Rational Functions

Variation

Review Queue

1. $y = \frac{1}{2}x$
2. $y = -3x$
3. $y = -\frac{2}{3}x + 3$

Direct Variation

1. $y = 5x; y = 60$
2. $y = -\frac{1}{3}x; y = -4$
3. $y = \frac{2}{3}x; y = 4$
4. $y = \frac{4}{5}x; x = \frac{5}{2}$
5. $y = \frac{1}{6}x; x = 12$
6. $y = -4x; x = -\frac{1}{2}$
7. no
8. yes
9. 351.6 calories; 30 miles
10. 844 miles
11. \$5.00; 32 more cars
12. \$25 per hour, \$1000

Inverse Variation

1. $y = \frac{12}{x}; y = \frac{4}{5}$
2. $y = \frac{2}{x}; y = \frac{2}{15}$
3. $y = \frac{6}{x}; y = \frac{2}{5}$
4. $y = \frac{4}{x}; x = 2$
5. $y = \frac{6}{x}; x = 3$
6. $y = \frac{36}{5x}; x = 36$
7. yes
8. no
9. 2 pieces; 3 pieces; $y = \frac{24}{x}$
10. 78 mph
11. 12 hours; 9 friends total
12. 1.3 degrees Celsius

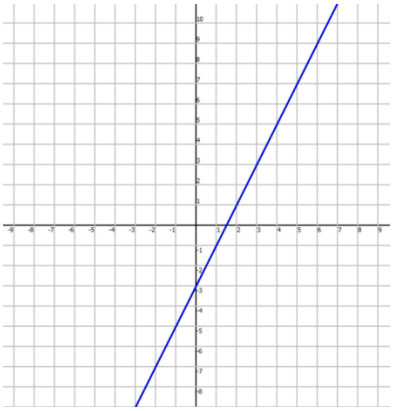
Joint Variation

1. $w = \frac{k}{xy}$
2. $r = \frac{k}{q^2}$
3. $z = \frac{kxy}{w}$
4. $a = \frac{kb}{c\sqrt{d}}$
5. $z = \frac{2}{3}xy; z = -8$
6. $z = -2xy; z = 7$
7. $z = -\frac{2}{3}xy; y = 4$
8. $z = \frac{1}{4}xy; x = -3$
9. 180 cars
10. $k = \frac{1}{5}$; 1 hour 15 min
11. $W = 9.8mh$; 2940 joules
12. $I = \frac{4}{d^2}$; 0.4 watts per m^2

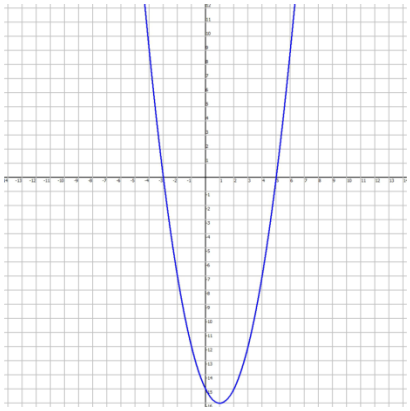
Graphing Rational Functions

Review Queue

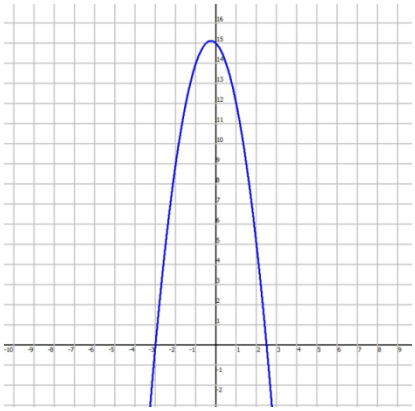
1. ans-0902-01



2. ans-0902-02

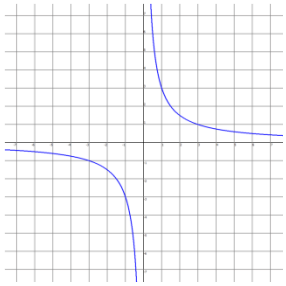


3. ans-0902-03



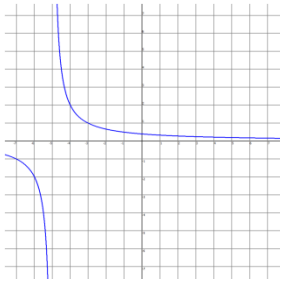
Graphing $f(x)=\frac{1}{x-h}+k$

1. VA: $x=-8$; HA: $y=-3$; yes
2. VA: $x=4$; HA: $y=6$
3. ans-0902-04



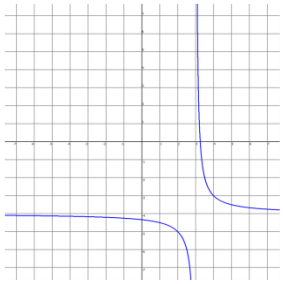
VA: $x=0$; HA: $y=0$;
Domain: all real numbers, $x\neq 0$;
Range: all real numbers, $y\neq 0$;
no intercepts

4. ans-0902-05



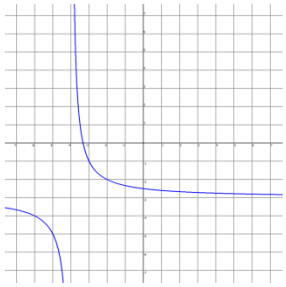
VA: $x=-5$; HA: $y=0$;
Domain: all real numbers, $x\neq -5$;
Range: all real numbers, $y\neq 0$;
$\left(0,\frac{2}{5}\right)$

5. ans-0902-06



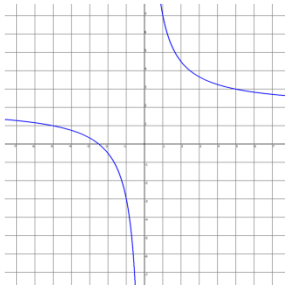
VA: $x=3$; HA: $y=-4$;
Domain: all real numbers, $x\neq 3$;
Range: all real numbers, $y\neq -4$;
$\left(0,-\frac{13}{3}\right),\left(\frac{13}{4},0\right)$

6. ans-0902-07



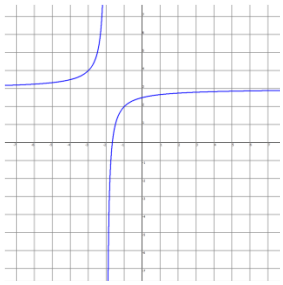
VA: $x=-4$; HA: $y=-3$;
Domain: all real numbers, $x\neq 4$;
Range: all real numbers, $y\neq -3$;
$\left(0,-\frac{5}{2}\right),\left(\frac{-10}{3},0\right)$

7. ans-0902-08



VA: $x=0$; HA: $y=2$;
Domain: all real numbers, $x\neq 0$;
Range: all real numbers, $y\neq 2$;
$\left(-\frac{5}{2},0\right)$

8. ans-0902-09



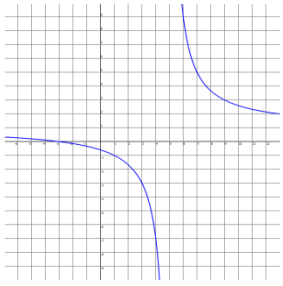
VA: $x=-2$; HA: $y=3$;
Domain: all real numbers, $x\neq -2$;
Range: all real numbers, $y\neq 3$;
$\left(0,\frac{5}{2}\right),\left(\frac{-5}{3},0\right)$

9. $y=\frac{1}{x-6}-4$

10. $y=3-\frac{1}{x+1}$

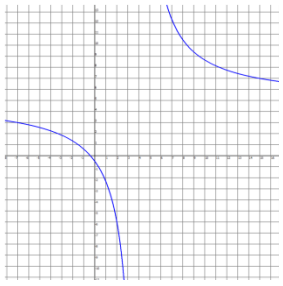
Graphing $f(x)=\frac{p(x)}{q(x)}$ When the Degrees are the Same

1. ans-0902-10



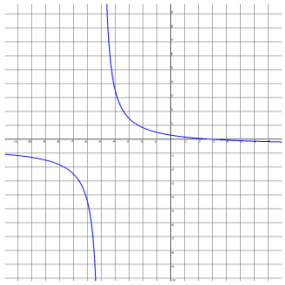
VA: $x=5$; HA: $y=1$;
Domain: all real numbers, $x\neq 5$;
Range: all real numbers, $y\neq 1$;
$\left(0,-\frac{3}{5}\right),(-3,0)$

2. ans-0902-11



VA: $x=4$; HA: $y=5$;
Domain: all real numbers, $x\neq 4$;
Range: all real numbers, $y\neq 5$;
$\left(0,-\frac{1}{2}\right),\left(-\frac{2}{5},0\right)$

3. ans-0902-12



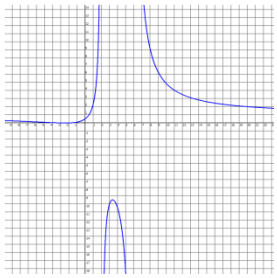
VA: $x=-5$; HA: $y=-\frac{1}{2}$;

Domain: all real numbers, $x\neq -5$;

Range: all real numbers, $y\neq -\frac{1}{2}$;

$\left(0,\frac{3}{10}\right),(3,0)$

4. ans-0902-13



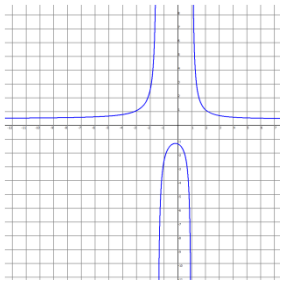
VA: $x=6, x=2$; HA: $y=1$;

Domain: all real numbers, $x\neq 6,2$;

Range: $(-\infty,-9.493)\cup(-0.007,\infty)$;

$\left(0,\frac{1}{2}\right),(-3,0),(-2,0)$

5. ans-0902-14



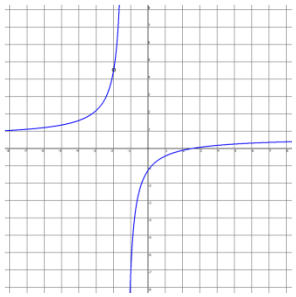
VA: $x=-\frac{3}{2}, x=1$; HA: $y=\frac{1}{2}$;

Domain: all real numbers, $x\neq -\frac{3}{2},1$;

Range: all real numbers, $y\neq \frac{1}{2}$;

$\left(0,-\frac{4}{3}\right)$, no x-intercepts

6. ans-0902-15



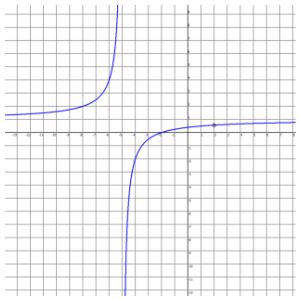
VA: $x=-\frac{4}{3}$; HA: $y=\frac{2}{3}$;

Domain: all real numbers, $x\neq -\frac{4}{3},-2$;

Range: all real numbers, $y\neq \frac{2}{3}$;

$\left(0,-\frac{5}{4}\right),\left(\frac{5}{2},0\right)$; hole at $\left(-2,\frac{9}{2}\right)$

7. ans-0902-16



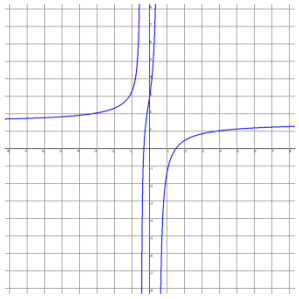
VA: $x=-5$; HA: $y=1$;

Domain: all real numbers, $x\neq -5,2$;

Range: all real numbers, $y\neq 1$;

$\left(0,\frac{2}{5}\right),(-2,0)$; hole at $\left(2,\frac{4}{7}\right)$

8. ans-0902-17



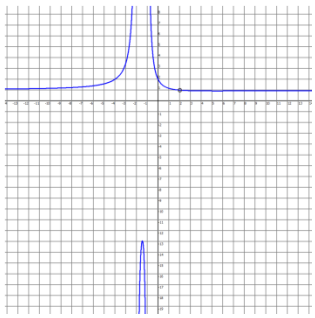
VA: $x=-\frac{1}{2}, x=\frac{1}{2}$; HA: $y=\frac{3}{2}$;

Domain: all real numbers, $x\neq -\frac{1}{2},\frac{1}{2}$;

Range: all real numbers;

$(0,3),\left(-\frac{1}{3},0\right),\left(\frac{3}{2},0\right)$

9. ans-0902-18



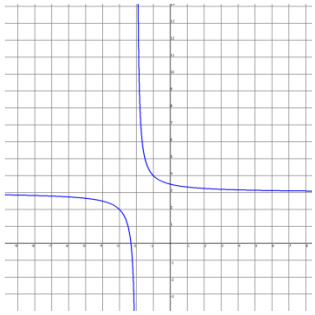
VA: $x=-2, x=-1$; HA: $y=1$;

Domain: all real numbers, $x\neq -2,-1,2$;

Range: $(-\infty,-12.928)\cup(0.928,\infty)$;

$(0,2)$, no x-intercepts

10. ans-0902-19

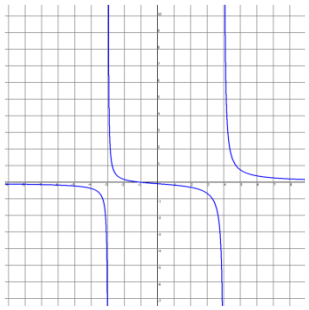


These are actually the same because

$$\frac{1}{x-2}+3=\frac{3x-5}{x-2}$$

Graphing $f(x)=\frac{p(x)}{q(x)}$ when the Degrees are Different

1. ans-0902-20

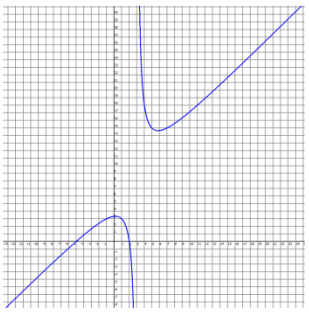


VA: $x=-3, x=4$

HA: $y=0$

Intercepts: $\left(0,-\frac{1}{12}\right)$, $(-1,0)$

2. ans-0902-21

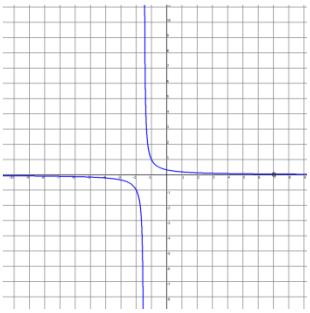


VA: $x=3$

SA: $y=x+6$

Intercepts: $\left(0,\frac{10}{3}\right)$, $(2,0)$, $(-5,0)$

3. ans-0902-22



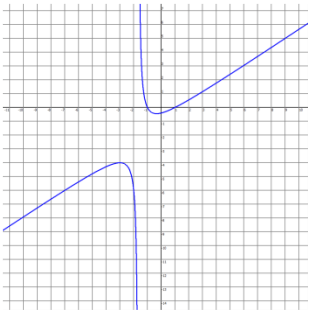
VA: $x=-\frac{3}{2}$

HA: $y=0$

Intercepts: $\left(0,\frac{1}{3}\right)$, $(7,0)$

Hole: $\left(7,\frac{1}{17}\right)$

4. ans-0902-23

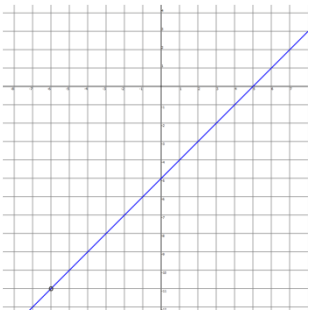


VA: $x=-\frac{5}{3}$

SA: $y=\frac{2}{3}x+\frac{10}{9}$

Intercepts: $\left(0,-\frac{2}{5}\right)$, $(1,0)$, $(-1,0)$

5. ans-0902-24

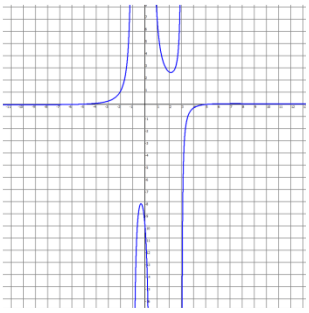


No asymptotes

Intercepts: $(0,-5)$, $(5,0)$

Hole: $(-6,-11)$

6. ans-0902-25

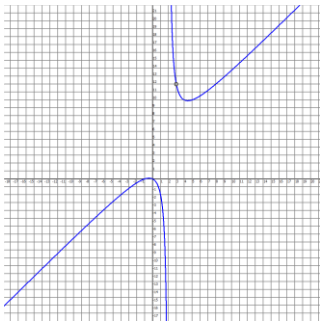


VA: $x=\frac{1}{2}, x=-1, x=3$

HA: $y=0$

Intercepts: $(0,-10)$, $(5,0)$, $(-6,0)$

7. ans-0902-26



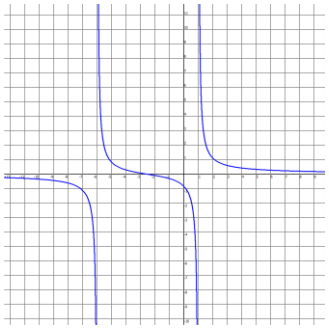
VA: $x=-\frac{3}{2}$

SA: $y=x+3$

Intercepts: $(0,0)$, $(-1,0)$, $(3,0)$

Hole: $(3,12)$

8. ans-0902-27

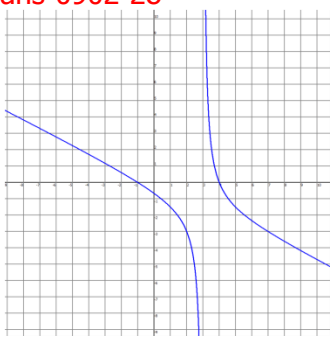


VA: $x=-6, x=1$

HA: $y=0$

Intercepts: $\left(0,-\frac{5}{6}\right)$, $\left(-\frac{5}{2},0\right)$

9. ans-0902-28

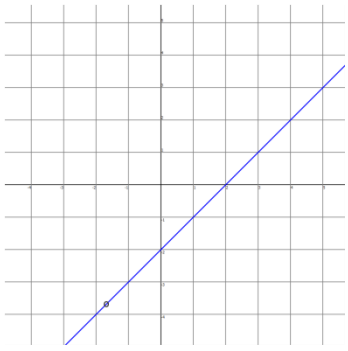


VA: $x=3$

SA: $y=-\frac{1}{2}x$

Intercepts: $\left(0,-\frac{2}{3}\right)$, $(-1,0)$, $(4,0)$

10. ans-0902-29



If we divide, $\frac{3x^2-x-10}{3x+5}=x-2$ this can also be found by factoring the numerator and denominator and cancelling the like factor of $3x+5$. We have seen before that when factors cancel, a hole is formed, not an asymptote.

Simplifying, Multiplying, and Dividing Rational Expressions

Review Queue

1. $\frac{2}{5}$ 2. $\frac{2x^2}{3y^3}$ 3. $\frac{a^4}{5b^3c^7}$ 4. $\frac{4}{9}$ 5. $\frac{8}{3}$ 6. 8

Simplifying Rational Expressions

1. $\frac{4x^2}{2x+3}$
2. $\frac{x+2}{x(x+5)}$
3. $\frac{x-3}{x-4}$
4. $\frac{x+7}{x(x-7)}$
5. $\frac{-2(2x+1)}{2x+3}$
6. $\frac{2(x-4)}{3(x-1)(x+1)}$
7. $\frac{3x+4}{3x+1}$
8. $\frac{x+2}{x}$
9. $\frac{x(2x+3)}{4}$
10. $x-2$
11. $\frac{1-3x}{x^2+1}$
12. $\frac{x+3}{2x+5}$

Multiplying Rational Expressions

1. $\frac{12y^5}{x^3}$
2. x^5y^8
3. $6x^5$
4. $\frac{3(y+2)}{2}$
5. $\frac{2(2x-1)}{x+1}$
6. $\frac{x+3}{x-1}$
7. -3
8. $\frac{2(x+5)}{4x-3}$
9. $\frac{-(x+5)}{x+1}$
10. $\frac{(x-3)(2x+3)}{x+8}$
11. $\frac{2x-3}{x-3}$

Dividing Rational Expressions

1. $\frac{b}{a}$
2. $\frac{2x^3}{y^6}$
3. $\frac{yz^5}{7}$
4. $\frac{4}{x}$
5. $\frac{x+3}{4}$
6. $\frac{-x(x+2)}{3(2x-5)}$
7. $\frac{x(x+2)}{x+3}$
8. $\frac{x(x+1)}{x+8}$
9. $\frac{2(x+5)(x^2-3x+9)}{x+7}$
10. $\frac{-(x+5)}{(2x-1)(7-x)}$

Adding & Subtracting Rational Expressions and Complex Fractions

Review Queue

1. $\frac{5}{4}$
2. $\frac{13}{21}$
3. $\frac{8}{15}$
4. $\frac{57}{72}$

Adding and Subtracting Rational Expressions with Like Denominators

1. $\frac{x+4}{x}$
2. 1
3. $\frac{x+5}{x-2}$
4. 4

5. $\frac{5}{x-5}$ 6. $\frac{3}{x+2}$ 7. $\frac{1}{x-2}$ 8. $\frac{2(x+1)}{x^2+1}$

9. $\frac{5}{2x-3}$ 10. $\frac{3}{x+4}$

Adding and Subtracting Rational Expressions where One Denominator is the LCD

1. $\frac{7}{4x}$ 2. $\frac{x^2+3x-1}{x^2+3x}$ 3. $\frac{1}{3}$ 4. $\frac{5}{x+2}$

5. $\frac{-1}{2x+3}$ 6. $\frac{10x-18}{3x^2+x-10}$ 7. $\frac{x-3}{2(2x-3)}$ 8. $\frac{x+3}{5x-3}$

9. $\frac{7x+19}{2x^2+x-15}$ 10. $\frac{3}{x-3}$ 11. $\frac{4}{x+3}$

Adding and Subtracting Rational Expressions with Unlike Denominators

1. $\frac{3x^2+10}{6x}$ 2. $\frac{2x+7}{7x^2}$ 3. $\frac{x^2-3x-7}{4x(x+2)}$ 4. $\frac{7x-9}{2(x+3)(x-1)}$

5. $\frac{x^2-14x+6}{x(2x-1)(x+6)}$ 6. $\frac{1}{5(x+1)}$ 7. $\frac{2(2x^2+5x+6)}{(x+2)(x-5)(x+3)}$

8. $\frac{x^2+3x+23}{x(x-2)(3x+5)}$ 9. $\frac{20x^2+42x-7}{5x(x-1)(x+1)}$ 10. $\frac{7x^3-12x^2+7x-10}{(2x-1)(3x+2)(x)}$

Complex Fractions

1. $\frac{2}{3x(x-3)}$ 2. $\frac{1}{5(x+3)}$ 3. $\frac{3(2x-5)}{2}$ 4. $\frac{-1}{x-2}$

5. $\frac{(x+1)(x+2)}{x}$ 6. $\frac{(x-8)(x+1)}{(3x+7)}$ 7. $\frac{(x-3)(2x^2-4x-11)}{3x^2+12x-21}$

8. $\frac{2(x^2+2x+15)}{8x^2-3x+15}$ 9. $-\frac{(7x+2)(x-1)}{(x-3)(x+4)}$ 10. $\frac{(x+1)(4x^2+5x-8)}{(x+2)(-x^2+6x+1)}$

Solving Rational Equations

Review Queue

1. x = -7, -2 2. x = 0 3. x = 2 4. x = 30

5. x = 1.33 6. x = 103

Using Cross-Multiplication

1. $x = 6, -2$
2. $x = -5, 2$
3. $x = 6, -1$
4. $x = 1, -1$
5. $x = \frac{1}{2}, -4$
6. $x = -3$
7. $x = -2, \frac{9}{2}$
8. $x = \frac{5}{28}$
9. $x = -40$
10. $x = 8, -1$

Using the LCD

1. $x = 2, -5$
2. $x = -\frac{7}{13}$
3. $x = \frac{1}{4}$
4. $x = \frac{2}{3}$
5. $x = -5, 2$
6. $x = \frac{15 \pm 3\sqrt{17}}{4}$
7. $x = 0, 28$
8. $x = 5, -3$
9. $x = -\frac{1}{4}$
10. $x = \frac{-7 \pm \sqrt{5}}{2}$