8.1 Circles and Similarity

Answers

1. Translate circle A one unit left and 11 units down. The, dilate about its center by a scale factor of $\frac{3}{4}$.

2. Translate circle A 11 units to the left and 2 units up. Then, dilate about its center by a scale factor of $\frac{5}{3}$.

3. Translate circle A two units to the right and 6 units down. Then, dilate about its center by a scale factor of $\frac{7}{2}$.

4. Translate circle A two units to the left and 5 units down. Then, dilate about its center by a scale factor of $\frac{4}{3}$.

5. Translate circle A 1 unit to the right and 14 units down. Then, dilate about its center by a scale factor of 2.

6. Dilate circle A about its center by a scale factor of $\frac{5}{4}$.

7. Translate circle A 5 units to the right and 10 units up. Then, dilate about its center by a scale factor of 4.

8. Translate circle A 6 units to left and one unit down. Then, dilate about its center by a scale factor of $\frac{8}{5}$.

9. Dilate circle A about its center by a scale factor of $\frac{6}{5}$.

10. Translate circle A 8 units to the right and 10 units down.

11. $\frac{2}{3}$

12. $\frac{\sqrt{6}}{1}$

13. $\frac{25}{81}$

14. $\frac{2\sqrt{3}}{\sqrt{5}}$

15. Any reflection or rotation on a circle could more simply be a translation. Therefore, reflections and rotations are not necessary when looking to prove that two circles are similar.

8.2 Area and Circumference of Circles

Answers

1. $\frac{360°}{2n} = \frac{180}{n}$

2. $2\sin\frac{180}{n}$

3. $\cos\frac{180}{n}$

4. $\sin\frac{180}{n} \cdot \cos\frac{180}{n}$

5. $n \cdot \sin\frac{180}{n} \cdot \cos\frac{180}{n}$

6. $2n\sin\frac{180}{n}$

7. $A = 3.141592; P = 6.2831852$

8. $A = 3.1415926; P = 6.2831853$. It makes sense that the area of a circle with radius 1 unit is π and the circumference of a circle with radius 1 unit is 2π.

9. The polygon gets closer and closer to the circle so its area gets closer and closer to the area of the circle.

10. The polygon gets closer and closer to the circle so its perimeter gets closer and closer to the circumference of the circle.

11. The scale factor for a circle with radius 1 and a circle with radius r is $k = r$. Therefore, the ratio of their circumferences is $r: 1$. Since the circumference of the circle with radius 1 is 2π, the circumference of the circle with radius r is $2\pi r$.

12. The ratio of the areas is $9: 25$. The ratio of the circumferences is $3: 5$.

13. 5 units.

14. 6 units.

15. $\approx 14.31\pi$

8.3 Central Angles and Chords

Answers

1. Answers vary.

2. They have the same measure.

3. Answers vary.

4. Diameter

5. $m\widehat{FE} = 30°$.

6. $m\widehat{CD} = 130°$.

7. $m\angle EAG = 75°$

8. $m\widehat{GB} = 75°$

9. $\overline{AG}$ is the perpendicular bisector of $\overline{EB}$.

10. $\overline{EF} \cong \overline{FD}$ by assumption, $\overline{EA} \cong \overline{AD}$ because they are both radii of the circle, and $\overline{AF} \cong \overline{AF}$ by the reflexive property. Therefore, $\Delta AFE \cong \Delta AFD$ by $SSS \cong$. $\angle AFE$ and $\angle AFD$ are both congruent (corresponding parts of congruent triangles) and supplementary, so they must both be right angles. Therefore, $m\angle AFD = 90°$.

11. ΔAFE and ΔAFD are both right triangles by assumption, $\overline{EA} \cong \overline{AD}$ because they are both radii of the circle, and $\overline{AF} \cong \overline{AF}$ by the reflexive property. Therefore, $\Delta AFE \cong \Delta AFD$ by $HL \cong$. $\overline{EF} \cong \overline{FD}$ because they are corresponding parts.

12. $DF = 8$

13. $AC = 12$

14. $AF \approx 8.94$

15. $CF \approx 3.06$

8.4 Inscribed Angles

Answers

1. If an inscribed angle and a central angle intercept the same arc, the measure of the inscribed angle will be half the measure of the central angle.

2. $120°$

3. $30°$

4. $60°$

5. $30°$

6. Equilateral

7. $x = 7.2$

8. $x = 50°$

9. $x = 3$

10. $x = 42.5°$

11. $x = 29°$

12. $x = 29°$

13. $x = 6\sqrt{2}$

14 . $x = 43°$

15. $\overline{BD} \parallel \overline{EC}$, so alternate interior angles are congruent. This means that $\angle BEC \cong \angle DCE$ and thus $m\angle BEC = m\angle DCE$. $m\widehat{BC} = 2m\angle BEC$ and $m\widehat{DE} = 2m\angle DCE$. By substitution, $m\widehat{BC} = m\widehat{DE}$ and $\widehat{BC} \cong \widehat{EC}$.

8.5 Inscribed and Circumscribed Circles of Triangles

Answers

1-3: Answers vary. See Examples A-C for help.

4. The third angle bisector will intersect in the same point of intersection as the first two angle bisectors. The third angle bisector does not provide any new information.

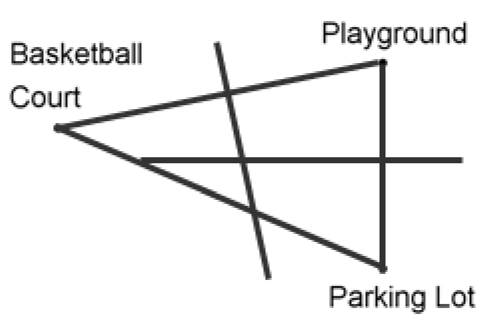
5. The distance between a point and a line is the length of the segment perpendicular to the line that passes through the point. The segments perpendicular to each of the sides of the triangle that pass through the incenter are radii of the inscribed circle. Therefore, the incenter is equidistant from each of the sides of the triangle.

6-7: Answers vary. See Guided Practice for help.

8. The circumcenter is the center of the circumscribed circle and each of the three vertices are on the circle. From the circumcenter to each of the vertices is a radius, so the distance from the circumcenter to each of the vertices is the same.

9. circumcenter

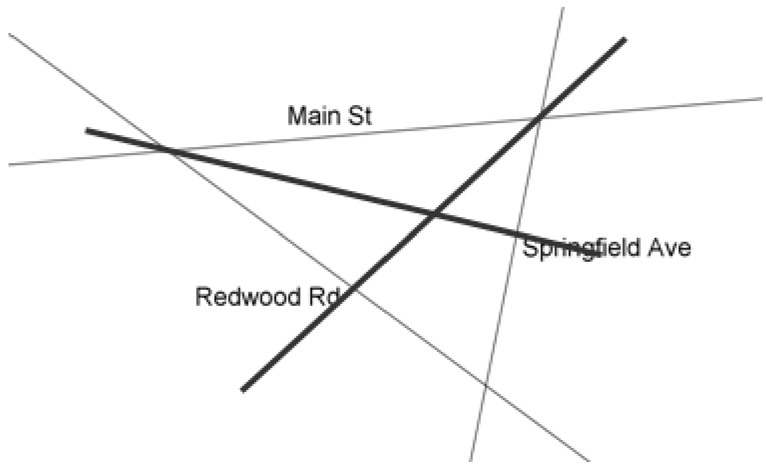
10. Construct the circumcenter by drawing a triangle and finding the point of intersection of the perpendicular bisectors.



11. Fold the map so the playground overlaps with the basketball court and make a crease. This should be the perpendicular bisector of the line segment connecting those two locations. Fold the map again so the playground overlaps with the parking lot and make a crease. The point where the creases intersect is the circumcenter.

12. incenter

13. Construct the incenter by following the steps from the Examples.



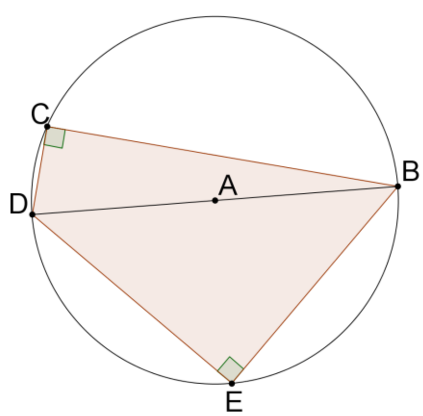
14. Fold the map so Main St overlaps with Redwood Rd and make a crease. This should be the angle bisector of the angle formed by those two roads. Fold the map again so Main St overlaps with Springfield Ave and make a crease. The point where the creases intersect is in incenter.

15. Any three non-collinear points define a triangle. All triangles have exactly one circumcenter and therefore all triangles have exactly one circumscribed circle. The circumscribed circle will be the circle that passes through the three points.

8.6 Quadrilaterals Inscribed in Circles

Answers

1. A cyclic quadrilateral is a quadrilateral that can be inscribed in a circle.
2. supplementary
3. $127°$
4. $92°$
5. $53°$
6. $60°$
7. $60°$
8. $60°$
9. $120°$
10. $x = 15$
11. $y = 109$
12. $x = 90$
13. $x = 90$
14. No, but two opposite vertices will create a diameter of the circle, and it will have two right angles.



15. First note that $m\widehat{CDE} + m\widehat{CBE} = 360°$ because these two arcs make a full circle. $2m\angle B = m\widehat{CDE}$ and $2m\angle D = m\widehat{CBE}$ because the measure of an inscribed angle is half the measure of its intercepted arc. By substitution, $2m\angle B + 2m\angle D = 360°$. Divide by 2 and you have $m\angle B + m\angle D = 180°$. Therefore, $\angle B$ and $\angle D$ are supplementary.

8.7 Tangent Lines to Circles

Answers

1. A tangent line is a line that intersects a circle exactly once.

2. $AP = 5$

3. $AC \approx 10.3$

4. $m\angle CAQ = 61°$

5. $QC = 9.62$

6. $AQ = 5.33$

7. $PC = 9.62$

8. $m\widehat{PQ} = 140°$

9. $m\widehat{PEQ} = 220°$

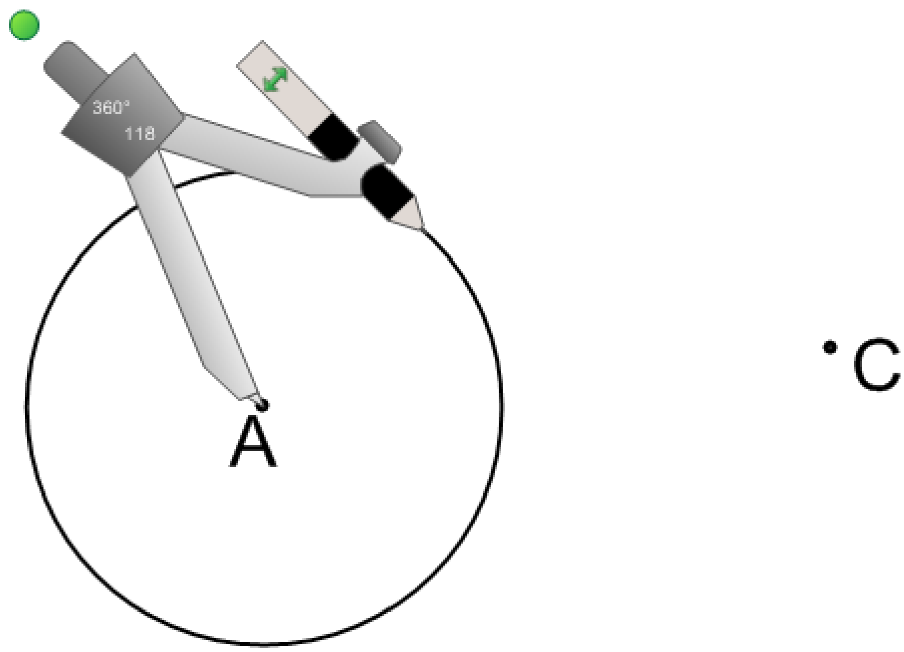
10. $223.2°$

11. $43.2°$

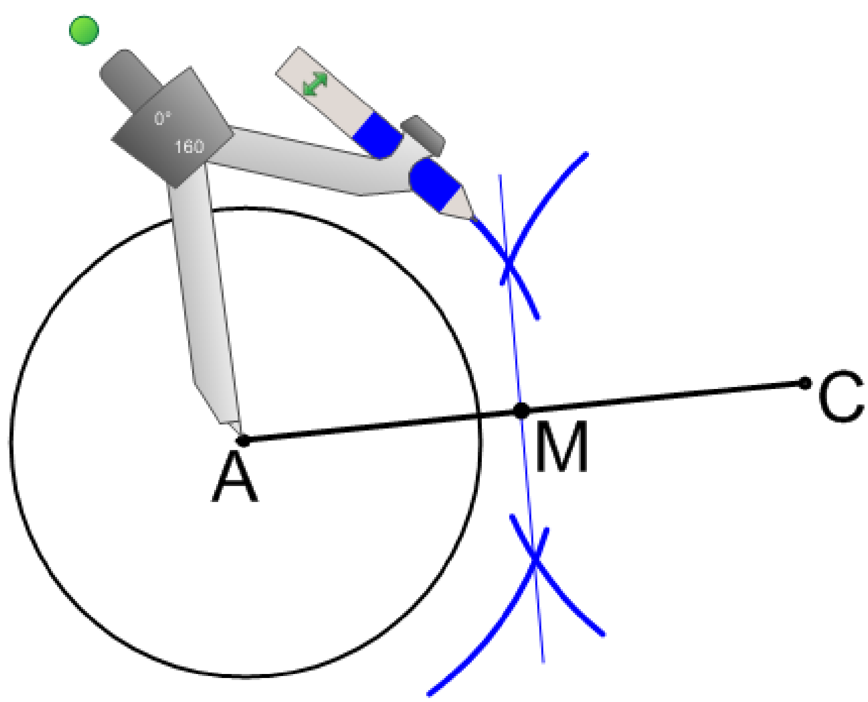
12. $\Delta ABI \sim \Delta HGI$

13. $\angle ABI$ and $\angle HGI$ are right angles, so $\angle ABI \cong \angle HGI$. $\angle AIB$ and $\angle HIG$ are vertical angles, so $\angle AIB \cong \angle HIG$. $\Delta ABI \sim \Delta HGI$ by $AA\sim$.

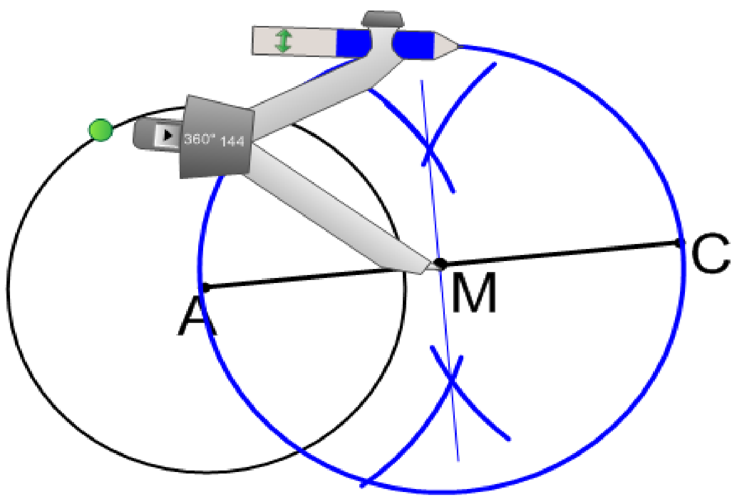
14.



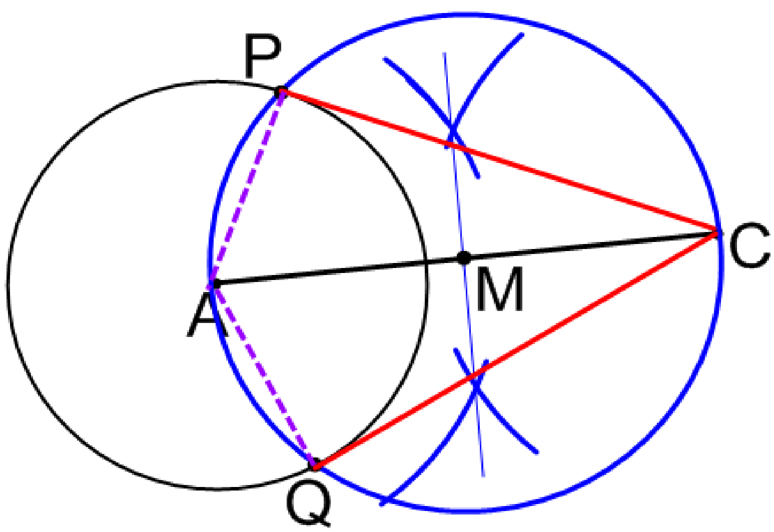
Construct the perpendicular bisector of $\overline{AC}$ in order to find its midpoint.



Then construct a circle centered at point M that passes through point C. The circle should also pass through point A.



Find the points of intersection and connect them with point C.



15. $\overline{AC}$ is a diameter of circle M, so it divides circle M into two semicircles. $\angle APC$ and $\angle AQC$ are inscribed angles of these semicircles, so they must be right angles. $\overline{PC}$ meets radius $\overline{AP}$ at a right angle, so $\overline{PC}$ is tangent to circle A. Similarly, $\overline{QC}$ meets radius $\overline{AQ}$ at a right angle, so $\overline{QC}$ is tangent to circle A.

8.8 Secant Lines to Circles

Answers

1. A secant intersects a circle in two points while a tangent intersects a circle in one point.
2. $\theta = 94°$
3. $x = 9.6$
4. $\theta = 50°$
5. $x \approx 7.3$
6. $\theta = 47.5°$
7. $x \approx 2.3$
8. Both angles are equal to $\frac{m\widehat{EB}}{2}$.
9. $\angle FEB \cong \angle EHB$ and both triangles share $\angle EFB$, so $\Delta EHF \sim \Delta BEF$ by $AA\sim$.
10. Because $\Delta EHF \sim \Delta BEF$, corresponding sides are proportional. This means $\frac{FB}{FE} = \frac{FE}{FH}$, so $FB \cdot FH = FE^2$.
11. $m\angle EBH = \frac{m\widehat{HGE}}{2}$ and $m\angle FEB = \frac{m\widehat{BE}}{2}$ because they are inscribed angles. $m\angle EBH = \angle FEB + \angle BFE$ because the measure of an exterior angle of a triangle is equal to the sum of the measures of the remote interior angles. By substitution, $\frac{m\widehat{HGE}}{2} = \frac{m\widehat{BE}}{2} + m\angle BFE$.
12. Solve the result from #11 for $m\angle BFE$ and rewrite. $\frac{m\widehat{HGE} - m\widehat{BE}}{2} = m\angle BFE$.
13. In both theorems, the measure of the angle of intersection is equal to half the difference of the measures of the intercepted arcs.
14. $x \approx 10.2$
15. $x = 47$

8.9 Arc Length

Answers

1. The measure of an arc is an angle measurement equal to the measure of the central angle. The length of an arc is a distance measurement that will depend on the size of the circle.

2. A radian is the measure of an arc with a length of 1 radius.

3. 2π radians is equal to $360°$.

4. You can translate, then rotate, then dilate the red sector to match the blue sector. A similarity transformation exists so the sectors must be similar.

5. If θ is in radians then θ is equal to the number of radii that fit around the arc. The number of radii that fit around the arc multiplied by the length of the radius will equal the length of the arc.

6. π

7. 2π

8. $\frac{\pi}{2}$

9. $\frac{\pi}{3}$

10. $\frac{\pi}{6}$

11. $m\widehat{CD} = 90°$. Length of $CD = \pi$ cm.

12. $m\widehat{CD} = 120°$. Length of $CD = \frac{16\pi}{3}$ cm.

13. $m\widehat{CD} = 60°$. Length of $CD = \frac{5\pi}{3}$ cm.

14. $m\widehat{CD} = 100°$. Length of $CD = \frac{60\pi}{9}$ cm.

15. When given the central angle in radians, multiply the length of the radius by the central angle. When given the central angle in degrees, first multiply it by $\frac{\pi}{180}$, and then multiply by the length of the radius.

8.10 Sector Area

Answers

1. The area of a whole circle is πr^2 and a sector with a central angle of θ represents $\frac{\theta}{2\pi}$ of the whole circle. Therefore, the area of the sector is:

$$\pi r^2 \cdot \frac{\theta}{2\pi} = \frac{r^2\theta}{2}$$

2. Convert the degrees to radians by multiplying by $\frac{\pi}{180}$, then multiplying by $\frac{r^2}{2}$. (Or, calculate $\frac{\pi r^2\theta}{360}$.)
3. $A \approx 33.3\ in^2$
4. $A \approx 20.94\ cm^2$
5. $A = \pi\ cm^2$
6. $A = \frac{64\pi}{3}\ cm^2$
7. $A = \frac{25\pi}{6}\ cm^2$
8. $A \approx 125.66\ cm^2$
9. $A \approx 153.28\ cm^2$
10. $A \approx 6.09\ in^2$
11. $A \approx 24.37$
12. $A = 8\pi - 16\ in^2 \approx 9.13\ in^2$
13. $A \approx 15.85\ cm^2$
14. $A \approx 61.52\ in^2$
15. $A = 18\pi\ in^2$