Here you’ll learn about inscribed quadrilaterals and how to use the Inscribed Quadrilateral Theorem to solve problems about circles.

What if you were given a circle with a quadrilateral inscribed in it? How could you use information about the arcs formed by the quadrilateral and/or the quadrilateral’s angle measures to find the measure of the unknown quadrilateral angles? After completing this Concept, you’ll be able to apply the Inscribed Quadrilateral Theorem to solve problems like this one.

**Guidance**

An **inscribed polygon** is a polygon where every vertex is on the circle, as shown below.

For inscribed quadrilaterals in particular, the opposite angles will always be supplementary.

**Inscribed Quadrilateral Theorem:** A quadrilateral can be inscribed in a circle if and only if the opposite angles are supplementary.

If $ABCD$ is inscribed in $\bigcirc E$, then $m\angle A + m\angle C = 180^\circ$ and $m\angle B + m\angle D = 180^\circ$. Conversely, if $m\angle A + m\angle C = 180^\circ$ and $m\angle B + m\angle D = 180^\circ$, then $ABCD$ is inscribed in $\bigcirc E$.

**Example A**

Find the values of the missing variables.

a)
b)

**Answers:**

a)

\[ x + 80^\circ = 180^\circ \]
\[ x = 100^\circ \]
\[ y + 71^\circ = 180^\circ \]
\[ y = 109^\circ \]

b)

\[ z + 93^\circ = 180^\circ \]
\[ z = 87^\circ \]
\[ x = \frac{1}{2}(58^\circ + 106^\circ) \]
\[ x = 82^\circ \]
\[ y + 82^\circ = 180^\circ \]
\[ y = 98^\circ \]

**Example B**

Find \( x \) and \( y \) in the picture below.

\[ (7x + 1)^\circ + 105^\circ = 180^\circ \]
\[ 7x + 106^\circ = 180^\circ \]
\[ 7x = 74 \]
\[ x = 10.57 \]

\[ (4y + 14)^\circ + (7y + 1)^\circ = 180^\circ \]
\[ 11y + 15^\circ = 180^\circ \]
\[ 11y = 165 \]
\[ y = 15 \]
Example C

Find the values of $x$ and $y$ in $\bigcirc A$.

Use the Inscribed Quadrilateral Theorem. $x^\circ + 108^\circ = 180^\circ$ so $x = 72^\circ$. Similarly, $y^\circ + 88^\circ = 180^\circ$ so $y = 92^\circ$.

Vocabulary

A circle is the set of all points that are the same distance away from a specific point, called the center. A radius is the distance from the center to the circle. A chord is a line segment whose endpoints are on a circle. A diameter is a chord that passes through the center of the circle. The length of a diameter is two times the length of a radius. A central angle is an angle formed by two radii and whose vertex is at the center of the circle. An inscribed angle is an angle with its vertex on the circle and whose sides are chords. The intercepted arc is the arc that is inside the inscribed angle and whose endpoints are on the angle.

Guided Practice

Quadrilateral $ABCD$ is inscribed in $\bigcirc E$. Find:

1. $m\angle A$
2. $m\angle B$
3. $m\angle C$
4. $m\angle D$

Answers:

First, note that $m\hat{A}D = 105^\circ$ because the complete circle must add up to $360^\circ$.

1. $m\angle A = \frac{1}{2}m\hat{BD} = \frac{1}{2}(115 + 86) = 100.5^\circ$
2. $m\angle B = \frac{1}{2}m\hat{AC} = \frac{1}{2}(86 + 105) = 95.5^\circ$
3. $m\angle C = 180^\circ - m\angle A = 180^\circ - 100.5^\circ = 79.5^\circ$
4. $m\angle D = 180^\circ - m\angle B = 180^\circ - 95.5^\circ = 84.5^\circ$
Practice

Fill in the blanks.

1. A(n) _______________ polygon has all its vertices on a circle.
2. The _______________ angles of an inscribed quadrilateral are _______________.

Quadrilateral $ABCD$ is inscribed in $\bigcirc E$. Find:

3. $m\angle DBC$
4. $m\overline{BC}$
5. $m\overline{AB}$
6. $m\angle ACD$
7. $m\angle ADC$
8. $m\angle ACB$

Find the value of $x$ and/or $y$ in $\bigcirc A$. 

9. $x^\circ$
10. $y^\circ$
11. $x^\circ$

Solve for $x$.

12. \[(4x+9)^\circ\]

13. \[\begin{align*}
(3x+3)^\circ \\
(3x-32)^\circ \\
(3x+2)^\circ
\end{align*}\]