Basic Algebra Flexbook Solution Key
Chapter 7
Systems of Equations and Inequalities; Counting Methods

Lesson 7.1
Linear Systems by Graphing

1. A **system** is a set of equations or inequalities with the same variables.
OR
- A **system** is a set of algebraic sentences joined by the word "and".

2. The solution to a system is the set of ordered pairs that are in common to each algebraic sentence.

3. To solve a system by graphing:
1) graph each equation in the system on the same set of axes
2) find the point(s) at which the graphs intersect
3) the point(s) of intersection are the solution to the system

4. One problem with using a graph to solve a system is that it requires very careful graphing, and even when that is done, it is likely that the graph can only give you an approximate value of the intersection (solution). If the intersection values are both integers, then it works, but in most cases another method would be necessary to find the exact solution.

*5.*The two main ways to write the solution to a system of equations are as an ordered pair (x, y) or as equivalent values: x=? and y=?

6. If the solution to a system is (4, -6) then the graphs of each equation will intersect at (4, -6). Each graph will cross through the point (4, -6).

7. The "**Intersection**" command is located in the "**Calculate"** menu of your calculator. When in the graphing mode, if you press "2^{nd}" then "**Trace**" you will see a list that includes "**Intersection**". If you select this command the calculator will show you at what x and y values the two graphs you have entered cross or intersect.

8. Peter will be further from the starting line at 19.99 seconds and Nadia will be further from the starting line at 20.02 seconds. Peter has a 20 second head start and Nadia catches up with him at 20 seconds. At any time before 20 seconds, Peter will be ahead. At any time after 20 seconds Nadia will be ahead.

9. To find which ordered pair is the solution, we must plug each one into both equations.

(1,4) $\begin{cases} y = 3x - 2 \quad 4 = 3(1) - 2 \quad 4 = 3 - 2 \quad 4 \neq 1 \\ y = -x \quad 4 = -(1) \quad 4 \neq -1 \end{cases}$ Does not work

(2,9) $\begin{cases} y = 3x - 2 \quad 9 = 3(2) - 2 \quad 9 = 6 - 2 \quad 9 \neq 4 \\ y = -x \quad 9 = -(2) \quad 9 \neq -2 \end{cases}$ Does not work

$\mathbf{(\frac{1}{2}, -\frac{1}{2})}$ $\begin{cases} y = 3x - 2 \quad -\frac{1}{2} = 3(\frac{1}{2}) - 2 \quad -\frac{1}{2} = \frac{3}{2} - 2 \quad -\frac{1}{2} = -\frac{1}{2} \\ y = -x \quad -\frac{1}{2} = -(\frac{1}{2}) \quad -\frac{1}{2} = -\frac{1}{2} \end{cases}$

This is the solution.

10. To find which ordered pair is the solution, we must plug each one into both equations.

(8,13) $\begin{cases} y = 2x - 3 \quad 13 = 2(8) - 3 \quad 13 = 16 - 3 \quad 13 = 13 \\ y = x + 5 \quad 13 = 8 + 5 \quad 13 = 13 \end{cases}$ **This is the solution.**

(-7, 6) $\begin{cases} y = 2x - 3 \quad 6 = 2(-7) - 3 \quad 6 = (-14) - 3 \quad 6 \neq -17 \\ y = x + 5 \quad 6 = -7 + 5 \quad 6 \neq -2 \end{cases}$ Does not work

(0, 4) $\begin{cases} y = 2x - 3 \quad 4 = 2(0) - 3 \quad 4 = 0 - 3 \quad 4 \neq -3 \\ y = x + 5 \quad 4 = 4 + 5 \quad 4 \neq 9 \end{cases}$ Does not work

11. To find which ordered pair is the solution, we must plug each one into both equations.

(-9, 1) $\begin{cases} 2x+y=8 \quad 2(-9)+1=8 \quad -18+1=8 \quad -17\neq 8 \\ 5x+2y=10 \quad 5(-9)+2(1)=8 \quad -45+2=8 \quad -43\neq 8 \end{cases}$ Does not work

(-6, 20) $\begin{cases} 2x+y=8 \quad 2(-6)+20=8 \quad -12+20=8 \quad 8=8 \\ 5x+2y=10 \quad 5(-6)+2(20)=10 \quad (-30)+(40)=10 \quad 10=10 \end{cases}$

This is the solution.

(14, 2) $\begin{cases} 2x+y=8 \quad 2(14)+2=8 \quad 28+2=8 \quad 30\neq 8 \\ 5x+2y=10 \quad 5(14)+2(2)=10 \quad 70+4=10 \quad 74\neq 10 \end{cases}$ Does not work

12. To find which ordered pair is the solution, we must plug each one into both equations.

$\left(\mathbf{3},-\frac{\mathbf{3}}{\mathbf{2}}\right)$ $\begin{cases} 3x+2y=6 \quad 3(3)+2(-\frac{3}{2})=6 \quad 9+(-\frac{6}{2})=6 \quad 9+(-3)=6 \quad 6=6 \\ y=\frac{x}{2}-3 \quad -\frac{3}{2}=\frac{3}{2}-3 \quad -\frac{3}{2}=\left(-\frac{3}{2}\right)-\left(\frac{6}{2}\right) \quad -\frac{3}{2}=-\frac{3}{2} \end{cases}$

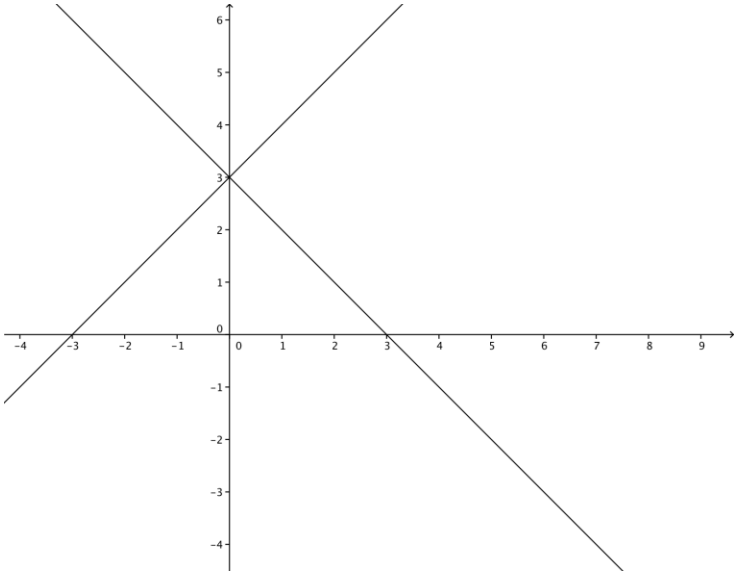
This is the solution.

(-4,3) $\begin{cases} 3x+2y=6 \quad 3(-4)+2(3)=6 \quad -12+6=6 \quad -6\neq 6 \\ y=\frac{x}{2}-3 \quad 3=\frac{-4}{2}-3 \quad 3=-2-3 \quad 3\neq -5 \end{cases}$ Does not work

$\left(\frac{1}{2},4\right)$ $\begin{cases} 3x+2y=6 \quad 3(\frac{1}{2})+2(4)=6 \quad \frac{3}{2}+8=6 \quad \frac{3}{2}+\frac{16}{2}=6 \quad \frac{19}{2}=9\frac{1}{2}\neq 6 \\ y=\frac{x}{2}-3 \quad 4=\frac{\frac{1}{2}}{2}-3 \quad 4=\frac{1}{4}-3 \quad 4\neq -2\frac{3}{4} \end{cases}$

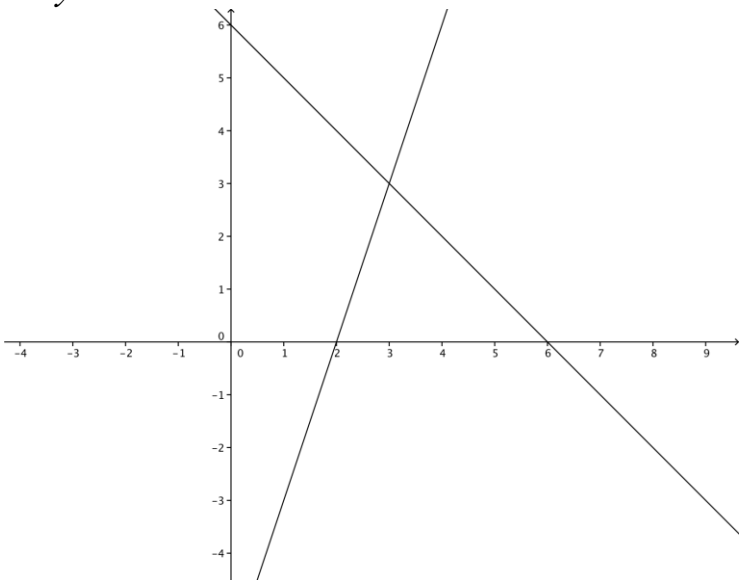
Does not work

13. $y = x + 3$
$y = -x + 3$



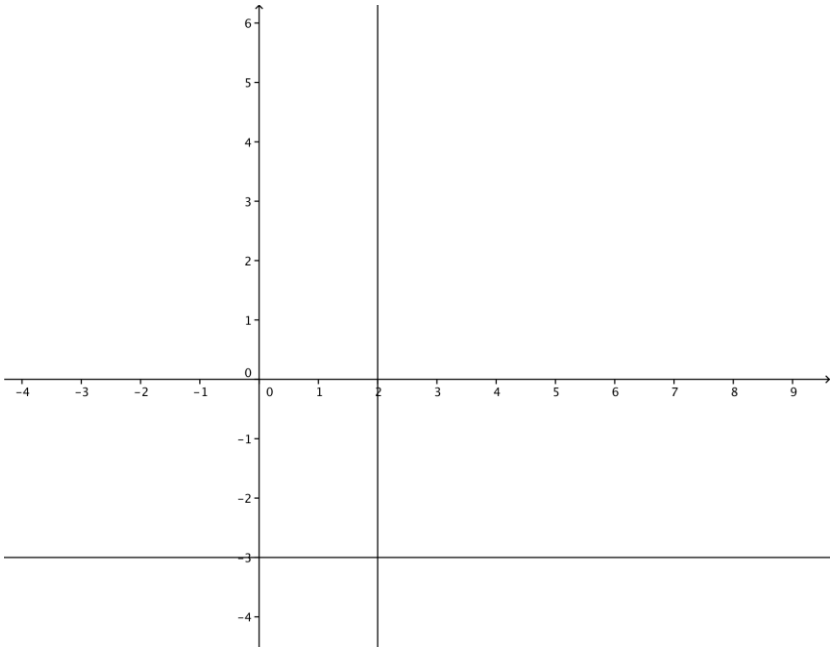
(0,3) is the solution for this set of equations, because that is where the two graphs intersect.

14. $y = 3x - 6$
$y = -x + 6$



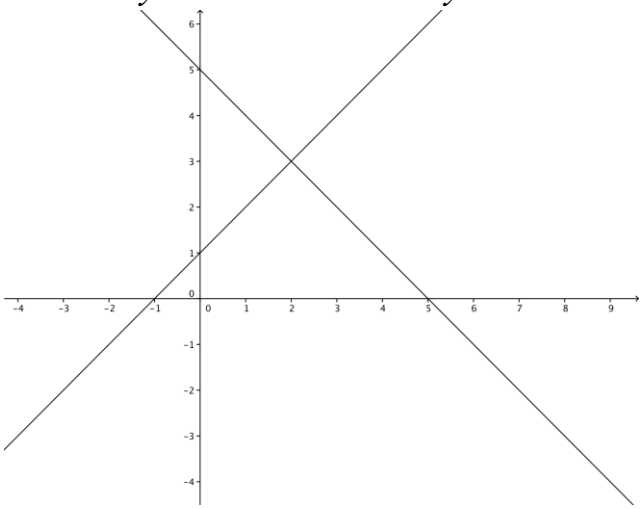
(3,3) is the solution for this set of equations, because that is where the two graphs intersect.

15. $\begin{matrix} 2x = 4 \\ y = -3 \end{matrix}$ is the same as $\begin{matrix} x = 2 \\ y = -3 \end{matrix}$



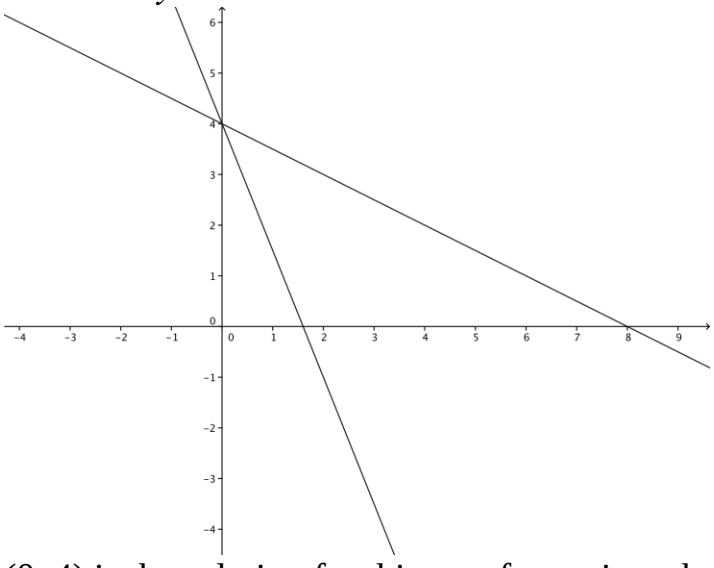
(2, -3) is the solution for this set of equations, because that is where the two graphs intersect.

16. $\begin{matrix} y=-x+5 \\ -x+y=1 \end{matrix}$ is the same as $\begin{matrix} y=-x+5 \\ y=x+1 \end{matrix}$



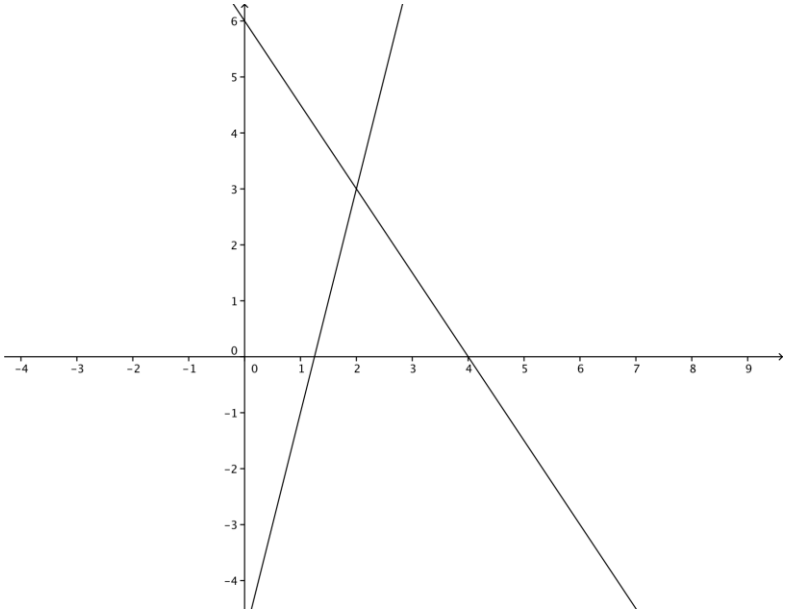
(2,3) is the solution for this set of equations, because that is where the two graphs intersect.

17. $\begin{matrix} x+2y=8 \\ 5x+2y=0 \end{matrix}$



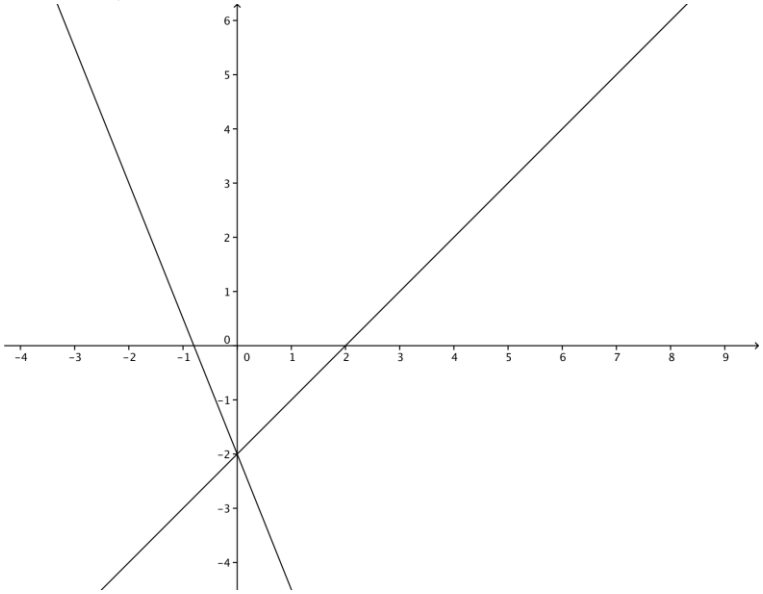
(0, 4) is the solution for this set of equations, because that is where the two graphs intersect.

18. $3x + 2y = 12$
$4x - y = 5$



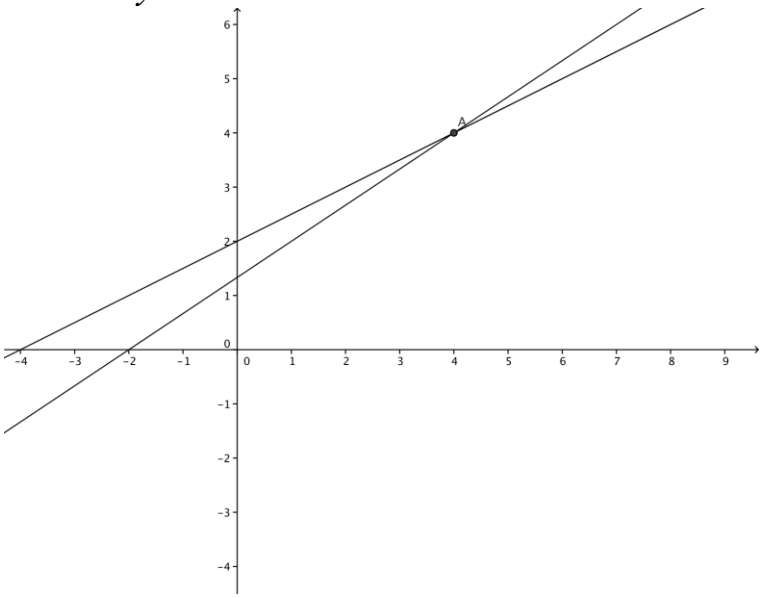
(2,3) is the solution for this set of equations, because that is where the two graphs intersect.

19. $5x + 2y = -4$
$x - y = 2$



(0, -2) is the solution for this set of equations, because that is where the two graphs intersect.

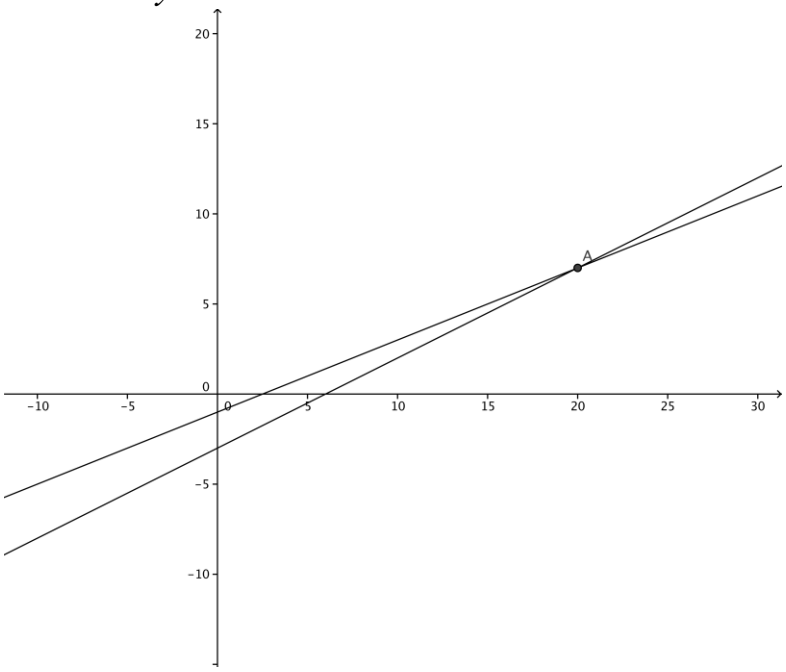
20. $2x = 4 = 3y$
$x - 2y + 4 = 0$



(4,4) is the solution for this set of equations, because that is where the two graphs intersect. (This is an approximate based on the graphs.)

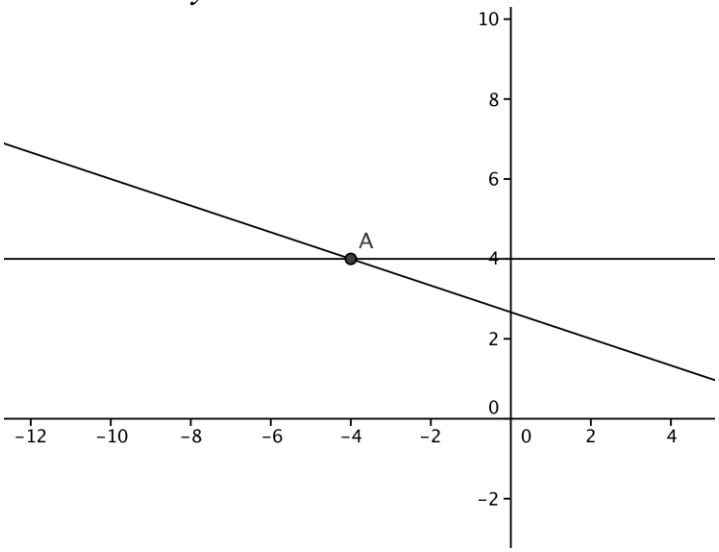
21. $y = \frac{x}{2} - 3$

$2x - 5y = 5$



(20,7) is the solution for this set of equations, because that is where the two graphs intersect. (This is an approximate based on the graphs.)

22. $y = 4$
$x = 8 - 3y$



(-4, 4) is the solution for this set of equations, because that is where the two graphs intersect. (This is an approximate based on the graphs.)

23.

Let's make X the total cost for each car.

Let's make Y the number of years.

Using the information in the problem and these two variables we can create two equations for what each car would cost over time.

Mary's car: x=1200 + 2000y

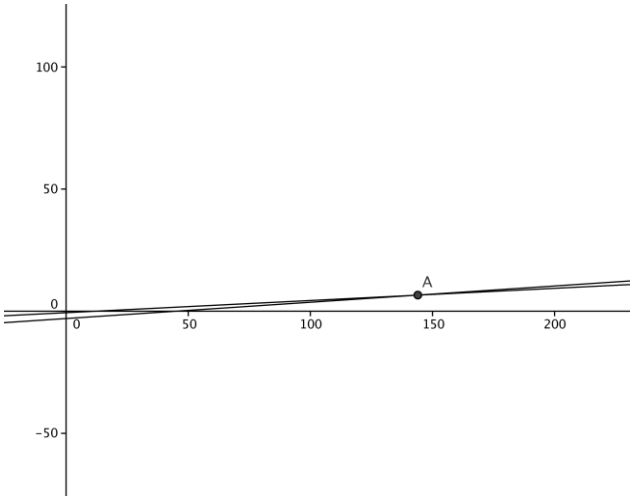
For Mary's car we add the cost of her current repair to the amount that gas will cost her multiplied by each year that goes by.

New car: x= 4500 + 1500y

For the new car we add the purchase cost to the amount that gas will cost her multiplied by each year that goes by.

We are looking to find at what year (y) the total costs (x) will be the same. We can do this by graphing each equation and finding the point of intersection.

(Note: I used $\begin{matrix} x = 12 + 20y \\ x = 45 + 15y \end{matrix}$ which are equivalent to the original equations, because the numbers were easier to work with on the graph.)

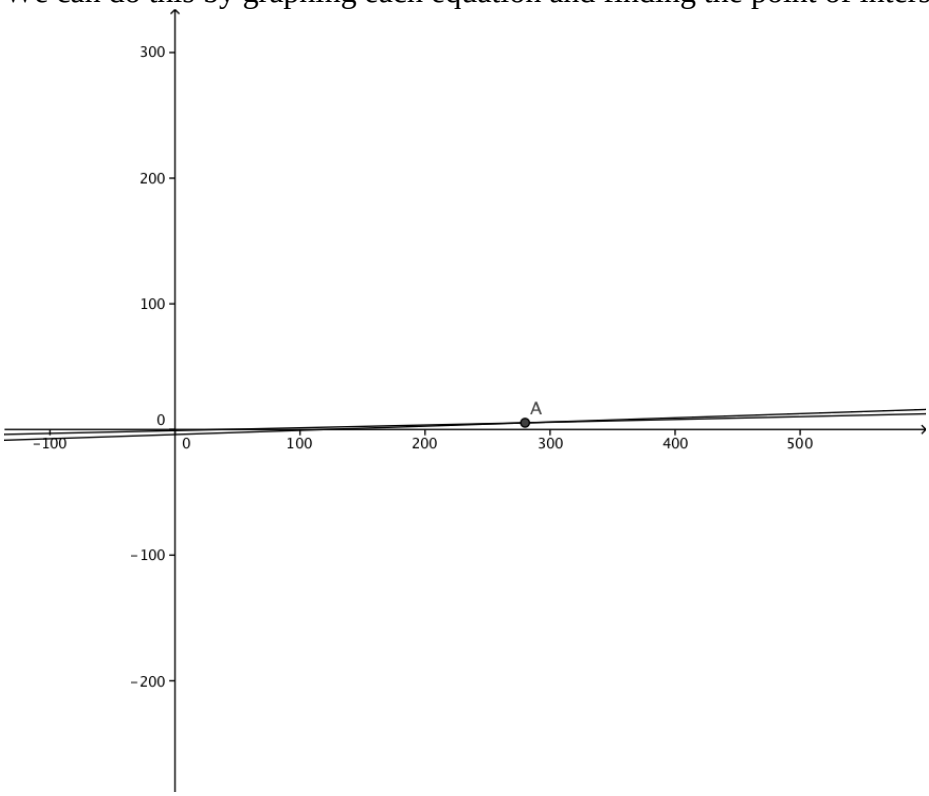


The graph shows that the two equations intersect at (14400, 6.6). This means that when the total cost for each car is x=14400, the number of years that will have passed is y=6.6. The solution is 6.6 years.

24.
Let's make X the total cost for each phone and plan.
Let's make Y the number of months.
Using the information in the problem and these two variables we can create two equations for what each cell phone plan would cost over time.

Plan #1: x=120 + 30y
For plan #1 we add the cost of the phone to the amount that the plan will cost him multiplied by each month that goes by.
Plan #2: x= 40 + 45y
For plan #2 we add the cost of the phone to the amount that the plan will cost him multiplied by each month that goes by.

We are looking to find at what month (y) the total costs (x) will be the same.
We can do this by graphing each equation and finding the point of intersection.

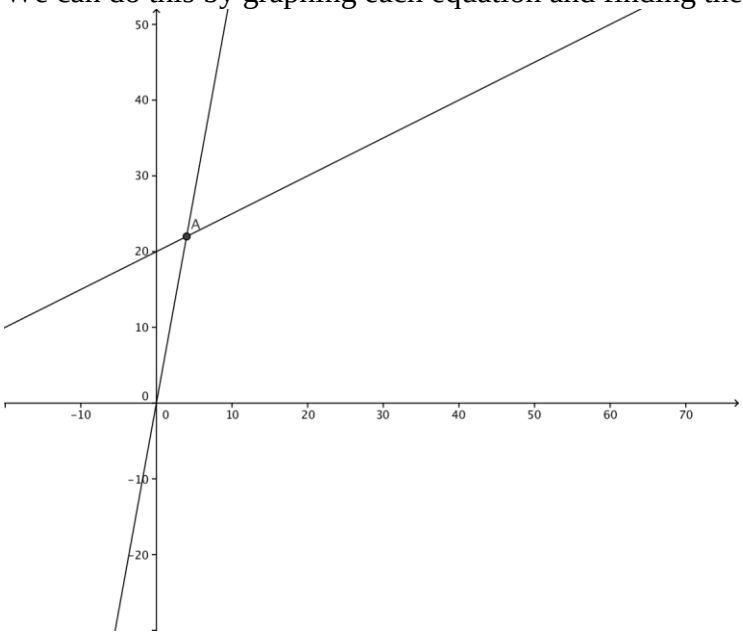


The graph shows that the two equations intersect at (280, 5.33). This means that when the total cost for each plan is x=280, the number of months that will have passed is y=5.33. The solution is 5.33 or 6 months.

25.
Let's make y the total distance run.
Let's make x the number of seconds.
Using the information in the problem and these two variables we can create two equations for how far each animal would go.

Tortoise: $y = 0.5x + 20$
For the tortoise we add the length of his lead to the rate at which he runs multiplied by each second that goes by.
Hare: $y = 5.5x$
For the Hare we take the rate at which he runs multiplied by each second that goes by.

We are looking to find at what second (x) the total distances (y) will be the same.
We can do this by graphing each equation and finding the point of intersection.



The graph shows that the two equations intersect at (4, 22). This means that when the total distance for each animal is y=22, the number of seconds that will have passed is x=4. The solution is 4 seconds.

26. Treat the inequality as if it were a regular equation and solve for h:

$25 = 2h + 5$

$20 = 2h$ Now we know this is actually an inequality so: $10 \geq h$

$10 = h$

Lets check:

h=9 (9 is less than or equal to 10)

$25 \geq |2h + 5|$

$25 \geq |(2)(9) + 5|$

$25 \geq |18 + 5|$

$25 \geq |23|$

The absolute value of 23 is 23 and 25 is greater than or equal to 23, so our answer checks out.

27.

In order to subtract fractions, they must have the same denominator, so we must find the lowest common denominator. In this example the easiest way is to multiply the two denominators. 3 x 2 =6

We must multiply the numerator by the same number we multiply the denominator by, in order to keep the fractions equivalent.

Once the fractions have been changed then we simply subtract the numerator and keep the same denominator.

$$\frac{4}{3} - \frac{1}{2}$$

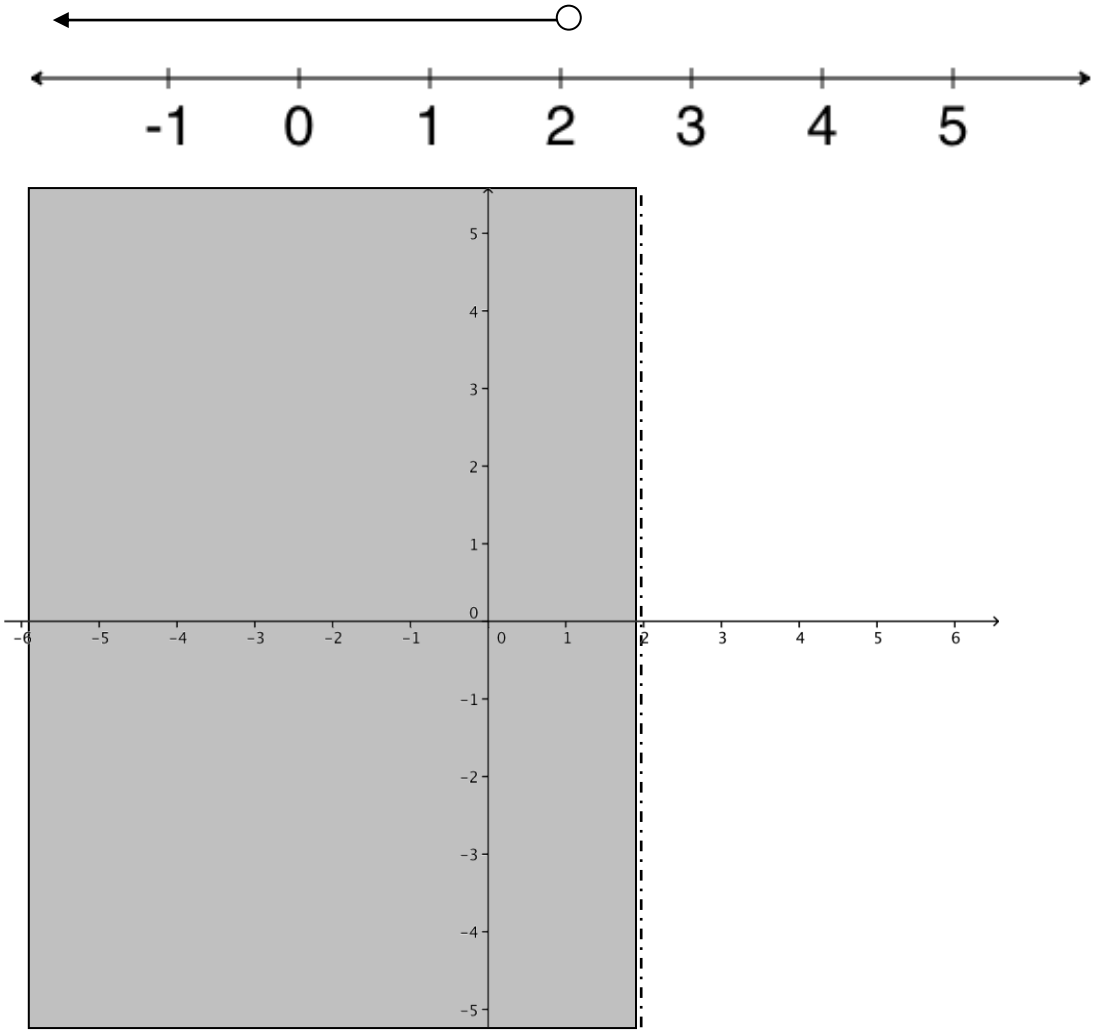
$$\frac{4 \times 2 = 8}{3 \times 2 = 6} - \frac{1 \times 3 = 3}{2 \times 3 = 6}$$

$$\frac{8}{6} - \frac{3}{6} = \frac{5}{6}$$

28.
(a) There are 8 pieces of paper in the hat and 3 of them are I. Therefore, the probability of drawing an I is 3 out of 8. **3:8**

(b) Once again there are 8 pieces of paper in the hat and this time 2 of them are L. The likelihood of drawing an L is 2 out of 8 or 1 out of 4. **1:4**

29.



30. Give an example of an ordered pair in quadrant II.

Any answer where the x value is negative and the y value is positive is correct.

31.

(a) First find the slope using the two points around the point you are trying to find.

$$m = \frac{68.2 - 62.9}{1950 - 1940} = \frac{5.3}{10} = 0.53$$

Next find the y-intercept by plugging in one of the two points and the slope.

$$y = mx + b$$

$$68.2 = (.53)(1950) + b$$

$$68.2 = 1033 + b$$

$$b = -964.8$$

The equation becomes:

$$y = 0.53x - 964.8$$

To estimate the life expectancy in 1943, substitute x=1943.

$$y = 0.53(1943) - 964.8$$

$$y = 1029.79 - 964.8$$

$$y = 64.99$$

(b) First find the slope using the last two points that you know.

$$m = \frac{76.7 - 75.8}{1998 - 1995} = \frac{0.9}{3} = .3$$

Next find the y-intercept by plugging in one of the two points and the slope.

$$y = mx + b$$

$$76.7 = 0.3(1998) + b$$

$$76.7 = 599.4 + b$$

$$b = -522.7$$

The equation becomes:

$$y = 0.3x - 522.7$$

To estimate the life expectancy in 2000, substitute x=2000.

$$y = 0.3(2000) - 522.7$$

$$y = 600 - 522.7$$

$$y = 77.3$$

Lesson 7.2
Solving Systems by Substitution

1)
When you have a system where at least one of the expressions is y=algebraic expression or x=algebraic expression, the algebraic expression can be substituted for that variable in the other expressions in the system. You then solve for the remaining variable. Once you have a value for that variable you can substitute it into any of the expressions in the system to find the value of the second variable.

2)
Systems are easier to solve using substitution if one of the expressions already has one of the variables isolated on one side of the equation or if it would be easy to isolate one variable on one side of the equation.

3) $\begin{cases} y = -3 \\ 6x - 2y = 0 \end{cases}$

$6x - 2(-3) = 0$

$6x + 6 = 0$

$6x = -6$

$x = -1$

Check:

$6x - 2y = 0$

$6(-1) - 2(-3) = 0$

$-6 + 6 = 0$

$0 = 0$

4) $\begin{cases} -3-3y=6 \\ y=-3x+4 \end{cases}$

$-3-3y=6$

$-3y=9$

$y=-3$

$y=-3x+4$

$-3=-3x+4$

$-7=-3x$

$x=\frac{7}{3}$

Check:

$-3-3y=6$

$-3-3(-3)=6$

$-3+9=6$

$6=6$

$y=-3x+4$

$-3=-3(\frac{7}{3})+4$

$-3=-7+4$

$-3=-3$

5) $\begin{cases} y = 3x + 16 \\ y = x + 8 \end{cases}$

$3x + 16 = x + 8$

$2x + 16 = 8$

$2x = -8$

$x = -4$

$y = x + 8$

$y = -4 + 8$

$y = 4$

Check:

$y = 3x + 16$

$4 = 3(-4) + 16$

$4 = -12 + 16$

$4 = 4$

$y = x + 8$

$4 = -4 + 8$

$4 = 4$

6) $\begin{cases} y = -6x - 3 \\ y = 3 \end{cases}$

$y = -6x - 3$

$3 = -6x - 3$

$6 = -6x$

$x = -1$

Check:

$y = -6x - 3$

$3 = -6(-1) - 3$

$3 = 6 - 3$

$3 = 3$

7) $\begin{cases} y = -2x + 5 \\ y = -1 - 8x \end{cases}$

$-2x + 5 = -1 - 8x$

$6x + 5 = -1$

$6x = -6$

$x = -1$

$y = -2(-1) + 5$

$y = 2 + 5$

$y = 7$

Check:

$y = -2x + 5$	$y = -1 - 8x$
$7 = -2(-1) + 5$	$7 = -1 - 8(-1)$
$7 = 2 + 5$	$7 = -1 + 8$
$7 = 7$	$7 = 7$

8) $\begin{cases} y = 6 + x \\ y = -2x - 15 \end{cases}$

$6 + x = -2x - 15$

$x = -2x - 21$

$3x = -21$

$x = -7$

$y = 6 + x$

$y = 6 + (-7)$

$y = -1$

Check:

$y = 6 + x$

$-1 = 6 + (-7)$

$-1 = -1$

$y = -2x - 15$

$-1 = -2(-7) - 15$

$-1 = 14 - 15$

$-1 = -1$

9) $\begin{cases} y = -2 \\ y = 5x - 17 \end{cases}$

$y = 5x - 17$

$-2 = 5x - 17$

$15 = 5x$

$x = 3$

Check:

$y = 5x - 17$

$-2 = 5(3) - 17$

$-2 = 15 - 17$

$-2 = -2$

10) $\begin{cases} x+y=5 \\ 3x+y=15 \end{cases}$ $\quad x+y=5 \quad y=5-x$ $\quad 3x+y=15 \quad y=15-3x$

$5-x=15-3x$

$5=15-2x$

$-10=-2x$

$x=5$

$x+y=5$

$5+y=5$

$y=0$

Check:

$x+y=5$

$5+0=5$

$5=5$

$3x+y=15$

$3(5)+0=15$

$15+0=15$

$15=15$

11) $\begin{cases} 12y-3x=-1 \\ x-4y=1 \end{cases}$

$x-4y=1$

$x=4y-1$

$12y-3x=-1$

$12y-3(4y-1)=-1$

$12y-12y+3=-1$

$3\neq -1$

$12y-3x=-1$

$-3x=-12y-1$

$x=4y+\frac{1}{3}$

$x-4y=1$

$(4y+\frac{1}{3})-4y=1$

$4y+\frac{1}{3}-4y=1$

$\frac{1}{3}\neq 1$

There is no solution set.
The solution is *undefined.*

12. $\begin{cases} x+2y=9 \\ 3x+5y=20 \end{cases}$

$x+2y=9$

$x=-2y+9$

$3x+5y=20$

$3(-2y+9)+5y=20$

$-6y+27+5y=20$

$-y+27=20$

$-y=-7$

$y=7$

$x+2y=9$

$x+2(7)=9$

$x+14=9$

$x=-5$

Check:

$x+2y=9$

$-5+2(7)=9$

$-5+14=9$

$9=9$

$3x+5y=20$

$3(-5)+5(7)=20$

$-15+35=20$

$20=20$

13) $\begin{cases} x-3y=10 \\ 2x+y=13 \end{cases}$ $\quad x-3y=10$ $\quad x=3y+10$

$2x+y=13$

$2(3y+10)+y=13$

$6y+20+y=13$

$7y+20=13$

$7y=-7$

$y=-1$

$x-3y=10$

$x-3(-1)=10$

$x+3=10$

$x=7$

Check:

$x-3y=10$ $\quad 2x+y=13$

$7-3(-1)=10$ $\quad 2(7)+(-1)=13$

$7+3=10$ $\quad 14-1=13$

$10=10$ $\quad 13=13$

14) $\begin{cases} y = \frac{1}{4}x - 14 \\ y = \frac{19}{8}x + 7 \end{cases}$ Solve the system by graphing and substitution

$$\frac{1}{4}x - 14 = \frac{19}{8}x + 7$$

$$\frac{1}{4}x = \frac{19}{8}x + 21$$

$$\frac{2}{8}x - \frac{19}{8}x = 21$$

$$-\frac{17}{8}x = 21$$

$$-2.125x = 21$$

$$x = -9.88$$

$$y = .25x - 14$$

$$y = .25(-9.88) - 14$$

$$y = -2.47 - 14$$

$$y = -16.47$$

Check:

$$y = .25x - 14$$

$$-16.47 = .25(-9.88) - 14$$

$$-16.47 = -2.47 - 14$$

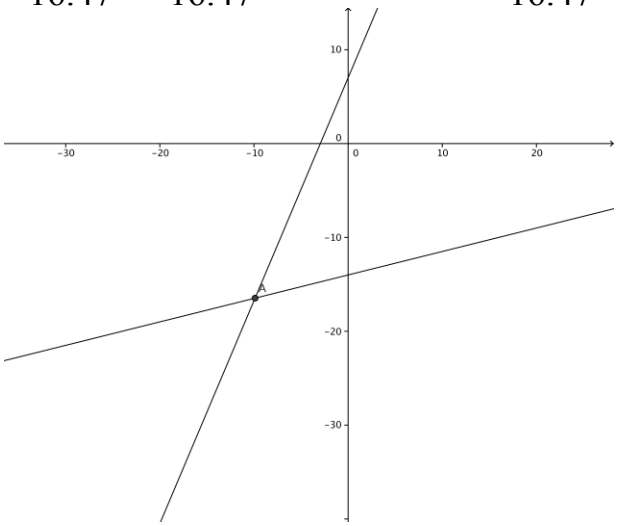
$$-16.47 = -16.47$$

$$y = 2.375x + 7$$

$$-16.47 = 2.375(-9.88) + 7$$

$$-16.47 = -23.465 + 7$$

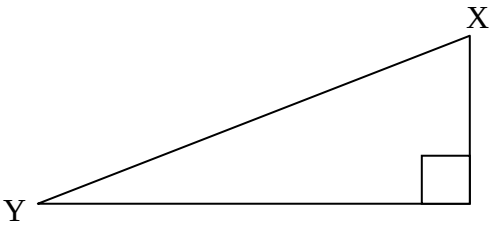
$$-16.47 = -16.47$$



(-9.88, -16.47) is the solution set for these two equations because that is where the two graphs intersect.

Which method do you prefer? Why? *Answers will vary.*

15)



We know that all three angles of any triangle add up to 180 degrees. We also know that a right angle is 90 degrees. That means that the twp angles left in the triangle must add up to 90 degrees. X+Y=90

We also know that one of the angles is twice the size of the other angle. X=2Y

Now we solve the system of equations.

$$\begin{cases} x + y = 90 \\ x = 2y \end{cases}$$

$$x + y = 90$$
$$2y + y = 90$$
$$3y = 90$$
$$y = 30$$

$$x = 2y$$
$$x = 2(30)$$
$$x = 60$$

Check:

$x + y = 90$ $\qquad$ $x = 2y$

$60 + 30 = 90$ $\qquad$ $60 = 2(30)$

$90 = 90$ $\qquad$ $60 = 60$

$\angle X = 60°$

$\angle Y = 30°$

16)
Let's call the numbers x and y. We know when added together they make 70.

$$x + y = 70$$

We also know the difference between the two numbers is 11. So the larger # minus 11 will equal the smaller number.

$$x - 11 = y$$

Now we solve the system to find the solution set.

$$\begin{cases} x + y = 70 \\ x - 11 = y \end{cases}$$

$$x + y = 70$$
$$x = -y + 70$$

$$x - 11 = y$$
$$x = y + 11$$

$$-y + 70 = y + 11$$
$$-2y + 70 = 11$$
$$-2y = -59$$
$$y = 29.5$$

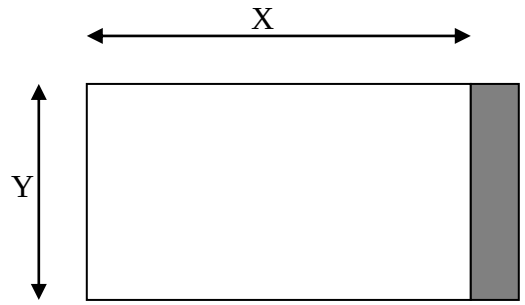
$$x + 29.5 = 70$$
$$x = 40.5$$

Check:

$x + y = 70$	$x - 11 = y$
$29.5 + 40.5 = 70$	$40.5 - 11 = 29.5$
$70 = 70$	$29.5 = 29.5$

The two numbers are 29.5 and 40.5

17)



We know that the total perimeter around all four sides is equal to 400 yards. We know that a rectangle has two long equal sides (x) and two shorter equal sides (y). If we combine all four sides we get the perimeter.

$$2x + 2y = 400$$

We also know that the length of the three sides of fence total 320 yards. By looking at the diagram we can figure out that those three sides are x, x, and y. If we add them all together, they equal 320.

$$2x + y = 320$$

Now we solve the system.

$$\begin{cases} 2x + 2y = 400 \\ 2x + y = 320 \end{cases} \qquad \begin{aligned} 2x + y &= 320 \\ y &= -2x + 320 \end{aligned}$$

$$2x + 2(-2x + 320) = 400$$

$$\begin{aligned} 2x - 4x + 640 &= 400 \\ -2x + 640 &= 400 \\ -2x &= -240 \\ 2x &= 240 \\ x &= 120 \end{aligned}$$

$$\begin{aligned} 2x + y &= 320 \\ 2(120) + y &= 320 \\ 240 + y &= 320 \\ y &= 80 \end{aligned}$$

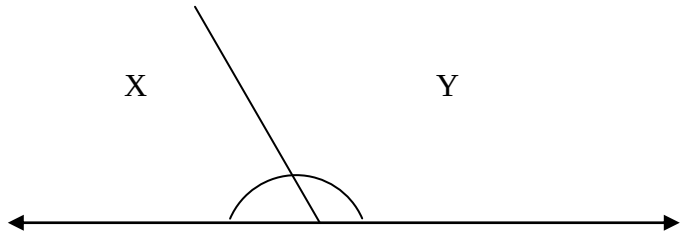
Check:

$$\begin{aligned} 2x + 2y &= 400 \\ 2(120) + 2(80) &= 400 \\ 240 + 160 &= 400 \\ 400 &= 400 \end{aligned}$$

$$\begin{aligned} 2x + y &= 320 \\ 2(120) + 80 &= 320 \\ 240 + 80 &= 320 \\ 320 &= 320 \end{aligned}$$

The dimensions of the field are 120 yards by 80 yards.

18)



We know that a straight angle (or straight line) is 180 degrees, therefore the two angles together must add up to 180 degrees.

$$x + y = 180$$

We also know that the difference between the angles is 18 degrees. If we subtract 18 from the larger angle, we will get the measure of the smaller angle.

$$x - 18 = y$$

Now we solve the system.

$$\begin{cases} x + y = 180 \\ x - 18 = y \end{cases}$$

$$x + y = 180$$
$$x + (x - 18) = 180$$
$$x + x - 18 = 180$$
$$2x - 18 = 180$$
$$2x = 198$$
$$x = 99$$

$$x - 18 = y$$
$$99 - 18 = y$$
$$y = 81$$

Check:

$x + y = 180$ $\qquad$ $x - 18 = y$

$99 + 81 = 180$ $\qquad$ $99 - 18 = 81$

$180 = 180$ $\qquad$ $81 = 81$

$\angle X = 99°$

$\angle Y = 81°$

19)
First we have to make the total poundage equal 5. If p is pounds of peanuts and c is pounds of cashews,

$$p + c = 5$$

Next we have to make sure the total cost is \$15.00. Peanuts are 2.2 times p (pounds of peanuts) and Cashews are 4.7 times c (pounds of cashews),

$$2.2p + 4.7c = 15$$

Now we solve the system.

$$\begin{cases} p + c = 5 \\ 2.2p + 4.7c = 15 \end{cases}$$

$$p + c = 5$$
$$p = -c + 5$$

$$2.2p + 4.7c = 15$$
$$2.2(-c + 5) + 4.7c = 15$$
$$-2.2c + 11 + 4.7c = 15$$
$$2.5c + 11 = 15$$
$$2.5c = 4$$
$$c = 1.6$$

$$p + c = 5$$
$$p + 1.6 = 5$$
$$p = 3.4$$

Check:

$$p + c = 5$$
$$3.4 + 1.6 = 5$$
$$5 = 5$$

$$2.2p + 4.7c = 15$$
$$2.2(3.4) + 4.7(1.6) = 15$$
$$7.48 + 1.52 = 15$$
$$15 = 15$$

You should by 1.6 pounds of peanuts and 3.4 pounds of cashews.

20) The total amount of the liquid needs to be 1 liter. If we call the 10% solution x and the 35% solution y then: $x + y = 1000\text{mL}$

The concentration needs to be 15% of the 1liter. If we change the liter to 1000mL then we figure out 15% we get 150 mL. So 150 mL of the total 1000mL needs to be sulfuric acid. We need to add how much of each solution with a total of 150.

Now we solve the system.

$$\begin{cases} x + y = 1000 \\ 0.1x + .35y = 150 \end{cases}$$

$$x + y = 1000$$
$$x = 1000 - y$$

$$0.1x + .35y = 150$$
$$0.1(1000 - y) + .35y = 150$$
$$100 - 0.1y + .35y = 150$$
$$100 + .25y = 150$$
$$.25y = 50$$
$$y = 200$$

$$x + y = 1000$$
$$x + 200 = 1000$$
$$x = 800$$

Check:

$$x + y = 1000$$
$$800 + 200 = 1000$$
$$1000 = 1000$$

$$0.1x + .35y = 150$$
$$0.1(800) + .35(200) = 150$$
$$80 + 70 = 150$$
$$150 = 150$$

The answer is that 800mL or *0.8L of the 10% solution* and 200mL or *0.2L of the 35% solution* need to be added to get the desired result.

21) The volume of silver used in Bachelle's bracelet is 10cc and the volume of gold used is 20 cc.

As the density of gold is 19.3 g/cc, the mass of gold used in her bracelet is 19.3x20 = 386 g. As the density of silver is 10.5 g/cc, the mass of silver used in her bracelet is 10.5x10 = 105 g.

The total mass of her bracelet is 105 + 386 = 491 g
The volume of her bracelet is 10 + 20 = 30 cc

This allows us to find the density using the formula: density = mass/volume. The density of the bracelet is 491/30 = 16.37 g/cc

The combined density of her bracelet is 16.37 g/cc

22) The total amount of berries must equal six pounds, if we call blueberries B and raspberries R, then:

$$B + R = 6$$

The total cost must be $11.60. We must multiply the per pound cost times the number of pounds of raspberries and add it to the per pound cost times the number of pounds of blueberries and the total must equal $11.60

$$\$1.00(B) + \$2.00(R) = \$11.60$$

$$1B + 2R = 11.60$$

Now lets solve the system:

$$\begin{cases} B + R = 6 \\ 1B + 2R = 11.60 \end{cases}$$

$$B + R = 6$$

$$B = 6 - R$$

$$1B + 2R = 11.60$$

$$1(6 - R) + 2R = 11.60$$

$$6 - R + 2R = 11.60$$

$$6 + R = 11.60$$

$$R = 5.6$$

$$B + R = 6$$
$$B + 5.6 = 6$$
$$B = .4$$

Check:

$$B + R = 6$$
$$.4 + 5.6 = 6$$
$$6 = 6$$

$$1B + 2R = 11.60$$
$$1(.4) + 2(5.6) = 11.60$$
$$.4 + 11.20 = 11.60$$
$$11.60 = 11.60$$

Jeffrey needs 0.4 pounds of blueberries and 5.6 pounds of raspberries.

23)

$$A = l^2$$
$$96 = l^2$$
$$l = \sqrt{96}$$
$$l = 9.7979589$$

24)

$$d = 2r$$
$$11 = 2r$$
$$r = \frac{11}{2}$$
$$r = 5.5$$

$$V = \frac{4}{3}\pi r^3$$
$$V = \frac{4}{3}\left(\frac{22}{7}\right)\left(\frac{11}{2}\right)^3$$
$$V = \frac{44}{21}$$
$$V = 2.1$$

25) The additive inverse is the number that when added to the number we have makes zero, therefore the additive inverse of 7.6 is -7.6.

The multiplicative inverse is the reciprocal of the number, or the number in a reversed fraction. 7.6 is the same as $\frac{76}{10}$ or $\frac{38}{5}$. The reciprocal is $\frac{5}{38}$, therefore the multiplicative inverse of 7.6 is $\frac{5}{38}$.

26) Solve for $x: \frac{1.5}{x} = 6$

$$\frac{1.5}{x} = 6$$
$$1.5 = 6x$$
$$x = \frac{1.5}{6}$$
$$x = .25$$

27)

1. The domain of the function is all numbers ≥ 0. (We can have any number of cricket chirps, including none, but you can't have negative chirps.)

2. The range of the function is all numbers ≥ 40. The function is essentially $F = x + 40$ (where x is the number of chirps you heard); therefore the smallest value you can ever get for F is 40 (when x is 0).

3. I would not expect to hear any crickets at 32°F. If the function is correct then we will not hear any crickets for any temperature less than 40. Again, the lowest value we can attain for this function is 40°F.

4. In order to answer this we must first convert the temperature from Celsius to Fahrenheit.

$$F = 1.8C + 32$$
$$F = 1.8(57) + 32$$
$$F = 102.6 + 32$$
$$F = 134.6$$

$F = x + 40$

$134.6 = x + 40$

$x = 94.6$

I would expect to hear 94.6 or about 95 chirps when the temperature is 57°C.

28)

$45 - 6x \le 18$

$45 - 6(4.5) \le 18$

$45 - 27 \le 18$

$18 \le 18$

Yes, 4.5 is a solution.

Lesson 7.3
Solving Linear Systems by Addition or Subtraction

1) The purpose of the elimination method is to eliminate a variable in the system to make solving certain systems faster and easier. The method is appropriate when both equations are in standard form and graphing or using substitution would require a great deal of work.

2) $\begin{cases} 6x + 3y = -51 \\ 8x - 3y = -19 \end{cases}$

$$6x + 3y = -51$$
$$\underline{+8x - 3y = -19}$$
$$14x + 0y = -70$$
$$14x = -70$$
$$x = -5$$

$$6x + 3y = -51$$
$$6(-5) + 3y = -51$$
$$-30 + 3y = -51$$
$$3y = -21$$
$$y = -7$$

Check:

$6x + 3y = -51$
$6(-5) + 3(-7) = -51$
$-30 - 21 = -51$
$-51 = -51$

$8x - 3y = -19$
$8(-5) - 3(-7) = -19$
$-40 + 21 = -19$
$-19 = -19$

3) $\begin{cases} 2x+8y=-18 \\ -2x-5y=12 \end{cases}$

$-2x-5y=12$

$\underline{+2x+8y=-18}$

$0x+3y=-6$

$3y=-6$

$y=-2$

$2x+8y=-18$

$2x+8(-2)=-18$

$2x-16=-18$

$2x=-2$

$x=-1$

Check:

$2x+8y=-18$

$2(-1)+8(-2)=-18$

$-2-16=-18$

$-18=-18$

$-2x-5y=12$

$-2(-1)-5(-2)=12$

$2+10=12$

$12=12$

4) $\begin{cases} -2x-5y=-10 \\ 2x+8y=16 \end{cases}$

$-2x-5y=-10$

$+2x+8y=16$

$0x+3y=6$

$3y=6$

$y=2$

$-2x-5y=-10$

$-2x-5(2)=-10$

$-2x-10=-10$

$-2x=0$

$x=0$

Check:

$-2x-5y=-10$ $\qquad$ $2x+8y=16$

$-2(0)-5(2)=-10$ $\qquad$ $2(0)+8(2)=16$

$0-10=-10$ $\qquad$ $0+16=16$

$-10=-10$ $\qquad$ $16=16$

5) $\begin{cases} 8x-24y=-80 \\ -8x+5y=-15 \end{cases}$

$-8x+5y=-15$

$+8x-24y=-80$

$0x-19y=-95$

$-19y=-95$

$y=5$

$8x-24y=-80$

$8x-24(5)=-80$

$8x-120=-80$

$8x=40$

$x=5$

Check:

$8x-24y=-80$ $\qquad$ $-8x+5y=-15$

$8(5)-24(5)=-80$ $\qquad$ $-8(5)+5(5)=-15$

$40-120=-80$ $\qquad$ $-40+25=-15$

$-80=-80$ $\qquad$ $-15=-15$

6) $\begin{cases} -x-6y=-18 \\ x-6y=-6 \end{cases}$

$-x-6y=-18$

$+x-6y=-6$

$0x-12y=-24$

$-12y=-24$

$12y=24$

$y=2$

$x-6(2)=-6$

$x-12=-6$

$x=6$

Check:

$-x-6y=-18$ $\qquad$ $x-6y=-6$

$-6-6(2)=-18$ $\qquad$ $6-6(2)=-6$

$-6-12=-18$ $\qquad$ $6-12=-6$

$-18=-18$ $\qquad$ $-6=-6$

7) $\begin{cases} 5x-3y=-14 \\ x-3y=2 \end{cases}$

$x-3y=2$

$-5x-3y=-14$

$-4x+0y=16$

$-4x=16$

$x=-4$

$5x-3y=-14$

$5(-4)-3y=-14$

$-20-3y=-14$

$-3y=6$

$y=-2$

Check:

$5x-3y=-14$

$5(-4)-3(-2)=-14$

$-20+6=-14$

$-14=-14$

$x-3y=2$

$-4-3(-2)=2$

$-4+6=2$

$2=2$

8) $\begin{cases} 3x+4y=2.5 \\ 5x-4y=25.5 \end{cases}$

$3x+4y=2.5$

$+5x-4y=25.5$

$8x+0y=28$

$8x=28$

$x=3.5$

$3x+4y=2.5$

$3(3.5)+4y=2.5$

$10.5+4y=2.5$

$4y=-8$

$y=-2$

Check:

$3x+4y=2.5$

$3(3.5)+4(-2)=2.5$

$10.5-8=2.5$

$2.5=2.5$

$5x-4y=25.5$

$5(3.5)-4(-2)=25.5$

$17.5+8=25.5$

$25.5=25.5$

9) $\begin{cases} 5x+7y=-31 \\ 5x-9y=17 \end{cases}$

$5x-9y=17$

$\underline{-5x+7y=-31}$

$0x-16y=48$

$-16y=48$

$y=-3$

$5x+7y=-31$

$5x+7(-3)=-31$

$5x-21=-31$

$5x=-10$

$x=-2$

Check:

$5x+7y=-31$

$5(-2)+7(-3)=-31$

$-10-21=-31$

$-31=-31$

$5x-9y=17$

$5(-2)-9(-3)=17$

$-10+27=17$

$17=17$

10) $\begin{cases} 3y-4x=-33 \\ 5x-3y=40.5 \end{cases}$

$-4x+3y=-33$

$+5x-3y=40.5$

$x+0y=7.5$

$x=7.5$

$3y-4x=-33$

$3y-4(7.5)=-33$

$3y-30=-33$

$3y=-3$

$y=-1$

Check:

$3y-4x=-33$

$3(-1)-4(7.5)=-33$

$-3-30=-33$

$-33=-33$

$5x-3y=40.5$

$5(7.5)-3(-1)=40.5$

$37.5+3=40.5$

$40.5=40.5$

11) First let's make the cost of a candy bar C and the cost of a fruit roll-up F.

The equation for Nadia's purchase is $3C + 4F = 2.84$
The equation for Peter's purchase is $3C + F = 1.79$

Now let's solve the system.

$$\begin{cases} 3C + 4F = 2.84 \\ 3C + F = 1.79 \end{cases}$$

$$3C + 4F = 2.84$$
$$\underline{-(3C + F = 1.79)}$$
$$0C + 3F = 1.05$$
$$3F = 1.05$$
$$F = .35$$

$$3C + 4F = 2.84$$
$$3C + 4(.35) = 2.84$$
$$3C + 1.40 = 2.84$$
$$3C = 1.44$$
$$C = .48$$

Check:

$3C + 4F = 2.84$
$3(.48) + 4(.35) = 2.84$
$1.44 + 1.40 = 2.84$
$2.84 = 2.84$

$3C + F = 1.79$
$3(.48) + (.35) = 1.79$
$1.44 + .35 = 1.79$
$1.79 = 1.79$

Candy bars cost \$0.48 and Fruit roll-ups cost \$0.35

12) Let's make the air-speed x and the wind speed y.
The first plane has its air speed plus the help of the wind. $x + y = 275$
The second plane is air speed minus the wind (it is flying against it) $x - y = 227$

Now let's solve the system.

$$\begin{cases} x + y = 275 \\ x - y = 227 \end{cases}$$

$$x + y = 275$$
$$+x - y = 227$$
$$2x + 0y = 502$$
$$2x = 502$$
$$x = 251$$

$$x + y = 275$$
$$251 + y = 275$$
$$y = 24$$

Check:

$x + y = 275$	$x - y = 227$
$251 + 24 = 275$	$251 - 24 = 227$
$275 = 275$	$227 = 227$

The wind speed is 24 miles per hour.

13)
Let's say that the pickup fee is x and the per mile rate is y.
We are always going to have to add the pickup fee to the price and multiply the per mile rate times the number of miles.

The equation for 12 miles is $x + 12y = 14.29$
The equation for 17 miles is $x + 17y = 19.91$

Now let's solve the system.

$$\begin{cases} x + 12y = 14.29 \\ x + 17y = 19.91 \end{cases}$$

$$x + 12y = 14.29$$
$$\underline{-(x + 17y = 19.91)}$$
$$0x - 5y = -5.62$$
$$-5y = -5.62$$
$$5y = 5.62$$
$$y = 1.12$$

$$x + 12y = 14.29$$
$$x + 12(1.12) = 14.29$$
$$x + 13.44 = 14.29$$
$$x = 0.85$$

(a) The pickup fee is \$0.85
(b) The per-mile rate is \$1.12

$$.85 + (1.12)7 = c$$

(c) The cost of a seven mile trip is $.85 + 7.84 = c$

$$c = \$8.69$$

(Note, when finding x and y you will have to round to 2 decimal places because we are working with money.)

14)
Let's call the rate for the first 5 minutes x and the rate for the additional minutes y. We would have to multiply the x rate times five and the y rate times the number of minutes over five. If we add these two together we get the total.

For a seven minute call we have 7-5=2 additional minutes so the equation is:
$5x + 2y = 4.25$
For a twelve minute call we have 12-5=7 additional minutes so the equation is:
$5x + 7y = 5.50$

Now let's solve the system.
$$\begin{cases} 5x + 2y = 4.25 \\ 5x + 7y = 5.50 \end{cases}$$

$$5x + 7y = 5.50$$
$$\underline{-(5x + 2y = 4.25)}$$
$$0x + 5y = 1.25$$
$$5y = 1.25$$
$$y = 0.25$$

$$5x + 2y = 4.25$$
$$5x + 2(0.25) = 4.25$$
$$5x + 0.50 = 4.25$$
$$5x = 3.75$$
$$x = 0.75$$

Check:

$5x + 2y = 4.25$	$5x + 7y = 5.50$
$5(.75) + 2(.25) = 4.25$	$5(.75) + 7(.25) = 5.50$
$3.75 + .50 = 4.25$	$3.75 + 1.75 = 5.50$
$4.25 = 4.25$	$5.50 = 5.50$

The rate for the first 5 minutes is \$0.75 a minute and the rate for each additional minute is \$0.25 a minute.

15)
Let x = the number of hours the plumber worked.
Let y = the number of hours the builder worked.

$35x represents the amount of money the plumber earned, and $28y represents the amount of money the builder earned. Together they earned $330.75. Therefore:
35x + 28y = 330.75
The plumber earned $106.75 more than the builder. Therefore:
35x - 28y = 106.75

Now let's solve the system.

$$\begin{cases} 35x + 28y = 330.75 \\ 35x - 28y = 106.75 \end{cases}$$

$$\begin{aligned} 35x + 28y &= 330.75 \\ \underline{35x - 28y &= 106.75} \\ 70x + 0y &= 437.50 \\ 70x &= 437.50 \\ x &= 6.25 \end{aligned}$$

$$\begin{aligned} 35x + 28y &= 330.75 \\ 35(6.25) + 28y &= 330.75 \\ 218.75 + 28y &= 330.75 \\ 28y &= 112 \\ y &= 4 \end{aligned}$$

Check

$35x + 28y = 330.75$	$35x - 28y = 106.75$
$35(6.25) + 28(4) = 330.75$	$35(6.25) - 28(4) = 106.75$
$218.75 + 112 = 330.75$	$218.75 - 112 = 106.75$
$330.75 = 330.75$	$106.75 = 106.75$

The plumber worked 6.25 hours and the builder worked 4 hours.

16)
Let's say that his hourly wage is equal to x and the bonus he makes for each warranty is y. We need to multiply his hourly wage (x) times 20 hours and his bonus (y) times the number of warranties he sold in the week. The total of these two is how much he earned in a week.

For the first week: $20x + 8y = 220$
For the second week: $20x + 13y = 280$

Now let's solve the system.

$$\begin{cases} 20x + 8y = 220 \\ 20x + 13y = 280 \end{cases}$$

$$20x + 13y = 280$$
$$-(20x + 8y = 220)$$
$$0x + 5y = 60$$
$$5y = 60$$
$$y = 12$$

$$20x + 8y = 220$$
$$20x + 8(12) = 220$$
$$20x + 96 = 220$$
$$20x = 124$$
$$x = 6.20$$

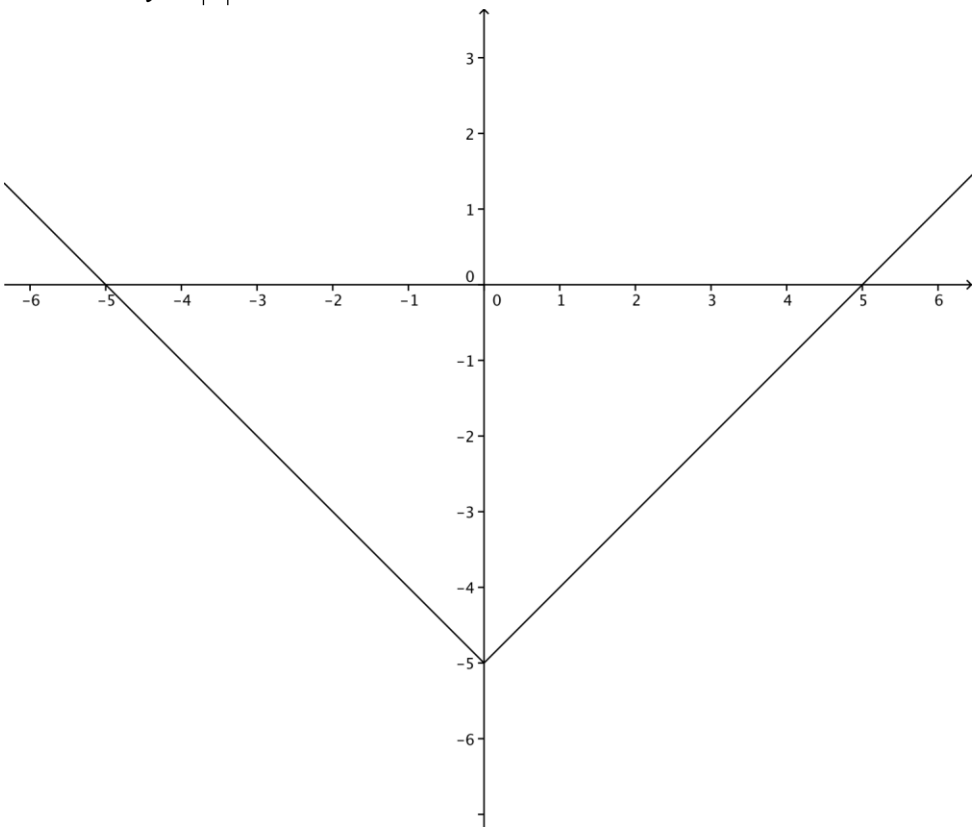
Check:

$20x + 8y = 220$
$20(6.20) + 8(12) = 220$
$124 + 96 = 220$
$220 = 220$

$20x + 13y = 280$
$20(6.20) + 13(12) = 280$
$124 + 156 = 280$
$280 = 280$

Paul makes an hourly wage of $6.20 and a bonus of $12 for each warranty he sells.

17) Graph $y = |x| - 5$



18) Solve:

$$-9 \geq \frac{c}{-4}$$

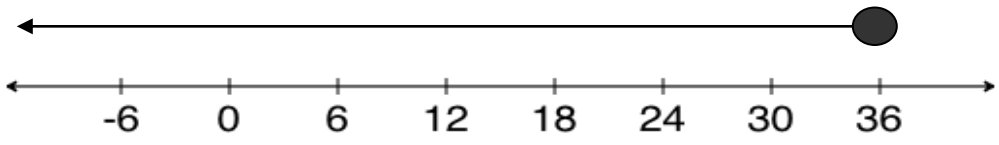
$$(-9)(-4) \geq c$$

$$36 \geq c$$

$$c \leq 36$$

Interval notation: $(-\infty, 36]$

Number Line:



re than its width. What are the dimensions of the rectangle?

$A = l \times w$

$1440 = 10w \times w$

$1440 = 10w^2$

$144 = w^2$

$\sqrt{144} = w$

$w = 12$

$l = 10w$

$l = 10(12)$

$l = 120$

The width of the rectangle is 12cm and the length is 120 centimeters.

20)

$f(x) = 8x^2 - 10$

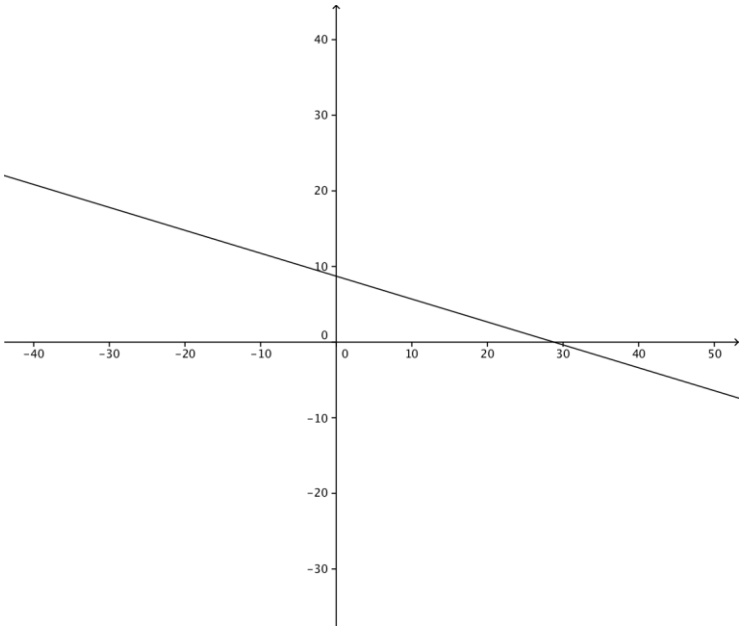
$f(-6) = 8(-6^2) - 10$

$f(-6) = 8(36) - 10$

$f(-6) = 288 - 10$

$f(-6) = 278$

21) First let's convert Torrey's total to square inches so we are working with all the same units. She has 16 x 144= 2304 square inches total. $2304 = 80x + 264y$



Lesson 7.4
Solving Linear Systems by Multiplication

Find the LCM of

1. 5 and 7 = 35

2. -11 and 6= 66

3. 15 and 18=90

4. 7 and 12 = 84

5. 2 and 17 = 34

6. -3 and 6 = 6

7. 6 and $\frac{1}{3}$ = 6

8. 3 and 111 = 111

9. 9 and 14 = 108

10. 5 and -5 = 5

Find the scalar needed to create the additive inverse.

11. -2

12. -7

13. 8

14. 9

15. -11

16. -4

Solve the systems using multiplication.

17. $\begin{cases} 5x-10y=15 \\ 3x-2y=3 \end{cases}$

$-5(3x-2y=3)$

$-15x+10y=-15$

$5x-10y=15$

$\underline{-15x+10y=-15}$

$-10x+0y=0$

$-10x=0$

$x=0$

$5x-10y=15$

$5(0)-10y=15$

$0-10y=15$

$-10y=15$

$y=-1.5$

Check

$5x-10y=15$

$5(0)-10(-1.5)=15$

$0+15=15$

$15=15$

$3x+2y=3$

$3(0)-2(-1.5)=3$

$0+3=3$

$3=3$

18. $\begin{cases} 5x - y = 10 \\ 3x - 2y = -1 \end{cases}$

$-2(5x - y = 10)$

$-10x + 2y = -20$

$-10x + 2y = -20$

$+\ 3x - 2y = -1$

$-7x + 0y = -21$

$-7x = -21$

$x = 3$

$5x - y = 10$

$5(3) - y = 10$

$15 - y = 10$

$-y = -5$

$y = 5$

Check

$5x - y = 10$	$3x - 2y = -1$
$5(3) - 5 = 10$	$3(3) - 2(5) = -1$
$15 - 5 = 10$	$9 - 10 = -1$
$10 = 10$	$-1 = -1$

19. $\begin{cases} 5x+7y=15 \\ 7x-3y=5 \end{cases}$

$7(5x+7y+15) \rightarrow 35x+49y=105$

$-5(7x-3y=5) \rightarrow -35x+15y=-25$

$35x+49y=105$

$\underline{-35x+15y=-25}$

$0x+64y=80$

$64y=80$

$y=1.25$

$5x+7y=15$

$5x+7(1.25)=15$

$5x+8.75=15$

$5x+6.25$

$x=1.25$

Check

$5x+7y=15$

$5(1.25)+7(1.25)=15$

$6.25+8.75=15$

$15=15$

$7x-3y=5$

$7(1.25)-3(1.25)=5$

$8.75-3.75=5$

$5=5$

20. $\begin{cases} 9x+5y=9 \\ 12x+8y=12.8 \end{cases}$

$4(9x+5y=9) \rightarrow 36x+20y=36$

$-3(12x+8y=12.8) \rightarrow -36x-24y=-38.4$

$36x+20y=36$

$\underline{-36x-24y=-38.4}$

$0x-4y=-2.4$

$-4y=-2.4$

$y=.6$

$y=\frac{3}{5}$

$9x+5y=9$

$9x+5\left(\frac{3}{5}\right)=9$

$9x+3=9$

$9x=6$

$x=\frac{2}{3}\approx 0.67$

Check

$9x+5y=9$

$9\left(\frac{2}{3}\right)+5\left(\frac{3}{5}\right)=9$

$6+3=9$

$9=9$

$12x+8y=12.8$

$12\left(\frac{2}{3}\right)+8\left(\frac{3}{5}\right)=12.8=12\frac{8}{10}$

$8+\frac{24}{5}=12\frac{8}{10}=12\frac{4}{5}$

$8+4\frac{4}{5}=12\frac{4}{5}$

$12\frac{4}{5}=12\frac{4}{5}$

21. $\begin{cases} 4x-3y=1 \\ 3x-4y=4 \end{cases}$

$4(4x-3y=1) \rightarrow 16x-12y=4$

$-3(3x-4y=4) \rightarrow -9x+12y=-12$

$16x-12y=4$

$\underline{-9x+12y=-12}$

$7x+0y=-8$

$7x=-8$

$x=-\frac{8}{7} \approx -1.14$

$4x-3y=1$

$4\left(-\frac{8}{7}\right)-3y=1$

$-\frac{32}{7}-3y=1$

$-3y=5\frac{4}{7}$

$y=5\frac{4}{7} \div -3$

$y=\frac{39}{7} \times -\frac{1}{3}$

$y=-\frac{39}{21}$

$y=-\frac{13}{7} \approx -1.86$

Check

$4x-3y=1$

$4\left(-\frac{8}{7}\right)-3\left(-\frac{13}{7}\right)=1$

$-\frac{32}{7}+\frac{39}{7}=1$

$\frac{7}{7}=1$

$1=1$

$3x-4y=4$

$3\left(-\frac{8}{7}\right)-4\left(-\frac{13}{7}\right)=4$

$-\frac{24}{7}+\frac{52}{7}=4$

$\frac{28}{7}=4$

$4=4$

22. $\begin{cases} 7x-3y=-3 \\ 6x+4y=3 \end{cases}$ $\quad 4(7x-3y=-3)\rightarrow 28x-12y=-12$ $\quad 3(6x+4y=3)\rightarrow 18x+12y=9$

$$28x-12y=-12$$
$$\underline{18x+12y=9}$$
$$46x+0y=-3$$
$$46x=-3$$
$$x=-\frac{3}{46}\approx -0.07$$

$$7x-3y=-3$$
$$7\left(-\frac{3}{46}\right)-3y=-3$$
$$-\frac{21}{46}-3y=-3$$
$$-3y=-3+\frac{21}{46}$$
$$-3y=-\frac{138}{46}+\frac{21}{46}$$
$$-3y=-\frac{117}{46}$$
$$y=-\frac{117}{46}\div -3$$
$$y=-\frac{117}{46}\times\frac{1}{3}$$
$$y=\frac{39}{46}\approx 0.85$$

Check

$7x-3y=-3$

$$7\left(-\frac{3}{46}\right)-3\left(\frac{39}{46}\right)=-3$$
$$-\frac{21}{46}-\frac{117}{46}=-3$$
$$-\frac{138}{46}=-3$$
$$-3=-3$$

$6x+4y=3$

$$6\left(-\frac{3}{46}\right)+4\left(\frac{39}{46}\right)=3$$
$$-\frac{18}{46}+\frac{156}{46}=3$$
$$\frac{138}{46}=3$$
$$3=3$$

23. $\begin{cases} x = 3y \\ x - 2y = -3 \end{cases}$

$x - 2y = -3$
$(3y) - 2y = -3$
$y = -3$

$x = 3y$
$x = 3(-3)$
$x = -9$

Check

$x = 3y$
$-9 = 3(-3)$
$-9 = -9$

$x - 2y = -3$
$-9 - 2(-3) = -3$
$-9 + 6 = -3$
$-3 = -3$

24. $\begin{cases} y = 3x + 2 \\ y = 2x + 7 \end{cases}$

$3x + 2 = -2x + 7$
$3x = -2x + 5$
$5x = 5$
$x = 1$

$y = 3x + 2$
$y = 3(1) + 2$
$y = 3 + 2$
$y = 5$

Check

$y = 3x + 2$
$5 = 3(1) + 2$
$5 = 3 + 2$
$5 = 5$

$y = -2x + 7$
$5 = -2(1) + 7$
$5 = -2 + 7$
$5 = 5$

25. $\begin{cases} 5x-5y=5 \\ 5x+5y=35 \end{cases}$

$5x+5y=35$

$\underline{+5x-5y=5}$

$10x+0y=40$

$10x=40$

$x=4$

$5x-5y=5$

$5(4)-5y=5$

$20-5y=5$

$-5y=-15$

$y=3$

Check

$5x-5y=5$

$5(4)-5(3)=5$

$20-15=5$

$5=5$

$5x+5y=35$

$5(4)+5(3)=35$

$20+15=35$

$35=35$

26. $\begin{cases} y = -3x - 3 \\ 3x - 2y + 12 = 0 \end{cases}$

$3x - 2y + 12 = 0$

$3x - 2(-3x - 3) + 12 = 0$

$3x + 6x + 6 + 12 = 0$

$9x + 18 = 0$

$9x = -18$

$x = -2$

$y = -3x - 3$

$y = -3(-2) - 3$

$y = 6 - 3$

$y = 3$

Check

$y = -3x - 3$

$3 = -3(-2) - 3$

$3 = 6 - 3$

$3 = 3$

$3x - 2y + 12 = 0$

$3(-2) - 2(3) + 12 = 0$

$-6 - 6 + 12 = 0$

$-12 + 12 = 0$

$0 = 0$

27. $\begin{cases} 3x-4y=3 \\ 4y+5x=10 \end{cases}$

$4y+5x=10 \rightarrow 5x+4y=10$

$3x-4y=3$

$\underline{5x+4y=10}$

$8x+0y=13$

$8x=13$

$x=\frac{13}{8}\approx 1.63$

$3x-4y=3$

$3\left(\frac{13}{8}\right)-4y=3$

$\frac{39}{8}-4y=3$

$-4y=3-\frac{39}{8}$

$-4y=\frac{24}{8}-\frac{39}{8}$

$-4y=-\frac{15}{8}$

$y=-\frac{15}{8}\div-\frac{4}{1}$

$y=-\frac{15}{8}\times-\frac{1}{4}$

$y=\frac{15}{32}\approx 0.47$

Check

$3x-4y=3$

$3\left(\frac{13}{8}\right)-4\left(\frac{15}{32}\right)=3$

$\frac{39}{8}-\frac{15}{8}=3$

$\frac{24}{8}=3$

$3=3$

$4y+5x=10$

$4\left(\frac{15}{32}\right)+5\left(\frac{13}{8}\right)=10$

$\frac{15}{8}+\frac{65}{8}=10$

$\underline{\frac{80}{8=10}}$

$10=10$

28. $\begin{cases} 9x - 2y = -4 \\ 2x - 6y = 1 \end{cases}$ $\qquad -3(9x - 2y = -4) \rightarrow -27x + 6y = 12$

$$-27x + 6y = 12$$
$$\underline{+\,2x - 6y = 1}$$
$$-25x + 0y = 13$$
$$-25x = 13$$
$$x = -0.52$$

$$2x - 6y = 1$$
$$2(-0.52) - 6y = 1$$
$$-1.04 - 6y = 1$$
$$-6y = 2.04$$
$$y = -0.34$$

Check

$9x - 2y = -4$
$9(-0.52) - 2(-0.34) = -4$
$-4.68 + 0.68 = -4$
$-4 = -4$

$2x - 6y = 1$
$2(-0.52) - 6(-0.34) = 1$
$-1.04 + 2.04 = 1$
$1 = 1$

29.First we need to create our two equations.
If angles A and B are supplementary then the equation is: $A + B = 180$

If we translate the third sentence into an equation we get: $A = 2B - 18$

Now we solve the system.

$$\begin{cases} A + B = 180 \\ A = 2B - 18 \end{cases}$$

$A + B = 180$
$(2B - 18) + B = 180$
$2B - 18 + B = 180$
$3B - 18 = 180$
$3B = 198$
$B = 66$

$A = 2B - 18$
$A = 2(66) - 18$
$A = 132 - 18$
$A = 114$

Angle A is 114° and Angle B is 66°.

30. Let's call the 5% solution *a* and the 15% solution *b*. The equation for the total amount of fertilizer is:

$a + b = 100$

Let's convert the percentages to decimals (.05, .15 and .12 for the total) Next we have to multiply the amount of solution *a* by its strength (.05a) and add that to solution *b* multiplied by its strength (.15b) with the total being .12:

$.05a + .15b = .12$

Now let's solve the system.

$$\begin{cases} a + b = 100 \\ 0.05a + 0.15b = .12 \end{cases}$$

$a + b = 100 \rightarrow a = 100 - b$

$0.05a + 0.15b = 0.12$

$0.05(100 - b) + 0.15b = 0.12$

$5 - 0.05b + 0.15b = 0.12$

$5 + 0.10b = 0.12$

$0.10b = -4.88$

$b = -48.8$

$a + b = 100$

$a + (-48.8) = 100$

$a = 148.8$

The farmer needs 148.8 liters of the 5% solution and 48.8 liters of the 15% solution.

31.

Let's call the pipe for the first field x and the pipe for the second field y. We know that the pipe for the two fields combined must equal 150 yards. Therefore: $x + y = 150$

We know that the length of the first piece is three yards less than twice the length of the second piece. Therefore: $x = 2y - 3$

Now let's solve the system. We'll use substitution.

$$\begin{cases} x + y = 150 \\ x = 2y - 3 \end{cases}$$

$$x + y = 150$$
$$(2y - 3) + y = 150$$
$$2y - 3 + y = 150$$
$$3y - 3 = 150$$
$$3y = 153$$
$$y = 51$$

$$x = 2y - 3$$
$$x = 2(51) - 3$$
$$x = 102 - 3$$
$$x = 99$$

The length of the first pipe is 99 yards. The length of the second pipe is 51 yards.

32. Let A represent the amount invested in Company A, and B represent the amount invested in company B.

Since we know that Mr. Stein invested a total of $100,000, it follows that A and B must add up to $100,000. Therefore: $A+B=100,000$

Next, we know that he made a return of 8%. 8% of $100,000 is $8,000. So 13% of the amount invested (his gain) in A minus 3% of the amount invested in B (his loss) must add up to $8,000. Therefore: $0.13A-0.03B=8000$

Now let's solve the system using substitution.

$$\begin{cases} A+B=100,000 \\ 0.13A-0.03B=8000 \end{cases}$$

$$A+B=100,000$$
$$A=100,000-B$$

$$0.13A-0.03B=8000$$
$$0.13(100,000-B)-0.03B=8000$$
$$13000-0.13B-0.03B=8000$$
$$-0.13B-0.03B=-5000$$
$$-0.16B=-5000$$
$$B=31250$$

$$A+B=100,000$$
$$A+31250=100,000$$
$$A=68750$$

Mr. Stein invested $68,750 in Company A and $31,250 in Company B.

33. First let's figure out how many cakes were sold. 120 to start with 3 left at the end of the day is 117 cakes sold. Well call plain cakes p and decorated cakes d. Therefore:
$p + d = 117$

The baker made $991 for all the cakes. If the plain cakes cost $7 and the decorated cakes cost $11 then the total is: $7p + 11d = 991$

Now let's solve the system. We'll use substitution.

$$\begin{cases} p + d = 117 \\ 7p + 11d = 991 \end{cases}$$

$p + d = 117 \rightarrow d = 117 - p$

$7p + 11d = 991$
$7p + 11(117 - p) = 991$
$7p + 1287 - 11p = 991$
$-4p + 1287 = 991$
$-4p = -296$
$p = 74$

$p + d = 117$
$(74) + d = 117$
$d = 43$

The baker sold 74 plain cakes and 43 decorated cakes.

34. Let's call John's age J and Claire's age C. Twice John's age plus five times Claire's age is 204. Therefore: $2J+5C=204$

Nine times John's age minus three times Claire's age is also 204. Therefore:
$9J-3C=204$

Now let's solve the system using multiplication and elimination.

$$\begin{cases} 2J+5C=204 \\ 9J-3C=204 \end{cases}$$

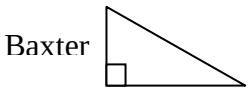
$3(2J+5C=204)\rightarrow 6J+15C=612$
$5(9J-3C=204)\rightarrow 45J-15C=1020$

$6J+15C=612$
$\underline{45J-15C=1020}$
$51J+0C=1632$
$J=32$

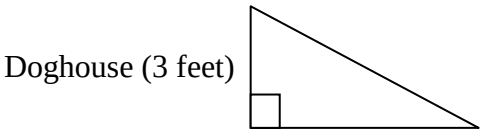
$2J+5C=204$
$2(32)+5C=204$
$64+5C=204$
$5C=140$
$C=28$

Claire is 28 years old and John is 32 years old.

35. Let's make a diagram. The small triangle represents Baxter. He is lying on the ground creating a right angle. His shadow is 3 feet long. The large triangle represents the doghouse. It also creates a right angle with the ground. Its shadow is 8 feet long. The two triangles are similar so we can create a proportion to find Baxter's height.



3 feet



8 feet

$$\frac{3}{8}=\frac{x}{3}$$
$$9=8x$$
$$x=\frac{9}{8}=1.125$$

The dog is 1.125 feet high.

36. First, let's say that x is the time of growth (in weeks) and y is the height of the lily (in inches).

Next, use the given information to write two points.
At 3 weeks, the lily was 4 inches tall. (3, 4)
Four weeks later, the lily was 21 inches tall. (7, 21)

Now use these points to calculate the slope of the line.
m = (y2 - y1) / (x2 - x1)
m = (21 - 4) / (7 - 3)
m = 17/4 = 4.25

Now substitute 4.25 in for m in the equation. Also, select one of the points and substitute 3 in for x and 4 in for y. Solve for b.
y = mx + b
4 = 4.25 * 3 + b
4 = 12.75 + b
-8.75 = b

The equation that represents the growth pattern of this plant is...
(a) y = 4.25x + -8.75

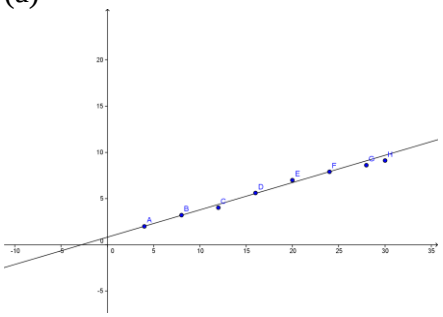
To find the height of the lily after 5.5 weeks, substitute 5.5 in for x and solve for y.
y = 4.25 * 5.5 + -8.75
y = 14.625
At the 5.5 mark, the height of the lily will be...
(b) 14.625 inches

There is a restriction on how high the plant will grow. A plant will not grow to an unlimited height. For example, after 75 weeks, the lily would reach a height of 310 inches. That's over 25 feet tall.
The equation does not show these restrictions. By definition, a linear equation continues indefinitely in both directions. Lilies grow to a maximum height of only a few feet.
(c) Yes, there is a restriction. No, the equation does not reflect this restriction.

37.

s (number of students in wave)	*t (time in seconds to complete on full round)*
4	*2*
8	*3.2*
12	*4*
16	*5.6*
20	*7*
24	*7.9*
28	*8.6*
30	*9.1*

(a)



The line of best fit is:

236x+800y=656
-236(18) + 800y=656
-4248 + 800y=656
800y=4904
y=6.13

(b) $m = \frac{y - y_1}{x - x_1}$

$m = \frac{7 - 5.6}{20 - 16}$

$m = \frac{1.4}{4}$

$m = 0.35$

$y = mx + b$

$y = (0.35)x + b$

$7 = (0.35)20 + b$

$7 = 7 + b$

$b = 0$

$y = (0.35)x + 0$

$y = (0.35)18 + 0$

$y = 6.3$

Quick Quiz

1. $\begin{cases} -3y = 3x + 6 \\ y = 3x + 4 \end{cases}$ (-3,-5)

$y = 3x + 4$
$-5 = 3(-3) + 4$
$-5 = -9 + 4$
$-5 = -5$

$-3y = 3x + 6$
$-3(-5) = 3(-3) + 6$
$15 = -9 + 6$
$15 \neq -3$

NO, (-3,-5) is not a solution to the system.

2. $\begin{cases} y = 6x + 17 \\ y = 7x + 20 \end{cases}$

$6x + 17 = 7x + 20$
$6x = 7x + 3$
$-x = 3$
$x = -3$

$y = 6x + 17$
$y = 6(-3) + 17$
$y = -18 + 17$
$y = -1$

Check

$y = 6x + 17$
$-1 = 6(-3) + 17$
$-1 = -18 + 17$
$-1 = -1$

$y = 7x + 20$
$-1 = 7(-3) + 20$
$-1 = -21 + 20$
$-1 = -1$

3. Let's call the number of cost of the daisies d and the cost of the carnations c.

Joann bought 10 daisies and 4 carnations for \$52.66. Therefore: $10d+4c=52.66$

Phyllis bought 3 daisies and 6 carnations for \$43.11. Therefore: $3d+6c=43.11$

Now let's solve the system using multiplication and elimination.

$$10d+4c=52.66$$
$$3d+6c=43.11$$

$$3(10d+4c=52.66)\rightarrow 30d+12c=157.98$$
$$-2(3d+6c=43.11)\rightarrow -6d-12c=-86.22$$

$$30d+12c=157.98$$
$$\underline{-6d-12c=-86.22}$$
$$24d+0c=71.76$$
$$24d=71.76$$
$$d=2.99$$

$$10d+4c=52.66$$
$$10(2.99)+4c=52.66$$
$$29.9+4c=52.66$$
$$4c=22.76$$
$$c=5.69$$

Daisies cost \$2.99 each and carnations cost \$5.69 each.

4. Let's say that m stands for mile and c stands for total cost.

Terry's Rental charges \$49 plus \$0.15 per mile. Therefore: $c = 49 + 0.15m$
Hurry-It-Up charges \$84 for the day. Therefore: $c = 84$

Now let's solve the system using substitution.

$$\begin{cases} c = 49 + 0.15m \\ c = 84 \end{cases}$$

$$84 = 49 + 0.15m$$
$$35 = 0.15m$$
$$m \approx 233.33$$

The two companies will charge the same when the mileage reaches 233.33.

Lesson 7.5
Special Types of Linear Systems

1. An *inconsistent system* is a system with no solutions. When graphed, the two equations will be parallel lines (with the same slope, and different y-intercepts).

2. The *three types of consistent systems* are (1) systems with one intersection, (2) systems with two or more intersections, and (3) systems with an infinite number of intersections.

3. You can conclude that the equations in the system are coincident lines that completely overlap. The system has an infinite number of intersections.

4. You can conclude that the equations make up a consistent system.

5. You can verify that the system will have no solution by solving the system algebraically and finding if the final equation has no variables and is untrue.

6. The system on the graph is a consistent system with only one solution.

7. $\begin{cases} 3x-4y=13 \\ y=-3x-7 \end{cases}$

$3x-4y=13$

$3x-4(-3x-7=13$ $\qquad$ $y=-3x-7$

$3x+12x+28=13$ $\qquad$ $y=-3(-1)-7$

$15x+28=13$ $\qquad$ $y=3-7$

$15x=-15$ $\qquad$ $y=-4$

$x=-1$

Check

$3x-4y=13$ $\qquad$ $y=-3x-7$

$3(-1)-4(-4)=13$ $\qquad$ $-4=-3(-1)-7$

$-3+16=13$ $\qquad$ $-4=3-7$

$13=13$ $\qquad$ $-4=-4$

The system is consistent.

8. $\begin{cases} 4x + y = 3 \\ 12x + 3y = 9 \end{cases}$

$-3(4x + y = 3) \rightarrow -12x - 3y = -9$

$12x + 3y = 9$
$\underline{-12x - 3y = -9}$
$0 + 0 = 0$
$0 = 0$

This system is a set of coincident lines with infinite solutions. It is consistent dependent.

9. $\begin{cases} 10x - 3y = 3 \\ 2x + y = 9 \end{cases}$

$2x + y = 9 \rightarrow y = 9 - 2x$

$10x - 3y = 3$
$10x - 3(9 - 2x) = 3$
$10x - 27 + 6x = 3$
$16x - 27 = 3$
$16x = 30$
$x = 1.875$

$2x + y = 9$
$2(1.875) + y = 9$
$3.75 + y = 9$
$y = 5.25$

Check

$10x - 3y = 3$	$2x + y = 9$
$10(1.875) - 3(5.25) = 3$	$2(1.875) + 5.25 = 9$
$18.75 - 15.75 = 3$	$3.75 + 5.25 = 9$
$3 = 3$	$9 = 9$

This system is consistent.

10. $\begin{cases} 2x-5y=2 \\ 4x+y=5 \end{cases}$

$4x+y=5$

$y=5-4x$

$2x-5y=2$

$2x-5(5-4x)=2$

$2x-25+20x=2$

$22x-25=2$

$22x=27$

$x=\frac{22}{27}$

$4x+y=5$

$4\left(\frac{27}{22}\right)+y=5$

$\frac{54}{11}+y=5$

$y=5-\frac{54}{11}$

$y=\frac{55}{11}-\frac{54}{11}$

$y=\frac{1}{11}$

Check

$2x-5y=2$

$2\left(\frac{27}{22}\right)-5\left(\frac{1}{11}\right)=2$

$\frac{27}{11}-\frac{5}{11}=2$

$\frac{22}{11}=2$

$2=2$

$4x+y=5$

$4\left(\frac{27}{22}\right)+\frac{1}{11}=5$

$\frac{54}{11}+\frac{1}{11}=5$

$\frac{55}{11}=5$

$5=5$

This system is consistent.

11. $\begin{cases} \frac{3x}{5} + y = 3 \\ 1.2x + 2y = 6 \end{cases}$

$\frac{3x}{5} + y = 3 \rightarrow 3x + y = 15 \rightarrow y = 15 - 3x$

$1.2x + 2y = 6$
$1.2x + 2(15 - 3x) = 6$
$1.2x + 30 - 6x = 6$
$-4.8x + 30 = 6$
$-4.8x = -24$
$x = 5$

$\frac{3x}{5} + y = 3$
$\frac{3(5)}{5} + y = 3$
$\frac{15}{5} + y = 3$
$3 + y = 3$
$y = 0$

Check

$\frac{3x}{5} + y = 3$
$\frac{3(5)}{5} + 0 = 3$
$\frac{15}{5} + 0 = 3$
$3 + 0 = 3$
$3 = 3$

$1.2x + 2y = 6$
$1.2(5) + 2(0) = 6$
$6 + 0 = 6 = 6 = 6$

This system is consistent.

12. $\begin{cases} 3x-4y=12 \\ y=-3x-7 \end{cases}$

$3x-4y=13$
$3x-4(-3x-7)=13$
$3x+12x+28=13$
$15x+28=13$
$15x=-15$
$x=-1$

$y=-3x-7$
$y=-3(-1)-7$
$y=3-7$
$y=-4$

Check

$3x-4y=13$	$y=-3x-7$
$3(-1)-4(-4)=13$	$-4=-3(-1)-7$
$-3+16=13$	$-4=3-7$
$13=13$	$-4=-4$

The system is consistent.

13. $\begin{cases} 3x-3y=3 \\ x-y=1 \end{cases}$

$x-y=1 \rightarrow x=1+y$

$3x-3y=3$
$3(1+y)-3y=3$
$3+3y-3y=3$
$3=3$

These equations are coincident lines and the system has infinite solutions. It is consistent dependent.

14. $\begin{cases} 0.5x - y = 30 \\ 0.5x - y = -30 \end{cases}$

$-1(0.5x - y = -30) \rightarrow -0.5x + y = 30$

$-0.5x + y = 30$
$\underline{0.5x - y = 30}$
$0x + 0y = 60$
$0 = 60$

This system has no solutions. It is inconsistent.

15. $\begin{cases} 4x - 2y = -2 \\ 3x + 2y = -12 \end{cases}$

$3x + 2y = -12$
$\underline{+\,4x - 2y = -2}$
$7x + 0y = -14$
$7x = -14$
$x = -2$

$4x - 2y = -2$
$4(-2) - 2y = -2$
$-8 - 2y = -2$
$-2y = 6$
$y = -3$

Check

$4x - 2y = -2$	$3x + 2y = -12$
$4(-2) - 2(-3) = -2$	$3(-2) + 2(-3) = -12$
$-8 + 6 = -2$	$-6 - 6 = -12$
$-2 = -2$	$-12 = -12$

This system is consistent.

16. $\begin{cases} 3x+2y=4 \\ -2x+2y=24 \end{cases}$

$-1(-2x+2y=24) \rightarrow 2x-2y=-24$

$3x+2y=4$

$\underline{2x-2y=-24}$

$5x+0y=-20$

$5x=-20$

$x=-4$

$3x+2y=4$

$3(-4)+2y=4$

$-12+2y=4$

$2y=16$

$y=8$

Check

$3x+2y=4$	$-2x+2y=24$
$3(-4)+2(8)=4$	$-2(-4)+2(8)=24$
$-12+16=4$	$8+16=24$
$4=4$	$24=24$

This system is consistent.

17. $\begin{cases} 5x-2y=3 \\ 2x-3y=10 \end{cases}$

$2(5x-2y=3) \rightarrow 10x-4y=6$

$-5(2x-3y=10) \rightarrow -10x+15y=-50$

$10x-4y=6$

$\underline{-10x+15y=-50}$

$0x+11y=-44$

$11y=-44$

$y=-4$

$5x-2y=3$

$5x-2(-4)=3$

$5x+8=3$

$5x=-5$

$x=-1$

Check

$5x-2y=3$	$2x-3y=10$
$5(-1)-2(-4)=3$	$2(-1)-3(-4)=10$
$-5+8=3$	$-2+12=10$
$3=3$	$10=10$

This system is consistent.

18. $\begin{cases} 3x-4y=13 \\ y=-3x-y \end{cases}$

$y=-3x-y \rightarrow 0=-3x-2y$

$3x-4y=13$

$\underline{-3x-2y=0}$

$0x-6y=13$

$-6y=13$

$y=-\frac{13}{6}$

$3x-4y=13$

$3x-4\left(-\frac{13}{6}\right)=13$

$3x+\frac{26}{3}=13$

$3x=13-\frac{26}{3}$

$3x=\frac{39}{3}-\frac{26}{3}$

$3x=\frac{13}{3}$

$x=\frac{13}{9}$

Check

$3x-4y=13$

$3\left(\frac{13}{9}\right)-4\left(-\frac{13}{6}\right)=13$

$\frac{13}{3}+\frac{26}{3}=13$

$\frac{39}{3}=13$

$13=13$

$y=-3x-y$

$-\frac{13}{6}=-3\left(\frac{13}{9}\right)-\left(-\frac{13}{6}\right)$

$-\frac{13}{6}=\left(-\frac{13}{3}\right)+\left(\frac{13}{6}\right)$

$-\frac{13}{6}=-\frac{26}{6}+\frac{13}{6}$

$-\frac{13}{6}=-\frac{13}{6}$

This system is consistent.

19. $\begin{cases} 5x-4y=1 \\ -10x+8y=-30 \end{cases}$ $2(5x-4y=1)\rightarrow 10x-8y=2$

$10x-8y=2$

$\underline{-10x+8y=-30}$

$0x+0y=-28$

$0\neq -28$

This system has no solutions. It is inconsistent.

$3x=6y+4.5\rightarrow 3x-6y=4.5$

20. $\begin{cases} 4x+5y=0 \\ 3x=6y+4.5 \end{cases}$ $3(4x+5y=0)\rightarrow 12x+15y=0$

$-4(3x-6y=4.5)\rightarrow -12x+24y=-18$

$12x+15y=0$

$\underline{-12x+24y=-18}$

$0x+39y=-18$

$39y=-18$

$y=-\frac{18}{39}$

$y=-\frac{6}{13}$

$4x+5y=0$

$4x+5\left(-\frac{6}{13}\right)=0$

$4x=\frac{30}{13}$

$x=-\frac{15}{26}$

Check

$4x+5y=0$

$4\left(\frac{15}{26}\right)+5\left(-\frac{6}{13}\right)=0$

$\frac{30}{13}-\frac{30}{13}=0$

$0=0$

$3x=6y+4.5$

$3\left(\frac{15}{26}\right)=6\left(-\frac{6}{13}\right)+4.5$

$\frac{45}{26}=-\frac{36}{13}+\frac{117}{26}$

$\frac{45}{26}=\frac{45}{26}$

This system is consistent.

21. $\begin{cases} -2y+4x=8 \\ y-2x=-4 \end{cases}$ $\qquad 2(y-2x=-4) \rightarrow 2y-4x=-8$

$$2y-4x=-8$$
$$\underline{-2y+4x=8}$$
$$0y+0x=0$$
$$0=0$$

These equations are coincident lines. The system has infinite solutions. It is consistent dependent.

22. $\begin{cases} x-\frac{y}{2}=\frac{3}{2} \\ 3x+y=6 \end{cases}$

$x-\frac{y}{2}=\frac{3}{2} \rightarrow x=\frac{3}{2}+\frac{y}{2}$

$3x+y=6$

$3\left(\frac{3}{2}+\frac{1}{2}y\right)+y=6$

$\frac{9}{2}+\frac{3}{2}y+y=6$

$\frac{3}{2}y+y=6-\frac{9}{2}$

$\frac{5}{2}y=\frac{3}{2}$

$y=\frac{3}{2}\div\frac{5}{2}$

$y=\frac{3}{2}\times\frac{2}{5}$

$y=\frac{3}{5}$

$x-\frac{y}{2}=\frac{3}{2}$

$x-\left(\frac{1}{2}\right)\left(\frac{3}{5}\right)=\frac{3}{2}$

$x-\frac{3}{10}=\frac{3}{2}$

$x=\frac{3}{2}+\frac{3}{10}$

$x=\frac{15}{10}+\frac{3}{10}$

$x=\frac{18}{10}$

$x=\frac{9}{5}$

Check

$x-\frac{y}{2}=\frac{3}{2}$

$\frac{9}{5}-\frac{1}{2}\left(\frac{3}{5}\right)=\frac{3}{2}$

$\frac{9}{5}-\frac{3}{10}=\frac{3}{2}$

$\frac{18}{10}-\frac{3}{10}=\frac{3}{2}$

$\frac{15}{10}=\frac{3}{2}$

$\frac{3}{2}=\frac{3}{2}$

$3x+y=6$

$3\left(\frac{9}{5}\right)+\frac{3}{5}=6$

$\frac{27}{5}+\frac{3}{5}=6$

$\frac{30}{5}=6$

$6=6$

This system is consistent.

23. $\begin{cases} 0.05x + 0.25y = 6 \\ x + y = 24 \end{cases}$

$x + y = 24 \rightarrow y = 24 - x$

$0.05x + 0.25y = 6$
$0.05x + 0.25(24 - x) = 6$
$0.05x + 6 - 0.25x = 6$
$0.05x - 0.25x = 0$
$-0.2x = 0$
$x = 0$

$x + y = 24$
$0 + y = 24$
$y = 24$

Check

$0.05x + 0.25y = 6$
$0.05(0) + 0.25(24) = 6$
$0 + 6 = 6$
$6 = 6$

$x + y = 24$
$0 + 24 = 24$
$24 = 24$

This system is consistent.

24. $\begin{cases} x+\dfrac{2y}{3}=6 \\ 3x+2y=2 \end{cases}$

$-3(x+\frac{2}{3}y=6) \rightarrow -3x-2y=-18$

$3x+2y=2$

$\underline{-3x-2y=-18}$

$0x+0y=-16$

$0 \neq -16$

OR

$x+\dfrac{2y}{3}=6 \rightarrow x=6-\dfrac{2y}{3}$

$3x+2y=2$

$3\left(6-\dfrac{2y}{3}\right)+2y=2$

$18-2y+2y=2$

$18 \neq 2$

This system has no solutions. It is inconsistent.

25. Let's call the number of apples a and the number of bananas b.

Peter buys 2 apples and 3 bananas for \$4. Therefore: $2a+3b=4$

Nadia buys 4 apples and six bananas for \$8. Therefore: $4a+6b=8$

Now let's solve the system using multiplication and elimination.

$\begin{cases} 2a+3b=4 \\ 4a+6b=8 \end{cases}$ $\qquad -2(2a+3b=4) \rightarrow -4a-6b=-8$

$4a+6b=8$

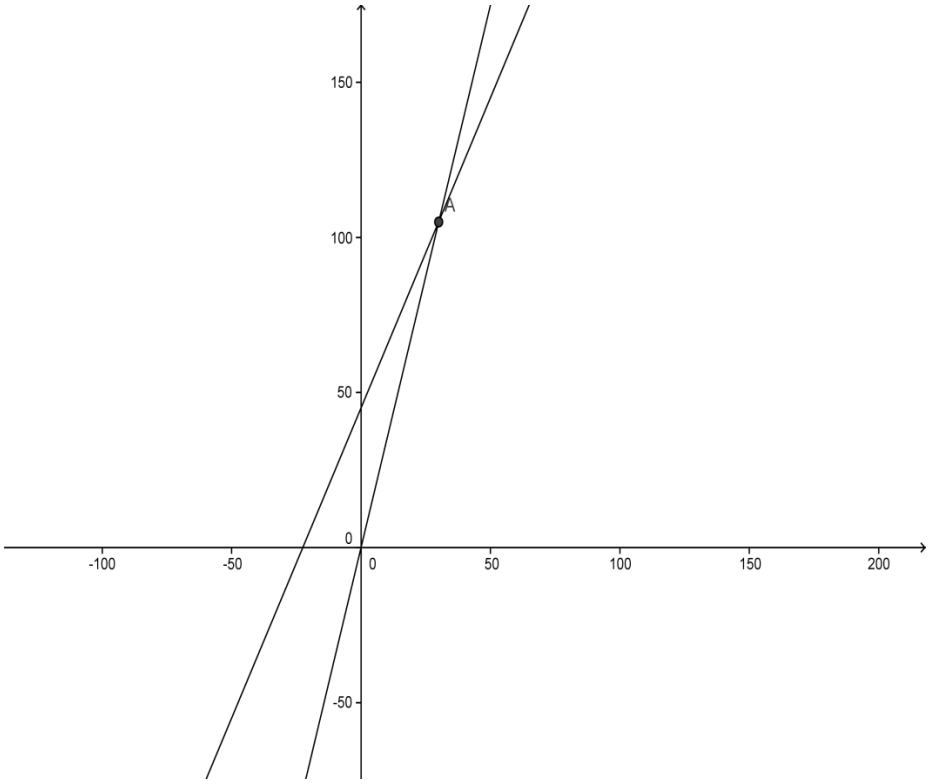
$\underline{-4a-6b=-8}$

$0a+0b=0$

$0=0$

These equations are coincident lines. The system has infinite solutions. It is consistent dependent.

26. The equation for the membership is y=45+2x and the equation for the second option is y=3.5x.
If we graph the equations they look like this.



The two lines intersect at (30,105). The two options will cost the same amount at $105(y) and when a customer has rented 30 movies. If we look at how the lines cross we can see that the membership will cost less after one more movie.

The membership will cost less at 31 movies.
This system is consistent.

27.
Let's call the number of adults a and the number of children c.

The number of adults plus the number of children equals the total number of people. Therefore: $a + c = 1200$

The total amount of money collected equals the adult ticket price times the number of the adults plus the children's ticket price times the number of children. Therefore:
$4.5c + 8a = 8375$

Now let's solve the system by using substitution.

$$\begin{cases} a + c = 1200 \\ 4.5c + 8a = 8375 \end{cases}$$

$a + c = 1200 \rightarrow c = 1200 - a$

$4.5c + 8a = 8375$
$4.5(1200 - a) + 8a = 8375$
$5400 - 4.5a + 8a = 8375$
$5400 + 3.5a = 8375$
$3.5a = 2975$
$a = 850$

$a + c = 1200$
$850 + c = 1200$
$c = 350$

The number of adults that day was 850 and the number of children was 350.
This system is consistent.

28. Let's call the per-item cost of the ties T and the per-item cost of the suspenders S.

Andrew's first purchase is 13 ties and 4 suspenders equals $487.
Therefore: $13T+4S=487$

Andrew's second purchase is 6 ties and 2 suspenders equals $232.
Therefore: 6T+2S=232

Now let's solve the system using multiplication and elimination.

$$\begin{cases} 13T+4S=487 \\ 6T+2S=232 \end{cases}$$

$$-2(6T+2S=232) \rightarrow -12T-4S=-464$$

$$13T+4S=487$$
$$\underline{-12T-4S=-464}$$
$$T+0S=23$$
$$T=23$$

$$6T+2S=232$$
$$6(23)+2S=232$$
$$138+2S=232$$
$$2S=94$$
$$S=47$$

Ties cost $23 and suspenders cost $47.
This system is consistent.

29. Let's call the speed of the airplane x and the speed of the jet-stream y.

On the first trip the plane takes four hours with help from the speed of the jet stream.
Therefore: $4x + 4y = 2400$
On the trip back the plane takes five hours and it is moving against the jet stream.
Therefore: $5x - 5y = 2400$

Now let's solve the system using elimination.

$$\begin{cases} 4x + 4y = 2400 \\ 5x - 5y = 2400 \end{cases}$$

$5(4x + 4y = 2400) \rightarrow 20x + 20y = 12000$

$4(5x - 5y = 2400) \rightarrow 20x - 20y = 9600$

$$\begin{array}{l} 20x + 20y = 12000 \\ \underline{20x - 20y = 9600} \\ 40x + 0y = 21600 \end{array}$$

$40x = 21600$

$x = 540$

$4x + 4y = 2400$

$4(540) + 4y = 2400$

$2160 + 4y = 2400$

$4y = 240$

$y = 60$

The speed of the airplane is 540 miles per hour and the speed of the jet-stream is 60 miles per hour. This system is consistent.

30.
Let's call the cost of an apple A and the cost of a banana B.
We need to find out the cost of one of each.

The first purchase was two apples and one banana for a total of $2.50.
Therefore: $2A+B=2.5$

The second purchase was four apples and two bananas for a total of $6.00.
Therefore: $4A+2B=6$

Now let's solve the system using substitution.

$$\begin{cases} 2A+B=2.5 \\ 4A+2B=6 \end{cases}$$

$$2A+B=2.5 \rightarrow B=2.5-2A$$

$$4A+2B=6$$
$$4A+2(2.5-2A)=6$$
$$4A+5-4A=6$$
$$5 \neq 6$$

This system has no solutions. It is inconsistent.

Mixed Review

31.
Let's call the number of regular seats R and the number of box seats B.

We know the total number of all kinds of seats is 10,413. Therefore: $R+B=10413$

We know that the number of regular seats is twelve times the number of box seats. Therefore: $R=12B$

Now let's solve the system using substitution.

$$\begin{cases} R+B=10413 \\ R=12B \end{cases}$$

$$R+B=10413$$
$$(12B)+B=10413$$
$$13B=10413$$
$$B=801$$

$$R=12B$$
$$R=12(801)$$
$$R=9612$$

There are 9,612 regular seats in the stadium and 801 box seats in the stadium.

32. $y = -\frac{3}{5}x - 8.5$

(2,7)

The slope of the line is -3/5 so the slope of a line perpendicular to it is 5/3 (the reciprocal with the opposite sign). Now let's use point slope form to find the equation of the line.

$$y - y_1 = m(x - x_1)$$
$$y - 7 = \frac{5}{3}(x - 2)$$
$$y - 7 = \frac{5}{3}x - \frac{10}{3}$$
$$y = \frac{5}{3}x - \frac{10}{3} + 7$$
$$y = \frac{5}{3}x - \frac{10}{3} + \frac{21}{3}$$
$$y = \frac{5}{3}x + \frac{11}{3}$$

33. $y = \frac{1}{6}x - 4$

Standard form is $ax+by=c$

$$y = \frac{1}{6}x - 4$$
$$4 + y = \frac{1}{6}x$$
$$4 = \frac{1}{6}x - y$$
$$\frac{1}{6}x - y = 4$$

34. $7\frac{2}{3} + \frac{4}{5}$

$$7\frac{2}{3} + \frac{4}{5} = \frac{23}{3} + \frac{4}{5} = \frac{115}{15} + \frac{12}{15} = \frac{127}{15} = 8\frac{7}{15}$$

35. $\frac{7}{8} \div -\frac{2}{3}$

$\frac{7}{8} \div -\frac{2}{3} = \frac{7}{8} \times -\frac{3}{2} = -\frac{21}{16}$

36. Yes the product of two rational numbers is always a rational number. Regardless of what answer you get, it can always be represented as a ratio of two integers, a rational number. If the answer is a whole number we can always put a one in the denominator to make it a ratio.

Lesson 7.6
Systems of Linear Inequalities

1. *Linear Programming* is the mathematical process of analyzing a system of inequalities to make the best decisions given the constraints of the situation.

2. The *feasible region* of a system of inequalities is the region on the graph that represents the solutions that fit all the inequalities of the system.

3. Constraints are the particular restrictions of a situation due to time, money or materials. They limit the possible solutions to a system of inequalities.

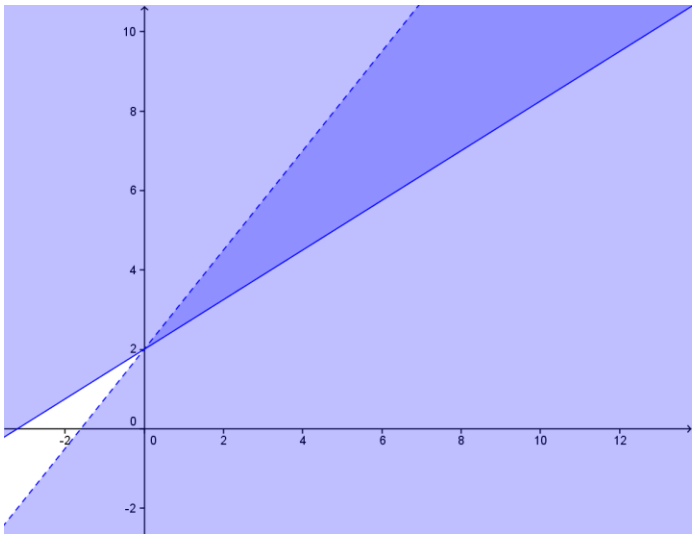
4. An *optimization equation* is an equation that allows you to find the maximum number of solutions. Its purpose is to find the best possible solution for the system of inequalities in your given situation (i.e. most money etc).

5. The *maximum or minimum points of the optimization equation* are located at the vertices of the feasible region, where the boundary lines intersect.

6. $\begin{cases} 4y-5x<8 \\ -5x\geq 16-8y \end{cases}$

$4y-5x<8 \rightarrow y<\frac{5}{4}x+2$

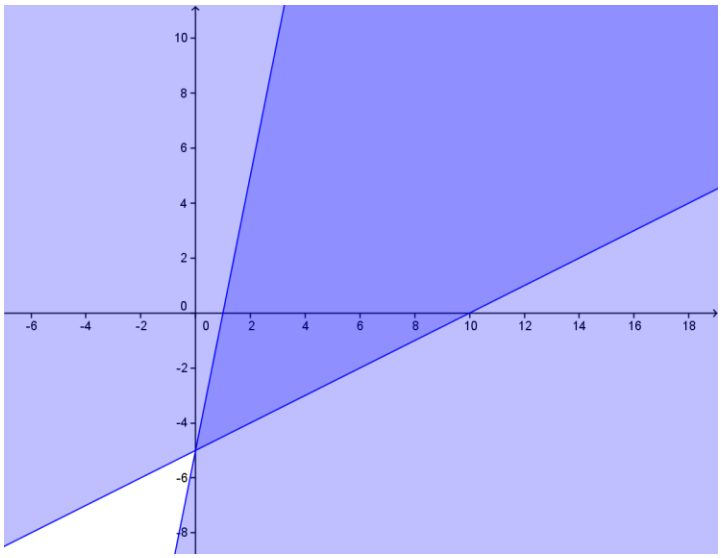
$-5x\geq 16-8y \rightarrow y\geq \frac{5}{8}x+2$



7. $\begin{cases} 5x - y \geq 5 \\ 2y - x \geq -10 \end{cases}$

$5x - y \geq 5 \rightarrow y \leq 5x - 5$

$2y - x \geq -10 \rightarrow y \geq \frac{1}{2}x - 5$

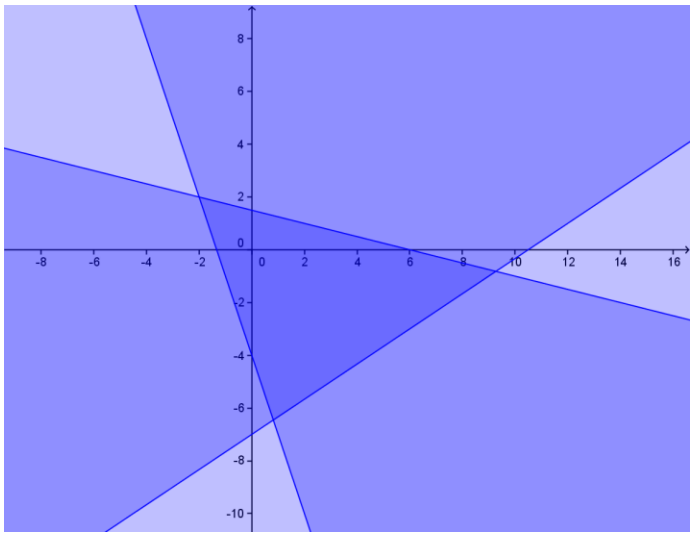


8. $\begin{cases} 2x - 3y \leq 21 \\ x + 4y \leq 6 \\ 3x + y \geq -4 \end{cases}$

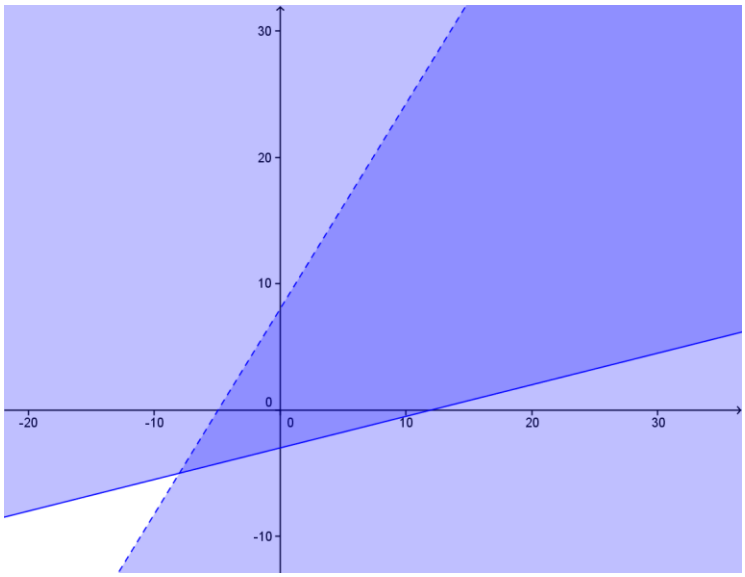
$2x - 3y \leq 21 \rightarrow y \geq \frac{2}{3}x - 7$

$x + 4y \leq 6 \rightarrow y \leq -\frac{1}{4}x + \frac{3}{2}$

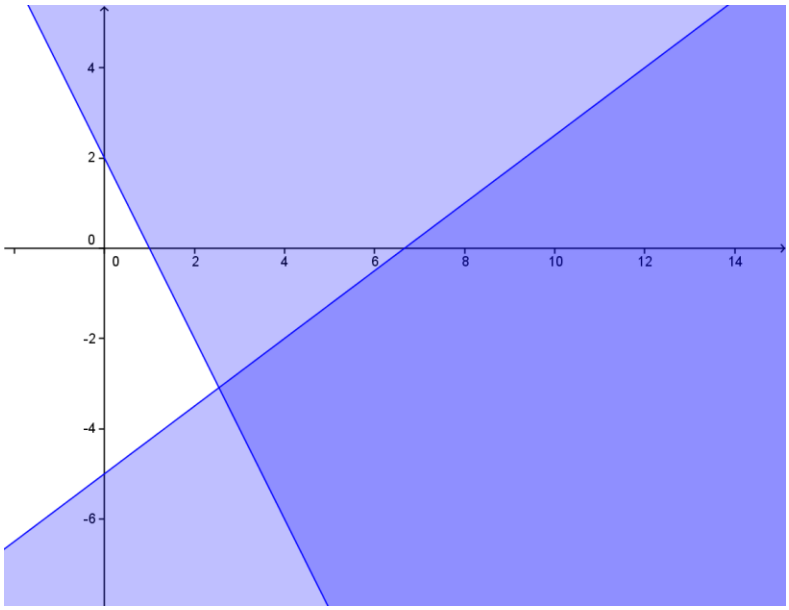
$3x + y \geq -4 \rightarrow y \geq -3x - 4$



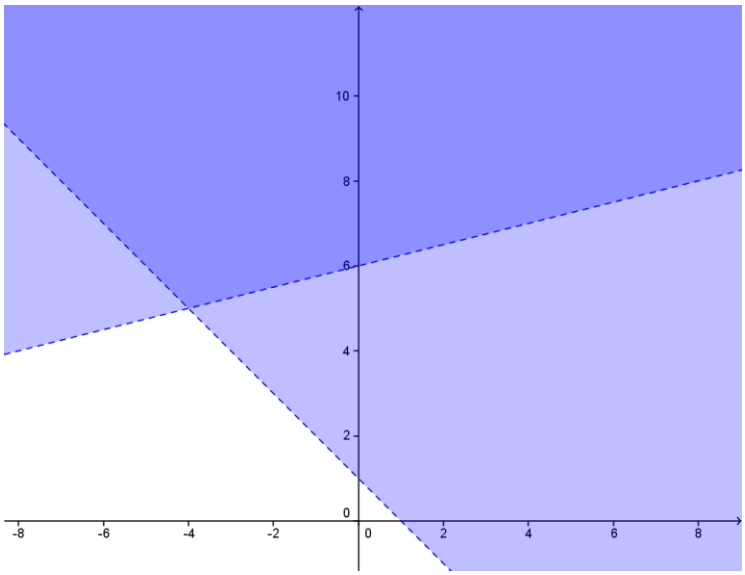
9. $\begin{cases} y \geq \frac{1}{4}x - 3 \\ y < \frac{13}{8}x + 8 \end{cases}$



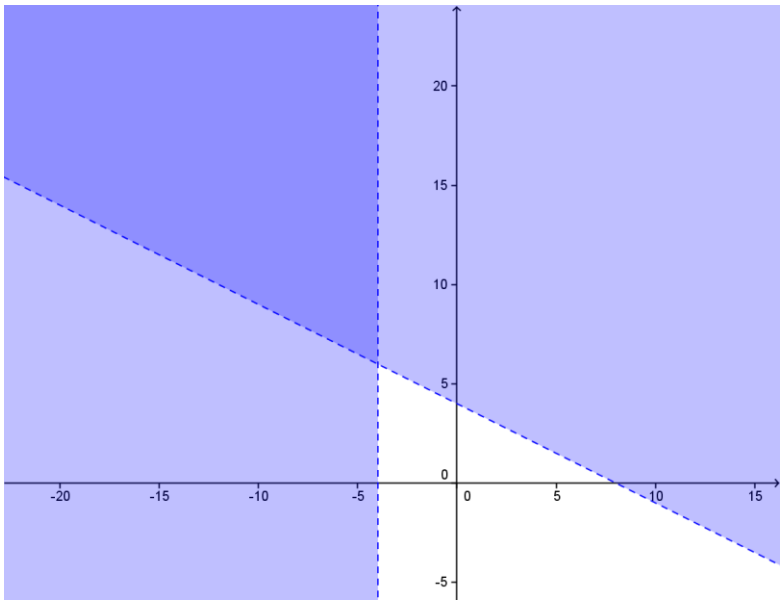
10. $\begin{cases} y \leq \frac{3}{4}x - 5 \\ y \geq -2x + 2 \end{cases}$



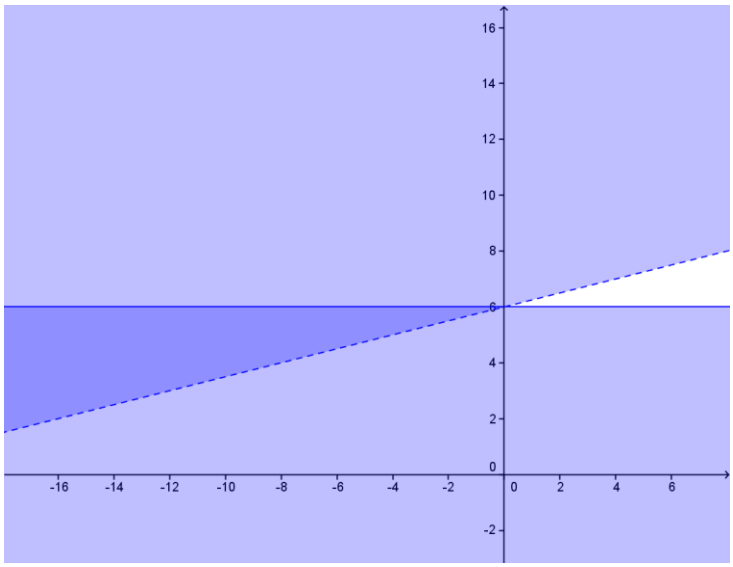
11. $\begin{cases} y > -x + 1 \\ y > \frac{1}{4}x + 6 \end{cases}$



12. $\begin{cases} y > -\frac{1}{2}x + 4 \\ x < -4 \end{cases}$



13. $\begin{cases} y \le 6 \\ y > \frac{1}{4}x + 6 \end{cases}$



14.

$x - y = -6$

$-y = -6 - x$

$y = 6 = x$

$y = x + 6$

$y > x + 6$

$2y = 3x + 17$

$y = \frac{3}{2}x + \frac{17}{2}$

$y \ge \frac{3}{2}x + \frac{17}{2}$

$y \ge \frac{3}{2}x + 8\frac{1}{2}$

$\begin{cases} y > x + 6 \\ y \ge \frac{3}{2}x + 8\frac{1}{2} \end{cases}$

15.

$x+4y=6$

$4y=-x+6$

$y=-\frac{1}{4}x+\frac{3}{2}$

$y=-\frac{1}{4}x+1\frac{1}{2}$

$y\le -\frac{1}{4}x+1\frac{1}{2}$

$2x-3y=21$

$-3y=-2x+21$

$y=\frac{2}{3}x-7$

$y\ge \frac{2}{3}x-7$

$3x+4y=-4$

$4y=-3x-4$

$y=-\frac{3}{4}x-1$

$y\ge -\frac{3}{4}x-1$

$$\begin{cases} y\le -\frac{1}{4}x+1\frac{1}{2} \\ y\ge \frac{2}{3}x-7 \\ y\ge -\frac{3}{4}x-1 \end{cases}$$

16.

$2x+y=12$

$y=-2x+12$

$y\le -2x+12$

$2x-8y=36$

$-8y=-2x+36$

$y=\frac{1}{4}x+4\frac{1}{2}$

$y\le \frac{1}{4}x+4\frac{1}{2}$

$12x-7y=120$

$-7y=-12x+120$

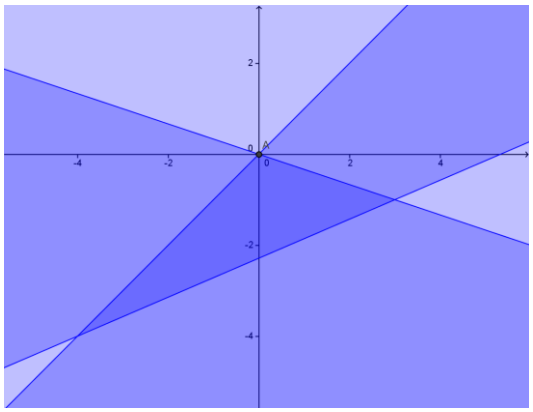
$y=\frac{12}{7}x-\frac{120}{7}$

$y>\frac{12}{7}x-\frac{120}{7}$

$$\begin{cases} y\le -2x+12 \\ y\le \frac{1}{4}x+4\frac{1}{2} \\ y>\frac{12}{7}x-\frac{120}{7} \end{cases}$$

17. $z = -x + 5y$

$x + 3y \le 0$
$x - y \ge 0$
$3x - 7y \le 16$



(0,0) (4,-4) (3,-1)

$z = -x + 5y$
$z = -0 + 5(0)$
$z = 0$

$z = -x + 5y$
$z = -(4) + 5(-4)$
$z = -4 - 20$
$z = -20$

$z = -x + 5y$
$z = -(3) + 5(-1)$
$z = -3 - 5$
$z = -8$

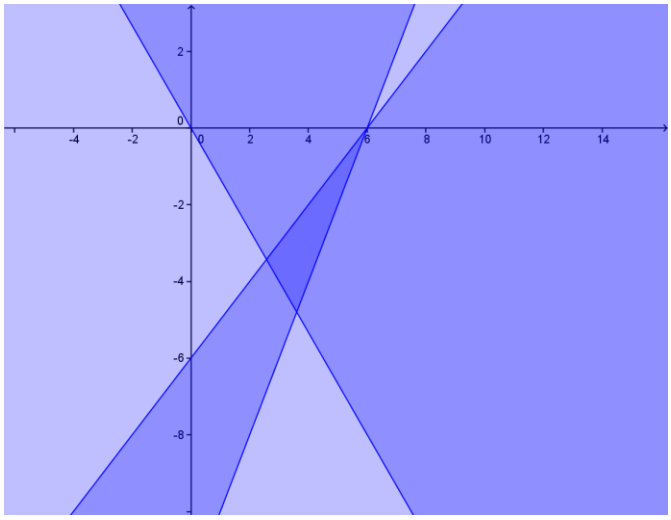
The maximum value of z is 0 and the minimum value of z is -20.

18. $z = 2x + 5y$

$2x - y \le 12$

$4x + 3y \ge 0$

$x - y \ge 6$



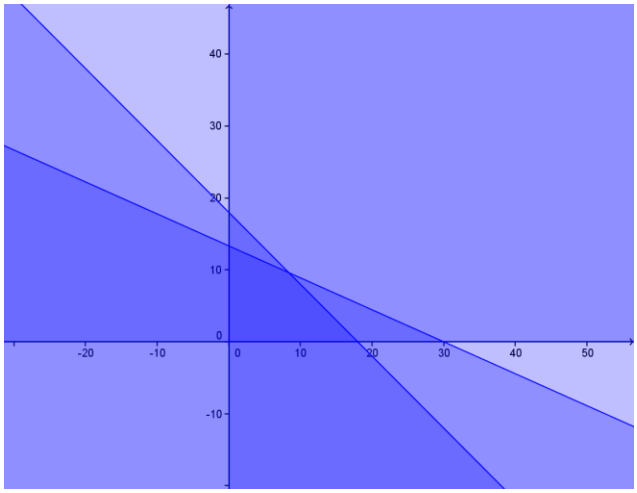
(6,0) (4-5) (2.5,-3)

$z = 2x + 5y$	$z = 2x + 5y$	$z = 2x + 5y$
$z = 2(6) + 5(0)$	$z = 2(4) + 5(-5)$	$z = 2(2.5) + 5(-3)$
$z = 12 + 0$	$z = 8 - 25$	$z = 5 - 15$
$z = 12$	$z = -17$	$z = -10$

The maximum value of z is 12 and the minimum value is -17.

19.

Let's call z the maximum profit, x the number of bookcases, and y the number of TV stands.
We are trying to find the maximum profit: z=60x+100y
His time constraint is: $x+y\le 18$
His money constraint is: $20x+45y\le 600$
Also: $\begin{matrix} x\ge 0 \\ y\ge 0 \end{matrix}$ because you cannot make negative furniture.



(0,0) (18,0) (8,10) (0,14)

$z=60x+100y$
$z=60(0)+100(0)$
$z=0$

$z=60x+100y$
$z=60(18)+100(0)$
$z=1080+0$
$z=1080$

$z=60x+100y$
$z=60(8)+100(10)$
$z=480+1000$
$z=1480$

$z=60x+100y$
$z=60(0)+100(14)$
$z=0+1400$
$z=1400$

Andrew should make 8 bookcases and 10 TV stands for a maximum profit of $1480.

20.
x = Amount of municipal bonds
y = Amount of CD's
z = Amount of high risk

Maximize 1.05x+1.07y+1.10z

$x + y + z = 10000$

$z \leq 1000$

$z \geq 0$

$x \geq 3y$

$x \geq 0$

$y \geq 0$

To maximize your return, you want to invest the maximum in high risk = $1000.

You now have $9000, you want to invest the maximum in CD's, but for tax purposes you need 3 times as much invested in municipal bonds than CD's.
So 3y + y = $9000, this solves to y = $2250.
And x = 3y = $6750.

So the answer is $6750 in municipal bonds, $2250 in CD's, and $1000 in high risk.

Mixed Review

21. *Solve by elimination* $\begin{cases} 12x+8y=24 \\ -6x+3y=9 \end{cases}$ $\quad 2(-6x+3y=9) \rightarrow -12x+6y=18$

$12x+8y=24$	$-6x+3y=9$
$\underline{-12x+6y=18}$	$-6x+3(3)=9$
$0x+14y=42$	$-6x+9=9$
$14y=42$	$-6x=0$
$y=3$	$x=0$

Check

$12x+8y=24$	$-6x+3y=9$
$12(0)+8(3)=24$	$-6(0)+3(3)=9$
$0+24=24$	$0+9=9$
$24=24$	$9=9$

22. *Solve* $36=\left|5t-6\right|$

$36=\left|5t-6\right|$

$36=5t-6$

$42=5t$

$t=\left|8.4\right|$

$t=8.4$

23. *Determine the intercepts of* $y = -\frac{5}{6}x - 3$

$$y = -\frac{5}{6}x - 3$$

$$0 = -\frac{5}{6}x - 3$$

$$3 = -\frac{5}{6}x$$

$$x = 3 \div -\frac{5}{6}$$

$$x = \frac{3}{1} \times -\frac{6}{5}$$

$$x = -\frac{18}{5} = -3\frac{3}{5} = -3.6$$

Since the equation is in y=mx+b form, we know the y-intercept is -3
x-intercept: (-3.6,0)
y-intercept: (0,-3)

24.
Assume that the x-axis is the axis of time and y-axis represents the axis of earnings.
We'll identify x1 = 3h, x2 = 9h, y1 = $145, y2 = $355

The point slope formula of the linear equation is:
$y - y_1 = m(x - x_1)$

The equation for m (slope) is:
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{355 - 145}{9 - 3}$$
$$m = \frac{210}{6}$$
$$m = 35$$

$$y - y_1 = m(x - x_1)$$
$$y - 145 = 35(x - 3)$$
$$y - 145 = 35x - 105$$
$$y = 35x - 105 + 145$$
$$y = 35x + 40$$

$$y = 35x + 40$$
$$y = 35(1.25) + 40$$
$$y = 43.75 + 40$$
$$y = 83.75$$

Jerry's aunt would charge $83.75 for 1.25 hours worth of work.

25.
Yoder is y, Kate is k and Dylan is d.
$$y = k - 4$$
$$k = d - 6$$
$$d = 20$$

If Dylan is 20 then Kate is 14 and Yoder is 10.

Lesson 7.7
Probability and Permutation

1. A *permutation* is an arrangement of objects in a specific order. It is the product of the counting numbers 1 through n.

2. $7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$

3. $10! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 3628800$

4. $1! = 1 \cdot 1 = 1$

5. $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

6. $9! = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 362880$

7. $3! = 3 \cdot 2 \cdot 1 = 6$

8.
$4!+4!$

$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

$4!+4! = 24 + 24 = 48$

9.
$16!-5!$

$16! = 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 20922789888000$

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

$16!-5! = 20922789888000 - 120 = 20922789887880$

10. $\frac{98!}{96!} = \frac{98 \cdot 97}{1} = 9506$

11. $\frac{11!}{2!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3}{1} = 19958400$

12. $\frac{301!}{300!} = \frac{301}{1} = 301$

13. $\frac{8!}{3!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{1} = 6720$

14.

$2!+9!$

$2! = 2 \cdot 1 = 2$

$9! = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 362880$

$2!+9! = 2 + 362880 = 362882$

15. ${}_{11}P_2 = \frac{11!}{(11-2)!} = \frac{11!}{9!} = 11 \cdot 10 = 110$

16. ${}_5P_5 = \frac{5!}{(5-5)!} = \frac{5!}{0!} = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

17. ${}_5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 5 \cdot 4 \cdot 3 = 60$

18. ${}_{15}P_{10} = \frac{15!}{(15-10)!} = \frac{15!}{5!} = 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 10897286400$

19.

${}_{60}P_{59} = \frac{60!}{(60-59)!} = \frac{60!}{1!} = 60! =$

$8320987112741390144276341183223364380754172606361245952449277696409600000000000000$

20. $14! = 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 87178291200$

21. ${}_{10}P_4 = \frac{10!}{(10-4)!} = \frac{10!}{6!} = 10 \cdot 9 \cdot 8 \cdot 7 = 5040$

22.

${}_{21}P_{13} = \frac{21!}{(21-13)!} = \frac{21!}{8!} = 21 \cdot 20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 = 1267136462592000$

23. $8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40320$

24.

(a)

The first place can be won by either of the 12 horses, this gives 12 ways for it to happen. Once the first place has been secured, the second place can be won by either of the remaining 11 horses. This can happen in 11 ways. In total we have 12*11 = 132 ways

(b)
Now let's find the number of ways in which all the horses finish the race. The 1st horse can finish at any of the 12 places. The 2nd can finish at any of the remaining 11 places. This goes on till there is one place left for the last horse. Therefore:
$12! = 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 479001600$

25. $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

26. It is given that Jerry, Kerry, Larry, and Mary are waiting at a bus stop. The number of ways that the 4 can get into the bus is given by

$${}_4P_4 = \frac{4!}{(4-4)!} = 4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

The number of ways in which Mary gets on to the bus first is

$${}_3P_3 = \frac{3!}{(3-3)!} = 3! = 3 \cdot 2 \cdot 1 = 6$$

The probability that Mary gets on the bus first is $\frac{6}{24} = \frac{1}{4}$

27. $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

28. For the word AMAZING, A is repeated twice. The number of permutations can be found by dividing the factorial of the total number of letters by the factorial of the number of times the letter A repeats.

$$\frac{7!}{2!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 = 2520$$

29. The number of ice cream cones possible is equal to the number of permutations possible with 3 elements in all with each permutation having all the three elements.
This is given by: ${}_3P_3 = 3! = 3 \cdot 2 \cdot 1 = 6$
The total number of cones possible is 6.

The number of permutations in which he Superman is on top is equal to 2. This gives the probability that a random ice cream cone has Superman on top as 2/6 = 1/3
The probability that Superman will be on top is 1/3

30. There are 4 jacks in a standard deck of 52 cards.
The chance of drawing a jack the first time is 4/52

If you draw a jack and the card is not replaced, there are now 3 jacks in a deck of 51.
So the chance of drawing a jack the second time is 3/51.

The probability of two events occurring is equal to the product of the individual events.
Therefore:

$$\frac{4}{52}\cdot\frac{3}{51}=\frac{12}{2652}=\frac{1}{221}\approx 0.452\%$$

31.
(a) If the town cannot host a Super Bowl two consecutive years, use the following permutation:

$${}_nP_k=\frac{n!}{(n-k)!}$$

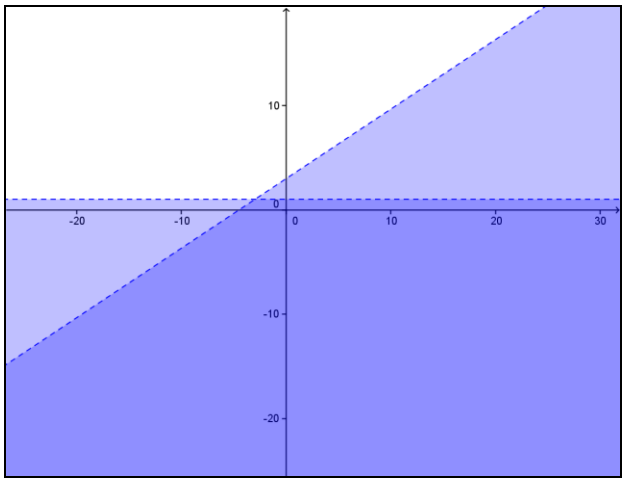
If the town cannot repeat a Super Bowl two consecutive years, then...
n = 9 (the number of events, in this case, the number of towns)
k = 2 (the number of sequences without repetition)

$${}_9P_2=\frac{9!}{(9-2)!}=\frac{9!}{7!}=\frac{362{,}880}{5{,}040}=72$$

(b) If the town can host a Super Bowl two consecutive years, you do not use a permutation because a permutation requires non-repetition. If you have m ways to make the first choice and n ways to make the second choice, then you have m*n ways to make both choices.
m = 9
n = 9
m * n = 9 * 9 = 81

Mixed Review

32. $\begin{cases}2x-3y>-9\\ y<1\end{cases}$



33.
There are about 3.28 feet in a meter.
24 * 3.28 $\approx$78.72 feet.
There are 60 seconds in a minute.
1*60 =60 seconds.

$\approx$78.72 feet / 60 seconds
Divide 78.72 $\div$60
$\approx 1.312\ feet / second$

34. The absolute value can never be a negative number so this can never be true. There is no solution.

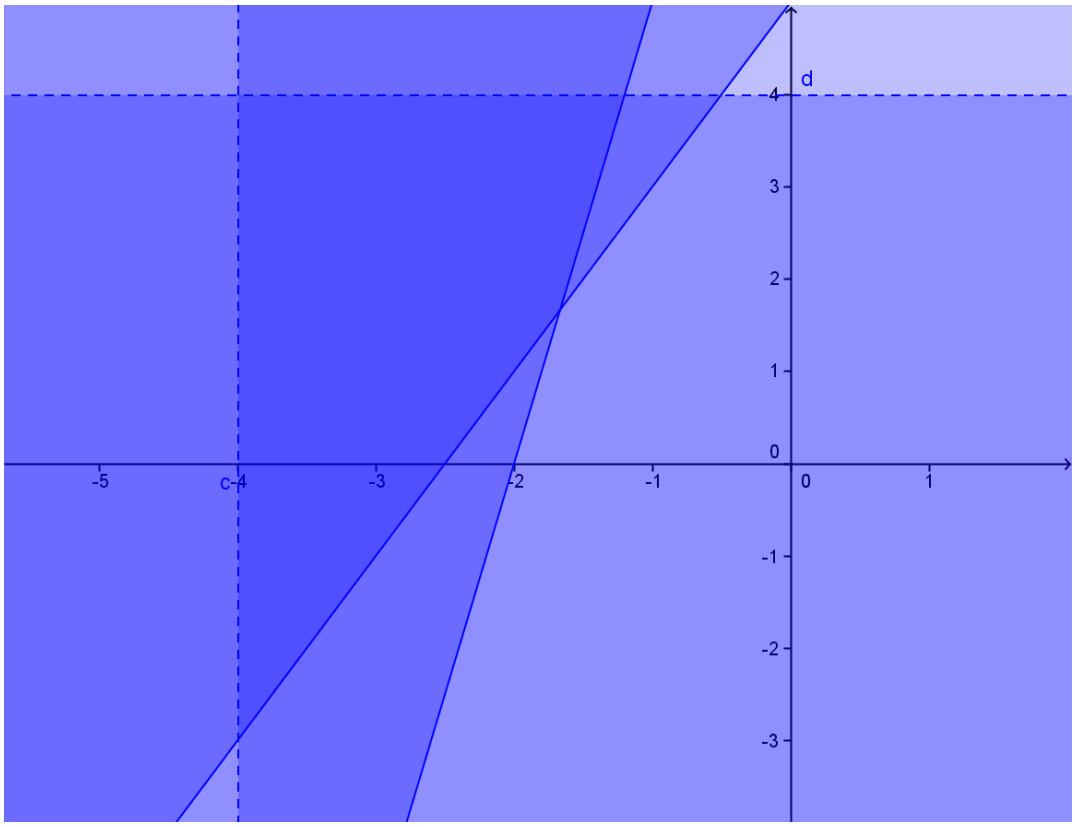
35. 6.16-(-9.86)= 16.02

36. *Which of the following vertices provides the minimum cost according to the equation* $2x + 20y = \cos t$ *: (3,6) (9,0) (6,2) (0,11)*

$2x + 20y = \cos t$	$2x + 20y = \cos t$	$2x + 20y = \cos t$	$2x + 20y = \cos t$
$2(3) + 20(6) = \cos t$	$2(9) + 20(0) = \cos t$	$2(6) + 20(2) = \cos t$	$2(0) + 20(11) = \cos t$
$6 + 120 = \cos t$	$18 + 0 = \cos t$	$12 + 40 = \cos t$	$0 + 220 = \cos t$
$\cos t = 126$	$\cos t = 18$	$\cos t = 52$	$\cos t = 220$

The point that provides the minimum cost is (9,0) with the cost being 18.

37.



$y \geq 5x + 10$

$y \geq 2x + 5$

$x > -4$

$y < 4$

Lesson 7.8
Probability and Combinations

1. A combination is an arrangement of objects in no particular order. It is different from a permutation because in a permutation the order of the objects is essential to finding the correct solution.

2. The number of ways you can choose k objects from n possibilities is $_nC_k = \frac{n!}{k!(n-k)!}$

3. $_3C_9$ is impossible to evaluate because we would have to work with a factorial of a negative number. Factorials of negative numbers are undefined.

4. $\binom{12}{2} = {}_{12}C_2 = \frac{12!}{2!(12-2)!} = 66$

5. $\binom{8}{5} = {}_8C_5 = \frac{8!}{5!(8-5)!} = 56$

6. $\binom{5}{1} = {}_5C_1 = \frac{5!}{1!(5-1)!} = 5$

7. $\binom{3}{0} = {}_3C_0 = \frac{3!}{0!(3-0)!} = 1$

8. $\binom{9}{9} = {}_9C_9 = \frac{9!}{9!(9-9)!} = 1$

9. $\binom{9}{4} = {}_9C_4 = \frac{9!}{4!(9-4)!} = 126$

10. $\binom{20}{10} = {}_{20}C_{10} = \frac{20!}{10!(20-10)!} = 184756$

11. $\binom{19}{18} = {}_{19}C_{18} = \frac{19!}{18!(19-18)!} = 19$

12. $\binom{20}{14} = {}_{20}C_{14} = \frac{20!}{14!(20-14)!} = 38760$

13. $\binom{13}{9} = {}_{13}C_{9} = \frac{13!}{9!(13-9)!} = 715$

14. ${}_{7}C_{3} = \frac{7!}{3!(7-3)!} = 35$

15. ${}_{11}C_{5} = \frac{11!}{5!(11-5)!} = 462$

16. ${}_{5}C_{4} = \frac{5!}{4!(5-4)!} = 5$

17. ${}_{13}C_{9} = \frac{13!}{9!(13-9)!} = 715$

18. ${}_{20}C_{5} = \frac{20!}{5!(20-5)!} = 15504$

19. ${}_{15}C_{15} = \frac{15!}{15!(15-15)!} = 1$

20. $\binom{6}{2} = {}_{6}C_{2} = \frac{6!}{2!(6-2)!} = 15$

21. (steak)! * (vegan)! * (seafood)!
4! * 2! * 1!
4 * 3 * 2 * 1 * 2 * 1 * 1 = 48
48 ways to order dinner

22. ${}_{10}C_{4} = \frac{10!}{4!(10-4)!} = 210$

23. ${}_{28}C_{2} = \frac{28!}{2!(28-2)!} = 378$

24. (wins)! * (losses)! * (ties)!
8! * 4! * 2!
8 * 7 * 6 * 5 * 4 * 3 * 2 *1 *4 *3 * 2* 1 * 2 * 1=1935360

25.
There are ${}_6C_3$ ways for all three to be red.
There are ${}_{10}C_3$ total possible outcomes.

$$P = \frac{{}_6C_3}{{}_{10}C_3}$$

$${}_6C_3 = \frac{6!}{3!(6-3)!} = 20$$
$${}_{10}C_3 = \frac{10!}{3!(10-3)!} = 120$$

$$P = \frac{20}{120} = \frac{1}{6}$$

The probability of all three marbles being red is one out of six.

26.
P(red) = 6/10 because there are 6 out of 10 red marbles
P(red) = 5/9 because there are 5 out of 9 red marbles
P(blue) = 4/8 because there are 4 out of 8 blue marbles
The probability that all three will occur is the product of these individual probabilities.
6/10 * 5/9 * 4/8 = 120/720 = 1/6
The probability is 1/6.

27.
There are 100 Senators, therefore: ${}_{100}C_2 = \frac{100!}{2!(100-2)!} = 4950$

28. ${}_7C_4 = \frac{7!}{4!(7-4)!} = 35$

29.
${}_{15}C_{11} = \frac{15!}{11!(15-11)!} = \frac{1307674368000}{958003200} = 1365$
(The fact that they are true/false questions does not matter.)

30.

1. There are 7 total applicants and we are picking any two.

$${}_7C_2 = \frac{7!}{2!(7-2)!} = \frac{5040}{240} = 21$$

2. The are 4 women applicants and we are picking any two.
The first position has 4 women to choose from and the second position has 3 women to choose from, therefore: 4 * 3 = 12

3.We are picking one man and one woman
The first position has 4 women to choose from and the second position has 3 men to choose from, therefore: 4 * 3 = 12

Mixed Review

31. 15! =15 * 14 * 13 * 12 * 11 * 10 *9 * 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1=1307674368000

32. You can divide. When you divide the numerator by the denominator you can cancel out all the number in each factorial that are the same. So in this case the numbers 1 through 297 appear on both the top and the bottom. If we cancel all these out, we are left with: $\frac{300*299*298}{1} = 300*299*298 = 26730600$

33.
There are 6 total outcomes on the die. Three of them are multiples of two.

$$P = \frac{{}_3C_1}{{}_6C_1} = \frac{3}{6} = \frac{1}{2}$$

The probability that the number rolled will be a multiple of 2 is one half or one out of 2.

34.

Let the number be x and y.

If the sum of the two numbers is 70.6, then: x+y=70.6
If the first number is one less than twice the second number, then: x=2y-1

Now let's solve the system using substitution.

$$\begin{cases} x + y = 70.6 \\ x = 2y - 1 \end{cases}$$

$$x + y = 70.6$$
$$(2y - 1) + y = 70.6$$
$$2y - 1 + y = 70.6$$
$$3y - 1 = 70.6$$
$$3y = 71.6$$
$$y \approx 23.8667$$

$$x = 2y - 1$$
$$x = 2(23.8667) - 1$$
$$x = 47.7334 - 1$$
$$x \approx 46.7334$$

Lesson 7.9
Chapter 7 Review

1. combination-an arrangement of objects in which order does not matter consistent-dependent system-a system with an infinite amount of solutions constraints-restrictions imposed by time, materials, or money feasible region-a solution set to system of inequalities inconsistent system-a system with no solutions
linear programming-a method used by businesses to determine the most profitable or least cost given constraints permutation-an arrangement of objects when order matters system-two or more algebraic systems joined by the word and

2. The solutions to a system are located on the graph where the lines for all the equations intersect.

3. If one equation of a system is in slope-intercept form (y=mx+b) and the other is in standard form (ax+by=c) the most effective way to solve the system is to use substitution (put mx+b in for y in the second equation). This is the most effective because it doesn't require any extra work to change the equations to able to work with them. You can solve the system just as it is.

4. $\begin{cases} 7x-4y=11 \\ x+2y=-19 \end{cases} (-3,-8)$

$7x-4y=11$	$x+2y=-19$
$7(-3)-4(-8)=11$	$-3+2(-8)=-19$
$-21+32=32$	$-3-16=-19$
$11=11$	$-19=-19$

Yes (-3,-8) is a solution to this system.

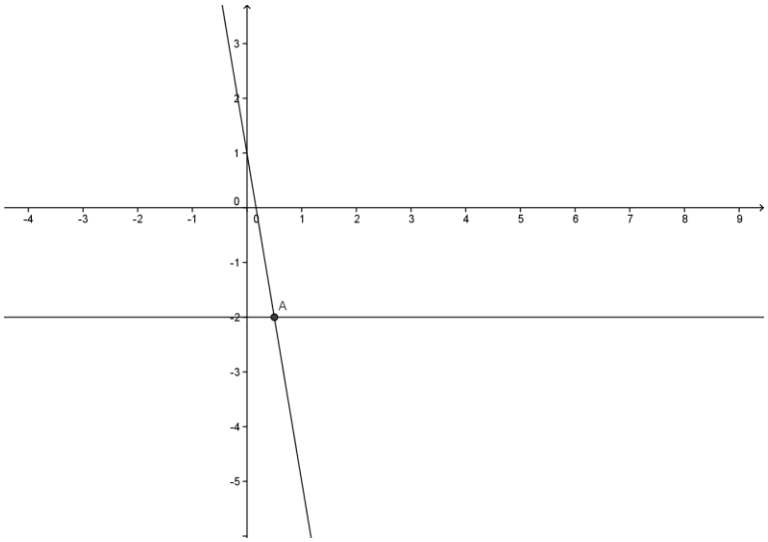
5. $\begin{cases} y=0 \\ 8x+7y=8 \end{cases} (-1,0)$

$8x+7y=8$

$8(-1)+7(0)=8$	$y=0$
$-8+0=8$	$0=0$
$-8\neq 8$	

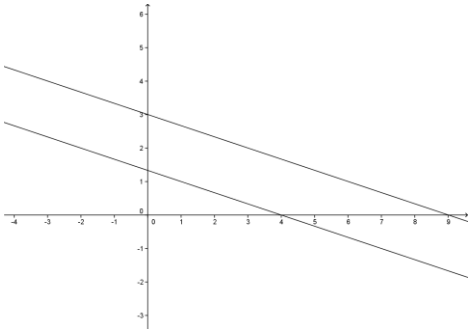
No (-1,0) is not a solution to this system.

6. $\begin{cases} y = -2 \\ y = -6x + 1 \end{cases}$



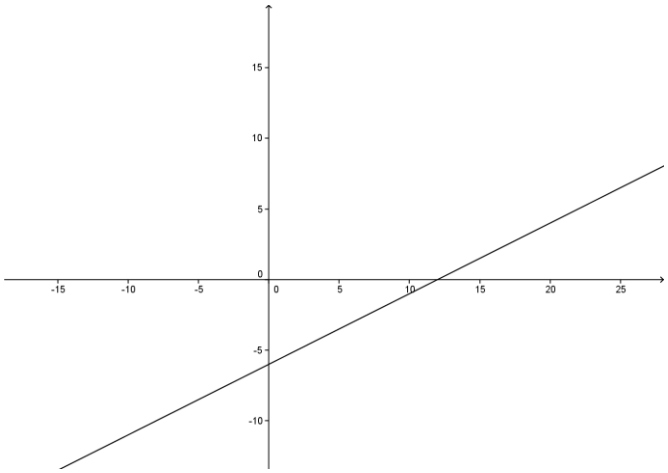
(0.5,-2)

7. $\begin{cases} y = 3 - \frac{1}{3}x \\ x + 3y = 4 \end{cases}$



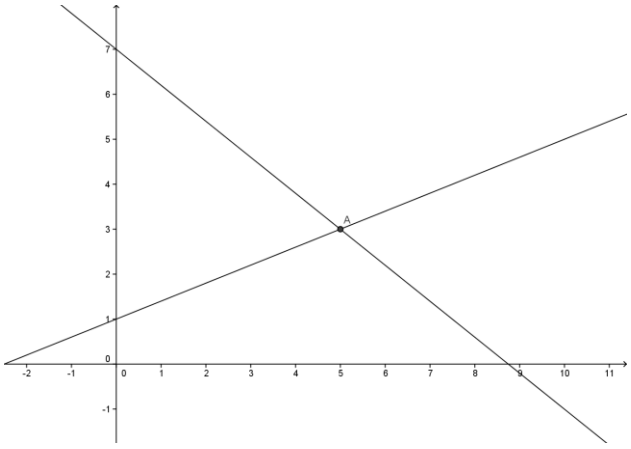
This system is inconsistent, it has no solutions (the lines are parallel).

8. $\begin{cases} y = \frac{1}{2}x - 6 \\ 4y = 2x - 24 \end{cases}$



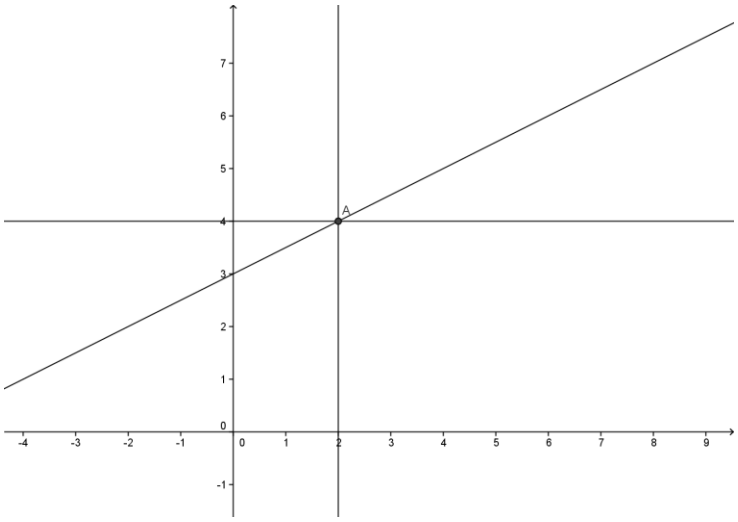
These lines are consistent-dependent. There are an infinite number of solutions.

9. $\begin{cases} y = -\frac{4}{5}x + 7 \\ y = \frac{2}{5}x + 1 \end{cases}$



(5,3)

10. $\begin{cases} x = 2 \\ y = 4 \\ y = \frac{1}{2}x + 3 \end{cases}$



(2,4)

11. $\begin{cases} y = 2x - 7 \\ y + 7 = 4x \end{cases}$

$y + 7 = 4x$

$(2x - 7) + 7 = 4x$

$2x - 7 + 7 = 4x$

$2x = 4x$

$-2x = 0$

$x = 0$

$y = 2x - 7$

$y = 2(0) - 7$

$y = 0 - 7$

$y = -7$

Check

$y = 2x - 7$

$-7 = 2(0) - 7$

$-7 = 0 - 7$

$-7 = -7$

$y + 7 = 4x$

$-7 + 7 = 4(0)$

$0 = 0$

12. $\begin{cases} y = -3x + 22 \\ y = -2x + 16 \end{cases}$

$-3x + 22 = -2x + 16$
$-x + 22 = 16$
$-x = -6$
$x = 6$

$y = -3x + 22$
$y = -3(6) + 22$
$y = -18 + 22$
$y = 4$

Check
$y = -3x + 22$
$4 = -3(6) + 22$
$4 = -18 + 22$
$4 = 4$

$y = -2x + 16$
$4 = -2(6) + 16$
$4 = -12 + 16$
$4 = 4$

13. $\begin{cases} y = 3 - \frac{1}{3}x \\ x + 3y = 4 \end{cases}$

$x + 3y = 4$
$x + 3(3 - \frac{1}{3}x) = 4$
$x + 9 - x = 4$
$9 \neq 4$
This system is inconsistent. It has no solutions. (The equations are parallel lines.)

14. $\begin{cases} 2x + y = -10 \\ y = x + 14 \end{cases}$

$2x + y = -10$

$2x + (x + 14) = -10$

$2x + x + 14 = -10$

$3x + 14 = -10$

$3x = -24$

$x = -8$

$y = x + 14$

$y = (-8) + 14$

$y = 6$

Check

$2x + y = -10$

$2(-8) + 6 = -10$

$-16 + 6 = -10$

$-10 = -10$

$y = x + 14$

$6 = -8 + 14$

$6 = 6$

15. $\begin{cases} y+19=-7x \\ y=-2x-9 \end{cases}$

$y+19=-7x$
$(-2x-9)+19=-7x$
$-2x-9+19=-7x$
$-2x+10=-7x$
$5x=-10$
$x=-2$

$y=-2x-9$
$y=-2(-2)-9$
$y=4-9$
$y=-5$

Check
$y+19=-7x$
$-5+19=-7(-2)$
$14=14$

$y=-2x-9$
$-5=-2(-2)-9$
$-5=4-9$
$-5=-5$

16. $\begin{cases} y=0 \\ 5x=15 \end{cases}$

$5x=15$
$x=3$

The solution to the system is x=3 and y=0.

17. $\begin{cases} y = 2x + 5 \\ y = \frac{1}{2}x + 2 \end{cases}$

$2x + 5 = \frac{1}{2}x + 2$

$2x = \frac{1}{2}x - 3$

$\frac{3}{2}x = -3$

$x = -\frac{3}{1} \div \frac{3}{2}$

$x = -\frac{3}{1} \times \frac{2}{3} = -2$

$x = -2$

$y = 2x + 5$

$y = 2(-2) + 5$

$y = -4 + 5$

$y = 1$

Check

$y = 2x + 5$

$1 = 2(-2) + 5$

$1 = -4 + 5$

$1 = 1$

$y = \frac{1}{2}x + 2$

$1 = \frac{1}{2}(-2) + 2$

$1 = -1 + 2$

$1 = 1$

18. $\begin{cases} 7x+3y=3 \\ y=8 \end{cases}$

$7x+3y=3$

$7x+3(8)=3$

$7x+24=3$

$7x=-21$

$x=-3$

$y=8$

Check

$7x+3y=3$

$7(-3)+3(8)=3$

$-21+24=3$

$3=3$

19. $\begin{cases} 2x+4y=-14 \\ -2x+4y=8 \end{cases}$

$2x+4y=-14$

$\underline{-2x+4y=8}$

$0x+8y=-6$

$8y=-6$

$y=-\frac{6}{8}=-\frac{3}{4}=-0.75$

$2x+4y=-14$

$2x+4\left(-\frac{3}{4}\right)=-14$

$2x-3=-14$

$2x=-11$

$x=-\frac{11}{2}=-5\frac{1}{2}=-5.5$

Check

$2x+4y=-14$

$2\left(-\frac{11}{2}\right)+4\left(-\frac{3}{4}\right)=-14$

$-11-3=-14$

$-14=-14$

$-2x+4y=8$

$-2\left(-\frac{11}{2}\right)+4\left(-\frac{3}{4}\right)=8$

$11-3=8$

$8=8$

20. $\begin{cases} 6x-9y=27 \\ 6x-8y=24 \end{cases}$

$6x-9y=27$

$\underline{-6x+8y=24}$

$0x-y=3$

$-y=3$

$y=-3$

$6x-9y=27$

$6x-9(-30=27$

$6x+27=27$

$6x=0$

$x=0$

Check

$6x-9y=27$

$6(0)-9(-3)=27$

$0+27=27$

$27=27$

$6x-8y=24$

$6(0)-8(-3)=24$

$0+24=24$

$24=24$

21. $\begin{cases} 3x - 2y = 0 \\ 2y - 3x = 0 \end{cases}$

$2y-3x=0 \rightarrow -3x+2y=0$

$3x - 2y = 0$

$\underline{-3x + 2y = 0}$

$0x + 0y = 0$

$0 = 0$

This system is consistent dependent. There are an infinite number of solutions.

22. $\begin{cases} 4x + 3y = 2 \\ -8x + 3y = 14 \end{cases}$

$-8x + 3y = 14$

$\underline{-4x - 3y = 2}$

$-12x + 0y = 12$

$-12x = 12$

$x = -1$

$4x + 3y = 2$

$4(-1) + 3y = 2$

$-4 + 3y = 2$

$3y = 6$

$y = 2$

Check

$4x + 3y = 2$

$4(-1) + 3(2) = 2$

$-4 + 6 = 2$

$2 = 2$

$-8x + 3y = 14$

$-8(-1) + 3(2) = 14$

$8 + 6 = 14$

$14 = 14$

23. $\begin{cases} -8x+8y=8 \\ 6x+y=1 \end{cases}$

$-8(6x+y=1) \rightarrow -48x-8y=-8$

$-48x-8y=-8$

$\underline{-8x+8y=8}$

$-56x=0$

$x=0$

$-8x+8y=8$

$-8(0)+8y=8$

$0+8y=8$

$8y=8$

$y=1$

Check

$-8x+8y=8$

$-8(0)+8(1)=8$

$0+8=8$

$8=8$

$6x+y=1$

$6(0)+1=1$

$0+1=1$

$1=1$

24. $\begin{cases} 7x-4y=11 \\ x+2y=-19 \end{cases}$

$2(x+2y=-19) \rightarrow 2x+4y=-38$

$7x-4y=11$

$\underline{2x+4y=-38}$

$9x+0y=-27$

$9x=-27$

$x=-3$

$7x-4y=11$

$7(-3)-4y=11$

$-21-4y=11$

$-4y=32$

$y=-8$

Check

$7x-4y=11$

$7(-3)-4(-8)=11$

$-21+32=11$

$11=11$

$x+2y=-19$

$-3+2(-8)=-19$

$-3-16=-19$

$-19=-19$

25. $\begin{cases} y = -2x - 1 \\ 4x + 6y = 10 \end{cases}$

$y = -2x - 1 \rightarrow 2x + y = -1$

$-2(2x + y = -1) \rightarrow -4x - 2y = 2$

$4x + 6y = 10$

$\underline{-4x - 2y = 2}$

$0x + 4y = 12$

$4y = 12$

$y = 3$

$y = -2x - 1$

$3 = -2x - 1$

$4 = -2x$

$x = -2$

Check

$y = -2x - 1$

$3 = -2(-2) - 1$

$3 = 4 - 1$

$3 = 3$

$4x + 6y = 10$

$4(-2) + 6(3) = 10$

$-8 + 18 = 10$

$10 = 10$

26. $\begin{cases} x-6y=20 \\ 2y-3x=-12 \end{cases}$

$2y-3x=-12 \rightarrow -3x+2y=-12$

$3(x-6y=20) \rightarrow 3x-18y=60$

$3x-18y=60$
$\underline{-3x+2y=-12}$
$0x-16y=48$
$-16y=48$
$y=-3$

$x-6y=20$
$x-6(-3)=20$
$x+18=20$
$x=2$

Check
$x-6y=20$
$2-6(-3)=20$
$2+18=20$
$20=20$

$2y-3x=-12$
$2(-3)-3(2)=-12$
$-6-6=-12$
$-12=-12$

27. $\begin{cases} -4x+4y=0 \\ 8x-8y=0 \end{cases}$

$2(-4x+4y=0) \rightarrow -8x+8y=0$

$-8x+8y=0$

$\underline{8x-8y=0}$

$0x+0y=0$

$0=0$

This system is consistent-dependent. It has infinite solutions.

28. $\begin{cases} -9x+6y=-27 \\ -3x+2y=-9 \end{cases}$

$-3(-3x+2y=-9) \rightarrow 9x-6y=27$

$-9x+6y=-27$

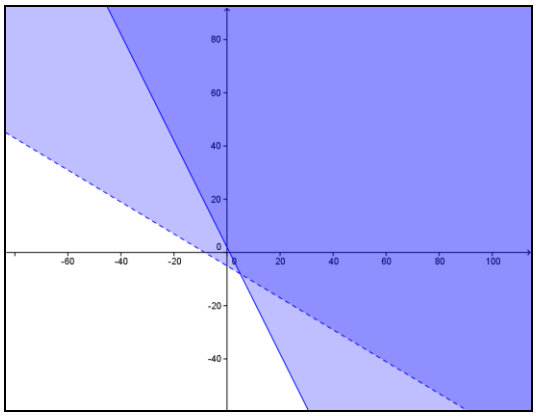
$\underline{9x-6y=27}$

$0x+0y=0$

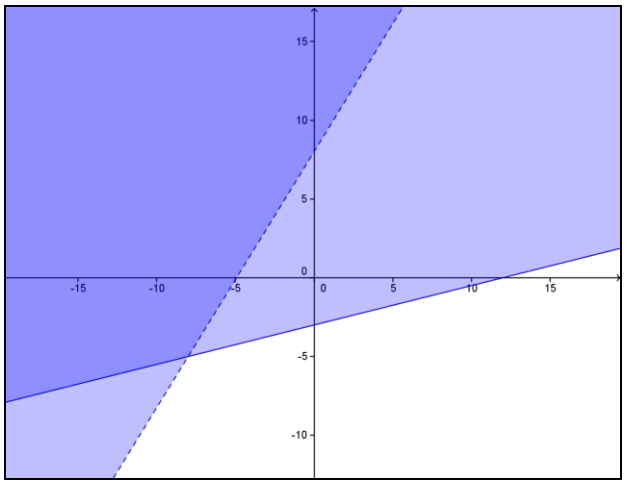
$0=0$

This system is consistent-dependent. It has infinite solutions.

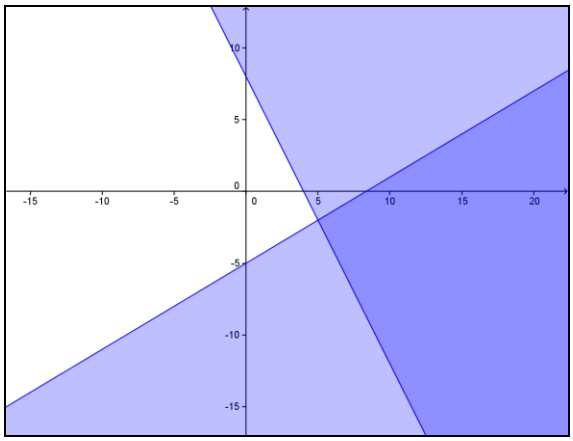
29. $\begin{cases} y>-\frac{3}{5}x-5 \\ y\geq -2x+2 \end{cases}$



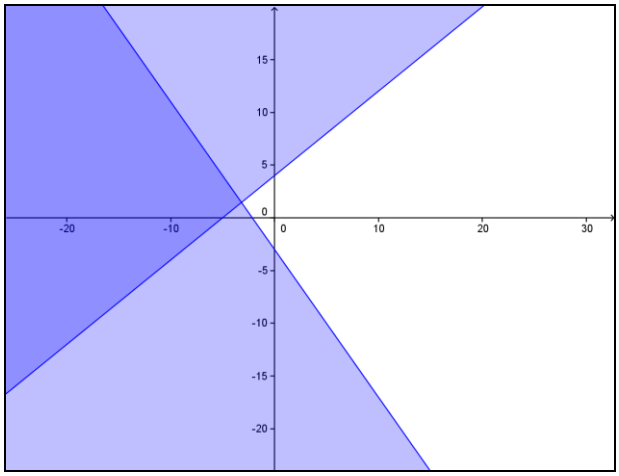
30. $\begin{cases} y > \frac{13}{8}x + 8 \\ y \geq \frac{1}{4}x - 3 \end{cases}$



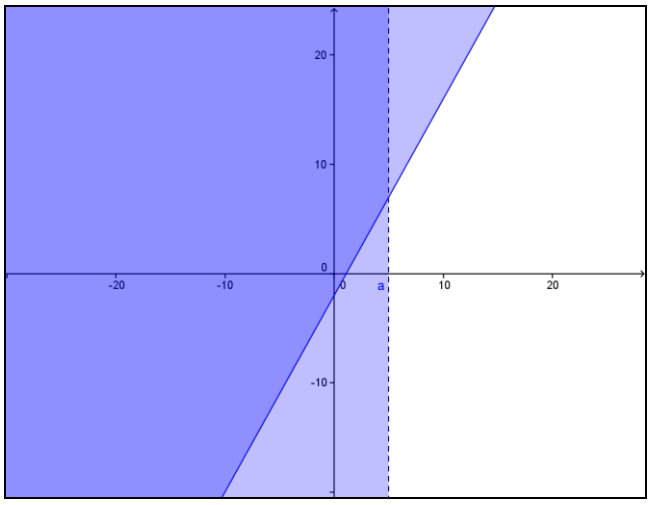
31. $\begin{cases} y \leq \frac{3}{5}x - 5 \\ y \geq -2x + 8 \end{cases}$



32. $\begin{cases} y \le -\frac{7}{5}x - 3 \\ y \ge \frac{4}{5}x + 4 \end{cases}$



33. $\begin{cases} x < 5 \\ y \ge \frac{9}{5}x - 2 \end{cases}$



34. Equation on the right side:
Let's use the slope intercept formula $\rightarrow y=mx+b$
y-intercept=(0,5)
b=5

Now let's find two points on the line so we can find the slope: (2,0) (0,5)

$$m=\frac{y_2-y_1}{x_2-x_1}$$
$$m=\frac{5-0}{0-2}=-\frac{5}{2}=-2.5$$
$$m=-2.5$$

If we look at the graph we can see that the shaded area is where y is less than or equal to the equation. Therefore: $y\leq -2.5x+5$

Equation on the left side:
Let's use the slope intercept formula again $\rightarrow y=mx+b$
y-intercept=(0,-5)
b= -5

Now let's find two points on the line so we can find the slope: (0,-5) (-2,0)

$$m=\frac{y_2-y_1}{x_2-x_1}$$
$$m=\frac{0-(-5)}{-2-0}=-\frac{5}{2}=-2.5$$
$$m=-2.5$$

If we look at the graph we can see that the shaded area is where y is greater than but not equal to the equation. Therefore: $y>-2.5x-5$

The system is: $\begin{cases} y\leq -2.5x+5 \\ y>-2.5x-5 \end{cases}$

35. Equation on the upper right side:
Let's use the slope intercept formula $\rightarrow y=mx+b$
y-intercept=(0,8)
b=8

Now let's find two points on the line so we can find the slope: (0,8) (5,4.5)

$$m=\frac{y_2-y_1}{x_2-x_1}$$
$$m=\frac{8-4.5}{0-5}=-\frac{3.5}{5}=-3.5\div 5=-0.7$$
$$m=-0.7$$

If we look at the graph we can see that the shaded area is where y is less than or equal to the equation. Therefore: $y\leq -0.7x+8$

Equation on the lower right side:
Let's use the slope intercept formula again $\rightarrow y=mx+b$
y-intercept=(0,0)
b= 0

Now let's find two points on the line so we can find the slope: (0,0) (5,4.5)

$$m=\frac{y_2-y_1}{x_2-x_1}$$
$$m=\frac{4.5-0}{5-0}=\frac{4.5}{5}=4.5\div 5=0.9$$
$$m=0.9$$

If we look at the graph we can see that the shaded area is where y is greater than but not equal to the equation. Therefore: $y>-0.9x+0\rightarrow y>-0.9x$

The system is also constrained by the y-axis which is $x\geq 0$

The system is: $\begin{cases} y\leq -0.7x+8 \\ y>0.9x \\ x\geq 0 \end{cases}$

36.

Let's say that x is the number of texts and y is the total cost.

Plan A is: $y = 39.99 + 0.08x$
Plan B is: $y = 69.99$

Now let's solve the system using substitution.

$$\begin{cases} y = 39.99 + 0.08x \\ y = 69.99 \end{cases}$$

$69.99 = 39.99 + 0.08x$

$30 = 0.08x$

$x = 375$

The two plans will charge the same at 375 texts.

My advice for Yolanda would be that if she uses more than 375 texts, she should get plan B, but if she uses less, she should get plan A.

37. Let's call the numbers x and y.

The difference of the two numbers is -21.3, therefore: $x - y = -21.3$

The second number is 3 more than one half the first number, therefore: $y = \frac{1}{2}x + 3$

Now let's solve the system using substitution.

$$\begin{cases} x - y = -21.3 \\ y = \frac{1}{2}x + 3 \end{cases}$$

$$x - y = -21.3$$
$$x - (\frac{1}{2}x + 3) = -21.3$$
$$x - \frac{1}{2}x - 3 = -21.3$$
$$\frac{1}{2}x = -18.3$$
$$x = -36.6$$

$$y = \frac{1}{2}x + 3$$
$$y = \frac{1}{2}(-36.6) + 3$$
$$y = -18.3 + 3$$
$$y = -15.3$$

Check

$$x - y = -21.3$$
$$(-36.6) - (-15.3) = -21.3$$
$$-36.6 + 15.3 = -21.3$$
$$-21.3 = -21.3$$

$$y = \frac{1}{2}x + 3$$
$$(-15.3) = (.5)(-36.6) + 3$$
$$-15.3 = -18.3 + 3$$
$$-15.3 = -15.3$$

The first number is -36.6 and the second number is -15.3.

38.
Let's call the cost of an apple pie A and the cost of a blueberry pie B.

If nine apple pies and six blueberry pies cost a total of \$126 then: $9A+6B=126$
If 12 apple pies and 12 blueberry cost a total of \$204 then: $12A+12B=204$

Now let's solve the system using multiplication and elimination.

$$\begin{cases} 9A+6B=126 \\ 12A+12B=204 \end{cases}$$

$$-2(9A+6B=126) \rightarrow -18A-12B=-252$$

$$12A+12B=204$$
$$\underline{-18A-12B=-252}$$
$$-6A+0y=-48$$
$$-6A=-48$$
$$A=8$$

$$12A+12B=204$$
$$12(8)+12B=204$$
$$96+12B=204$$
$$12B=108$$
$$B=9$$

Check

$9A+6B=126$	$12A+12B=204$
$9(8)+6(9)=126$	$12(8)+12(9)=204$
$72+54=126$	$96+108=204$
$126=126$	$204=204$

The cost of one apple pie is \$8. The cost of two blueberry pies is \$9(2)=\$18.

39. Let's call the speed of the jet x and the speed of the wind y.

On the trip there, the jet is traveling with the help of the wind. Therefore: $7x+7y=784$
On the trip back, the jet is traveling against the wind. Therefore: $14x-14y=784$

Now let's solve the system using elimination.

$$\begin{cases} 7x+7y=784 \\ 14x-14y=784 \end{cases}$$

$$2(7x+7y=784) \rightarrow 14x+14y=1568$$

$$14x+14y=1568$$
$$\underline{14x-14y=784}$$
$$28x+0y=2352$$
$$28x=2352$$
$$x=\frac{2352}{28}=84$$

$$7x+7y=784$$
$$7(84)+7y=784$$
$$588+7y=784$$
$$7y=196$$
$$y=28$$

Check

$7x+7y=784$	$14x-14y=784$
$7(84)+7(28)=784$	$14(84)-14(28)=784$
$588+196=784$	$1176-392=784$
$784=784$	$784=784$

The jet travels at approximately 84 miles per hour and the wind speed is approximately 28 miles per hour.

40.
Let's call the speed of the canoe x and the speed of the river y.

On the trip downstream the canoe travels for one hour and has the help of the river.
Therefore: $x+y=7$

On the trip upstream the canoe travels for 10.5 hours and has to work against the river.
Therefore: $10.5x-10.5y=7$

Now let's solve the system using elimination.

$$\begin{cases} x+y=7 \\ 10.5x-10.5y=7 \end{cases}$$

$$10.5(x+y=7)\rightarrow 10.5x+10.5y=73.5$$

$$10.5x-10.5y=7$$
$$\underline{10.5x+10.5y=73.5}$$
$$21x+0y=80.5$$
$$21x=80.5$$
$$x=80.5\div 21=\frac{805}{10}\times\frac{1}{21}=\frac{115}{30}=\frac{23}{6}$$
$$x=\frac{23}{6}\approx 3.83$$

$$x+y=7$$
$$\frac{23}{6}+y=7$$
$$y=7-\frac{23}{6}=\frac{42}{6}-\frac{23}{6}=\frac{19}{6}$$
$$y=\frac{19}{6}\approx 3.167$$

The speed of the canoe is approximately 3.83 miles per hour and the speed of the river is approximately 3.167 miles per hour.

41. Let call the price of a student ticket S and the price of an adult ticket A.

On Saturday we have 120 times the price of student tickets plus 45 times the price of adult tickets which equals 1102.50. Therefore: $120S + 45A = 1102.50$

On Sunday we have 35 times the price of student tickets plus 80 times the price of adult tickets which equals 890. Therefore: $35S + 80A = 890$

Now let's solve the system using multiplication and elimination.

$$\begin{cases} 120S + 45A = 1102.50 \\ 35S + 80A = 890 \end{cases}$$

$$16(120S + 45A = 1102.50) \rightarrow 1920S + 720A = 17640$$
$$-9(35S + 80A = 890) \rightarrow -315S - 720A = -8010$$

$$1920S + 720A = 17640$$
$$\underline{-315S - 720A = -8010}$$
$$1605S + 0A = 9630$$
$$1605S = 9630$$
$$S = 6$$

$$35S + 80A = 890$$
$$35(6) + 80A = 890$$
$$210 + 80A = 890$$
$$80A = 680$$
$$A = 8.5$$

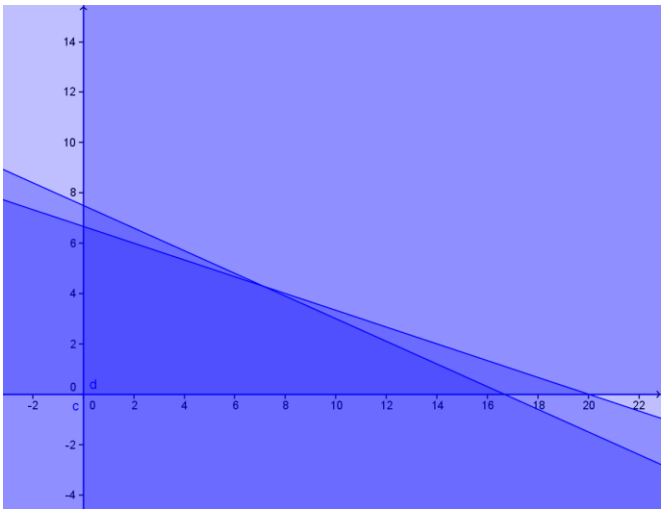
Student tickets cost $6 and adult tickets cost $8.50.

42.

1. Let's call the number of bracelets x and the number of necklaces y. Based on the information in the problem we can create an inequality that represents the constraint on beads and an inequality that represents the constraint on time. In addition Rihanna cannot make negative necklaces or bracelets so both x and y must be greater than or equal to 0.

$$\begin{cases} 36x+80y \le 600 \\ x+3y \le 20 \\ x \ge 0 \\ y \ge 0 \end{cases}$$

2.



(0,0) (17,0) (7,5) (0,6.5)

3. Profit=8x+7y

Looking at the vertices we can work out where Rihanna will make her maximum profit. We know it is not at (0,0) because if she makes zero pieces of jewelry, her profit will be zero.

Let's try out the rest.

$8x+7y=P$	$8x+7y=P$	$8x+7y=P$
$8(17)+7(0)=P$	$8(7)+7(5)=P$	$8(0)+7(6.5)=P$
$136+0=P$	$56+35=P$	$0+45.5=P$
$P=136$	$P=91$	$P=45.5$

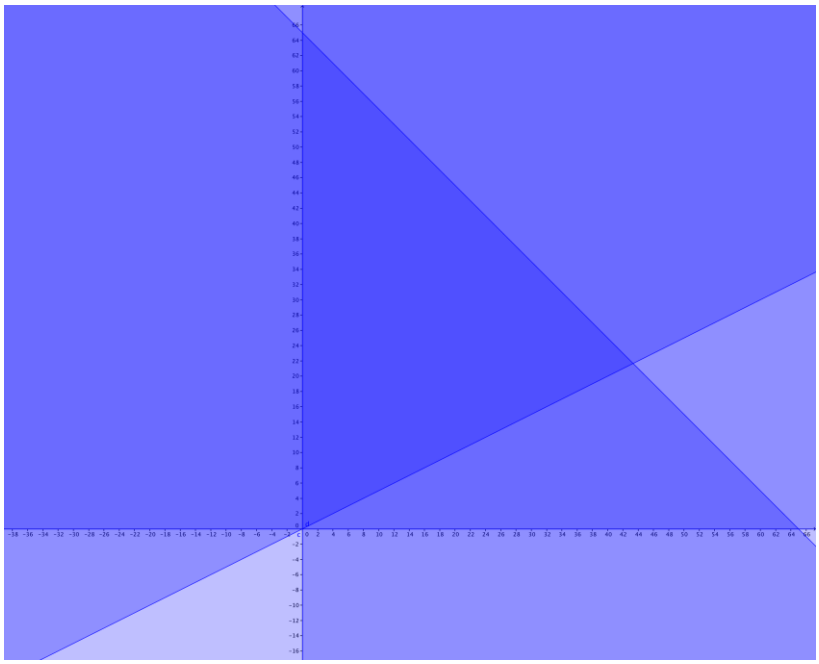
Rihanna will make her maximum profit if she uses her beads and time to make 17 bracelets and 0 necklaces.

43.

1. Let's call the number of acres of soybeans x and the number of acres of wheat y. Based on the information we know the constraints on space and that the farmer has a requirement for the ratio of soybeans to wheat. In addition, it is impossible to plant negative crops so both x and y must be greater than or equal to zero.

$$\begin{cases} x + y \le 65 \\ x \le 2y \\ x \ge 0 \\ y \ge 0 \end{cases}$$

2.



(0,0) (0,65) (43.333, 21.666)

3. To minimize the cost the farmer should plant no crops. Assuming he wants to plant something, he should plant 43.333 soybeans and 21.666 wheat.
Since the crops cost the same thing, that information is unnecessary.

44. $10! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 3628800$

45. $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

46. $\frac{100!}{97!} = 100 \cdot 99 \cdot 98 = 970200$

47.
Here is a visual way to do it. Imagine each blank is a player's position:
___ ___ ___ ___ kicker ___ ___ ___ ___
The kicker position is taken, the other 8 can fit into the other spaces.
Use a permutation of 8 instead of 9 because the kicker position is already taken.
8! = 40,320 arrangements

48. $${}_{15}C_1 = \frac{15!}{1!(15-1)!} = 15$$

49. $${}_{15}C_3 = \frac{15!}{3!(15-3)!} = 455$$

50. There are 6 teams (T1, T2, ..., T6)
$${}_6C_2 = \frac{6!}{6!(6-2)!} = 30$$

51. For the 1st, 2nd, 3rd, .. 6th places, the total number of combination is
$${}_6C_6 = 6! = 720$$

52. $$\binom{14}{12} = {}_{14}C_{12} = \frac{14!}{12!(14-12)!} = 91$$

53. $$\binom{8}{8} = {}_8C_8 = \frac{8!}{8!(8-8)!} = 1$$
This problem is a combination of eight items or choices chosen 8 at a time.

54. $${}_9C_4 = \frac{9!}{4!(9-4)!} = 126$$

55.

$$P = \frac{{}_6C_2}{{}_{11}C_2}$$

$${}_6C_2 = \frac{6!}{2!(6-2)!} = \frac{720}{48} = 15$$

$${}_{11}C_2 = \frac{11!}{2!(11-2)!} = \frac{39916800}{725760} = 55$$

$$P = \frac{15}{55} = \frac{3}{11}$$

The probability that both marbles will be green is 3 out of 11.

56.

Teachers= $${}_{22}C_4 = \frac{22!}{4!(22-4)!} = 7315$$

Students= $${}_{200}C_6 = \frac{200!}{6!(200-6)!} = 82408626300$$

Total= C(teachers) * C(students)=7315 * 82408626300=602819101384500

There are 602,819,101,384,500 possible committees.

Lesson 7.10
Chapter 7 Test

1. *False* $\binom{n}{k}$ is a shorter way to write a combination, NOT a permutation.

2. $\begin{cases} y = \frac{1}{17}x + 18 \\ y = -\frac{21}{17}x - 4 \end{cases}$ $(-17,17)$

$y = \frac{1}{17}x + 18$

$(17) = \frac{1}{17}(-17) + 18$

$17 = -1 + 18$

$17 = 17$

Yes (-17,17) is a solution to the system.

$y = -\frac{21}{17}x - 4$

$(17) = -\frac{21}{17}(-17) - 4$

$17 = 21 - 4$

$17 = 17$

3. *The primary difference between a combination and a permutation* is that the order of the items chosen is essential to a permutation, while the order makes no difference in a combination.

4. Let's call the plane's speed x and the wind speed y.

Traveling against the wind the trip took 12.5 hours, therefore: 12.5x-12.5y=1150
Traveling with the wind the trip took 11 hours, therefore: 11x+11y=1150

Now let's solve the system using multiplication and elimination.

$$\begin{cases} 12.5x - 12.5y = 1150 \\ 11x + 11y = 1150 \end{cases}$$

$$11(12.5x - 12.5y = 1150) \rightarrow 137.5x - 137.5y = 12650$$
$$12.5(11x + 11y = 1150) \rightarrow 137.5x + 137.5y = 14375$$

$$137.5x - 137.5y = 12650$$
$$\underline{137.5x + 137.5y = 14375}$$
$$275x + 0y = 27025$$
$$275x = 27025$$
$$x \approx 98.27$$

$$11x + 11y = 1150$$
$$11(98.27) + 11y = 1150$$
$$1080.97 + 11y = 1150$$
$$11y = 69.03$$
$$y \approx 6.2755$$

The speed of the plane is approximately 98.27 miles per hour. The speed of the wind is approximately 6.2755 miles per hour.

5. *If a solution set to a system of inequalities has two dashed boundary lines then you can conclude that the coordinates on the boundaries* are NOT included in the solution to the system.

6. In order for the system to be consistent-dependent, the equations have to be equal. They must have the same slope and y-intercept. Let's change the first equation into point-slope form.

$$5x + 2y = 20$$
$$2y = 20 - 5x$$
$$2y = -5x + 20$$
$$y = -\frac{5}{2}x + 10$$

Now let's do the same with the second equation.

$$15x + ky = 60$$
$$ky = 60 - 15x$$
$$ky = -15x + 60$$
$$y = -\frac{15}{k}x + \frac{60}{k}$$

Now that both equations are in point-slope form, let's find k.

$$-\frac{15}{k} = -\frac{5}{2}$$
$$\frac{15}{k} = \frac{5}{2}$$
$$30 = 5k$$
$$k = 6$$

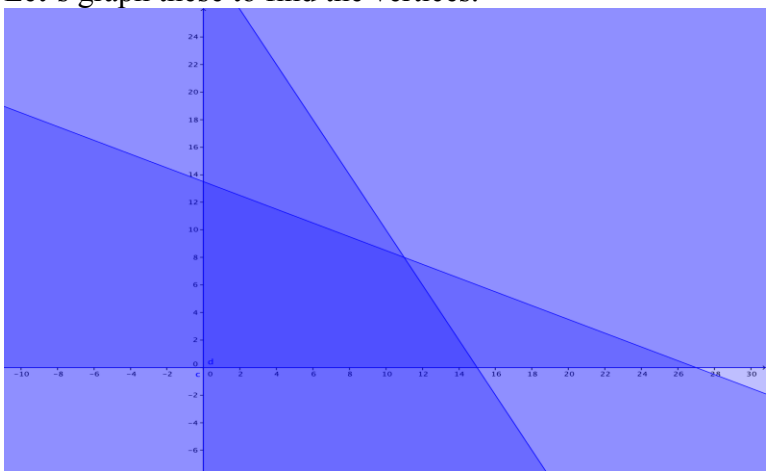
$$\frac{60}{k} = 10$$
$$10k = 60$$
$$k = 6$$

We can see that k must equal 6 so that both equations have the same slope and y-intercept to make the equations consistent-dependent.

7. Let's call Mother's Day arrangements x and Graduation arrangements y.
Joy Lynn has constraints on the number of roses and the number of lilies.
Each Mother's Day arrangement takes 8 roses, each Graduation arrangement takes 4 roses and the total must not equal more than 120, therefore: $8x+4y\le 120$
Each Mother's Day arrangement takes 6 lilies, each Graduation arrangement takes 12 lilies and the total must not equal more than 162, therefore: $6x+12y\le 162$
In addition, Joy Lynn cannot make negative arrangements so both x and y must be greater than or equal to zero.
The constraints are:

$$\begin{cases} 8x+4y\le 120 \\ 6x+12y\le 162 \\ x\ge 0 \\ y\ge 0 \end{cases}$$

Let's graph these to find the vertices.



(0,0) (15,0) (11,8) (0,13.5)

The Maximum Revenue= 32.99x+27.99y. Let's substitute the vertices to find max R. (We know we can eliminate (0,0) because that will not produce any revenue. We can also eliminate the last point of intersection because we know that 13.5 of the product that produces $27.99 each is not going to create more revenue than 15 of the product that produces 32.99 each.)

$P=32.99x+27.99y$	$P=32.99x+27.99y$
$P=32.99(15)+27.99(0)$	$P=32.99(11)+27.99(8)$
$P=494.85+0$	$P=362.89+223.92$
$P=494.85$	$P=586.81$

To maximize revenue Joy Lynn should make 11 Mother's Day arrangements and 8 Graduation arrangements. Her maximum revenue will be $586.81.

8. $\begin{cases} -6x + y = -1 \\ -7x - 2y = 2 \end{cases}$

$-6x + y = -1 \rightarrow y = 6x - 1$

$-7x - 2y = 2$
$-7x - 2(6x - 1) = 2$
$-7x - 12x + 2 = 2$
$-19x = 0$
$x = 0$

$-6x + y = -1$
$-6(0) + y = -1$
$0 + y = -1$
$y = -1$

Check

$-6x + y = -1$	$-7x - 2y = 2$
$-6(0) + (-1) = -1$	$-7(0) - 2(-1) = 2$
$0 - 1 = -1$	$0 + 2 = 2$
$-1 = -1$	$2 = 2$

9. $\begin{cases} y = 0 \\ 8x + 7y = 8 \end{cases}$

$8x + 7y = 8$
$8x + 7(0) = 8$
$8x + 0 = 8$
$8x = 8$
$x = 1$

$y = 0$

10. $\begin{cases} y = x + 8 \\ y = 3x + 16 \end{cases}$

$x + 8 = 3x + 16$
$x - 3x = 16 - 8$
$-2x = 8$
$x = -4$

$y = x + 8$
$y = (-4) + 8$
$y = 4$

Check

$y = x + 8$
$4 = (-4) + 8$
$4 = 4$

$y = 3x + 16$
$4 = 3(-4) + 16$
$4 = -12 + 16$
$4 = 4$

11. $\begin{cases} y = -2x - 2 \\ y = -2x + 17 \end{cases}$

$-2x - 2 = -2x + 17$
$-2x + 2x = 17 + 2$
$0 \neq 19$

This system is inconsistent. It has NO solutions. (The equations are parallel lines.)

12.

1. There are 7 letters in the word violent, so all the letter can be pulled from the bag 7!
7! = 7*6*5*4*3*2*1=5040

2. There are 4 consonants in the word violent. 6!*4= 6*5*4*3*2*1*4=2880
$P = \frac{2880}{5040} = \frac{4}{7}$
The probability that the last letter will be a consonant is 4 out of 7.

13. $_{12}C_5 = \frac{12!}{5!(12-5)!} = \frac{479001600}{604800} = 792$

14. 13! = 13*12*11*10*9*8*7*6*5*4*3*2*1=6227020800