6.1 Dilations

Answers

1. To perform a dilation, draw rays starting at the center of dilation through each point. Move each point along the ray according to the scale factor.

2. In general, dilations do not preserve distance so they are not rigid transformations. Dilations cause the size of the shape to change.

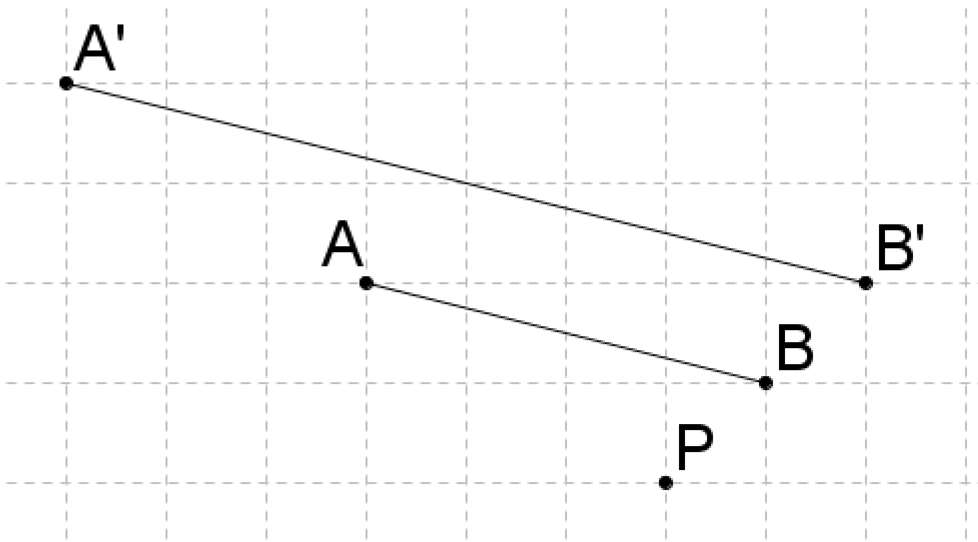
3. True

4. The image is larger than the original shape. The distance from the center point to each point has been increased by a factor of $\frac{3}{2}$.

5. Larger

6. Smaller

7.

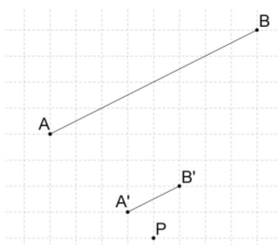


8. $AB = \sqrt{17}$ and $A'B' = \sqrt{68} = 2\sqrt{17}$.

9. The slope of each segment is $-\frac{1}{4}$, so the line segments are parallel.

10. It does not move.

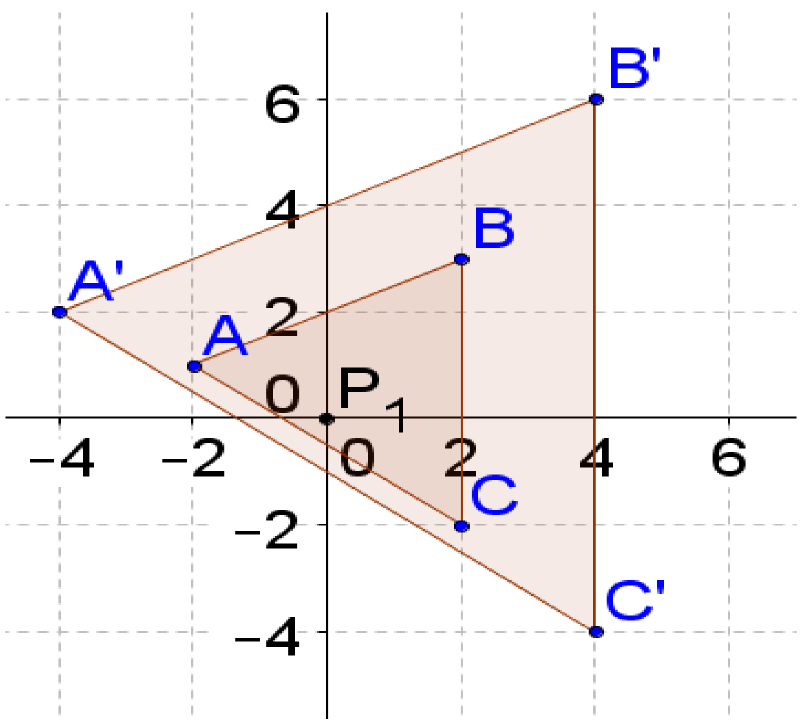
11.



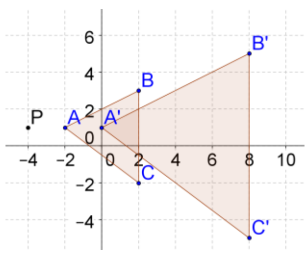
12. $AB = \sqrt{80} = 4\sqrt{5}$ and $A'B' = \sqrt{5}$.

13. The slope of each segment is $\frac{1}{2}$, so the line segments are parallel.

14. Answers vary. Possible answer:



15. Answers vary. Possible answer:



16. The two images are congruent, but in different locations.

6.2 Definition of Similarity

Answers

1. No. They will only be congruent if the scale factor is 1.
2. Yes. Their corresponding angles will be congruent and their corresponding sides will be proportional with a scale factor of 1.
3. True. Corresponding sides are proportional when triangles are similar.
4. False. $\frac{AC}{DF} \neq \frac{EF}{BC}$. This would be correct if one of the fractions was changed to its reciprocal. For example, $\frac{AC}{DF} = \frac{BC}{EF}$.
5. True. Corresponding angles are congruent.
6. $\Delta ABC \sim \Delta DEF$. Reflect ΔABC across $\overline{BC}$. Then dilate the image about point D by a scale factor of $\frac{1}{2}$.
7. The triangles are not similar because the sides are not proportional. $\frac{AC}{DF} = \frac{8}{4} = 2$ while $\frac{BC}{EF} = \frac{5}{\sqrt{5}} = \sqrt{5}$.
8. $\Delta ABC \sim \Delta FED$. The corresponding angles are congruent and the corresponding sides are proportional with a scale factor of $\frac{1}{3}$.
9. $\Delta ABC \sim \Delta EDC$. Rotate ΔABC $180°$ around point C. Dilate about point C with a scale factor of $\frac{1}{2}$.
10. $m\angle F = 45°$
11. $CB = 7.5$
12. $DE = 1$
13. $GF = \frac{16}{7}$
14. $JF = 4$
15. $ED = \frac{20}{7}$
16. $m\angle J = 63°$.
17. 1) Check to see if a similarity transformation would carry one triangle to the other.

    2) Check to see if all corresponding angles are congruent and all corresponding sides are proportional.

6.3 AA Triangle Similarity

Answers

1. AA stands for Angle Angle and it refers to the fact that two triangles are similar if two pairs of corresponding angles are congruent.

2. Answers vary. Triangles should have two pairs of congruent angles.

3. $\Delta ACB \sim \Delta DCF$ due to the fact that $\angle A \cong \angle D$ and $\angle ACB \cong \angle DCF$ (vertical angles).

4. ΔABC~ΔDBE due to the fact that $\angle B \cong \angle B$ and $\angle BED \cong \angle BCA$.

5. $\Delta AEB \sim \Delta CED$. Because each triangle is isosceles, its base angles are congruent. Because $\angle AEB \cong \angle CED$ (vertical angles), all four base angles must be congruent.

6. $\Delta ABC \sim \Delta FDE$. Note that $m\angle B = 30°$ because the sum of the measures of the interior angles of a triangle is $180°$. Therefore, there are two pairs of congruent angles.

7. Not enough information. You only have one pair of angles and one pair of sides congruent.

8. Not enough information. You only have one pair of angles congruent.

9. $\Delta ABC \cong \Delta FDE$ by $ASA \cong$. Note that $m\angle E = 80°$.

10. $\Delta ABC \sim \Delta DEF$. Note that $m\angle A = 66°$.

11. $\Delta AEB \sim \Delta DEC$. Parallel lines create congruent alternate interior angles so $\angle BAE \cong \angle CDE$ and $\angle ABE \cong \angle DCE$.

12. $\Delta ABC \sim \Delta DBE$. Parallel lines create congruent corresponding angles so $\angle BED \cong \angle BCA$ and $\angle BDE \cong \angle BAC$.

13. $\Delta DBF \sim \Delta DCE$. Parallel lines create congruent alternate interior angles so $\angle CDB \cong \angle CBF$ and $\angle CDB \cong \angle ECD$. This means $\angle CBF \cong \angle ECD$ and gives you two pairs of congruent angles.

14. No, for other shapes you would need more information than just two pairs of angles.

15. Possible answer: You can always dilate the smaller triangle using the ratio between the two given triangles to create a new triangle that is congruent to the original larger triangle by $ASA \cong$. This means that there will always exist a similarity transformation between the two triangles.

6.4 SAS Triangle Similarity

Answers

1. SAS stands for side angle side. If two pairs of sides of two triangles are proportional and their included angles are congruent then the triangles are similar.
2. SSA stands for side side angle. This is *not* a criterion for triangle similarity.
3. Answers vary.
4. Answers vary.
5. $\Delta AEB \sim \Delta DEC$ by $AA\sim$. Not enough information to use SAS~.
6. Not enough information to know if they are similar. Marked sides and angles are not corresponding.
7. $\Delta ABC \sim \Delta EBD$ by $SAS\sim$.
8. $\Delta AEB \sim \Delta CED$ by $SAS\sim$.
9. $\Delta ABC \sim \Delta EFD$ by $SAS\sim$.
10. $\Delta AEB \cong \Delta DEC$ by $SAS \cong$.
11. You would need to know that $\frac{AC}{ED} = 3$..
12. You would need to know that $\frac{DE}{AE} = 4$.
13. You would need to know that $EB = 6$.
14. Yes, but they are more than enough information. With AAS or ASA you know two pairs of angles are congruent. This is enough information to show that two triangles are similar by AA~. Therefore, the additional information about the sides would be extra and not necessary.
15. Dilate ΔDEF by a scale factor of $\frac{AC}{FD}\left(=\frac{CB}{DE}\right)$. Then, show that the resulting triangle is congruent to ΔCBA by $SAS \cong$. Therefore, a similarity transformation exists between ΔDEF and ΔCBA so the triangles are similar.

6.5 SSS Triangle Similarity

Answers

1. SSS stands for Side Side Side. If three pairs of sides of two triangles are proportional then the triangles are similar.
2. Answers vary.
3. Neither
4. $\Delta ABC \cong \Delta DEF$ by $SSS \cong$
5. $\Delta DEF \sim \Delta ABC$ by $SSS \sim$
6. Not enough information
7. $\Delta ABC \sim \Delta DEF$ by $SSS \sim$
8. Equilateral triangles are always similar.
9. The ratio of the perimeters is the same as the ratio of the corresponding sides.
10. $18, 20, and\ 25$
11. 3
12. 6, 12, 12
13. $CB = \sqrt{c^2 - b^2}$ and $EF = \sqrt{(kc)^2 - (kb)^2} = \sqrt{k^2(c^2 - b^2)} = k\sqrt{c^2 - b^2}$
14. $\frac{DF}{AB} = \frac{kc}{c} = k.$ $\frac{DE}{AC} = \frac{kb}{b} = k.$ $\frac{EF}{CB} = \frac{k\sqrt{c^2-b^2}}{\sqrt{c^2-b^2}} = k.$ Because three pairs of sides are proportional, the triangles are similar by $SSS\sim$. This means in general if you have two right triangles and a pair of legs and the hypotenuses are proportional, the triangles are similar.
15. Dilate ΔDEF by a factor of k. The side lengths of $\Delta D'E'F'$ will be ka, kb, and kc. $\Delta D'E'F \cong \Delta ACB$ by $SSS \cong$. Therefore, a similarity transformation must exist between ΔDEF and ΔACB, so $\Delta DEF \sim \Delta ACB$.

6.6 Theorems Involving Similarity

Answers

1. $x = 1.75$
2. $x = 6$
3. $x = 9.6$
4. $x = \frac{36}{7}$
5. $x \approx 12.22$
6. $x = \frac{33}{7}$
7. $x = 10$
8. $x = 2$
9. $z = \sqrt{5}$
10. $y = 2\sqrt{5}$
11. $\frac{b}{a} = \frac{d}{c} \to \frac{b}{a} + 1 = \frac{d}{c} + 1 \to \frac{b}{a} + \frac{a}{a} = \frac{d}{c} + \frac{c}{c} \to \frac{b+a}{a} = \frac{d+c}{c}$
12. $\Delta YST \sim \Delta YXZ$ by $SAS\sim$ because two pairs of sides are proportional as shown in #11 and their included angles are shared and thus congruent.
13. Because $\Delta YST \sim \Delta YXZ$, corresponding angles are similar. This means $\angle YST \cong \angle YXZ$. Because corresponding angles are congruent, lines must be parallel. Therefore, $\overline{ST} \parallel \overline{XZ}$.
14. Consider the picture from #11 with $b = a$ and $d = c$. Then, $\frac{YS}{YX} = \frac{a}{2a} = \frac{1}{2}$ and $\frac{YT}{YZ} = \frac{c}{2c} = \frac{1}{2}$. Two pairs of sides are proportional, and it follows that $\overline{ST} \parallel \overline{XZ}$ as shown in #12 and #13.
15. Look at Guided Practice #1-#3 for help.

6.7 Applications of Similar Triangles

Answers

1. They have two pairs of congruent angles so they are similar by $AA\sim$.

2. $1:\sqrt{3}:2$

3. The missing sides are 6 and $6\sqrt{3}$.

4. The missing sides are $8\sqrt{3}$ and 16.

5. The missing sides are $\frac{8}{\sqrt{3}}$ and $\frac{16}{\sqrt{3}}$.

6. They have two pairs of congruent angles so they are similar by $AA\sim$.

7. $1:1:\sqrt{2}$

8. The missing sides are both length 3.

9. The missing sides are 7 and $7\sqrt{2}$.

10. The missing sides are both $\frac{7}{\sqrt{2}}$.

11. Due to parallel lines, $\angle CBA \cong \angle DFA$ and $\angle CAB \cong \angle DEB$. The triangles are similar by $AA\sim$.

12. The scale factor is 2. $x = 8$.

13. $m\angle ADB = 45°$. ΔADB is an isosceles right triangle.

14. $BD = AB = 3$.

15. $AC^2 = 3^2 + 5^2$. $AC = \sqrt{34}$.