Chapter 11 – Sequences and Series

11.1 Finding the Next Term in a Sequence

Answers

1. 39, 45, 51
2. -324, 972, -2916
3. 35, 31, 27
4. 0.01, 0.001, 0.0001
5. 16, 32, 64
6. \(-1, -3, -5\)
\(\frac{6}{7}, \frac{7}{8}\)
7. 9; 36
8. \(\frac{4}{5}, \frac{6}{7}\)
9. 5; 20
10. 8; 216
11. 6; 416
12. 15; 35
13. 3; 8
14. Problems 1 and 3.
15. Problems 2, 4 and 5.
11.2 Describing a Pattern and Writing a Recursive Rule for a Sequence

Answers

1. Each term is multiplied by -2 to get the next term: $a_n = -2a_{n-1}$
2. Each term is increased by 6 to get the next term: $a_n = a_{n-1} + 6$
3. Subtract 5 from each term to find the next term: $a_n = a_{n-1} - 5$
4. Multiply each term by 4 to find the next term: $a_n = 4a_{n-1}$
5. Each term is 9 more than the previous term: $a_n = a_{n-1} + 9$
6. Each term is 25 less than the previous term: $a_n = a_{n-1} - 25$
7. Each term is two thirds the previous term: $a_n = \frac{2}{3}a_{n-1}$
8. Each term is multiplied by $\frac{3}{4}$ to get the next term: $a_n = \frac{3}{4}a_{n-1}$
9. First add 4, then 5, then 6, etc (add one more each time): $a_n = a_{n-1} + (n + 2)$
10. Each term is the sum of the two previous terms: $a_n = a_{n-1} + a_{n-2}$
11. Add the term number to the previous term to get each term: $a_n = a_{n-1} + n$
12. Add twice the previous term number to the previous term to get the term: $a_n = a_{n-1} + 2(n - 1)$
13. Add 5 to get the next term: $a_n = a_{n-1} + 5$
14. The numerator increases by 1, as does the denominator, but the denominator is one more than the numerator: $a_n = \frac{a_n}{a_{n+1}}$
15. Add the previous two terms to get the next term: \( a_n = a_{n-1} + a_{n-2} \)
11.3 Using and Writing Nth Term Rules for Sequences

Answers

1. 9, 11, 13, 15, 17; 27
2. -6, -11, -16; -251
3. 1, 3, 7; 1023
4. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}; \frac{1}{256}$
5. 1, 3, 6, 10; 210
6. 1, 5, 9, 13, 17
7. $\frac{9}{2}, \frac{4}{3}, \frac{7}{3}, \frac{5}{2}$
8. $\frac{5}{3}, \frac{13}{9}, \frac{35}{27}, \frac{97}{81}, \frac{275}{243}$
9. 0, 4, 12, 24, 40
10. 1, 5, 14, 30, 55
11. $2n + 1$
12. $3^n - 2$
13. $n(n + 5)$
14. $-n + 7$
15. $\frac{n(n + 3)}{2}$
11.4 Series and Summation Notation

Answers

1. \(2 + 4 + 6 + 8 + 10 = 30\)
2. \(8 + 9 + 10 + 11 = 38\)
3. \(70 + 88 + 108 + 130 + 154 + 180 = 730\)
4. \(3 + 6 + 10 + 15 + 21 = 55\)
5. \(4 + 5 + 7 + 11 + 19 + 35 = 81\)
6. \(55.5\)
7. \(0\)
8. \(31\)
9. \(654\)
10. \(2525\)
11. \(20,100\)
12. a. \(5 + 7 + 9 + 11 + 13 = 45\)
   b. \(3(5) + (2 + 4 + 6 + 8 + 10) = 45\)
13. a. \(1 + 3 + 6 + 10 + 15 = 35\)
   b. \(\frac{1}{2} (2 + 6 + 12 + 20 + 30) = 35\)
14. a. \(4 + 32 + 108 + 256 + 500 = 900\)
   b. \(4(1) + 4(8) + 4(27) + 4(64) + 4(125) = 900\)
15. Answers will vary.
16. The sum of the first 200 numbers.
11.5 Arithmetic Sequences and Finding the Nth Term Given the Common Difference and a Term

Answers

1. arithmetic; \( a_n = n + 1 \)
2. not arithmetic
3. arithmetic; \( a_n = -5n + 10 \)
4. not arithmetic
5. arithmetic; \( a_n = 3n - 3 \)
6. arithmetic; \( -n + 14 \)
7. \( a_n = -8n + 25 \)
8. \( a_n = a + 2(n - 1) \)
9. \( a_n = -8n + 22 \)
10. \( a_n = \frac{1}{2}n - \frac{21}{2} \)
11. \( a_n = -2n + 30 \)
12. \( a_n = 3n - 18 \)
13. \( a_n = -11n + 95 \)
14. \( a_n = 7n - 17 \)
15. \( a_n = 3n + 8 \)
11.6 Finding the Nth Term Given Two Terms

Answers

1. \( a_n = -3n + 4 \)

2. \( a_n = 2n + 1 \)

3. \( a_n = \frac{1}{3}n - 7 \)

4. \( a_n = \frac{5}{2}n + 4 \)

5. \( a_n = -5n + 3 \)

6. \( a_n = 6n + 13 \)

7. \( a_n = -2n + 6 \)

8. \( a_n = 7n + 2 \)

9. \( a_n = -n - 1 \)

10. \( a_n = 8n - 15 \)

11. \( a_n = -\frac{4}{5}n + 6 \)

12. \( a_n = \frac{3}{4}n + 2 \)

13. \( a_n = 4n + 34 \)

14. \( a_n = -3n - \frac{9}{2} \)

15. Answers will vary.
11.7 Finding the Sum of an Arithmetic Sequence

Answers

1. 1356
2. -84
3. 2499
4. 13
5. 875
6. 861
7. 240
8. 9900
9. 91
10. -1860
11. 361
12. 180
13. 1207
14. -483
15. 63
16. 630
17. 1378
Answers

1. arithmetic

2. geometric

3. neither

4. geometric

5. arithmetic

6. neither

7. \(a_n = 32 \left( \frac{3}{2} \right)^{n-1} \); \{32, 48, 72, 108, 162\}

8. \(a_n = -81 \left( -\frac{1}{3} \right)^{n-1} \); \{-81, 27, -9, 3, -1\}

9. \(a_n = 7 \left( 2 \right)^{n-1} \); \{7, 14, 28, 56, 112\}

10. \(a_n = \frac{8}{125} \left( -\frac{5}{2} \right)^{n-1} \); \left\{ \frac{8}{125}, \frac{-4}{25}, \frac{-2}{5}, -1, \frac{5}{2} \right\} \)
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Answer Key

11. \[ a_n = 162 \left( \frac{2}{3} \right)^{n-1} \]

12. \[ a_n = -625 \left( \frac{3}{5} \right)^{n-1} \]

13. \[ a_n = \frac{9}{4} \left( -\frac{2}{3} \right)^{n-1} \]

14. \[ a_n = 3(5)^{n-1} \]

15. \[ a_n = 5(2)^{n-1} \]

16. \[ a_n = \frac{1}{2} (-4)^{n-1} \]

17. $81,445

18. $29,647
11.9 Finding the Nth Term Given the Common Ratio and any Term or Two Terms

Answers

1. 
   \[ a_n = 54 \left( \frac{2}{3} \right)^{n-1} \]

2. 
   \[ a_n = 160 \left( -\frac{3}{4} \right)^{n-1} \]

3. 
   \[ a_n = \frac{125}{72} \left( \frac{6}{5} \right)^{n-1} \]

4. 
   \[ a_n = 320 \left( -\frac{1}{2} \right)^{n-1} \]

5. 
   \[ a_n = \left( \frac{6}{7} \right)^n \]

6. 
   \[ a_n = \frac{11}{8} (2)^{n-1} \]

7. 
   \[ a_n = 24 \left( \frac{3}{2} \right)^{n-1} \]

8. 
   \[ a_n = 48 \left( \frac{1}{4} \right)^{n-1} \]

9. 
   \[ a_n = \frac{343}{216} \left( \frac{6}{7} \right)^{n-1} \]
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Answer Key

10. \( a_n = 2(3)^{n-1} \)

11. \( a_n = 128\left(\frac{3}{2}\right)^{n-1} \)

12. \( a_n = \frac{8}{27}\left(\frac{3}{2}\right)^{n-1} \)

13. \( a_n = \frac{1}{12}(-2)^{n-1} \)

14. $34,000

15. $93,000
11.10 Finding the Sum of a Finite Geometric Series

Answers

1. \( \frac{844}{9} \)
2. 99
3. 5
4. \( \frac{1031}{50} \)
5. \( \frac{15}{8} \)
6. \( \frac{5461}{128} \)
7. \( \frac{11529}{2000} \)
8. \( \frac{43}{2} \)
9. \( \frac{63}{8} \)
10. \( a_n = 3(-2)^{n-1} \)
11. \( a_n = 216 \left( \frac{5}{6} \right)^{n-1} \)
12. \( a_n = 2(-3)^{n-1} \)
13. \( a_n = 96 \left( -\frac{1}{2} \right)^{n-1} \)
14. $9799
15. $1500
11.11 Partial Sums

Answers

1. \( S_1 = 5 \; ; \; S_2 = 7.5 \; ; \; S_3 = 8.75 \; ; \; S_4 = 0.625 + 8.75 = 9.375 \; ; \; S_5 = 9.6875 \; ; \)
\( S_{50} = 10 \; ; \)

This infinite series converges to a sum of 10.

2. \( S_1 = 2 \; ; \; S_2 = 3.5 \; ; \; S_3 = 4.625 \; ; \; S_4 = 5.46875 \; ; \; S_5 = 6.1016 \; ; \; S_{50} = 7.999 \; ; \)
\( S_{100} = 8 \; ; \)

This infinite series converges to a sum of 8.

3. \( S_1 = 10 \; ; \; S_2 = 19 \; ; \; S_3 = 27.1 \; ; \; S_4 = 34.39 \; ; \; S_5 = 40.95 \; ; \; S_{50} = 99.485 \; ; \)
\( S_{100} = 99.997 \; ; \; S_{500} = 100 \)

This infinite series converges to a sum of 100.

4. \( S_1 = 8 \; ; \; S_2 = 16.24 \; ; \; S_3 = 24.727 \; ; \; S_4 = 33.469 \; ; \; S_5 = 42.473 \; ; \; S_{50} = 902.375 \; ; \)
\( S_{100} = 4858.302 \; ; \)

This infinite series will continue to grow without bound.

5. \( S_1 = 0.5 \; ; \; S_2 = 1.5 \; ; \; S_3 = 3 \; ; \; S_4 = 5 \; ; \; S_5 = 7.5 \; ; \; S_{50} = 637.5 \; ; \; S_{100} = 2525 \; ; \)

This infinite series will continue to grow without bound.

6. \( S_1 = 10 \; ; \; S_2 = 15 \; ; \; S_3 = 18.333 \; ; \; S_4 = 20.833 \; ; \; S_5 = 22.833 \; ; \; S_{50} = 44.992 \; ; \)
\( S_{100} = 51.874 \; ; \)

This infinite series will continue to grow without bound.

7. \( S_1 = 0.5 \; ; \; S_2 = 0.875 \; ; \; S_3 = 1.156 \; ; \; S_4 = 1.367 \; ; \; S_5 = 1.525 \; ; \; S_{50} = 2 \; ; \; S_{100} = 2 \; ; \)
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Answer Key

This infinite series converges to a sum of 2.

8. \( S_1 = 1; \quad S_2 = 1.25; \quad S_3 = 1.361; \quad S_4 = 1.424; \quad S_5 = 1.464; \quad S_{50} = 1.625; \quad S_{100} = 1.635; \quad S_{500} = 1.643; \quad S_{750} = 1.644; \) This infinite series appears to approach a sum of approximately 1.644.

9. \( S_1 = 6; \quad S_2 = 6.6; \quad S_3 = 6.66; \quad S_4 = 6.666; \quad S_5 = 6.667; \quad S_{50} = 6.667; \) This infinite series converges to a sum of approximately 6.667.

10. \( S_1 = 5.01; \quad S_2 = 10.03; \quad S_3 = 15.06; \quad S_4 = 20.1; \quad S_5 = 25.15; \quad S_{50} = 262.75; \) This infinite series will continue to grow without bound.

11. \( S_1 = 2; \quad S_2 = 3.75; \quad S_3 = 5.281; \quad S_4 = 6.621; \quad S_5 = 7.793; \quad S_{50} = 15.980; \quad S_{100} = 16; \) This infinite series converges to a sum of 16.

12. The series in problems 5 and 10 are both arithmetic. No.

13. The series in problems 1, 2, 3, 4, 7, 9, and 11 are all geometric. All but #4 has a finite sum.


15. Answers will vary.
11.12 Finding the Sum of an Infinite Geometric Series

Answers

1. 15

2. no sum, \( \left| \frac{-4}{3} \right| \geq 1 \)

3. \( \frac{3}{2} \)

4. no sum, \( |1.1| \geq 1 \)

5. 10

6. \( \frac{7}{8} \)

7. 2

8. no sum, \( |1.05| \geq 1 \)

9. 4

10. 180

11. no sum, \( |\frac{3}{2}| \geq 1 \)

12. 125

13. no sum, \( |1.25| \geq 1 \)

14. \( \frac{50}{3} \)

15. \( \frac{-48}{7} \)