Exponential and Logarithmic Functions

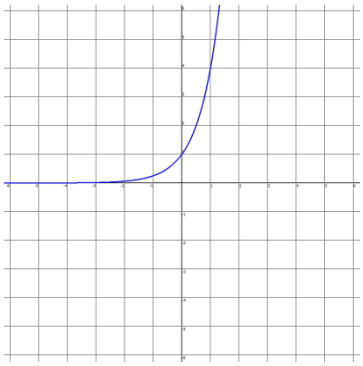
Exponential Growth and Decay

Review Queue Answers

1. $12x^6$ 2. $\frac{1}{3x^2y^4}$ 3. $27x^6y^{15}$ 4. $x=5$

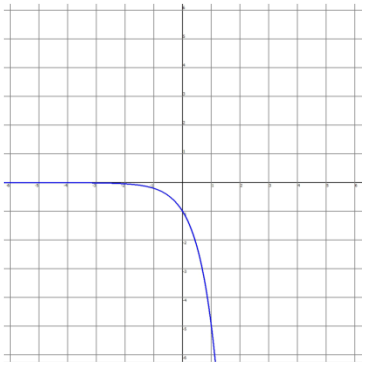
Exponential Growth Function

1. ans-0801-01



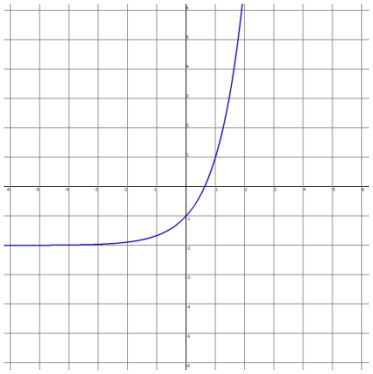
(0, 1), $y=0$, $x\in\mathbb{R}$, $y>0$

2. ans-0801-02



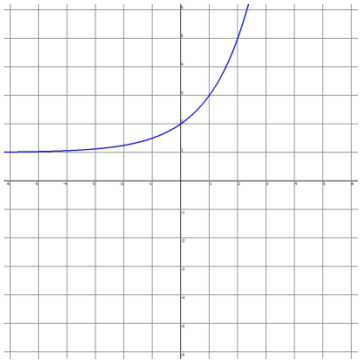
(0, -1), $y=0$, $x\in\mathbb{R}$, $y<0$

3. ans-0801-03



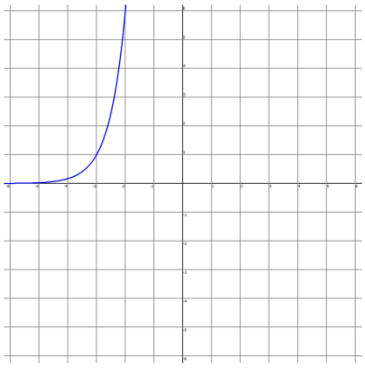
(0, -1), $y=-2$, $x\in\mathbb{R}$, $y>-2$

4. ans-0801-04



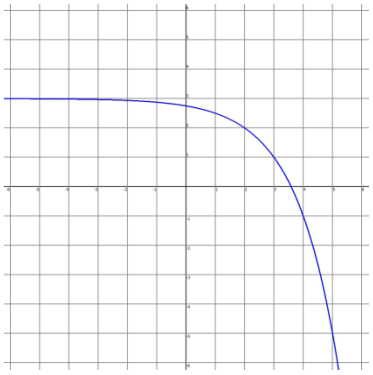
(0, 2), $y=1$, $x\in\mathbb{R}$, $y>1$

5. ans-0801-05



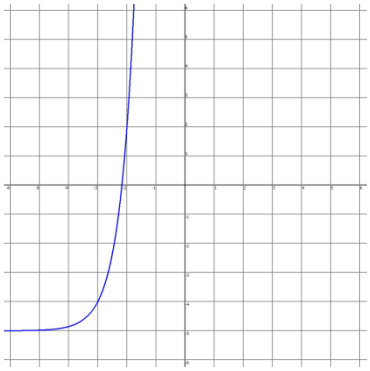
(0, 216), $y=0$, $x\in\mathbb{R}$, $y>0$

6. ans-0801-06



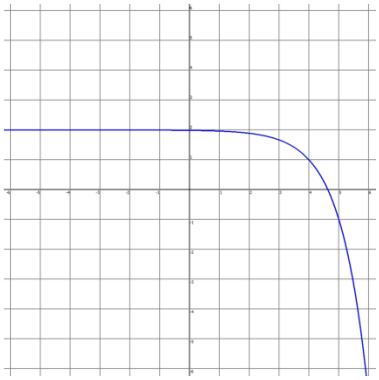
(0, 2.75), $y=3$, $x\in\mathbb{R}$, $y<3$

7. ans-0801-07



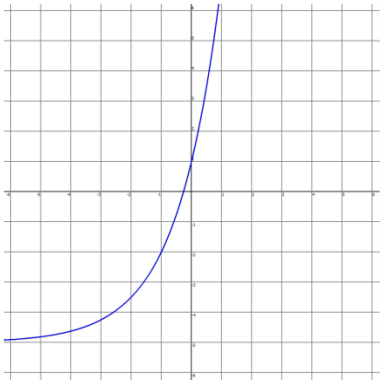
(0, 343), $y=-5$, $x\in\mathbb{R}$, $y>-5$

8. ans-0801-08



(0, 1.99), $y=2$, $x\in\mathbb{R}$, $y<2$

9. ans-0801-09



(0, 1), $y=-5$, $x\in\mathbb{R}$, $y>-5$

10. $16,289

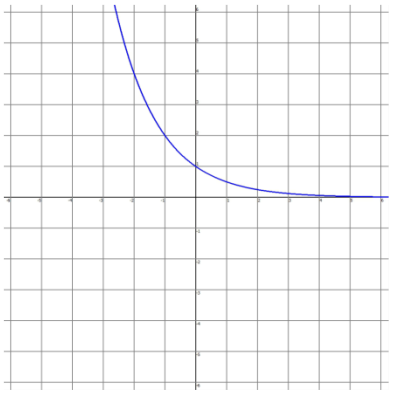
Exponential Decay Function

1. exponential decay

2. exponential growth

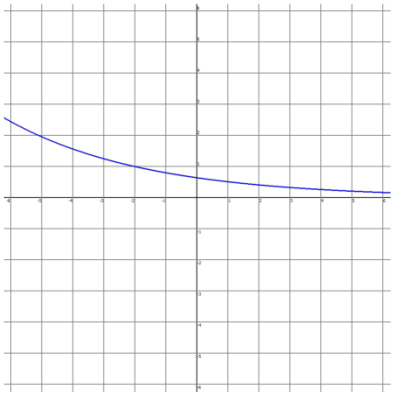
3. exponential growth

4. ans-0801-10



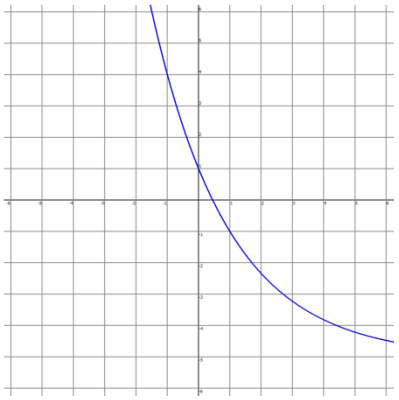
(0, 1), $y=0$, $x\in\mathbb{R}$, $y>0$

5. ans-0801-11



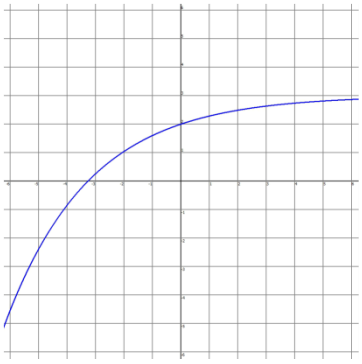
(0,0.64), $y=0$, $x\in\mathbb{R}$, $y>0$

6. ans-0801-12



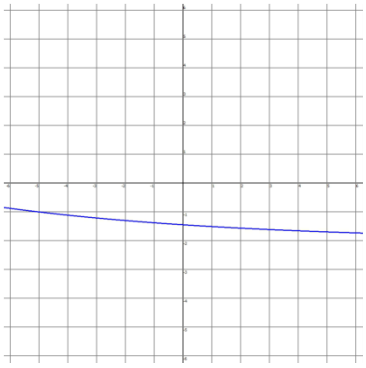
(0, 1), $y=-5$, $x\in\mathbb{R}$, $y>-5$

7. ans-0801-13



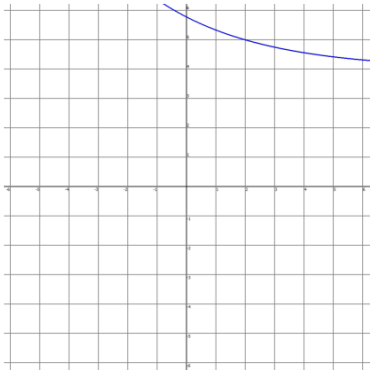
(0, 2), $y=3$, $x\in\mathbb{R}$, $y<3$

8. ans-0801-14



(0, -1.45), $y=-2$, $x\in\mathbb{R}$, $y>-2$

9. ans-0801-15



(0, 5.78), $y=4$, $x\in\mathbb{R}$, $y>4$

10. a. $P=50(0.9)^x$ b. $29.52 c. 7 weeks

Using Exponential Growth and Decay Models

1.	$61,600	2.	$8,089	3.	Yes, week 15 is 20.9 mi	4.	$31,200
5.	$143,000	6.	$377,000	7.	168,156 children	8.	15 years
9.	$17,890	10.	$10,759	11.	$40,870	12.	$18,600

The Number e

1. growth, $e > 1$
2. decay, $e^{-x} = \left(\frac{1}{e}\right)^x$ and $\frac{1}{e} < 1$
3. decay, $\frac{1}{e} < 1$
4. decay, $\left(\frac{3}{e}\right)^{-x} = \left(\frac{e}{3}\right)^x$ and $\frac{e}{3} < 1$

5.	$\frac{5}{e^7}$	6.	$6e$	7.	$\frac{9}{16e^6}$
8.	a. 173,325	b.	2025	9.	a. 19,666
	b. 7530	10.	a. $A = Pe^{0.045t}$		b. $171
	c. $10,750				

Logarithmic Functions

Review Queue Answers

1.	$y = 2x + 10$	2.	$y = x^2 - 5$	3.	$y = \pm\sqrt{\frac{x-1}{6}}$

Defining Logarithms

1.	$\log_3 5 = x$	2.	$\log_a b = x$	3.	$\log_5\left(\frac{5}{2}\right) = x$	4.	$2^x = 32$
5.	$\left(\frac{1}{3}\right)^{-2} = x$	6.	$a^b = y$	7.	2	8.	-3
9.	-1	10.	6	11.	1.857	12.	2.079
13.	3.585	14.	2				

Inverse Properties of Logarithmic Functions

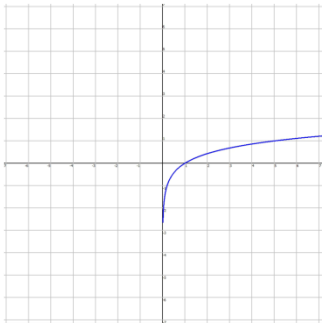
1.	3x	2.	-x	3.	-5x	4.	x + 3
5.	2x – 2	6.	3x	7.	x – 7	8.	-6x
9.	5x – 3	10.	$y = \ln\left(\frac{x}{3}\right) - 2$	11.	$f^{-1}(x) = 7\ln(5x)$		

12. $y=\dfrac{\ln(x-2)+3}{2}$ 13. $f^{-1}(x)=\dfrac{3}{\log_7(x+5)-1}$

14. $y=2\log_6\left(\dfrac{x}{2}\right)+5$ 15. $f^{-1}(x)=2\left[\log_8(3x)+5\right]$

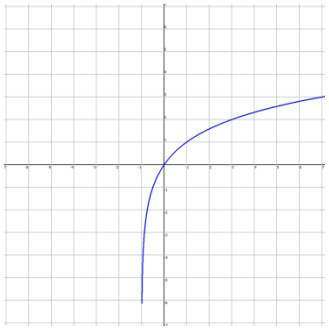
Graphing Logarithmic Functions

1. ans-0802-01



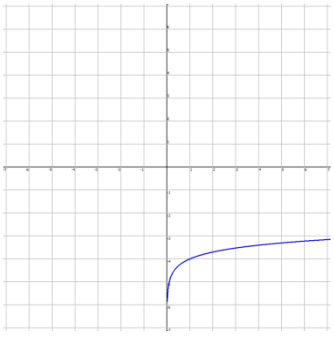
$x=0$; D: $x>0$; R: $y\in\mathbb{R}$

2. ans-0802-02



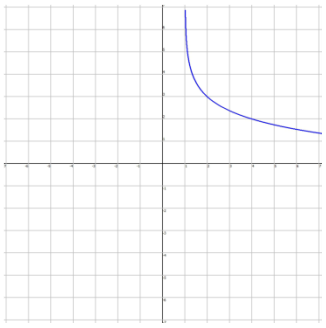
$x=-1$; D: $x>-1$; R: $y\in\mathbb{R}$

3. ans-0802-03



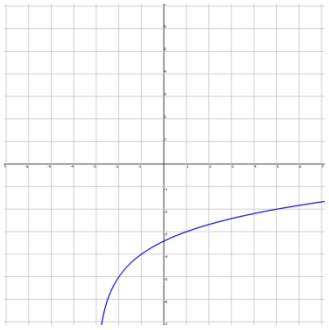
$x=0$; D: $x>0$; R: $y\in\mathbb{R}$

4. ans-0802-04



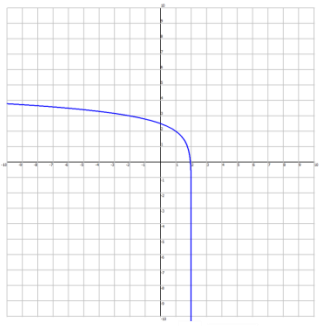
$x=1$; D: $x>1$; R: $y\in\mathbb{R}$

5. ans-0802-05



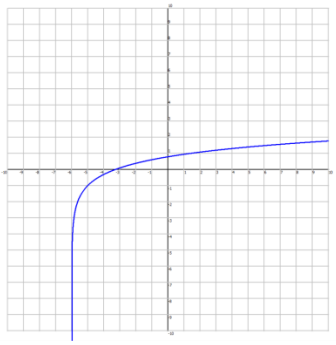
$x=-3$; D: $x>-3$; R: $y\in\mathbb{R}$

6. ans-0802-06b



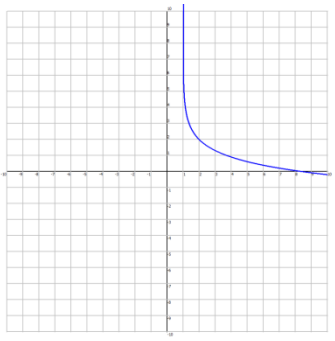
$x=1$; D: $x>1$; R: $y\in\mathbb{R}$

7. ans-0802-07b



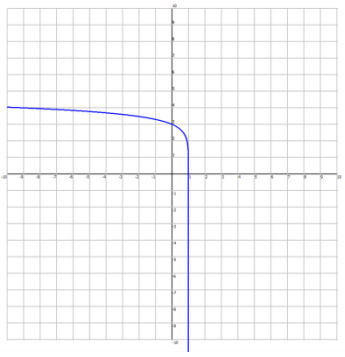
10. yes

8. ans-0802-08b



11. no

9. ans-0802-09b



12. yes

Properties of Logarithms

Review Queue Answers

1. $81x^2$ 2. $40xy^7$ 3. $\frac{5xy^{10}}{2z^7}$

Product and Quotient Properties

1. $\log_3\left(\frac{3y}{2}\right)$ 2. $\log\left(\frac{12y^2}{x}\right)$ 3. $\log_6\left(\frac{x}{y}\right)$
4. $\ln 4$ 5. $\ln 5$ 6. $\log_{11} 2$
7. $\log_3 a+\log_3 b+\log_3 c$ 8. $\log a^2-\log b$ 9. $\log_9 x+\log_9 y-\log_9 5$
10. $\log_{15} 2+\log_{15} x-\log_{15} y$ 11. $\log 8+\log x^2-\log 15$ 12. $\log_4 5-\log_4 9-\log_4 y$
13. Given $\log_a x-\log_a y$; $\log_a x=m$ and $\log_a y=n$. By converting to exponential form we get $a^m=x$ and $a^n=y$; dividing the respective sides we get $\frac{a^m}{a^n}=\frac{x}{y}$. Now, $a^{m-n}=\frac{x}{y}$. Taking the log of both sides we get $\log_a a^{m-n}=\log_a\left(\frac{x}{y}\right)$ and by the inverse property of logarithms $m-n=\log_a\left(\frac{x}{y}\right)$. Substituting for m and n we can conclude $\log_a x-\log_a y=\log_a\left(\frac{x}{y}\right)$.

Power Property of Logarithms

1. $3\left(\log_4 9+\log_4 x\right)$ 2. $2\log 3+2\log x-2\log y$ 3. $3\log_8 x+2\log_8 y-4\log_8 z$
4. $4+8\log_5 x-2\log_5 y$ 5. $-2\ln 6-2\ln x+6\ln y$ 6. $30-12\ln x-18\ln y$
7. $\log_6\left(x^2y^5\right)$ 8. $\log\left(\frac{x}{y}\right)^3$ 9. $\log\left(\frac{\sqrt{x+1}}{y^3}\right)$
10. $\log_2\left(y^4x\right)$ 11. $\log_2\frac{(x-3)^2}{\sqrt[5]{y}}+1$ or $\log_2\frac{2(x-3)^2}{\sqrt[5]{y}}$
12. $\log_3\frac{y^2}{x^{4/3}z^4}$

Solving Exponential and Logarithmic Equations

Review Queue Answers

1. x = 5 2. x = 4, 5 3. x = 69 4. $x=\frac{7}{3}$

Solving Exponential Equations

1. $x \approx 2.594$
2. $x \approx 6.492$
3. $x \approx 0.285$
4. $x \approx 1.165$
5. $x \approx 3.142$
6. $x \approx 4.869$
7. $x = 3/2$
8. $x = 1$
9. $x = 3$
10. $x = 6$
11. $x = 3$
12. $x = 2$

Solving Logarithmic Equations

1. $x = 170$
2. $x \approx 1.132$
3. $x = \sqrt[3]{3}$
4. $x = 2$
5. $x = 1$
6. $x = 8$
7. $x = 27$
8. $x = 2$
9. $x \approx 3.272$
10. $x = 4/3$
11. $x = 2 + 2\sqrt{11} \approx 8.633$
12. $x = 6$