# Introduction to Probability

## The Fundamental Counting Principle

### Review Queue Answers

1. $a_n = 5n + 16$
2. $a_n = -16\left(-\frac{1}{2}\right)^{n-1}$
3. 139
4. 10

### Applying the Fundamental Counting Principle

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | 144 | 2. | 24 | 3. | 120 | 4. | 12 | 5. | 24 |
| 6. | yes, 42 | 7. | 72 | 8. | 288 | 9. | 48 | 10. | 54 |

### Using the Fundamental Counting Principle with and without Repetition

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | 136,080 | 2. | 9000 | 3. | 45,697,600 | 4. | 750,000 | 5. | 456,976 |
| 6. | 1,413,720 | 7. | 362,880 | 8. | 1728; 1320 | 9. | 676,000,000 | 10. | 36,000 |

## Permutations and Combinations

### Review Queue Answers

1. 72
2. 900,000
3. 5040

### Define and Apply Permutations and Factorials

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | 10 | 2. | 360 | 3. | $\frac{8}{3}$ | 4. | $\frac{10}{11}$ | 5. | 362,880 |
| 6. | 720 | 7. | 479,001,600 | 8. | 40,320 | 9. | 10,080 | 10. | 720 |
| 11. | 1,728 | 12. | 138,240 | | | | | | |

### Permutations of Subsets and Permutations with Repetition

1. 6,720
2. 6,652,800
3. ${}_5P_5 = \frac{5!}{(5-5)!} = \frac{5!}{0!} = 5! = 120$; ${}_5P_0 = \frac{5!}{(5-0)!} = 1$ ${}_5P_1 = \frac{5!}{(5-1)!} = \frac{5!}{4!} = 5$
4. 20
5. 55,440
6. 259,459,200
7. 120
8. 720
9. 34,650
10. 12,870
11. 2,520

## Define and Apply Combinations

1. 286 2. 210 3. 43,758
4. Algebraically, they are the same: ${}_9C_4 = \frac{9!}{4!5!}$ and ${}_9C_5 = \frac{9!}{5!4!}$.
5. a and c are permutations; b, d and e are combinations
6. 35 7. 120; 604,800 8. 252
9. 220 10. 27,405; 657,720
11. a. 167,960 b. 46,200 c. 7,759,752,000

# Binomial Theorem

### Review Queue Answers

1. 5040 2. 45,360 3. 495

## Pascal's Triangle and the Coefficients in the Expansion of Binomials

1. 1 6 15 20 15 6 1
2. 1 12 78 286 715 1287 1716 1716 1287 715 286 78 12 1
3. $x^4 - 24x^3 + 216x^2 - 864x + 1296$
4. $64x^6 + 960x^5 + 6000x^4 + 20000x^3 + 37500x^2 + 37500x + 15625$
5. $2187 - 5103x + 5103x^2 - 2835x^3 + 945x^4 - 189x^5 + 21x^6 - 1$
6. $x^6 - 6x^4 + 12x^2 - 8$
7. $x^5 + 20x^4 + 160x^3 + 640x^2 + 1280x + 1024$
8. $16 - 32x^3 + 24x^6 - 8x^9 + x^{12}$
9. $a^6 - 6a^5b + 15a^4b^2 - 2a^3b^3 + 15a^2b^4 - 6ab^5 + b^6$
10. $x^{10} + 10x^9 + 45x^8 + 120x^7 + 210x^6 + 252x^5 + 210x^4 + 120x^3 + 45x^2 + 10x + 1$

## Using the Binomial Theorem

1. $x^7 - 7x^6a + 21x^5a^2 - 35x^4a^3 + 35x^3a^4 - 21x^2a^5 + 7xa^6 - a^7$
2. $16a^4 + 96a^3 + 216a^2 + 216a + 81$
3. $-3920x^2$ 4. $40824x^5$ 5. $10500a^6$ 6. $4320x^3y^3$ 7. $20412x^5$
8. $700y^6$ 9. $-960a^3b^7$ 10. $3240x^4$ 11. 405 12. 700

# Introduction to Probability

### Review Queue Answers

1. $-40x^2y^3$ 2. $m^4 - 4m^3n + 6m^2n^2 - 4mn^3 + n^4$ 3. $336x^4$

## Finding the Probability of an Event

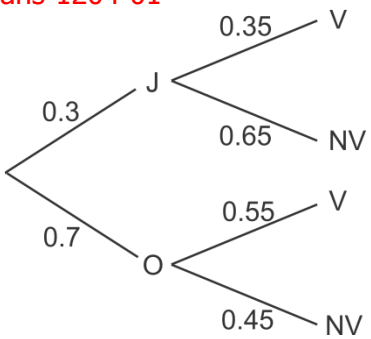
1. $\frac{1}{13}$ 2. $\frac{1}{2}$ 3. $\frac{3}{13}$ 4. $\frac{1}{26}$ 5. $\frac{5}{13}$
6. $\frac{5}{18}$ 7. $\frac{1}{6}$ 8. $\frac{1}{4}$ 9. $\frac{5}{12}$ 10. $\frac{1}{2}$
11. $\frac{5}{23}$ 12. $\frac{10}{23}$ 13. $\frac{1}{3}$

## Calculating Probabilities of Combined Events

1. $\frac{1}{24} \approx 0.0417$ 2. $\frac{1}{4} = 0.25$ 3. $\frac{11}{4165} \approx 0.0026$
4. $\frac{11}{1105} \approx 0.0100$ 5. $\frac{5}{144} \approx 0.0347$ 6. $\frac{1}{64} \approx 0.0156$
7. $\frac{1}{4} = 0.25$ 8. 0.3251 9. 0.0020
10. 0.2743 11. 0.5865 12. 0.1229

## Tree Diagrams and Probability Distributions

1. ans-1204-01




2. 0.315 3. 0.490 4. 0.344 5. 0.0011 6. 0.0673
7. 0.0155 8. 0.0110 9. 0.9173
10.

| Number of high cards | Probability |
|---|---|
| 0 | 0.1451 |
| 1 | 0.3626 |
| 2 | 0.3297 |
| 3 | 0.1357 |
| 4 | 0.0252 |
| 5 | 0.0017 |

11. 0.8549 12. 0.4923

## Basic Geometric Probability

1. $\frac{1}{4}$ 2. $\frac{3}{25}$ 3. $\frac{13}{100}$ 4. $\frac{3}{4}$ 5. $\frac{63}{100}$
6. $\frac{160}{361} \approx 0.443$ 7. $\frac{1}{361} \approx 0.003$ 8. 0.240 9. 0.0274 10. 1080

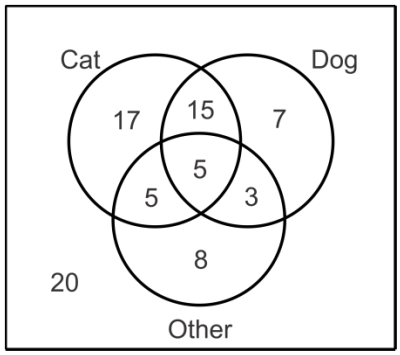
# Venn Diagrams and Independence

### Review Queue Answers

1. $\frac{25}{1296} \approx 0.0193$ 2. $\frac{20}{27} \approx 0.741$ 3. $\frac{1}{4} = 0.25$; $\frac{4}{17} \approx 0.235$

## Union and Intersection of Sets

1. ans-1205-01




2. 15 3. 7 4. 20
5. 8 6. $x$ 7. $y+x$
8. $x+y+z$ 9. $y$ 10. $w+y+z$
11. $w$ 12. $w+z$ 13. $w+y+x$

## Probability Using a Venn Diagram and Conditional Probability

1. 0.4 2. 0.25 3. 0.8
4. 0 5. 1 6. 0.75
7. 0.3 8. 0.6 9. 0.5
10. 0.6
11. a. $\frac{16}{105} \approx 0.152$ b. $\frac{41}{89} \approx 0.461$ c. $\frac{21}{26} \approx 0.808$ d. $\frac{33}{105} \approx 0.314$

## Independent, Conditional and Mutually Exclusive Events

1. $P(A \mid B)$ refers to the probability of A occurring given that B has occurred. For mutually exclusive events, this isn't possible since only one can occur. Thus, $P(A \mid B) = 0$.
2. $P(B \mid A) = \frac{P(B \cap A)}{P(A)} = \frac{P(B)P(A)}{P(A)} = P(B)$.
3. Independent

4. Not Independent
5. Independent
6. Not Independent
7. $P(B) = 0.4$; P(A or B but not both) = 0.56
8. $P(B) = 0.4$; $P(A \cup B) = 0.94$
9. $P(A) = 0.8$; $P(A' \cup B') = 0.36$
10. $P(A) = 0.4$; $P(B) = 0.7$