7.1 Tangent Ratio

Answers

1. Right triangles with $40°$ angles have two pairs of congruent angles and therefore are similar. This means that the ratio of the opposite leg to adjacent leg is constant for all $40°$ right triangles, and this is the tangent ratio of $40°$.

2. $\tan 40° \approx 0.839$

3. $x = \frac{22}{\tan 40°} \approx 26.22$

4. $\tan 80° \approx 5.671$

5. $x = 2\tan 80° \approx 11.343$

6. $\tan 10° = 0.176$

7. $x = \frac{2}{\tan 10°} \approx 11.343$

8. The triangle was the same. If a right triangle has an $80°$ angle then it also has a $10°$ angle.

9. $\tan 27° \approx 0.51$

10. $\mathrm{x} = 13\tan 27° \approx 6.624$

11. $\tan 42° \approx 0.9$

12. $x \approx 45$

13. $A = 11.26\, in^2$

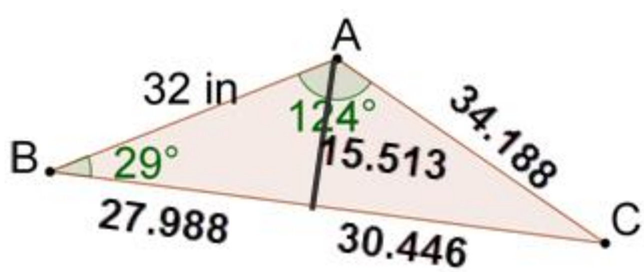
14. $\tan 30° \approx 0.577.\ \frac{1}{\sqrt{3}} \approx 0.577$ so it matches.

15. As the angle increases the length of the opposite side will increase. Therefore, $\frac{opposite\ leg}{adjacent\ leg}$ will increase.

7.2 Sine and Cosine Ratios

Answers

1. $\sin E = \frac{12}{13}$
2. $\cos E = \frac{5}{13}$
3. $\tan E = \frac{12}{5}$
4. $\sin F = \frac{5}{13}$
5. $\cos F = \frac{12}{13}$
6. $\tan F = \frac{5}{12}$
7. sine, 10.765
8. tangent, 17.968
9. cosine, 2.736
10. sine, 7.713
11. cosine, 5.26
12. tangent, 17.138
13. $m\angle C = 27°$
14.



15. The perimeter is approximately 125 inches.
16. True. The trigonometric ratios only give you the ratios between the sides, not the lengths of the sides.
17. Sine, cosine, and tangent.
18. The trigonometric ratios exist because all right triangles with a given angle are similar.

7.3 Sine and Cosine of Complementary Angles

Answers

1. They are complementary.
2. Sine and cosine of complementary angles are equal.
3. Tangents of complementary angles are reciprocals.
4. $\tan B = 2$
5. $\cos B = \frac{7}{10}$
6. $\sin B = \frac{1}{4}$
7. $\cos B = \frac{3}{5}$
8. $\sin A$
9. $\tan B = \frac{3}{2}$
10. $\tan A = 5$. $\angle A$ is bigger, because the tangent of $\angle A$ is bigger.
11. $\theta = 60°$
12. $\theta = 15°$
13. $\theta = 48°$
14. $\theta = 72°$
15. $\theta = 41°$

7.4 Inverse Trigonometric Ratios

Answers

1. $\sin^{-1}$ is inverse sine and allows you to find a missing angle while sin is sine and allows you to find a missing side length.
2. You use regular trigonometric ratios to find the length of a missing side and inverse trigonometric ratios to find the measure of a missing angle.
3. $23.58°$
4. $78.69°$
5. $64.85°$
6. $53.13°$
7. $60.07°$
8. $53.13°$
9. $30°$
10. You could have noticed that the hypotenuse is twice the length of one of the legs, making the triangle a 30-60-90 triangle.
11. $m\angle A = 60.95°, m\angle B = 29.05°, AB = 10.30$
12. $AC = 10.20, m\angle A = 67.81°, m\angle B = 22.2°$
13. $AB = 37.74, m\angle A = 32°, m\angle B = 58°$
14. Their sum will be $90°$ because the angles will be complementary.
15. Their sum will be $90°$ because the angles will be complementary.

7.5 Sine to find the Area of a Triangle

Answers

1. $A \approx 15.73\ un^2$
2. $A \approx 19.99\ un^2$
3. $A \approx 44.12\ un^2$
4. $A \approx 49.50\ un^2$
5. Because $\sin 45° = \sin 135°$ and the lengths of the sides are the same.
6. $A \approx 17.13\ un^2$
7. $A \approx 55.51\ un^2$
8. $A \approx 44.52\ un^2$
9. $\sin 90° = 1$
10. The two formulas are both the same calculation of $A = \frac{1}{2}(12)(10)$ since $\sin 90° = 1$. $A = 60\ un^2$.
11. $A \approx 200.29\ un^2$
12. $A = absin\theta$ where a and b are the two sides of the parallelogram and θ is any of the angles.
13. 72°
14. If b is the base, then $\sin C = \frac{h}{a}$. This means $h = \text{a} \sin C$ and $Area = \frac{1}{2}(b)(\text{a} \sin C)$. Therefore, $= \frac{1}{2}ab \sin C$.
15. Because in general, $\sin\theta = \sin(180 - \theta)$. While the acute exterior angle of $180 - \theta$ is more closely related to the height of the triangle (see Example C), because $\sin\theta = \sin(180 - \theta)$, you can use the given obtuse angle and obtain the same area.

7.6 Law of Sines

Answers

1. $AC = 20, m\angle C = 40°, AB = 16.1$
2. $AC = 20.05, m\angle C = 106°, AB = 32.79$
3. $AC = 38.2, m\angle C = 26°, AB = 16.96$
4. $AC = 38.24, m\angle C = 51°, AB = 38.8$
5. $AC = 28.5, m\angle C = 100°, AB = 35.7$
6. $AC = 21.33, m\angle C = 37°, AB = 13$
7. $AC = 27.2, m\angle C = 29°, AB = 14.04$
8. $AC = 23.79, m\angle C = 37°, AB = 14.35$
9. When using the Law of Sines to solve for a missing angle, you are presented with SSA. In these types of problems, you are given two sides and a non-included angle and are asked to find the measure of one of the other angles in the triangle.
10. $m\angle A = 110°, m\angle C = 42°$
11. $m\angle A = 48°, m\angle C = 105°$
12. $m\angle A = 49°, m\angle C = 49°$
13. $h = c\sin A$ and $h = a\sin C$ so $c\sin A = a\sin C$. Therefore, $\frac{\sin A}{a} = \frac{\sin C}{c}$.
14. Given two angles and one of the opposite sides, solve for all other angles and sides. Or, given two sides and one opposite angle, solve for other angles (have to be careful with this type).
15. You are not given an opposite angle/side pair.

7.7 Law of Cosines

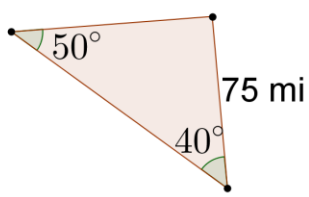
Answers

1. $a^2 + b^2 - 2ab\cos C = c^2$
2. C must be the included angle of sides a and b. c is opposite angle C.
3. $a^2 + b^2 - 2ab\cos 90° = c^2$. $\cos 90° = 0$, so this reduces to $a^2 + b^2 = c^2$.
4. $x = 28.6$
5. $x = 9.19$
6. $\theta = 26.6°$
7. $x = 17.4$
8. $x = 13.6$ or $x = 15$. You get two possible solutions because the given information is SSA and there are multiple triangles that fit the SSA criterion (remember that it is not a method for proving triangles are congruent).
9. $\theta = 31.1°$
10. $\cos 30 \approx 0.87$ and $\cos 150 = -0.87$
11. $\cos 80 \approx 0.17$ and $\cos 100 = -0.17$
12. The cosine of supplementary angles are opposites. $\cos\theta = \cos(180 - \theta)$.
13. $h^2 + x^2 = a^2$ and $(x + b)^2 + h^2 = c^2$
14. Solve both equations for h^2 and set them equal to each other. $a^2 - x^2 = c^2 - (b + x)^2$. Expand and rewrite to get $a^2 + b^2 + 2bx = c^2$.
15. $\cos\theta = \frac{x}{a}$, so $x = \text{a}\cos\theta$. θ and $\angle C$ are supplementary angles, so $\theta = 180 - C$. $x = a\cos(180 - \theta)$. $a^2 + b^2 + 2ab\cos(180 - C) = c^2$
16. $a^2 + b^2 + 2ab\,(-\cos C) = c^2$ -> $a^2 + b^2 - 2ab\cos C = c^2$

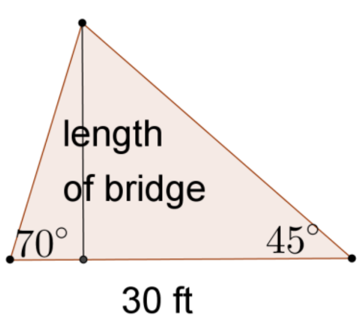
7.8 Triangles in Applied Problems

Answers

1. Pictures vary.
2. You have a right triangle. With alternate interior angles you know one angle in the triangle is $25°$. You also know the height of the triangle. You can use tangent to find the length of the base of the triangle.
3. $428.9\ ft$
4.



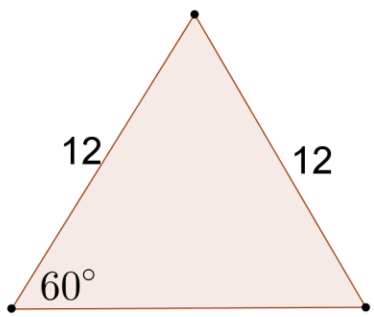
5. You are looking for the sum of the lengths of the two missing sides. Find the missing angle and then use the Law of Sines to find the missing sides.
6. The missing angle turns out to be $90°$ so you can use basic trigonometric ratios or the Law of Sines to find the missing sides. The two sides are $62.9\ mi$ and $97.9\ mi$. The length of the detour was $160.8\ mi$.
7.



8. Find the missing angle and then use Law of Sines to find the lengths of the missing sides. Use trigonometric ratios to find the length of the bridge.

9. From one point the end of the bridge is 23.4 feet away and from the second point the bridge is 31.1 feet away. The bridge is 22 feet long.

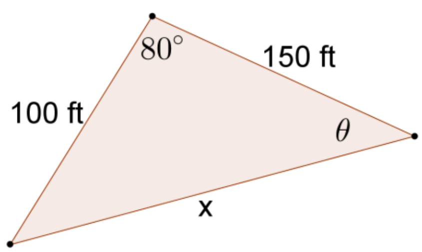
10.



11. The triangle must be equilateral so all angles are $60°$ and all sides are length 12.

12. The area is $36\sqrt{3} \approx 62.4\ un^2$

13.



14. You can use the Law of Cosines to find the measure of x and then the Law of Sines to find the measure of θ.

15. She must walk 165.2 ft to get back to her starting position. She must turn right at a $36.4°$ angle.