## Chapter 6 – Normal Distribution Curves *Answer Key*

# 6.1 Standard Deviation of a Normal Distribution

**Answers**

1. "The typical measurement is approximately 65 kilograms, give or take 2 kilograms."

2. "The typical measurement is approximately 25.0 miles per hour, give or take 0.5 miles per hour."

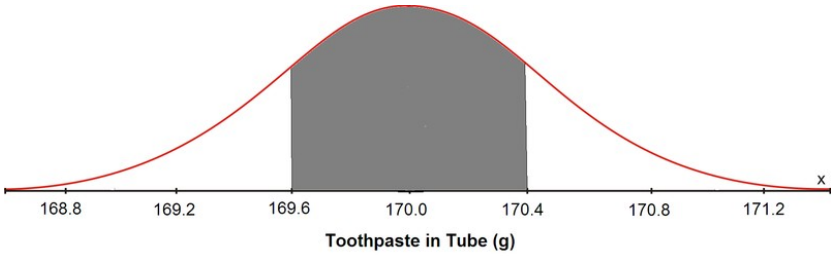
3. "The typical measurement is approximately 375 feet, give or take 5 feet."

4. The values within 1 standard deviation of the mean are from 16.8 – 0.7 years to 16.8 + 0.7 years, or from 16.1 years to 17.5 years.

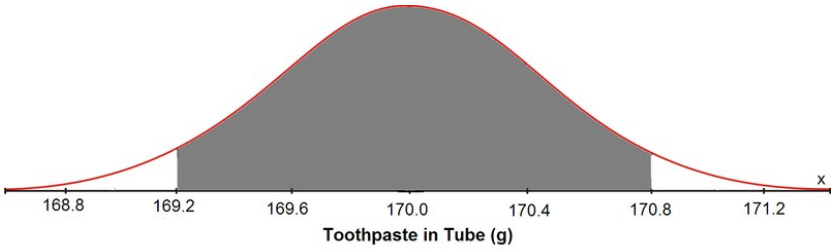
5. The values within 2 standard deviations of the mean are from 16.8 – (2 × 0.7) years to 16.8 + (2 × 0.7) years, or from 15.4 years to 18.2 years.

6. The values within 3 standard deviations of the mean are from 16.8 – (3 × 0.7) years to 16.8 + (3 × 0.7) years, or from 14.7 years to 18.9 years.

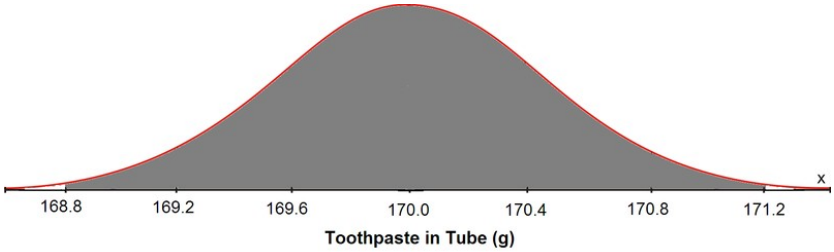
7. The graph should appear as follows:




8. The graph should appear as follows:




9. The graph should appear as follows:




10. Answers will vary, but the actual percentages are 68% for the values within 1 standard deviation of the mean, 95% for the values within 2 standard deviations of the mean, and 99.7% for the values within 3 standard deviations of the mean.

## 6.2 Variance of a Data Set

**Answers**

1. Answer:

TABLE 6.1:

| | Data $(x)$ | Mean $(\mu)$ | Data − Mean $(x-\mu)$ | Square of Data − Mean $(x-\mu)^2$ |
|---|---|---|---|---|
| | 5 | 6.375 | −1.375 | 1.8906 |
| | 8 | 6.375 | 1.625 | 2.6406 |
| | 9 | 6.375 | 2.625 | 6.8906 |
| | 10 | 6.375 | 3.625 | 13.1406 |
| | 4 | 6.375 | −2.375 | 5.6406 |
| | 3 | 6.375 | −3.375 | 11.3906 |
| | 7 | 6.375 | 0.625 | 0.3906 |
| | 5 | 6.375 | −1.375 | 1.8906 |
| $\sum$ | 51 | | | 43.8748 |

$$\mu = \frac{\sum x}{n} = \frac{51}{8} = 6.375$$
$$\sigma^2 = \frac{\sum (x-\mu)^2}{n}$$
$$\sigma^2 = \frac{43.8748}{8}$$
$$\sigma^2 = 5.48435$$

2. Answer:

$$\bar{x} = \frac{\sum x}{n} = \frac{51}{8} = 6.375$$
$$s^2 = \frac{\sum (x-\bar{x})^2}{n-1}$$
$$s^2 = \frac{43.8748}{7}$$
$$s^2 = 6.27$$

3. Answer:

TABLE 6.2:

| Data $(x)$ | Mean $(\mu)$ | Data − Mean $(x-\mu)$ | Square of Data − Mean $(x-\mu)^2$ |
|---|---|---|---|
| 11 | 14.7 | −3.7 | 13.69 |
| 15 | 14.7 | 0.3 | 0.09 |
| 16 | 14.7 | 1.3 | 1.69 |
| 12 | 14.7 | −2.7 | 7.29 |

| | | | | |
|---|---|---|---|---|
| | 19 | 14.7 | 4.3 | 18.49 |
| | 17 | 14.7 | 2.3 | 5.29 |
| | 14 | 14.7 | −0.7 | 0.49 |
| | 18 | 14.7 | 3.3 | 10.89 |
| | 15 | 14.7 | 0.3 | 0.09 |
| | 10 | 14.7 | −4.7 | 22.09 |
| $\Sigma$ | 147 | | | 80.1 |

$$\mu = \frac{\Sigma x}{n} = \frac{147}{10} = 14.7$$
$$\sigma^2 = \frac{\Sigma (x-\mu)^2}{n}$$
$$\sigma^2 = \frac{80.1}{10}$$
$$\sigma^2 = 8.01$$

4. Answer:

$$\bar{x} = \frac{\Sigma x}{n} = \frac{147}{10} = 14.7$$
$$s^2 = \frac{\Sigma (x-\bar{x})^2}{n-1}$$
$$s^2 = \frac{80.1}{9}$$
$$s^2 = 8.9$$

5. Answer:

TABLE 6.3:

| | **Data** $(x)$ | **Mean** $(\mu)$ | **Data − Mean** $(x-\mu)$ | **Square of Data − Mean** $(x-\mu)^2$ |
|---|---|---|---|---|
| | 55 | 56.2 | −1.2 | 1.44 |
| | 54 | 56.2 | −2.2 | 4.84 |
| | 48 | 56.2 | −8.2 | 67.24 |
| | 52 | 56.2 | −4.2 | 17.64 |
| | 69 | 56.2 | 12.8 | 163.84 |
| | 60 | 56.2 | 3.8 | 14.44 |
| | 47 | 56.2 | −9.2 | 84.64 |
| | 66 | 56.2 | 9.8 | 96.04 |
| | 50 | 56.2 | −6.2 | 38.44 |
| | 61 | 56.2 | 4.8 | 23.04 |
| $\Sigma$ | 562 | | | 511.6 |

$$\mu = \frac{\Sigma x}{n} = \frac{562}{10} = 56.2$$
$$\sigma^2 = \frac{\Sigma (x-\mu)^2}{n}$$

$$\sigma^2 = \frac{511.6}{10}$$
$$\sigma^2 = 51.16$$

6. Answer:

$$\bar{x} = \frac{\sum x}{n} = \frac{562}{10} = 56.2$$
$$s^2 = \frac{\sum (x - \bar{x})^2}{n - 1}$$
$$s^2 = \frac{511.6}{9}$$
$$s^2 = 56.84$$

7. Answer:

TABLE 6.4:

| | Data $(x)$ | Mean $(\bar{x})$ | Data – Mean $(x - \bar{x})$ | Square of Data – Mean $(x - \bar{x})^2$ |
|---|---|---|---|---|
| | 26 | 25.5 | 0.5 | 0.25 |
| | 30 | 25.5 | 4.5 | 20.25 |
| | 20 | 25.5 | −5.5 | 30.25 |
| | 27 | 25.5 | 1.5 | 2.25 |
| | 23 | 25.5 | −2.5 | 6.25 |
| | 33 | 25.5 | 7.5 | 56.25 |
| | 19 | 25.5 | −6.5 | 42.25 |
| | 26 | 25.5 | 0.5 | 0.25 |
| $\sum$ | 204 | | | 158 |

$$\bar{x} = \frac{\sum x}{n} = \frac{204}{8} = 25.5$$
$$s^2 = \frac{\sum (x - \bar{x})^2}{n - 1}$$
$$s^2 = \frac{158}{7}$$
$$s^2 = 22.57$$

8. Answer:

$$\mu = \frac{\sum x}{n} = \frac{204}{8} = 25.5$$
$$\sigma^2 = \frac{\sum (x - \mu)^2}{n}$$
$$\sigma^2 = \frac{158}{8}$$

$\sigma^2 = 19.75$

9. Answer:

TABLE 6.5:

| | Data $(x)$ | Mean $(\bar{x})$ | Data − Mean $(x-\bar{x})$ | Square of Data − Mean $(x-\bar{x})^2$ |
|---|---|---|---|---|
| | 85 | 94.25 | −9.25 | 85.5625 |
| | 99 | 94.25 | 4.75 | 22.5625 |
| | 89 | 94.25 | −5.25 | 27.5625 |
| | 90 | 94.25 | −4.25 | 18.0625 |
| | 104 | 94.25 | 9.75 | 95.0625 |
| | 82 | 94.25 | −12.25 | 150.0625 |
| | 95 | 94.25 | 0.75 | 0.5625 |
| | 110 | 94.25 | 15.75 | 248.0625 |
| $\sum$ | 754 | | | 647.5 |

$$\bar{x} = \frac{\sum x}{n} = \frac{754}{8} = 94.25$$
$$s^2 = \frac{\sum (x-\bar{x})^2}{n-1}$$
$$s^2 = \frac{647.5}{7}$$
$$s^2 = 92.5$$

10. Answer:

$$\mu = \frac{\sum x}{n} = \frac{754}{8} = 94.25$$
$$\sigma^2 = \frac{\sum (x-\mu)^2}{n}$$
$$\sigma^2 = \frac{647.5}{8}$$
$$\sigma^2 = 80.9375$$

## 6.3 Standard Deviation of a Data Set

**Answers**

1. Answer:

TABLE 6.6:

| Data $(x)$ | Mean $(\mu)$ | Data − Mean $(x-\mu)$ | Square of Data − Mean $(x-\mu)^2$ |
|---|---|---|---|
| 71 | 74.4 | −3.4 | 11.56 |
| 73 | 74.4 | −1.4 | 1.96 |
| 77 | 74.4 | 2.6 | 6.76 |
| 69 | 74.4 | −5.4 | 29.16 |
| 80 | 74.4 | 5.6 | 31.36 |
| 67 | 74.4 | −7.4 | 54.76 |

| | | | | |
|---|---|---|---|---|
| | 78 | 74.4 | 3.6 | 12.96 |
| | 77 | 74.4 | 2.6 | 6.76 |
| | 70 | 74.4 | −4.4 | 19.36 |
| | 82 | 74.4 | 7.6 | 57.76 |
| Σ | 744 | | | 232.4 |

$$\mu = \frac{\Sigma x}{n} = \frac{744}{10} = 74.4$$
$$\sigma = \sqrt{\frac{\Sigma(x-\mu)^2}{n}}$$
$$\sigma = \sqrt{\frac{232.4}{10}}$$
$$\sigma = \sqrt{23.24}$$
$$\sigma = 4.82$$

2. Answer:

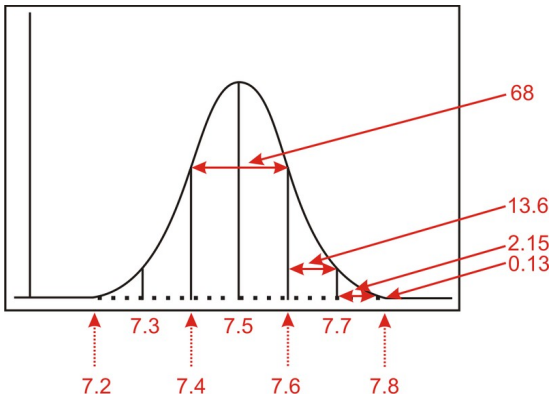
TABLE 6.7:

| | Data ($x$) | Mean ($\bar{x}$) | Data − Mean ($x - \bar{x}$) | Square of Data − Mean $(x-\bar{x})^2$ |
|---|---|---|---|---|
| | 55 | 57 | −2 | 4 |
| | 48 | 57 | −9 | 81 |
| | 65 | 57 | 8 | 64 |
| | 70 | 57 | 13 | 169 |
| | 48 | 57 | −9 | 81 |
| | 59 | 57 | 2 | 4 |
| | 67 | 57 | 10 | 100 |
| | 44 | 57 | −13 | 169 |
| Σ | 456 | | | 672 |

$$\bar{x} = \frac{\Sigma x}{n} = \frac{456}{8} = 57$$
$$s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$$
$$s = \sqrt{\frac{672}{7}}$$
$$s = \sqrt{96}$$
$$s = 9.80$$

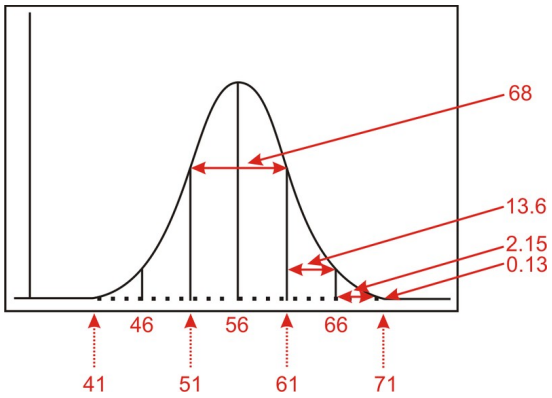
3. B
4. D
5. B
6. D
7. C
8. B

9. Answer:




a. 68% of the volumes can be found between 7.4 oz and 7.6 oz.
b. 95% of the volumes can be found between 7.3 oz and 7.7 oz.
c. 99.7% of the volumes can be found between 7.2 oz and 7.8 oz.

10. Answer:




a. 68% of the heights can be found between 51" and 61".
b. 95% of the heights can be found between 46" and 66".
c. 99.7% of the heights can be found between 41" and 71".

## 6.4 Applications of Variance and Standard Deviation

**Answers**

1. D

2. A

3. D

4. B

5. C

```
1-Var Stats
 x̄=40.4
 Σx=202
 Σx²=8242
 Sx=4.50555213
 σx=4.029888336
↓n=5
```

The variance is 4.0298883362 = 16.24.

6. D

```
1-Var Stats
 x̄=123.2
 Σx=616
 Σx²=76226
 Sx=9.148770409
 σx=8.182909018
↓n=5
```

The variance is 9.1487704092 = 83.7.

7. A

```
1-Var Stats
 x̄=6.55
 Σx=26.2
 Σx²=223.8
 Sx=4.170931151
 σx=3.612132334
↓n=4
```

8. B

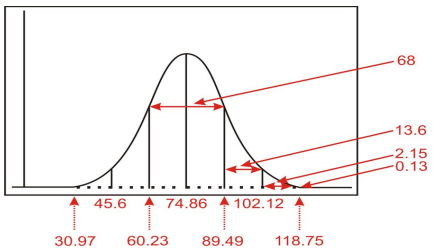
```
1-Var Stats
 x̄=37.75
 Σx=151
 Σx²=5802.3
 Sx=5.832380875
 σx=5.050990002
↓n=4
```

9. Answer:

```
1-Var Stats
 x̄=74.86363636
 Σx=1647
 Σx²=128013
 Sx=14.98029008
 σx=14.63586956
↓n=22
```

```
1-Var Stats
↑n=22
 minX=48
 Q1=64
 Med=77
 Q3=89
 maxX=98
```

a. The mean is 74.86.
b. The standard deviation is 14.63.
c. The variance is 14.635869562 = 214.21.
d. The normal distribution curve is shown below:

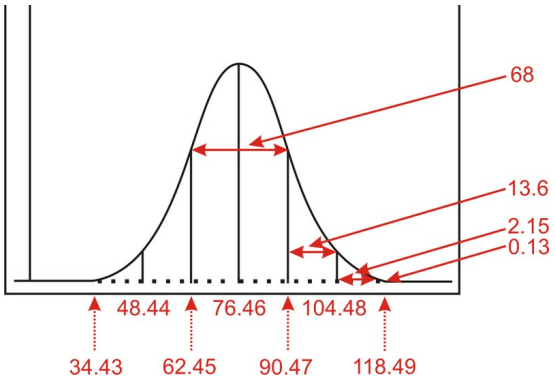



10. Answer:

```
1-Var Stats
 x̄=76.46153846
 Σx=1988
 Σx²=157112
 Sx=14.29190196
 σx=14.01436288
↓n=26
```

```
1-Var Stats
↑n=26
 minX=38
 Q1=69
 Med=80.5
 Q3=87
 maxX=93
```

a. The mean is 76.46.
b. The standard deviation is 14.01.
c. The variance is 14.014362882 = 196.40.
d. The normal distribution curve is shown below:




## 6.5 Empirical Rule

**Answers**

1. The answer can be calculated as follows: 34% + 34% + 13.5% = 81.5%

2. The answer can be calculated as follows: 2.35% + 13.5% + 34% + 34% = 83.85%

3. The answer can be calculated as follows: 2.35% + 0.15% = 2.5%

4. The answer can be calculated as follows: 13.5% + 34% + 34% + 13.5% + 2.35% = 97.35%

5. The answer can be calculated as follows: 2.35% + 13.5% + 34% = 49.85%

6. The answer can be calculated as follows: 13.5% + 2.35% + 0.15% = 16%

7. The answer can be calculated as follows: 34% + 13.5% = 47.5%

8. a. The percentage of students waiting more than 11.5 minutes would be 68% + 13.5% + 2.35% +

   0.15% = 84% of the students surveyed. 84% of 200 students = $0.84 \times 200 = 168$ students

   b. The percentage of students waiting more than 18.5 minutes would be 13.5 + 2.35 + 0.15 = 16% of the students surveyed. 16% of 200 students = $0.16 \times 200 = 32$ students

   c. The percentage of students waiting between 11.5 and 18.5 minutes would be 68% of the students surveyed. 68% of 200 students = $0.68 \times 200 = 136$ students

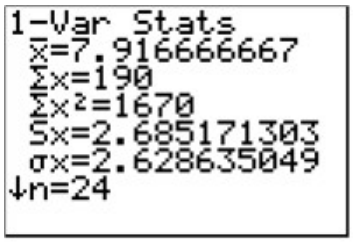
surveyed. 68% of 200 students = 0.68 × 200 = 136 students

9. a. The percentage of babies weighing more than 7.3 lbs would be 13.5 + 2.35 + 0.15 = 16% of the babies in the survey. 16% of 350 babies = 0.16 × 350 = 56 babies

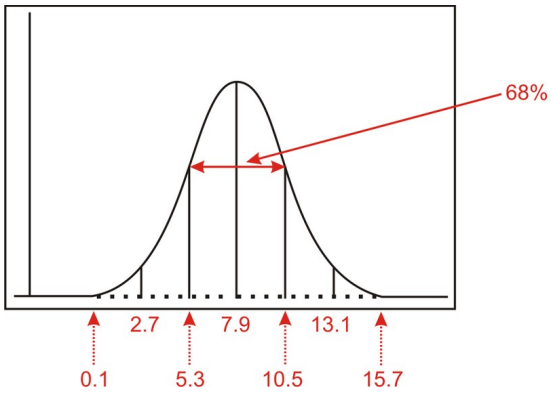
b. The percentage of babies weighing more than 7.8 lbs would be 2.35 + 0.15 = 2.5% of the babies in the survey. 2.5% of 350 babies = 0.025 × 350 = 9 babies

c. The percentage of babies weighing between 6.3 and 7.8 lbs minutes would be 68% + 13.5% = 81.5% of the babies in the survey. 81.5% of 350 babies = 0.815 × 350 = 285 babies

10. Answer:




You can use the data from the 1-Var Stats calculation to draw the normal distribution curve.




The range of the differences in heights of the seedlings for the middle 68% of the data is 5.3 inches to 10.5 inches.