14.1 Limit Notation

Answers

1. $\lim_{x\to\infty}\frac{2x^4+4x^2-1}{5x^4+3x+9}=\lim_{x\to-\infty}\frac{2x^4+4x^2-1}{5x^4+3x+9}=\frac{2}{5}$

2. $\lim_{x\to\infty}\frac{8x^3+4x^2-1}{2x^3+4x+7}=\lim_{x\to-\infty}\frac{8x^3+4x^2-1}{2x^3+4x+7}=4$

3. $\lim_{x\to\infty}\frac{x^2+2x^3-3}{5x^3+x+4}=\lim_{x\to-\infty}\frac{x^2+2x^3-3}{5x^3+x+4}=\frac{2}{5}$

4. $\lim_{x\to\infty}\frac{4x+4x^2-5}{2x^2+3x+3}=\lim_{x\to-\infty}\frac{4x+4x^2-5}{2x^2+3x+3}=2$

5. $\lim_{x\to\infty}\frac{3x^2+4x^3+4}{6x^3+3x^2+6}=\lim_{x\to-\infty}\frac{3x^2+4x^3+4}{6x^3+3x^2+6}=\frac{2}{3}$

6. $\lim_{x\to 3}2x^2+1=19$

7. $\lim_{x\to-\infty}e^x=0$

8. $\lim_{x\to\infty}\frac{1}{x}=0$

9. $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{1}{4}\right)^i=\frac{1}{3}$

10. $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{i}$ does not exist

11. $\lim_{n\to\infty}\sum_{i=0}^{n}\left(\frac{1}{2}\right)^i=2$

12. $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{9}{10^i}=1$

13. The limit of $f(x)=\frac{5x^2-4}{x+1}$ as x approaches 0 is -4.

14. The limit of $y=\frac{x^3-1}{x-1}$ as x approaches 1 is 3.

15. Yes, it's possible as long as a does not equal positive or negative infinity. #6 is an example of this.

14.2 Graphs to Find Limits

Answers

1. $\lim_{x \to -\infty} f(x) = 0$
2. $\lim_{x \to \infty} f(x) = DNE$
3. $\lim_{x \to 2} f(x) = DNE$
4. $\lim_{x \to 0} f(x) = 1$
5. $f(0) = 2$
6. $f(2) = 6$
7. $\lim_{x \to -\infty} g(x) = DNE$
8. $\lim_{x \to \infty} g(x) = 1$
9. $\lim_{x \to 2} g(x) = -2$
10. $\lim_{x \to 0} g(x) = DNE$
11. $\lim_{x \to 4} g(x) = 0$
12. $g(0) = 2$
13. $g(2) = 4$
14. Answers vary
15. Answers vary

14.3 Tables to Find Limits

Answers

1. 10

2. -5

3. DNE

4. $\frac{1}{2\sqrt{2}} \approx .35355$

5. DNE

6. 9

7. -6

8. $\frac{1}{2\sqrt{5}} \approx .2236$

9. $\frac{1}{6}$

10. 5

11. -6

12. $\frac{1}{4}$

13. $\frac{1}{4}$

14. $\frac{2}{15}$

15. DNE

14.4 Substitution to Find Limits

Answers

1. 10

2. -5

3. -7

4. $-\frac{1}{5}$

5. 3

6. 9

7. -6

8. $-\frac{1}{3}$

9. 7

10. 5

11. -6

12. DNE

13. 6

14. $\frac{2}{15}$

15. DNE

14.5 Rationalization to Find Limits

Answers

1. $\frac{1}{6}$

2. $\frac{1}{4}$

3. $\frac{1}{4}$

4. $\frac{1}{2\sqrt{3}}$

5. $\frac{5}{8}$

6. $-\frac{1}{4}$

7. $\frac{1}{2\sqrt{7}}$

8. 8

9. $4\sqrt{3}$

10. $\frac{1}{6}$

11. $\frac{1}{2}$

12. 2

13. 64

14. -144

15. If the function is a rational expression with a square root somewhere, there is a good chance that rationalizing will help you to evaluate the limit.

14.6 One Sided Limits and Continuity

Answers

1. 104
2. ∞ (the limit does not exist)
3. $-\infty$ (the limit does not exist)
4. 1
5. -1
6. 2
7. 1
8. 1
9. Yes
10. 11
11. 11
12. No because $g(-2) \neq 11$.
13. -3
14. -3
15. No because $h(0) \neq -3$.

14.7 Intermediate and Extreme Value Theorems

Answers

1. $f(-1) = \cos(-1) - 1 = -.4596$ and $f(1) = \cos(1) + 1 = 1.54$; therefore, there must exist a c such that $f(c) = 0$ because $-.4596 < 0 < 1.54$.

2. $f(1) = \ln(1) - e^{-1} - 1 = -1.37$ and f(3) $= \ln(3) - e^{-3} - 1 = 0.4883$; therefore, there must exist a c such that $f(c) = 0$ because $-1.37 < 0 < 0.4883$.

3. $f(1) = 2(1)^3 - 5(1)^2 - 10(1) + 5 = -8$ and $f(0) = 2(0)^3 - 5(0)^2 - 10(0) + 5 = 5$; therefore, there must exist a c such that $f(c) = 0$ because $-8 < 0 < 5$.

4. $f(x) = x^3 - x - 1$. $f(0) = -1$ *and* $f(2) = 5$; therefore, there must exist a c such that $f(c) = 0$ because $-1 < 0 < 5$.

5. $f(-2) = (-2)^2 - \cos(-2) = 4.4$ and $f(0) = (0)^2 - \cos(0) = -1$; therefore, there must exist a c such that $f(c) = 0$ because $-1 < 0 < 4.4$.

6. $f(x) = x^5 - 2x^3 - 2$. $f(1) = -3$ and $f(2) = 14$; therefore, there must exist a c such that $f(c) = 0$ because $-3 < 0 < 14$.

7. $f(x) = 3x^2 + 4x - 11$. $f(1) = -4$ and $f(2) = 9$; therefore, there must exist a c such that $f(c) = 0$ because $-4 < 0 < 9$.

8. $f(x) = 5x^4 - 6x^2 - 1$. $f(1) = -2$ and $f(2) = 55$; therefore, there must exist a c such that $f(c) = 0$ because $-2 < 0 < 55$.

9. $f(x) = 7x^3 - 18x^2 - 4x + 1$. $f(-1) = -20$ and $f(0) = 1$; therefore, there must exist a c such that $f(c) = 0$ because $-20 < 0 < 1$.

10. $f(1) = \frac{1}{3}$ and $f(2) = -1$. Therefore, there must exist a c such that $f(c) = 0$ because $-1 < 0 < \frac{1}{3}$.

11. $f(-1) = -1$ and $f(0) = \frac{1}{4}$. Therefore, there must exist a c such that $f(c) = 0$ because $-1 < 0 < \frac{1}{4}$.

12. False

13. True

14. True

15. Functions must be continuous over given intervals in order for the theorems to apply.

14.8 Instantaneous Rate of Change

Answers

1. The slope appears to be 2.

2. The limit is 2, which is the same as what the slope appeared to be in #1.

3. The slope appears to be 6.

4. The limit is 6, which is the same as what the slope appeared to be in #3.

5. The slope appears to be 3.

6. The limit is 3, which is the same as what the slope appeared to be in #5.

7. The slope appears to be 6.

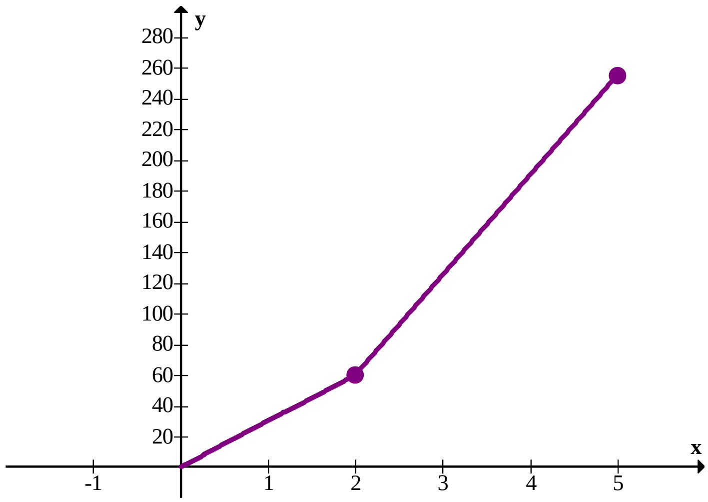
8. $\lim_{x \to 1} \left(\frac{2x^3-1-1}{x-1} \right)$

9. The slope at 0 is 0. The slope at $\frac{\pi}{2}$ is -1. The slope at π is 0. The slope at $\frac{3\pi}{2}$ is 1. The slope at 2π is 0.

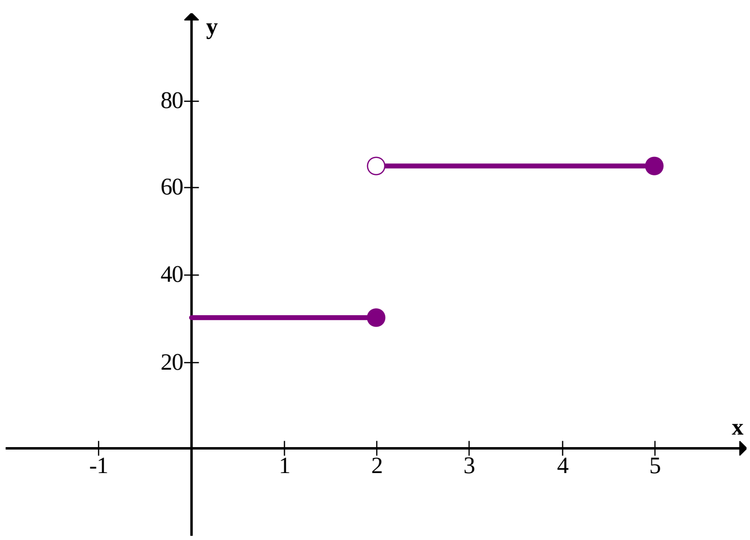
10. The derivative of the cosine function is the negative sine function.

11. The slope is 2 at every point. The derivative of the function is $y = 2$.

12. Distance vs. Time:



Rate vs. Time:



13. A tangent line is a line that "just touches" a curve. The slope of the tangent line at a given point is the derivative of the function at that point.

14. Instantaneous rate of change is the speed at a given point. Speed is shown as slope in functions; therefore, the slope of the tangent line will be the speed or instantaneous rate of change at that point.

15. We can't calculate a slope with a denominator of 0, but we can use limits to find the limit of the slope as the denominator approaches 0.

14.9 Area Under a Curve

Answers

1. 176

2. 60

3. 8.79

4. 8.86

5. -0.33

6. -0.59

7. -0.72

8. The car is going at a constant speed of 25 mph for 3 hours and then instantly starts going 65 mph for the next 2 hours.

9. 205 miles

10. The car accelerates steadily from 0 to 75 meters per second in the first 3 seconds and then stays at 75 meters per second for the next 2 seconds.

11. 262.5 feet

12. The runner increases in speed from 0 feet per second to 16 feet per second, then slows back down to 0 feet per second.

13. The exact answer is $\frac{256}{3} = 85.33$

14. The integral of the derivative of a function gives points on the original function. For example, the area under the curve of a rate vs. time graph gives points on the distance vs. time graph.

15. Integrals are areas under a curve. They can be calculated by finding the sum of the areas of an infinite number of rectangles.