Learning Objectives

Here you’ll learn how to solve equations that take several steps to isolate the unknown variable.

Multi-Step Equations

We’ve seen that when we solve for an unknown variable, it can take just one or two steps to get the terms in the right places. Now we’ll look at solving equations that take several steps to isolate the unknown variable. Such equations are referred to as multi-step equations.

In this section, we’ll simply be combining the steps we already know how to do. Our goal is to end up with all the constants on one side of the equation and all the variables on the other side. We’ll do this by collecting like terms. Don’t forget, like terms have the same combination of variables in them.

Solving for Unknown Values

Solve \( \frac{3x+4}{3} - 5x = 6 \).

Before we can combine the variable terms, we need to get rid of that fraction.

First let’s put all the terms on the left over a common denominator of three: \( \frac{3x+4}{3} - \frac{15x}{3} = 6 \).

Combining the fractions then gives us \( \frac{3x+4-15x}{3} = 6 \).

Combining like terms in the numerator gives us \( \frac{4-12x}{3} = 6 \).

Multiplying both sides by 3 gives us \( 4-12x = 18 \).

Subtracting 4 from both sides gives us \( -12x = 14 \).

And finally, dividing both sides by -12 gives us \( x = -\frac{14}{12} \), which reduces to \( x = -\frac{7}{6} \).

Solving Multi-Step Equations Using the Distributive Property

You may have noticed that when one side of the equation is multiplied by a constant term, we can either distribute it or just divide it out. If we can divide it out without getting awkward fractions as a result, then that’s usually the better choice, because it gives us smaller numbers to work with. But if dividing would result in messy fractions, then it’s usually better to distribute the constant and go from there.
Using the Distributive Property

1. Solve $7(2x - 5) = 21$.

The first thing we want to do here is get rid of the parentheses. We could use the Distributive Property, but it just so happens that 7 divides evenly into 21. That suggests that dividing both sides by 7 is the easiest way to solve this problem.

If we do that, we get $2x - 5 = \frac{21}{7}$ or just $2x - 5 = 3$. Then all we need to do is add 5 to both sides to get $2x = 8$, and then divide by 2 to get $x = 4$.

2. Solve $17(3x + 4) = 7$.

Once again, we want to get rid of those parentheses. We could divide both sides by 17, but that would give us an inconvenient fraction on the right-hand side. In this case, distributing is the easier way to go.

Distributing the 17 gives us $51x + 68 = 7$. Then we subtract 68 from both sides to get $51x = -61$, and then we divide by 51 to get $x = \frac{-61}{51}$. (Yes, that’s a messy fraction too, but since it’s our final answer and we don’t have to do anything else with it, we don’t really care how messy it is.)

3. Solve $4(3x - 4) - 7(2x + 3) = 3$.

Before we can collect like terms, we need to get rid of the parentheses using the Distributive Property. That gives us $12x - 16 - 14x - 21 = 3$, which we can rewrite as $(12x - 14x) + (-16 - 21) = 3$. This in turn simplifies to $-2x - 37 = 3$.

Next we add 37 to both sides to get $-2x = 40$.

And finally, we divide both sides by -2 to get $x = -20$.

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Example

Example 1

Solve the following equation for $x$: $0.1(3.2 + 2x) + \frac{1}{3} (3 - \frac{3}{2}) = 0$

This function contains both fractions and decimals. We should convert all terms to one or the other. It’s often easier to convert decimals to fractions, but in this equation the fractions are easy to convert to decimals—and with decimals we don’t need to find a common denominator!

In decimal form, our equation becomes $0.1(3.2 + 2x) + 0.5(3 - 0.2x) = 0$.

Distributing to get rid of the parentheses, we get $0.32 + 0.2x + 1.5 - 0.1x = 0$.

Collecting and combining like terms gives us $0.1x + 1.82 = 0$.

Then we can subtract 1.82 from both sides to get $0.1x = -1.82$, and finally divide by 0.1 (or multiply by 10) to get $x = -18.2$. 
Review

Solve the following equations for the unknown variable.

1. \(3(x - 1) - 2(x + 3) = 0\)
2. \(3(x + 3) - 2(x - 1) = 0\)
3. \(7(w + 20) - w = 5\)
4. \(5(w + 20) - 10w = 5\)
5. \(9(x - 2) - 3x = 3\)
6. \(12(t - 5) + 5 = 0\)
7. \(2(2d + 1) = \frac{2}{3}\)
8. \(2 \left(5a - \frac{1}{3}\right) = \frac{2}{7}\)
9. \(\frac{2}{9} \left(i + \frac{2}{3}\right) = \frac{2}{9}\)
10. \(4 \left(v + \frac{1}{4}\right) = \frac{35}{2}\)
11. \(\frac{s}{16} = \frac{6}{3}\)
12. \(\frac{m + 2}{2} - \frac{m}{4} = \frac{1}{3}\)
13. \(5 \left(\frac{k}{3} + 2\right) = \frac{32}{3}\)
14. \(\frac{3}{z} = \frac{2}{5}\)
15. \(\frac{7}{3} + 2 = \frac{10}{3}\)
16. \(\frac{12}{z} = \frac{3 + z}{3}\)

Review (Answers)

To view the Review answers, open this PDF file and look for section 3.7.