5.1 Normal Distributions

Answers

1. a) Yes, if the sample was large enough there would be spread in the data. There are restrictions in that the spanrestrictions in that the span cannot be zero and there is a maximum length.

   b) No, there would be salary distribution for the casuals and middle level managers then a huge gap and then a salary distribution for the higher end managers like the CEO, CFO, etc… It would most likely be a right skewed distribution.

   c) No. Some of the CEOs would make larger salaries and some really large salaries.

   d) No. Most pennies are newer pennies as the older ones are recycled. The data would be clustered from the past decade or so with a few outliers. It would most likely be a left skewed distribution.

2. -2.54; 0.317; 1.905; 3.016
3. a) 16%

   b) 95%

   c) 0.15%
4. a) 84%

   b) 16%

   c) 0

   d) 1

   e) 2.5%

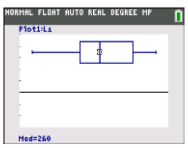
   order: c, e, b, a, d
5. c
6. You can use normal distributions to make meaningful conclusions using the normal distribution curve. If the sample is indicative of the population, the sample mean and the population mean will be close. This means that you can infer that 68% of the sample will record results to the experiment within 1 standard deviation from the mean. 95% of the sample will record results to the experiment within 2 standard deviations from the mean. And 99% of the sample will record results to the experiment within 3 standard deviations from the mean.

7. To calculate probabilities beyond the empirical rule, use the z-score calculation.

8. Other types of distributions are skewed left distributions, skewed right distributions, and bimodal distributions.

9. 2.5%

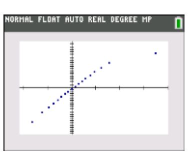
10. a)



The shape of the box plot is symmetric, the center of the distribution is at 260, and the spread of the distribution is from 133 to 366.

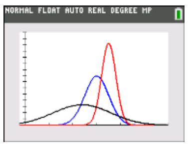
b) $\mu = 262, \sigma = 65.96$

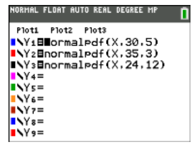
c)



d) The plot is strongly linear so the conclusion is that the data is reasonable normally distributed.

11.





12. a) 55
    b) 3
    c) N (55, 3)

13. $\mu = 36, \sigma = 3$

14. $\mu = 9, \sigma = 6$

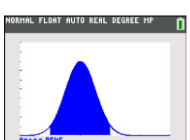
15. a) 2

    b) -1.2

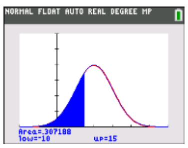
    c) -2.25

    d) -0.7857

16. a)



b)



17. The empirical rule states that for normal distributions, 68% of the area under the curve lies between $(\mu-\sigma, \mu+\sigma)$, 95% of the area under the curve lies between $(\mu-2\sigma, \mu+2\sigma)$, and 99.7% of the area under the curve lies between $(\mu-3\sigma, \mu+3\sigma)$.

18. 0.15%

19. a) 34%

    b) 0.15%

    c) 84%

    d) 99.85%

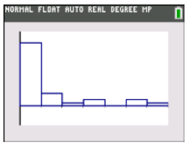
20. 47.5%

21. 94.7%

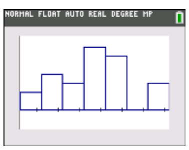
22. 40

23. $P(15.3 < x >< 115.3) < /x >$

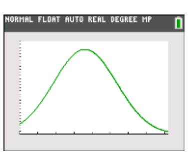
24. No, the histogram is right-skewed.

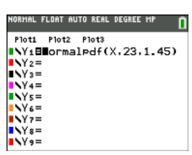


25. Yes, this data looks more like a normal distribution.



Graphing the normal distribution curve using the mean and standard deviation for the data, you get:

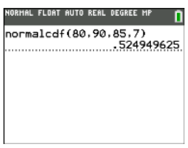




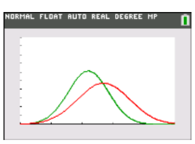
5.2 Density Curve of the Normal Distribution

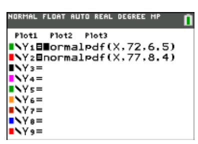
Answers

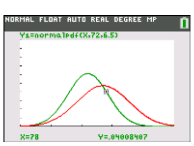
1. 3
2. a) 21.48%
   b) 68.26%
   c) 56.61%
3. She assumed the mean was 0 and the standard deviation was 1.

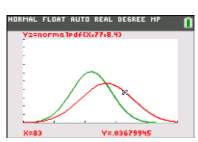


4.



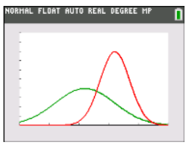


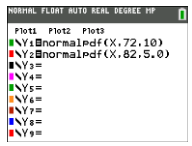




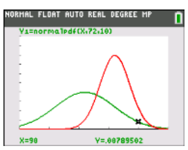
The z-score for 0.0401 is 0.6554 meaning 65.54% of the marks were below 78. The z-score for 0.03680 is 0.6443 so 64.43% of the marks were below 83. Therefore 78 would be a better grade.

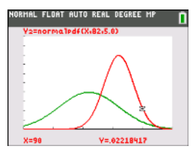
5.





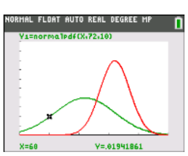
a)

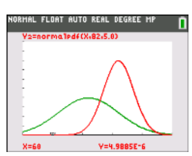




The z-score for 0.0079 is 0.5040 meaning 50.40% of the marks were below 90. The z-score for 0.0221 is 0.5080 so 50.80% of the marks were below 90. Therefore Teacher A has a more impressive 90.

b)





The z-score for 0.0194 is 0.492 meaning 49.2% of the marks were above 60. The z-score for 0.0 is 0.50 so 50% of the marks were above 60. Therefore Teacher B has a more discouraging 60.

6. When the data is not normally distributed the z-scores will give you misleading answers.

7. When given the mean, standard deviation, and actual values, use the formula to find the z-scores: $z = \frac{x - \mu}{\sigma}$. Given just the actual value, x, use normcdf(-3, x). Given

8. a) z = 0, probability distribution above the score is 50%.

   b) z = -0.45, probability distribution above the score is 67.4%.
   c) z = 0.45, probability distribution above the score is 32.6%.
   d) z = -1.96, probability distribution above the score is 97.5%
   e) z = 2.33, probability distribution above the score is 1%.
   f) z = -2.58, probability distribution above the score is 99.5%.
   g) z = -1.65, probability distribution above the score is 95.1%.

9. a) 0 (median and mean would be equal for a normal distribution)

   b) 0 (median and mean would be equal for a normal distribution)

10. a) z = 0.67

    b) z = -1.96
    c) z = -2.17 to z = 2.34

11. $\mu=31.52$

    $\sigma=21.96$

5.3 Applications of Normal Distribution

Answers

1. (e)

2.

Mean	Standard deviation	X	P
85	4.5	87.1	0.68
17.64	1	16	0.05
73	8.96	85	0.91
93	5	99.41	0.90

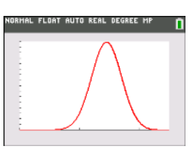
3. Z = -0.674

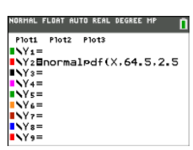
4. 0.00106

5. a) 0.04
   b) 0.49
   c) 2.29 ounces
   d) 2.254 ounces
   e) 0.023 ounces

6. When you know the probability, you can use invNorm to find the z-score of the variable. This will give you what percentage the variable can be found (or the likelihood that the variable will be in this range) on the normal distribution curve.

7.a)





   b) 84.1%, 2.3%, 2.3%

8. a) 50%
   b) 2.3%
   c) 15.9%
   d) 34.5%
   e) 65.5%
   f) 8.1%
   g) 168

9.Mary

10 a) 1.2
   b) 1.0 (Reference should be made to problem 8 – not problem 2)
   c) Joan

11. a) 1.6%
   b) 97.6%
   c) 74.2%

12. Answers will vary. Use $z = \frac{x-65}{2.5}$ where x is your height if you are a female and $z = \frac{x-79}{3}$ where x is your height if you are a male.

13. a) 41.7%
   b) 99.9%
   c) 20.3%

14. a) 5.5%
   b) 15.9%
   c) 67.3%
   d) 0%

15. You would be happier if the standard deviation was 5 because only 2.3% of the population scored higher than you but with a standard deviation of 15, 25.1% of the population scored higher than you.

16. 115.6

17. a) $\mu = 82.4, \sigma = 10.1$

All of the scores lie within 3 standard deviations from the mean. 54 is the only score beyond 2 standard deviations from the mean. Scores of 74, 75, 77,79, 80, 83, 86, 87, and 90 lie within 1 standard deviation from the mean.

b) A = 92.5 and better
B = 84.2 – 92.5
C = 72.3 – 82.4
D = 62.2 – 72.3
F = below 62.2

c) A = 15.9%
B = 34.1%
C=34.1%
D = 13.6%
F = 2.3%

d) No because 20 of the 27 students scored within one standard deviation of the mean.

18. 60.3%; $\mu = 63.96$