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Printed: May 3, 2019
What happens when an object moves very fast?

The theory of relativity refers to two different elements of the same theory: special relativity and general relativity. The theory of special relativity was first introduced by Albert Einstein in 1905 and was later (1916) considered to be a special case of the more comprehensive theory of general relativity.

The special theory of relativity was accepted reasonably quickly by physicists considering that it was introduced in 1905 and widely accepted by 1920. The theory of general relativity was not accepted as quickly. Some physics historians insist that considerable resistance to the theory existed even into the 1950’s and 1960’s. At least part of the problem for the acceptance of the theory was that some conclusions from the theory went against common sense and also, there was very little experimental evidence to support the theory.

The special theory of relativity essentially explains how to interpret motion between different inertial frames of reference, that is, places that are moving at constant speeds relative to each other. Special relativity is based on two postulates:
1. The laws of physics are the same for all observers within their own inertial reference frame.
2. The speed of light in a vacuum is the same for all observers regardless of their relative motion or the motion of the source of the light.

An inertial reference frame is one in which Newton’s first law, the law of inertia, is valid. That is, if an object experiences no net force due to other bodies, the object either remains at rest or remains in motion with constant velocity in a straight line.

A Closer Look at Postulate 1

Consider dropping a child’s toy block in a railroad boxcar.

The block will fall straight down and come to rest on a spot directly underneath the position from which it was dropped. For an observer inside a stationary boxcar, there will be a measurable distance the block fell, a time required to fall and an average velocity for the fall. If the boxcar is moving at a constant horizontal velocity 10 m/s, the observer inside the boxcar will find all measurements to be exactly the same as when the boxcar was stationary, including the spot where the block came to rest. For an observer inside the boxcar, the frame of reference is the same whether the car is moving or not and all the laws of physics will be the same in both cases.

Consider the block falling in the moving boxcar as observed by a stationary observer outside the boxcar. As observed in this frame of reference, the block does NOT fall straight down but rather follows a parabolic path. The distance it falls is NOT the same and the average velocity calculated is NOT the same. For the outside observer, the block has a constant horizontal velocity equal to the velocity of the boxcar and the vector sum of the horizontal velocity and
the vertical fall result in the parabolic path sketched. For the inside observer, there is no horizontal velocity. For the outside observer, the distance traveled along the parabolic path is longer than the path straight down but the time for the fall is the same. For the outside observer, the average velocity is greater.

Inside each frame of reference, all the laws of physics hold, but the measurements are not the same between the two frames of reference.

A Closer Look at Postulate 2

Suppose you are sitting on the hood of a stationary car and your brother is standing alongside the road some 50 feet ahead of the car. If you throw a ball to your brother with a velocity of 10 m/s, it will travel, relative to you, at a velocity of 10 m/s and it will travel, relative to your brother, as 10 m/s. Suppose then, that you repeat the throw except this time, you toss the ball while the car is moving toward your brother at 10 m/s. This time, the ball will move, relative to you, with a velocity of 10 m/s but it will move, relative to your brother, with a velocity of 20 m/s. In this case, the velocity of the source of the ball is added to the velocity of the ball to get the velocity relative to a stationary observer.

Common sense would tell us that if we did this same experiment with light, that is, shine a flashlight off the hood of a moving car in the same direction the car is moving, the velocity of the light relative to a stationary observer would be increased by the velocity of the car. In fact, such an increase in the speed of light has never been found. In fact, in experiments carried out to test for the effect of the movement of the source on the speed of light (Michelson-Morley), the results indicate that the speed of light is completely unaffected by the motion of the source. It appears that the speed of light in a vacuum is constant regardless of relative motion. Hence, the suggestion in postulate 2. The speed of light in a vacuum is the same for all observers regardless of their relative motion or the motion of the source of the light.

The special theory of relativity copes with the results of the Michelson-Morley experiments much better than does classical mechanics, but it also has some surprising consequences. For example, according to the theory of special relativity,

- Two events that occurred simultaneously for one observer were not simultaneous for another observer if the two observers had relative motion to each other. (Relativity of simultaneity).
- Clocks in a moving frame of reference tick more slowly than an observer’s “stationary” clock. (Time dilation).
- Objects are measured to be shorter in the direction that they are moving with respect to a stationary observer. (Length contraction).
- \( E = mc^2 \), energy and mass are equivalent and transmutable. (Mass-energy equivalence).
- No physical object can travel faster than the speed of light in a vacuum. (Maximum speed is finite).

Launch the PLIX Interactive below observe two particles moving relative to each other:
Time Dilation

Suppose we are in a rocket ship sitting at rest on the earth and we turn on an overhead light. The light will travel downward (and all other directions as well) and land on the table below. The observer in the rocket ship can measure the distance traveled to the table, the time required for the light to arrive on the table and an average velocity for the light. If the experiment is repeated in the rocket ship while the rocket is flying past the earth at a constant horizontal velocity, the observer inside will find all the measurements and calculations to be exactly the same as when the rocket was at rest.

Suppose now that the rocket ship is flying past the earth at the same constant horizontal velocity and that the observer is stationary on the earth. For the observer in the rocket ship, the light falls down on the table traveling at the regular speed of light, \( c \). For the observer on the earth, the light travels horizontally with the ship and also falls down onto the table. For the observer on the earth, the path appears to be longer since the light not only went downward but also horizontally.

However, since we have postulate 2, it is not allowed for the light to travel farther in the same time and therefore have a greater average velocity. According to postulate 2, the speed of the light as observed inside the space ship by that observer must be \( 3 \times 10^8 \) m/s and the speed of light observed by the observer on the earth (the light moving diagonally) must ALSO be \( 3 \times 10^8 \) m/s.

So, how is it possible for both observers to measure the speed of light as the same number when the distance traveled is clearly NOT the same? There is an unstated assumption involved here that indicates the passage of time in the two reference frames is the same and that is where the special theory of relativity changes our ideas. The special theory of relativity tells us that the observer on earth will see the clocks on the rocket ship ticking more slowly than his own clocks. So, while the observer on the rocket ship sees the light travel a distance \( x \) meters in 1.00 second as measured on his clock, the observer on the earth sees the light travel \( 2x \) meters in 2.00 seconds as measured by his clock. In both cases, the speed of light is measured to be \( c \). It is important to note that this is not an optical illusion or some strange effect on the mechanical operation of the clock, the actual time on the ship slows down compared to the time of the stationary observer. This is referred to as time dilation.

The equation for time dilation is \( \Delta T = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}} \)

where \( \Delta t \) is the time interval between two events in the moving reference frame and \( \Delta T \) is the time interval as measured in a stationary frame of reference. “\( v \)” is the relative velocity of the moving reference frame and \( c \) is the speed of light in a vacuum.
It should be clear from the equation that if the relative velocity between the two frames of reference is zero, then \( \Delta T = \Delta t \) and there is no time dilation. We can also use the equation to show that for relative speeds like 100 m/s, which seems very fast to us, the comparison to the speed of light would show no noticeable time dilation.

\[
\Delta T = \Delta t \sqrt{1 - \frac{v^2}{c^2}} = \frac{10 \text{ s}}{\sqrt{1 - \frac{(0.100 \text{ m/s})^2}{(3 \times 10^8 \text{ m/s})^2}}} = 10 \text{ s}
\]

100 m/s is so slow compared to the speed of light, that it makes no difference in the time dilation formula. In order for any noticeable effect to occur, the relative velocity of the reference frames must be a significant fraction of the speed of light.

**Example 1**

A muon has a rest lifetime of \( 2.2 \times 10^{-6} \text{ s} \). If it travels with a speed of 0.95c relative to you, how far will you see it travel before it decays?

Muon’s lifetime according to your reference frame

\[
\Delta T = \Delta t \sqrt{1 - \frac{v^2}{c^2}} = \frac{2.2 \times 10^{-6} \text{ s}}{\sqrt{1 - (0.95c)^2}} = 7.0 \times 10^{-6} \text{ s}
\]

distance = \( (3.0 \times 10^8 \text{ m/s})(0.95)(7.0 \times 10^{-6} \text{ s}) = 2.0 \times 10^3 \text{ meters} \)

Launch the PLIX Interactive below to compare how two clocks behave - one stationary on the ground and another moving rapidly in a rocket:

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**The Twin Paradox**

Shortly after Einstein proposed the special theory of relativity, an apparent paradox was pointed out. This paradox involved a pair of twins, one of whom traveled away from the earth and returned at very high speeds. The other twin remained at home on earth. Since one twin was traveling at very high speed, time for him was running slower than for the twin who remained on earth. Thus the traveling twin would return home younger than the twin who remained on earth.

The paradox comes about when each twin thinks that their frame of reference was at rest and the other twin’s frame of reference was moving at high speed. Therefore, each twin would find the other one to be younger. The resolution lies in the fact that the traveling twin must accelerate at the beginning and end of the trip and this acceleration guarantees that this twin is traveling and his clock is actually running slower. The traveling twin will return home younger than his twin brother.

This result was tested in 1971 with a pair of very precise clocks. One clock was sent around the world in high speed jet planes while the matched clock remained at rest. When the traveling clock was returned and placed next to the other clock, the traveling clock showed that less time had passed.
Length Contraction

In a similar manner, an observer on the rocket ship and an observer on the earth will not measure the length of the rocket ship to be the same length in the direction of its motion. The observer on the ship takes out his meter stick and measures the rocket ship to be 15 meters long while the ship sits at rest on the earth. An observer outside the ship, standing on the earth, will also measure the ship to be 15 m with his meter stick. When the rocket ship flies past the earth at a significant fraction of the speed of light, the observer on the ship takes out his meter stick and measures the length of the ship and again finds it to be 15 m. The “stationary” observer on the earth with his meter stick measures the moving rocket ship to be less than 15 m. Just for the sake of clarity, let’s say he measures the rocket ship to be 7.5 meters.

How is it possible that the two observers measure the same rocket ship to be two different lengths? When the rocket ship is at rest on the earth, the on-ship meter stick and the off-ship meter stick are exactly the same but when the rocket ship flies past the earth at significant fraction of the speed of light, the on-ship meter stick as seen by the on earth observer has shrunk. When the observer on ship says the ship is still 15 m long, the observer on earth says, “Nope, your meter stick has shrunk and so has your ship. The ship now measures 7.5 m long using my meter stick.” The on-ship meter stick shrinks by the same percentage that the ship shrinks.

It is important to note that the shortening of the moving object does not produce just a smaller object of the same shape. The object is only shortened in the direction of motion. Therefore, a long, slender rocket ship would NOT become a smaller version of itself, but rather, would become a short, stubby rocket ship.

At some point in history, the length contraction was known as the Fitzgerald contraction and physicists have been known to quote an applicable limerick.

There once was a young man named Fisk,
Whose fencing was exceedingly brisk,
So fast was his action,
The Fitzgerald contraction reduced his sword to a disk.

The equation for length contraction is \( L = L_o \sqrt{1 - \frac{v^2}{c^2}} \) where \( L_o \) is the length measured on the moving body, \( L \) is the length measured on the stationary body, \( v \) is the relative speed of the reference frames, and \( c \) is the speed of light.
You can see by analysis of the equation that when the relative velocity is zero, the two lengths are the same, when the relative speed is less than 1000 m/s, the effect is too small to notice, and only when the relative speed is a significant fraction of the speed of light is the contraction measurable.

Example 2

A spaceship passes the earth at a speed \( v = 0.80 \, c \).

a. What is the length of a meter stick laying on a table in the ship and pointing in the direction of motion of the ship as measured by a person on the ship?

b. What is the length of a meter stick laying on a table in the ship and pointing in the direction of motion of the ship as measured by a person on the earth?

a. relative to a person on the ship, the meter stick is at rest and therefore its length is 1.0 m

\[ L = \frac{L_0}{\sqrt{1 - \frac{v^2}{c^2}}} = (1.0 \, \text{m}) \sqrt{1 - \frac{(0.80 \, c)^2}{c^2}} = (1.0 \, \text{m}) \sqrt{1 - 0.64} = 0.60 \, \text{m} \]

Relativistic Mass

The three basic mechanical quantities are length, time, and mass. Since length and time have been shown to be relative (their value depends on the reference frame from which they are measured), it might be expected that mass is also relative. Einstein showed that the mass of an object increases as its speed increases according to the formula

\[ M = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \]

where \( M \) is the mass of the moving body, \( m_0 \) is the mass of the body at rest (or rest mass), \( v \) is the velocity of the body and \( c \) is the velocity of light.

For many years it was conventional to enter the discussion of dynamics through derivation of the relativistic mass and this is probably still the dominant mode in textbooks. More recently, however, it has been increasingly recognized that relativistic mass is a troublesome and dubious concept. Many physicists reject the concept of relativistic mass and oppose teaching the concept. Instead, they prefer to approach relativism through momentum rather than through relativistic mass.

If momentum is the preferred place to express relativistic dynamics, the equation is

\[ p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}} \]

Where \( p \) is momentum, \( m_0 \) is rest mass, \( v \) is the velocity of the body and \( c \) is the velocity of light.

Example 3

An electron has a rest mass of \( 9.1 \times 10^{-31} \, \text{kg} \). If the electron were traveling at 0.50 \( c \) relative to an observer, what electron mass would the observer measure?

\[ M = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{9.1 \times 10^{-31} \, \text{kg}}{\sqrt{1 - \frac{(0.50 \, c)^2}{c^2}}} = \frac{9.1 \times 10^{-31} \, \text{kg}}{\sqrt{1 - 0.25}} = 1.1 \times 10^{-30} \, \text{kg} \]
The Ultimate Speed Limit

A result of the special theory of relativity is that no physical object can equal or exceed the speed of light. From the equation for relativistic mass, it can be seen that as the object is accelerated faster and faster, its mass becomes greater and greater. The greater mass would require an even greater force to accelerate it. If the velocity of the mass ever reached the speed of light, the denominator of the equation would become zero and the mass would become infinite. The energy required to accelerate an infinite mass would also be infinite. The fact that light itself travels at the speed $c$, implies that light has a zero rest mass. Of course, light is never at rest.

The Equivalence of Mass and Energy

The special theory of relativity is also the origin of Einstein’s most famous equation, $E = mc^2$, and the concept that mass and energy are different forms of the same thing. Einstein himself described the equivalence of mass and energy as the “most important upshot of the special theory of relativity”. The idea is not that mass and energy can be mathematically related but that they two are, in fact, simply different forms of the same thing. Each may be converted into the other and the conversion factor is the speed of light squared. Launch the PLIX Interactive below to further explore mass-energy equivalence:

Example 4

How much energy would be released if a $\pi$ meson (rest mass $= 2.4 \times 10^{-28}$ kg) was transformed by decay completely into energy?

$E = mc^2 = (2.4 \times 10^{-28}$ kg)$ $(3.0 \times 10^8$ m/s)$^2 = 2.2 \times 10^{-11}$ joules

The Impact of the Theory of Special Relativity

A great many experiments have been performed to test the predictions of special relativity. No contradictions have been found. Scientists have therefore accepted special relativity as an accurate description of nature. When the relative velocities of objects are considerably less than the speed of light, the formulas for relativistic time, length, and mass all reduce to the classical formulas. It is required that the two theories correspond where they overlap at speeds much less than $c$. Special relativity does not contradict classical mechanics. Rather, it is a more general theory needed for object speeds approaching the speed of light.

Further Reading

- Time Dilation
- Length Contraction
Summary

• The special theory of relativity essentially explains how to interpret motion between different inertial frames of reference, that is, places that are moving at constant speeds relative to each other.
• Special relativity is based on two postulates:
  1. The laws of physics are the same for all observers within their own inertial reference frame.
  2. The speed of light in a vacuum is the same for all observers regardless of their relative motion or the motion of the source of the light.
• The special theory of relativity explains the unchangeable speed of light better than classical mechanics, but it also has some surprising consequences. For example, according to the theory of special relativity,
  – Two events that occurred simultaneously for one observer were not simultaneous for another observer if the two observers had relative motion to each other. (Relativity of simultaneity).
  – Clocks in a moving frame of reference tick more slowly than an observer’s “stationary” clock. (Time dilation).
  – Objects are measured to be shorter in the direction that they are moving with respect to a stationary observer. (Length contraction).
  – The mass of a moving object will be greater as measured by an observer at rest.
  – \( E = mc^2 \), energy and mass are equivalent and transmutable. (Mass-energy equivalence).
  – No physical object can travel faster than the speed of light in a vacuum. (Maximum speed is finite).

Review

1. A woman stands on top of a moving railroad car and tosses a ball straight up in the air. If there is no air resistance, where will the ball come back down?
   a. behind the railroad car
   b. ahead of the railroad car
   c. into the woman’s hand

2. If you were inside a windowless car that was traveling perfectly smoothly at a constant velocity, you could determine the speed of the car by dropping a ball.
   a. True
   b. False

3. Does time dilation mean that time actually passes more slowly in a moving reference frame or that it only seems to pass more slowly?

4. A young looking woman astronaut has just arrived home from a long trip at near the speed of light. She rushes up to an old gray-haired man and refers to him as her son. Is this possible?

5. If you were traveling away from the earth at a speed of 0.5 c, how would your heartbeat, length, and mass change? What would observers from earth say about their measurements of your heartbeat, length, and mass?

6. A person on another planet shines a flashlight at you. The planet and the earth are both in the same reference frame and are not moving relative to each other. At the same instant that the person shined the flashlight at you, a person on a spaceship passing that planet and moving toward you at 0.5 c also shined a flashlight at you. Which light pulse will reach you first?
   a. the light from the person on the planet
   b. the light from the flashlight on the spaceship
   c. the two light pulses will reach you at the same time

7. If a spaceship will shrink when it travels at a speed of 0.75 c, do we need to make design changes to accommodate passengers and crew?

8. A beam of particles travel at a speed of \( 2.85 \times 10^8 \) m/s. At this speed, the particles average lifetime is measured to be \( 2.50 \times 10^{-8} \) s. What is a particle’s lifetime when they are at rest?
9. A spaceship passes you at a speed of 0.80 \( c \). You measure its length to be 90.0 m. How long would this spaceship be at rest?

10. If you were to travel to a planet 36 light years from earth at a speed of 0.98 \( c \), what would you measure the distance to be?

11. If the rest mass of a proton is \( 1.67 \times 10^{-27} \) kg, what is its mass when traveling at 0.85 \( c \)?

12. At what speed will the relativistic mass of an object be exactly double its rest mass?

13. How much energy would be produced if 1.00 milligram of mass were completely converted into energy?

**Explore More**

Use this resource to answer the questions that follow.

1. What is relative motion?
2. The speed of light is a _________ across the universe. What letter represents this?
3. To maintain a uniform speed of light, does the behavior of space or time change? Why?

- **theory of special relativity**: The theory proposed in 1905 by Einstein, which assumes that the laws of physics are equally valid in all non-accelerated frames of reference and that the speed of electromagnetic radiation in free space has the same value for all observers regardless of relative motion.
- **theory of general relativity**: The theory of gravitation, developed by Einstein in 1916, extending the special theory of relativity to include acceleration and leading to the conclusion that gravitational forces are equivalent to forces caused by acceleration and results in curved space.
- **reference frames**: An inertial reference frame can be defined as either of the following:
  - A reference frame in which Newton’s law of inertia is valid.
  - A reference frame which isn’t accelerating.
  - A reference frame in which all three of Newton’s laws are valid.
- **time dilation**: The relativistic effect of the slowing of a clock with respect to an observer. In Special Relativity, a clock moving with respect to an observer appears to run more slowly than to an observer moving with the clock. In General Relativity, time dilation is also caused by gravity; clocks on the earth’s surface, for example, run more slowly than clocks at high altitudes, where gravitational forces are weaker.
- **length contraction**: One of the aspects of Einstein’s theory of special relativity is that the length of objects moving at relativistic speeds undergoes a contraction along the dimension of motion. An observer at rest (relative to the moving object) would observe the moving object to be shorter in length.
- **mass and energy equivalence**: The physical principle that a measured quantity of energy is equivalent to a measured quantity of mass. Another way of stating it is that mass and energy are different forms of the same thing. The equivalence is expressed by Einstein’s equation,

\[
E = mc^2
\]
\[ E = \frac{mc^2}{2} \]

where \( E \) represents energy, \( m \) the equivalent mass, and \( c \) the speed of light.

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**References**

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