

10.1 Chi-Square Test

Answers

1. C: the chi-square test

2. A: the goodness-of-fit test

3. B: gender

4. A: 125

5.
$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

6. a) The observed frequency value for the Science Museum category is 29.

b) The expected value for the Sporting Event is 33.3.

c) There is no preference between the types of field trips that students prefer.

d) The chi-square statistic value for the research question is 20.

e) Since the chi-square statistic value is greater than the chi-square level of significance, you would reject the null hypothesis.

7.

Deaths in Western Australia in 1982			
Cause	Males	Females	Total
heart disease	1317	854	2171
cancer	1119	828	1947
cerebral vascular disease	371	460	831
accidents	346	147	493
Total	3153	2289	5442

8. a) $p = 0.05$

b) $p = 0.15$

c) $p = 2.01 \times 10^{-6}$ or 0.000002

9. a) The critical value is 3.84.

b) The critical value is 5.99.

c) The critical value is 15.51

10. a) no
 b) yes, critical value is less than at the critical value at 0.05.
11. Situation b is statistically significant.
12. a) The expected count for each category is 100.
 b) The expected count for each category is 250, 250 and 500.
13. a) The chi-square statistic is never negative since $(O - E)^2$ results in a positive value.
 b) The chi-square statistic is zero when the observed values and the expected values are identical.
- 14.

Face value	Observed	Expected	$\frac{(O - E)^2}{E}$
1	29	20	27.03
2	15	20	11.27
3	15	20	9.6
4	16	20	9.0
5	15	20	6.67
6	30	20	19.2
			82.77

$$E = \left(\frac{1}{6}\right)120 = 20$$

The critical value for a chi-square statistic at the 0.05 level of significance is 11.07. Since $\chi^2 > 11.07$, then we can reject the null hypothesis and conclude that the die is not fair.

15. True
16. True
17. True
18. False, the probability is 20 (19.98)
19. In a goodness of fit test, if the p-value is 0.0113, this statement is incomplete.

20. a) The p value is not small enough to reject the null hypothesis so there is a relationship between gender and book preference (fiction and nonfiction).
- b) The smaller the p-value, the stronger the evidence for the alternative hypothesis. Therefore there is no relationship between gender and book preference. The null hypothesis that there is a relationship between gender and book preference (fiction or nonfiction) is rejected.

21.

Color	Observed	Expected	$\frac{(O - E)^2}{E}$
Silver	25	37	3.89
Black	59	37	13.08
White	27	37	2.70
			19.67

$$E = \frac{111}{3} = 37$$

$$H_0 : p_{\text{silver}} = p_{\text{black}} = p_{\text{white}} = \frac{1}{3}, n = 111$$

H_a : at least one probability is different

The chi-square statistic is 19.67. The critical value for a chi-square statistic at the 0.05 level of significance is 5.99. Since $\chi^2 > 11.07$, then we can reject the null hypothesis and conclude that the colors are not equally preferred.

22. Answers will vary.

10.2 Test of Independence

Answers

1. The chi-square test of independence is used to see if two factors are related.
2. True
3. A: Expected Frequency = $\frac{(\text{Row Total})(\text{Column Total})}{\text{Total Number of Observations}}$
4.
 - a) The total number of females in the sample is 782.
 - b) The total number of observations is 1 062.
 - c) The expected frequency for the number of males who did not study abroad is 161.
 - d) There is one degree of freedom in this sample.
 - e) True
 - f) The chi-square statistic is 1.60.
5. B: fail to reject the null hypothesis.
6. True
7. B: the test for independence
8. The incidence of malaria is not independent of the region.
9. Class attendance and course performance are not independent.
10. Bloating is independent of the cracker.
11.
 - a) MISSING ANSWER
 - b)MISSING ANSWER
 - c)The age category and the educational attainment are not independent.

12. a) No

b) Null Hypothesis: $H_o : O = E$ There is no relationship between the two questions.

Alternative hypothesis: $H_o : O \neq E$ There is a relationship between the answer to the first question and the answer to the second question.

c) $\chi^2 = 20.49$. The p-value is low so the alternative hypothesis would be accepted and conclude that there is a relationship between the answers to the two questions.

d) Null hypothesis: $H_o : O = E$ There is no relationship between the expected values with the two questions.

Alternative hypothesis: $H_o : O \neq E$ There is a relationship between the expected values in the two questions.

e) $\chi^2 = 0.01$. The p-value is greater than 0.05 so the null hypothesis is accepted.

13. The course grade and the number of modules completed are not independent.

14. The level of skier is just as likely to be the same at one ski area as at another.

15. For this data we will fail to reject the null hypothesis and conclude that an individuals with a larger family size are not significantly more likely to own a larger vehicle than those individuals with a smaller family size.

16. Represent the data from the table in Matrix A. These are the observed values.

NORMAL FLOAT AUTO REAL RADIAN MP

MATRIX[A] 5 x3

5	20	5
10	40	50
10	15	15
10	25	25
15	40	15

[A](1,1)= 5

Do the chi-square Test.

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χ^2 -Test

$\chi^2=24.4682282$
 $p=.0019119187$
 $df=8$

Check the values in Matrix B. These are the expected values. Check to make sure that all values are greater than one and also that not more than 20% of the values are less than 5. These are the observed values

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MATRIX[B] 5 x3

5	14	11
16.667	46.667	36.667
6.6667	18.667	14.667
10	28	22
11.667	32.667	25.667

The college major of students and their starting salaries are not independent.

17. False: The degrees of freedom for a test of independence are equal to the product of the number of rows minus 1 and the number of columns minus 1.
18. True
19. True

10.3 Test of Single Variance

Answers

In the example for a 95% confidence level, the critical chi square values should be under 0.025 and 0.975. They have the same values for both a 90% and a 95 % confidence interval (0.05 and 0.95)

1. D: all of the above
2. False
3. A: equal with respect to variance
4. C: the hypothesized population variance
5. D: no additional information is needed
6. $H_0 = 3.22$ The variance of the weighed bars of Dial soap is equal to that of the soap from the general factory.
7. $\chi_{29}^2 = 10.90$
8. We would fail to reject the null hypothesis.
9. $0.825 \leq \sigma^2 \leq 1.982$
10. We are 90 % confident that the variance of the population from which the sample was taken is between 0.825 and 1.982.
11. The result of this test is that the value of σ^2 at the $\alpha = 0.05$ level is 0.318 or 0.32.
12. The p-value is 0.95.
13. We are 90 % confident that the variance of the population from which the sample is taken is between 0.32 and 2.07.
14. Lower critical value – 3.247 and Upper critical value – 20.483
15. This solution was calculated assuming the commas in the mean and standard deviation were supposed to be decimals. This assumption was made since larger numbers are no longer written using commas, instead spaces are used.

$$102\ 606.19 \leq \sigma^2 \leq 197\ 917.53$$

16. The probability of observing a value this low or lower is 0.25.

17. The critical boundaries for rejecting the null hypothesis are $8.319 < \chi^2 < 21.064$.
18. No. $p > 0.05$ so the null hypothesis is accepted and the machine does not need to be calibrated.
19. We believe that the standard deviation is between \$10.70 and \$16.50.
20. The standard deviation of the data is greater than 0.75. It is 0.78.
21. $\chi^2 = \frac{(n-1)(s^2)}{\sigma^2}$ where n is the sample size, s^2 is the sample variance and σ^2 is the population variance.
22. a) The chi square statistic is 13.5.
- $H_0 : \sigma^2 = 16$
- b) $H_a : \sigma^2 > 16$
- c) The decision is to reject the null hypothesis. Therefore the company must improve its batteries or become truthful regarding their longevity.
23. The probability that the standard deviation would be greater than 6 minutes is 0.96.
24. $0.73 \leq \sigma^2 \leq 2.322$ is the 95 % confidence interval.