4.1 Triangle Sum Theorem

Answers

1. \( m\angle 1 = 41^\circ \)
2. \( m\angle 1 = 86^\circ \)
3. \( m\angle 1 = 61^\circ \)
4. \( m\angle 1 = 51^\circ \)
5. \( m\angle 1 = 13^\circ \)
6. \( m\angle 1 = 60^\circ \)
7. \( m\angle 1 = 70^\circ \)
8. \( 84^\circ \)
9. \( 57^\circ \)
10. \( 21^\circ \)
11. \( x = 14^\circ \)
12. \( x = 9^\circ \)
13. \( x = 22^\circ \)
14. \( x = 17^\circ \)
15. \( x = 12^\circ \)
4.2 Exterior Angles Theorem

Answers
1. \( m \angle 1 = 118^\circ \)
2. \( m \angle 1 = 68^\circ \)
3. \( m \angle 1 = 116^\circ \)
4. \( m \angle 1 = 161^\circ \)
5. \( m \angle 1 = 141^\circ \)
6. \( m \angle 1 = 135^\circ \)
7. \( 180^\circ \)
8. \( 360^\circ \)
9. \( 360^\circ \)
10. \( x = 30^\circ \)
11. \( x = 25^\circ \)
12. \( x = 7^\circ \)
4.3 Congruent Triangles

**Answers**

1. Yes
2. Yes
3. Yes
4. No
5. No
6. Yes
7. No
8. Yes
9. No
10. Yes
11. No
12. Yes
13. Yes
14. No
15. No
4.4 Congruence Statements

Answers
1. \( \Delta FGH \cong \Delta KLM \)

2. No, we do not know if \( \overline{AC} \cong \overline{BD} \), so we cannot say that the triangles are congruent.

3. \( \Delta ABE \cong \Delta DCE \)

4. No, we only know that one pair of sides and one pair of angles are congruent. This is not enough information to determine if the triangles are congruent.

5. Line up the congruent sides: \( \Delta BCD \cong \Delta ZYX \)

6. Corresponding Parts of Congruent Triangles are Congruent or CPCTC.

7. \( \angle T \cong \angle F, \angle B \cong \angle A, \angle S \cong \angle M, \overline{TB} \cong \overline{FA}, \overline{BS} \cong \overline{AM}, \overline{TS} \cong \overline{FM} \)

8. \( \angle P \cong \angle S, \angle A \cong \angle T, \angle M \cong \angle E, \overline{PA} \cong \overline{ST}, \overline{AM} \cong \overline{TE}, \overline{PM} \cong \overline{SE} \)

9. \( \angle I \cong \angle W, \angle N \cong \angle E, \angle T \cong \angle B, \overline{IN} \cong \overline{WE}, \overline{NT} \cong \overline{EB}, \overline{IT} \cong \overline{WB} \)

10. \( \angle E \)
4.5 Third Angle Theorem

Answers

1. 88°
2. 42°
3. 50°
4. 42°
5. 47°
6. 43°
7. 47°
8. 37°
9. Lines are not marked parallel, we cannot assume that \( m\angle HIJ = 108° \).
10. 35°
11. Lines are not marked parallel, we cannot assume that \( m\angle IHJ = 37° \).
12. 55°
13. 63°
14. 62°
15. 63°
4.6 SSS Triangle Congruence

Answers

1. Yes, $\triangle DEF \cong \triangle HJI$, SSS

2. No, one triangle is SSS and the other is SAS.

3. Yes, $\triangle ABC \cong \triangle FED$, SSS

4. Yes, $\triangle ATD \cong \triangle ETD$, SSS

5. $\overline{AB} \cong \overline{HI}$

6. $\overline{AB} \cong \overline{JL}$

7. |
<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. B is the midpoint of $\overline{DC}$, $\overline{AD} \cong \overline{AC}$</td>
<td>Given</td>
</tr>
<tr>
<td>2. $\overline{DB} \cong \overline{BC}$</td>
<td>Definition of a Midpoint</td>
</tr>
<tr>
<td>3. $\overline{AB} \cong \overline{AB}$</td>
<td>Reflexive PoC</td>
</tr>
<tr>
<td>4. $\triangle ABD \cong \triangle ABC$</td>
<td>SSS</td>
</tr>
</tbody>
</table>

8. Triangle #1: (-8, 0), (0, 3), and (-5, 9), Side #1 = $\sqrt{73}$, Side #2 = $\sqrt{90}$, Side #3 = $\sqrt{61}$
   Triangle #2: (2, 5), (10, 2), and (5, -4), Side #1 = $\sqrt{90}$, Side #2 = $\sqrt{73}$, Side #3 = $\sqrt{61}$
   The triangles are congruent by SSS.
9. Triangle #1: (-7, 2), (1, 6), and (4, 5), Side #1 = $\sqrt{80}$, Side #2 = $\sqrt{130}$, Side #3 = $\sqrt{10}$

Triangle #2: (-1, -10), (-5, -2), and (-8, -1), Side #1 = $\sqrt{80}$, Side #2 = $\sqrt{130}$, Side #3 = $\sqrt{10}$

The triangles are congruent by SSS.

10. $\Delta ABC$: $AB = \sqrt{18}$, $AC = \sqrt{58}$, $BC = \sqrt{52}$, $\Delta DEF$: $DF = \sqrt{17}$, $DE = \sqrt{58}$, $EF = \sqrt{137}$

The triangles are not congruent because not all the corresponding sides are congruent.

11. $\Delta ABC$: $AB = \sqrt{37}$, $AC = \sqrt{45}$, $BC = \sqrt{34}$, $\Delta DEF$: $DE = \sqrt{37}$, $DF = \sqrt{45}$, $EF = \sqrt{34}$

The triangles are congruent by SSS.
4.7 SAS Triangle Congruence

Answers

1. No, these are both SSA, which is not a congruence postulate.

2. Yes, $\triangle ABC \cong \triangle YXZ$, SAS

3. No, these are both SSA, which is not a congruent postulate

4. $\angle C \cong \angle G$

5. $\angle C \cong \angle K$

6. $AB \cong ON$

7. 

<table>
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<tbody>
<tr>
<td>1. B is a midpoint of $\overline{DC}$, $\overline{AB} \perp \overline{DC}$</td>
<td>Given</td>
</tr>
<tr>
<td>2. $\overline{DB} \cong \overline{BC}$</td>
<td>Definition of a midpoint</td>
</tr>
<tr>
<td>3. $\angle ABD$ and $\angle ABC$ are right angles</td>
<td>$\perp$ lines create 4 right angles</td>
</tr>
<tr>
<td>4. $\angle ABD \cong \angle ABC$</td>
<td>All right angles are $\cong$</td>
</tr>
<tr>
<td>5. $\overline{AB} \cong \overline{AB}$</td>
<td>Reflexive PoC</td>
</tr>
<tr>
<td>6. $\triangle ABD \cong \triangle ABC$</td>
<td>SAS</td>
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</table>
8.

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<tr>
<th>Statement</th>
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</tr>
</thead>
<tbody>
<tr>
<td>1. ( AB ) is an angle bisector of ( \angle DAC ), ( AD \cong AC )</td>
<td>Given</td>
</tr>
<tr>
<td>2. ( \angle DAB \cong \angle BAC )</td>
<td>Definition of an Angle Bisector</td>
</tr>
<tr>
<td>3. ( AB \cong AB )</td>
<td>Reflexive PoC</td>
</tr>
<tr>
<td>4. ( \triangle ABD \cong \triangle ABC )</td>
<td>SAS</td>
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</table>

9.

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<tr>
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<tbody>
<tr>
<td>1. ( B ) is the midpoint of ( DE ) and ( AC ), ( \angle ABE ) is a right angle</td>
<td>Given</td>
</tr>
<tr>
<td>2. ( DB \cong BE, AB \cong BC )</td>
<td>Definition of a Midpoint</td>
</tr>
<tr>
<td>3. ( m\angle ABE = 90^\circ )</td>
<td>Definition of a Right Angle</td>
</tr>
<tr>
<td>4. ( m\angle ABE = m\angle DBC )</td>
<td>Vertical Angle Theorem</td>
</tr>
<tr>
<td>5. ( \triangle ABE \cong \triangle CBD )</td>
<td>SAS</td>
</tr>
</tbody>
</table>

10.

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<tbody>
<tr>
<td>1. ( DB ) is the angle bisector of ( \angle ADC ), ( AD \cong DC )</td>
<td>Given</td>
</tr>
<tr>
<td>2. ( \angle ADB \cong \angle BDC )</td>
<td>Definition of an Angle Bisector</td>
</tr>
<tr>
<td>3. ( DB \cong DB )</td>
<td>Reflexive PoC</td>
</tr>
<tr>
<td>4. ( \triangle ABD \cong \triangle CBD )</td>
<td>SAS</td>
</tr>
</tbody>
</table>
4.8 ASA and AAS Triangle Congruence

Answers

1. Yes, AAS, $\triangle ABC \cong \triangle FDE$

2. Yes, ASA, $\triangle ABC \cong \triangle IHG$

3. No, the triangles have congruent parts that would be SSA. This is not a congruence theorem.

4. $\angle DBC \cong \angle DBA$ because they are both right angles and created by perpendicular lines.

5. $\angle CDB \cong \angle ADB$

6. $\overline{DB} \cong \overline{DB}$ from the Reflexive Property. We have enough to say that the triangles are congruent by ASA.

7. 

<table>
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<tbody>
<tr>
<td>1. $\overline{DB} \perp \overline{AC}$, $\overline{DB}$ is the angle bisector of $\angle CDA$</td>
<td>Given</td>
</tr>
<tr>
<td>2. $\angle DBC$ and $\angle ADB$ are right angles</td>
<td>Definition of perpendicular</td>
</tr>
<tr>
<td>3. $\angle DBC \cong \angle ADB$</td>
<td>All right angles are $\cong$</td>
</tr>
<tr>
<td>4. $\angle CDB \cong \angle ADB$</td>
<td>Definition of an angle bisector</td>
</tr>
<tr>
<td>5. $\overline{DB} \cong \overline{DB}$</td>
<td>Reflexive PoC</td>
</tr>
<tr>
<td>6. $\triangle CDB \cong \triangle ADB$</td>
<td>ASA</td>
</tr>
</tbody>
</table>
8. \( \angle C \cong \angle A \) by CPCTC (corresponding parts of congruent triangles are congruent)

9. \( \angle L \cong \angle O \) and \( \angle P \cong \angle N \) by the Alternate Interior Angles Theorem.

10. \( \angle LMP \cong \angle NMO \) by the Vertical Angles Theorem.

11. There is more than one correct proof, this is one possible answer.

<table>
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</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{LP} \parallel \overline{NO}, \overline{LP} \cong \overline{NO} )</td>
<td>Given</td>
</tr>
<tr>
<td>2. ( \angle L \cong \angle O, \angle P \cong \angle N )</td>
<td>Alternate Interior Angles Theorem</td>
</tr>
<tr>
<td>3. ( \triangle LMP \cong \triangle OMN )</td>
<td>ASA</td>
</tr>
</tbody>
</table>

12. CPCTC

13. Start with the proof from #11 and continue.

<table>
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<tr>
<td>1. ( \overline{LP} \parallel \overline{NO}, \overline{LP} \cong \overline{NO} )</td>
<td>Given</td>
</tr>
<tr>
<td>2. ( \angle L \cong \angle O, \angle P \cong \angle N )</td>
<td>Alternate Interior Angles</td>
</tr>
<tr>
<td>3. ( \triangle LMP \cong \triangle OMN )</td>
<td>ASA</td>
</tr>
<tr>
<td>4. ( \overline{LM} \cong \overline{MO} )</td>
<td>CPCTC</td>
</tr>
<tr>
<td>5. ( M ) is the midpoint of ( \overline{PN} ).</td>
<td>Definition of a midpoint</td>
</tr>
</tbody>
</table>

14. \( \angle A \cong \angle N \)

15. \( \angle C \cong \angle M \)

16. \( \overline{PM} \cong \overline{MN} \)

17. \( \overline{LM} \cong \overline{MO} \) or \( \overline{LP} \cong \overline{NO} \)
4.9 **HL Triangle Congruence**

**Answers**

1. $\overline{NM} \cong \overline{ZX}$
2. $\overline{FG} \cong \overline{RS}$ or $\overline{FE} \cong \overline{RQ}$
3. $\overline{VW} \cong \overline{DC}$ and $\overline{WX} \cong \overline{CB}$ or $\overline{VX} \cong \overline{DB}$
4. Given/Definition of Perpendicular Lines
5. $\overline{AC} \cong \overline{CD}$
6. $\overline{ED} \cong \overline{AB}$
7. HL Congruence
8. No
9. Yes, by SAS
10. No
11. Yes, by HL
12. No
13. No
14. Yes, by SSS
15. No
4.10 Isosceles Triangles

Answers

1. \( x = 13^\circ \)
2. \( y = 16^\circ \)
3. \( x = 1 \)
4. \( y = 3 \)
5. \( x = 4^\circ, y = 11^\circ \)
6. True
7. False, only in an isosceles right triangle.
8. False, only in the case of an equilateral triangle.
9. True
10.

<table>
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<tr>
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</tr>
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<tbody>
<tr>
<td>1. Isosceles ( \triangle CIS ), with base angles ( \angle C ) and ( \angle S ) ( \overrightarrow{IO} ) is the angle bisector of ( \angle CIS )</td>
<td>Given</td>
</tr>
<tr>
<td>2. ( \angle C \cong \angle S )</td>
<td>Base Angles Theorem</td>
</tr>
<tr>
<td>3. ( \triangle CIO \cong \triangle SIO )</td>
<td>Definition of an Angle Bisector</td>
</tr>
<tr>
<td>4. ( \overrightarrow{IO} \cong \overrightarrow{IO} )</td>
<td>Reflexive PoC</td>
</tr>
<tr>
<td>5. ( \triangle CIO \cong \triangle SIO )</td>
<td>ASA</td>
</tr>
<tr>
<td>6. ( \overrightarrow{CO} \cong \overrightarrow{OS} )</td>
<td>CPCTC</td>
</tr>
<tr>
<td>7. ( \angle IOC \cong \angle IOS )</td>
<td>CPCTC</td>
</tr>
<tr>
<td>8. ( \angle IOC ) and ( \angle IOS ) are supplementary</td>
<td>Linear Pair Postulate</td>
</tr>
<tr>
<td>9. ( m\angle IOC = m\angle IOS = 90^\circ )</td>
<td>Congruent Supplements Theorem</td>
</tr>
<tr>
<td>10. ( \overrightarrow{IO} ) is the perpendicular bisector of ( \overline{CS} )</td>
<td>Definition of a ( \perp ) bisector (Steps 6 and 9)</td>
</tr>
</tbody>
</table>
11. **Answer Key**

<table>
<thead>
<tr>
<th>Statement</th>
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</tr>
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<tbody>
<tr>
<td>1. Isosceles $\triangle ICS$ with $\angle C$ and $\angle S$, $IO$ is the perpendicular bisector of $CS$</td>
<td>Given</td>
</tr>
<tr>
<td>2. $\angle C \cong \angle S$</td>
<td>Base Angle Theorem</td>
</tr>
<tr>
<td>3. $CO \cong OS$</td>
<td>Definition of a $\perp$ bisector</td>
</tr>
<tr>
<td>4. $m\angle IOC = m\angle IOS = 90^\circ$</td>
<td>Definition of a $\perp$ bisector</td>
</tr>
<tr>
<td>5. $\triangle CIO \cong \triangle SIO$</td>
<td>ASA</td>
</tr>
<tr>
<td>6. $\angle CIO \cong \angle SIO$</td>
<td>CPCTC</td>
</tr>
<tr>
<td>7. $IO$ is the angle bisector of $\angle CIS$</td>
<td>Definition of an Angle Bisector</td>
</tr>
</tbody>
</table>

12. Side #1 = $\sqrt{18}$, Side #2 = $\sqrt{18}$, Side #3 = 6
   This is an isosceles triangle because Side #1 and Side #2 are equal.

13. Side #1 = $\sqrt{17}$, Side #2 = $\sqrt{40}$, Side #3 = $\sqrt{9}$
   No sides are equal, so this is a scalene triangle.

14. Side #1 = $\sqrt{72}$, Side #2 = $\sqrt{234}$, Side #3 = $\sqrt{162}$
   No sides are equal, so this is a scalene triangle.

15. Side #1 = $\sqrt{104}$, Side #2 = $\sqrt{208}$, Side #3 = $\sqrt{104}$
   This is an isosceles triangle because Side #1 and Side #3 are equal.

16. Side #1 = 8, Side #2 = $\sqrt{65}$, Side #3 = $\sqrt{65}$
    This is an isosceles triangle because Side #2 and Side #3 are equal.
Chapter 4 – Triangles and Congruence

4.11 Equilateral Triangles

Answers

1. \( x = 60^\circ \)
2. \( y = 68^\circ \)
3. \( x = 1.5 \)
4. \( y = 17 \)
5. \( z = 17 \)
6. \( n = 25^\circ \)
7. \( x = \pm 4\sqrt{2} \)
8. \( x = 3, y = 2 \)
9. \( x = 2, y = 5 \)
10. \( z = 4 \)
11. \( a = -1, 6 \)
12. \( m = -4^\circ, 15^\circ \)
13. \( x = 4, -1 \)
14. \( x = \pm 2\sqrt{14} \)
15. \( x = 25^\circ, y = 19^\circ \)