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Learning Objectives

Here you will learn about graphing more complex types of functions easily by applying horizontal and vertical shifts to the graphs of parent functions. If you are not familiar with parent functions or function families, it would be a good idea to review the lessons on those topics before proceeding.

Horizontal and vertical transformations are two of the many ways to convert the basic parent functions in a function family into their more complex counterparts.

What vertical and/or horizontal shifts must be applied to the parent function of \( y = x^2 \) in order to graph \( g(x) = (x - 3)^2 + 4 \)?

Vertical and Horizontal Transformations

Have you ever tried to draw a picture of a rabbit, or cat, or dog? Unless you are talented, even the most common animals can be a bit of a challenge to draw accurately (or even recognizably!). One trick that can help even the most "artistically challenged" to create a clearly recognizable basic sketch is demonstrated in nearly all "learn to draw" courses: start with basic shapes. By starting your sketch with simple circles, ellipses, rectangles, etc., the basic outline of the more complex figure is easily arrived at, then details can be added as necessary, but the figure is already recognizable for what it is.

The same trick works when graphing equations. By learning the basic shapes of different types of function graphs, and then adjusting the graphs with different types of transformations, even complex graphs can be sketched rather easily. This lesson will focus on two particular types of transformations: vertical shifts and horizontal shifts.

We can express the application of vertical shifts this way:

Formally: For any function \( f(x) \), the function \( g(x) = f(x) + c \) has a graph that is the same as \( f(x) \), shifted \( c \) units vertically. If \( c \) is positive, the graph is shifted up. If \( c \) is negative, the graph is shifted down.

Informally: Adding a positive number after the \( x \) outside the parentheses shifts the graph up, adding a negative (or subtracting) shifts the graph down.

We can express the application of horizontal shifts this way:

Formally: given a function \( f(x) \), and a constant \( a > 0 \), the function \( g(x) = f(x - a) \) represents a horizontal shift \( a \) units to the right from \( f(x) \). The function \( h(x) = f(x + a) \) represents a horizontal shift \( a \) units to the left.

Informally: Adding a positive number after the \( x \) inside the parentheses shifts the graph left, adding a negative (or subtracting) shifts the graph right.

MEDIA

Click image to the left or use the URL below.

URL: http://www.ck12.org/flx/render/embeddedobject/187147
Examples

Example 1

Earlier, you were given a question about applying vertical and/or horizontal shifts to a parent function in order to graph a different function in the same function family.

What transformations must be applied to \( y = x^2 \), in order to graph \( g(x) = (x - 3)^2 + 4 \)?

The graph of \( g(x) = (x - 3)^2 + 4 \) is the graph of \( y = x^2 \) shifted 3 units to the right, and 4 units up.

Example 2

What must be done to the graph of \( y = x^2 \) to convert it into the graphs of \( y = x^2 - 3 \), and \( y = x^2 + 4 \)?

At first glance, it may seem that the graphs have different widths. For example, it might look like \( y = x^2 + 4 \), the uppermost of the three parabolas, is thinner than the other two parabolas. However, this is not the case. The parabolas are congruent.

If we shifted the graph of \( y = x^2 \) up four units, we would have the exact same graph as \( y = x^2 + 4 \). If we shifted \( y = x^2 \) down three units, we would have the graph of \( y = x^2 - 3 \).
Example 3

Identify the transformation(s) involved in converting the graph of \( f(x) = |x| \) into \( g(x) = |x - 3| \).

From the examples of vertical shifts above, you might think that the graph of \( g(x) \) is the graph of \( f(x) \), shifted 3 units to the left. However, this is not the case. The graph of \( g(x) \) is the graph of \( f(x) \), shifted 3 units to the right.

The direction of the shift makes sense if we look at specific function values.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( g(x) = \text{abs}(x - 3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

From the table we can see that the vertex of the graph is the point (3, 0). The function values on either side of \( x = 3 \) are symmetric, and greater than 0.

Example 4

What transformations must be applied to \( y = x^2 \), in order to graph \( g(x) = (x + 2)^2 - 2 \)?
The graph of \( g(x) = (x + 2)^2 - 2 \) is the graph of \( y = x^2 \) shifted 2 units to the left, and 2 units down.

**Example 5**

Use the parent function \( f(x) = x^2 \) to graph \( f(x) = x^2 + 3 \).

The function \( f(x) = x^2 \) is a parabola with the vertex at \((0, 0)\).

Adding *outside the parenthesis* shifts the graph vertically.

Therefore, \( f(x) = x^2 + 3 \) will be a parabola with the vertex 3 units up.

Example 6

Use the parent function \( f(x) = |x| \) to graph \( f(x) = |x - 4| \).

The graph of the absolute value function family parent function \( f(x) = |x| \) is a large "V" with the vertex at the origin.

Adding or subtracting *inside the parenthesis* results in horizontal movement.

Recall that the horizontal shift is right for negative numbers, and left for positive numbers.

Therefore \( f(x) = |x - 4| \) is a large "V" with the vertex 4 units to the right of the origin.
Review

1. Graph the function \( f(x) = 2|x - 1| - 3 \) without a calculator.
2. What is the vertex of the graph and how do you know?
3. Does it open up or down and how do you know?
4. For the function: \( f(x) = |x| + c \) if \( c \) is positive, the graph shifts in what direction?
5. For the function: \( f(x) = |x| + c \) if \( c \) is negative, the graph shifts in what direction?
6. The function \( g(x) = |x - a| \) represents a shift to the right or the left?
7. The function \( h(x) = |x + a| \) represents a shift to the right or the left?
8. If a graph is in the form \( a \cdot f(x) \). What is the effect of changing the \( a \)?

Describe the transformation that has taken place for the parent function \( f(x) = |x| \).

9. \( f(x) = |x| - 5 \)
10. \( f(x) = 5|x + 7| \)

Write an equation that reflects the transformation that has taken place for the parent function \( g(x) = \frac{1}{x} \), for it to move in the following ways:

11. Move two spaces up
12. Move four spaces to the right
13. Stretch it by 2 in the y-direction

Write an equation for each described transformation.

14. a V-shape shifted down 4 units.
15. a V-shape shifted left 6 units
16. a V-shape shifted right 2 units and up 1 unit.

The following graphs are transformations of the parent function \( f(x) = |x| \) in the form of \( f(x) = a|x - h| = k \). Graph or sketch each to observe the type of transformation.
17. \( f(x) = |x| + 2 \). What happens to the graph when you add a number to the function? (i.e. \( f(x) + k \)).

18. \( f(x) = |x| - 4 \). What happens to the graph when you subtract a number from the function? (i.e. \( f(x) - k \)).

19. \( f(x) = |x - 4| \). What happens to the graph when you subtract a number in the function? (i.e. \( f(x - h) \)).

20. \( f(x) = |x + 2| \). What happens to the graph when you add a number in the function? (i.e. \( f(x + h) \)).

Practice: Graph the following.

21. \( f(x) = 2|x| \)
22. \( f(x) = \frac{3}{2}|x| \)
23. \( f(x) = \frac{1}{2}|x| \)
24. \( f(x) = \frac{3}{5}|x| \)
25. Let \( f(x) = x^2 \). Let \( g(x) \) be the function obtained by shifting the graph of \( f(x) \) two units to the right and then up three units. Find a formula for \( g(x) \) and then draw its graph.

Suppose \( H(t) \) gives the height of high tide in Hawaii (H) on a Tuesday, \( t \) of the year. Use shifts of the function \( H(t) \) to find formulas of each of the following functions:

26. \( F(t) \), the height of high tide on Fiji on Tuesday \( t \), given that high tide in Fiji is always one foot higher than high tide in Hawaii.
27. \( S(d) \), the height of high tide in Saint Thomas on Tuesday \( t \), given that high tide in Saint Thomas is the same height as the previous day’s height in Hawaii.

**Review (Answers)**

To see the Review answers, open this [PDF file](#) and look for section 1.12.