09p_1$$

$$-270 = -0.375p_1$$

$$p_1 = 7,200$$

That means that  $p_2 = 8000 - 7200 = 800$ .

So, Andrew invested \$7,200 in the account with 5.25% interest rate and \$800 in the 9% account.

8. We will have to work backwards to solve this item. First, we need to know “16 is

$\frac{2}{3}$  of what number?”

$$16 = \frac{2}{3}x$$

$$x = 16\left(\frac{3}{2}\right) = 24.$$

Now, we need to know “24 is  $\frac{4}{5}$  of what number?”

$$24 = \frac{4}{5}x$$

$$x = 24\left(\frac{5}{4}\right) = 30$$

Now we need to know “30 is  $\frac{3}{4}$  of what number?”

$$30 = \frac{3}{4}x$$

$$x = 30\left(\frac{4}{3}\right) = 40$$

Finally, we need to know “40 is  $\frac{5}{6}$  of what number?”

$$40 = \frac{5}{6}x$$

$$x = 40\left(\frac{6}{5}\right) = 48$$

There were 48 candies in the bowl at the beginning of the day.

9. First, let  $t$  = the amount of time it will take Nadia and Peter to mow the lawn together. Since it takes Nadia 30 minutes to mow, she can mow  $\frac{1}{30}$  of the lawn each minute. Peter can mow it in 45 minutes, so he mows  $\frac{1}{45}$  of the lawn each minute. Together, then can mow  $\frac{1}{t}$  of the lawn each minute. Therefore, to find  $t$ , we add their rates together.

$$\frac{1}{30} + \frac{1}{45} = \frac{1}{t}$$

$$\frac{1(3)+1(2)}{90} = \frac{3+2}{90} = \frac{5}{90} = \frac{1}{t}$$

$$\text{So, } t = \frac{90}{5} = 18 \text{ minutes.}$$

Together, they can mow the lawn in 18 minutes.

10.  $\sqrt{500} = \sqrt{100(5)} = 10\sqrt{5}$

11.  $\frac{-2}{13}$  = rational number, real number

12.  $\frac{1}{2}|19 - 65| - 14 = \frac{1}{2}(46) - 14 = 23 - 14 = 9$

13. Two properties are actually being used. The order was changed, so that is the commutative property of addition. The grouping was changed, so that is the associative property of addition.

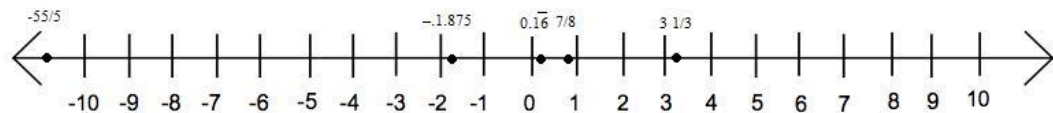
14. This relation is not a function because every element in the domain is not mapped to only element in the range.

15.  $y = \frac{1}{12}x - 5$  because  $f(x) = \frac{1}{12}x - 5$

16.  $\frac{36}{4} = 9$ . Each case is \$9.

## Lesson 2.9 Chapter 2 Review

- $7 > -11$
- $\frac{4}{5} = 0.8$  and  $\frac{11}{16} = 0.6875$ , so  $\frac{4}{5} > \frac{11}{16}$
- If one reduces  $\frac{10}{15}$ , then it becomes  $\frac{10 \div 5}{15 \div 5} = \frac{2}{3}$ . Therefore, these two fractions are equal.
- $\frac{31}{32} = 0.96875$ . Since  $0.985 > 0.96875$ , then  $0.985 > \frac{31}{32}$ .
- $-\frac{300}{9} \approx -33.\bar{3}$ . Since  $-16.12 > -33.\bar{3}$ , then  $-16.12 > -\frac{300}{9}$ .
- Convert all three fractions to decimals to compare them.  
 $\frac{8}{11} \approx 0.\overline{7272}$ ,  $\frac{7}{10} = 0.7$ ,  $\frac{5}{9} \approx 0.\overline{5555}$ . Therefore, the correct order is  $\frac{5}{9}, \frac{7}{10}, \frac{8}{11}$ .
- Convert all the fractions to decimals to compare them.  
 $\frac{2}{7} \approx 0.2857$ ,  $\frac{1}{11} \approx 0.\overline{0909}$ ,  $\frac{8}{13} \approx 0.6154$ ,  $\frac{4}{7} \approx 0.5714$ ,  $\frac{8}{9} \approx 0.\overline{8888}$ . Therefore, the correct order is  $\frac{1}{11}, \frac{2}{7}, \frac{4}{7}, \frac{8}{13}, \frac{8}{9}$ .
- 12.



- $6n(-2 + 5n) - n(-3n - 8) = -12n + 30n^2 + 3n^2 + 8n = 33n^2 - 4n$
- $7x + 2(-6x + 2) = 7x - 12x + 4 = -5x + 4$
- $-7x(x + 5) + 3(4x - 8) = -7x^2 - 35x + 12x - 24 = -7x^2 - 13x - 24$

$$16. -3(-6r - 5) - 2r(1 + 6r) = 18r + 15 - 2r - 12r^2 = -12r^2 + 16r + 15$$

$$17. 1 + 3(p + 8) = 1 + 3p + 24 = 3p + 25$$

$$18. 3(1 - 5k) - 1 = 3 - 15k - 1 = -15k + 2$$

$$19. \sqrt{26} \approx 5.10$$

$$20. \sqrt{330} \approx 18.17$$

$$21. \sqrt{625} = 25.00$$

$$22. \sqrt{121} = 11.00$$

$$23. \sqrt{225} = 15.00$$

$$24. \sqrt{11} \approx 3.32$$

$$25. \sqrt{8} \approx 2.83$$

$$26. \sqrt{50} = \sqrt{25(2)} = 5\sqrt{2}$$

$$27. \sqrt{8} = \sqrt{4(2)} = 2\sqrt{2}$$

$$28. \sqrt{80} = \sqrt{16(5)} = 4\sqrt{5}$$

$$29. \sqrt{32} = \sqrt{16(2)} = 4\sqrt{2}$$

$$30. 8 + b + 1 - 7b = -6b + 9$$

$$31. 9n + 9n + 17 = 18n + 17$$

$$32. 7h - 3 + 3 = 7h$$

$$33. 9x + 11 - x - 3 + 5x + 2 = 13x + 10$$

$$34. \frac{8}{5} - \frac{4}{3} = \frac{8(3) - 4(5)}{15} = \frac{24 - 20}{15} = \frac{4}{15}$$

$$35. \frac{4}{3} - \frac{1}{2} = \frac{4(2) - 1(3)}{6} = \frac{8 - 3}{6} = \frac{5}{6}$$

$$36. \frac{1}{6} + 1\frac{5}{6} = \frac{1}{6} + \frac{11}{6} = \frac{12}{6} = 2$$

$$37. \frac{-5}{4} \times \frac{1}{3} = \frac{-5}{12} \text{ OR } -\frac{5}{12}$$

$$38. \frac{4}{9} \times \frac{7}{4} = \frac{7}{9}$$

$$39. -1\frac{5}{7} \times -2\frac{1}{2} = \frac{-12}{7} \times \frac{-5}{2} = \frac{60}{14} = \frac{30}{7} \text{ OR } 4\frac{2}{7}$$

$$40. \frac{1}{9} \div \left(-1\frac{1}{3}\right) = \frac{1}{9} \div \left(-\frac{4}{3}\right) = \frac{1}{9} \left(-\frac{3}{4}\right) = -\frac{1}{12}$$

$$41. \frac{-3}{2} \div \frac{-10}{7} = \frac{-3}{2} \left(\frac{-7}{10}\right) = \frac{21}{20}$$

$$42. -3\frac{7}{10} \div 2\frac{1}{4} = \frac{-37}{10} \div \frac{9}{4} = \frac{-37}{10} \left(\frac{4}{9}\right) = -\frac{74}{45} \text{ OR } -1\frac{29}{45}$$

$$43. 1\frac{1}{5} - \left(-3\frac{3}{4}\right) = \frac{6}{5} + \frac{15}{4} = \frac{6(4) + 15(5)}{20} = \frac{24 + 75}{20} = \frac{99}{20} \text{ OR } 4\frac{19}{20}$$

$$44. 4\frac{2}{3} + 3\frac{2}{3} = \frac{14}{3} + \frac{11}{3} = \frac{25}{3} \text{ OR } 8\frac{1}{3}$$

$$45. 5.4 + (-9.7) = -4.3$$

$$46. (-7.1) + (-0.4) = -7.5$$

$$47. (-4.79) + (-3.63) = -8.42$$

$$48. (-8.1) - (-8.9) = 0.8$$

$$49. 1.58 - (-13.6) = 15.18$$

$$50. (-13.6) + 12 - (-15.5) = 13.9$$

$$51. (-5.6) - (-12.6) + (-6.6) = 0.4$$

$$52. 19.4 + 24.2 = 43.6$$

$$53. 8.7 + 3.8 + 12.3 = 24.8$$

54.  $9.8 - 9.4 = 0.4$
55.  $2.2 - 7.3 = -5.1$
56. 10.9 is a real number and a rational number.
57.  $\frac{-9}{10}$  is a real number and a rational number.
58.  $3\pi$  is a real number and an irrational number.
59.  $\frac{\pi}{2} - \frac{\pi}{2} = 0$  is a real number, a rational number, and an integer.
60.  $-21$  is a real number, a rational number, and an integer.
61. 8 is a real number, a rational number, and an integer.
62.  $6.78 + (-6.78) = 0$  – additive inverse property
63.  $9.8 + 11.2 + 1.2 = 9.8 + 1.2 + 11.2$  – the order changed, so this is the commutative property of addition
64.  $3a + (4a + 8) = (3a + 4a) + 8$  – the grouping changed, so this is the associative property of addition
65.  $\frac{4}{3} - \left(-\frac{5}{6}\right) = \frac{4}{3} + \frac{5}{6}$  - additive inverse property
66.  $(1)j = j$  – multiplicative identity
67.  $8(11)\left(\frac{1}{8}\right) = 8\left(\frac{1}{8}\right)(11)$  - commutative property of multiplication
68.  $18 + 132\left(\frac{1}{12}\right) = 18 + 11 = 29$  feet
69.  $\frac{12}{\frac{3}{8}} = 12\left(\frac{8}{3}\right) = 32$  cookies
70.  $11(20) - 11(0.01) = 220 - 0.11 = \$219.89$

71.  $d = rt$ , so We need to solve  $14 = \frac{3}{8}t$  where  $t =$  time in hours.

$$t = 14\left(\frac{8}{3}\right) = 37.\bar{3} \text{ hours}$$

72.  $a = 4.8 + 1.2 + 68.48 + 32.45 + 0.16 = \$107.09$

73.  $A = s^2$ , so  $145 = s^2$

$$\sqrt{145} = \sqrt{s^2}$$

$$s = \sqrt{145} \text{ meters}$$

**Lesson 2.10 Chapter 2 Test**

1.  $-3 + 7(3a - 2) = -3 + 21a - 14 = 21a - 17$
2.  $8(3 + 2q) + 5(q + 3) = 24 + 16q + 5q + 15 = 21q + 39$
3.  $8p - 5p = 3p$
4.  $9z + 33 - 2z - 15 = 7z + 18$
5.  $-\frac{9}{5} \div 2 = -\frac{9}{5} \left( \frac{1}{2} \right) = -\frac{9}{10}$
6.  $1\frac{6}{7} \times 5\frac{3}{4} = \frac{13}{7} \left( \frac{23}{4} \right) = \frac{299}{28}$  OR  $10\frac{19}{28}$
7.  $\frac{1}{2} - 3\frac{2}{3} = \frac{1}{2} - \frac{11}{3} = \frac{1(3) - 11(2)}{6} = \frac{3 - 22}{6} = -\frac{19}{6}$
8.  $\frac{3}{14} + \frac{15}{8} = \frac{3(8) + 15(14)}{112} = \frac{24 + 210}{112} = \frac{234}{112} = \frac{117}{56}$  OR  $2\frac{5}{56}$
9.  $3.5 - 5 - 10.4 = -11.9$
10.  $\frac{1}{6} - (-6.5) \times \frac{6}{5} = \frac{1}{6} - \left( -\frac{13}{2} \right) \times \frac{6}{5} = \frac{1}{6} - \left( -\frac{39}{5} \right) = \frac{1(5) + 39(6)}{30} = \frac{5 + 234}{30} = \frac{239}{30}$  OR  $7\frac{29}{30}$
11.  $\sqrt{125} = \sqrt{25(5)} = 5\sqrt{5}$
12.  $\sqrt{18} = \sqrt{9(2)} = 3\sqrt{2}$
13. The multiplicative inverse for any number  $x$  is  $\frac{1}{x}$ . Their product is 1. The additive inverse for any number  $x$  is  $-x$ . Their sum is 0.
14.  $A = s^2$ , so  $145 = s^2$   
 $\sqrt{168} = \sqrt{s^2}$   
 $s = 13.0$  miles
15. Troy could create 16 piles. Each pile would contain 14 candles, and he would have 4 candles left over since  $16(14) + 4 = 228$ .

16.  $-15 - 46.78 + 678.12 + x = \$1123.45$ , where  $x$  = the original amount in Laura's account.

$$616.34 + x = 1123.45$$

$$x = 1123.45 - 616.34 = \$507.11$$

17. The area of a circle will always be an irrational number since one has to multiply by  $\pi$ .

18. You would use the Commutative Property of Multiplication in order to make some problems easier to simplify. For example, if you had  $3(4)\left(\frac{2}{3}\right)$ , you could reorder the fractions to simplify the fraction first.

$$3(4)\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right)(4) = 1(2)(4) = 8$$