Quadratic Functions

Solving Quadratics by Factoring

Review Queue Answers

1. x = 7
2. x = -9
3. x = 5, y = -2
4. 8 and 2

Factoring When the First Coefficient Equals 1

1. $x^2 - 6x - 8$
2. $x^2 - 10x + 9$
3. $x^2 + 10x + 21$
4. (x – 2)(x + 1)
5. (x + 6)(x – 4)
6. x(x – 6)
7. (x + 3)(x + 3)
8. not factorable
9. (x – 6)(x – 5)
10. (x + 15)(x –2)
11. (x + 7)(x + 4)
12. (x – 6)(x – 2)
13. (x – 11)(x + 4)
14. (x – 10)(x + 2)
15. (x + 4)(x + 3)
16. not factorable
17. (x – 9)(x + 4)
18. x(x + 1)
19. The sum will be 2 and the other factor is 6.
20. The other factor is -8 and the product is -40.

Factoring When the First Coefficient Doesn't Equal 1

1. $2x^2 + 9x - 5$
2. $6x^2 - 5x - 6$
3. $16x^2 - 1$
4. (x + 3)(5x + 3)
5. 3x(2x – 7)
6. (5x – 3)(2x +1)
7. (x + 2)(3x – 4)
8. (2x + 3)(2x + 1)
9. not factorable
10. (2x – 1)(8x + 1)
11. 5(x – 3)(2x + 1)
12. (2x + 1)(x + 3)
13. not factorable
14. 2(x – 2)(4x + 1)
15. (x + 3)(10x – 3)
16. (2x + 3)(2x + 3)
17. 5x(3x + 7)
18. (2x – 3)(3x – 5)
19. b = 0, (x + 5)(x – 5)
20. b = 0, a and c are square numbers, (3x – 4)(3x + 4)

Factoring Special Quadratics

1. 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196
2. The sum of any two squares is not factorable, therefore, there is no formula.
3. (x – 1)(x + 1)
4. $(x + 2)^2$
5. $(4x - 3)^2$

6. $-3(x-6)^2$ 7. $(12x-7)(12x+7)$ 8. $(14x+5)^2$

9. not factorable 10. $2(9x+2)^2$ 11. $(15-x)(15+x)$

12. $(11-6x)^2$ 13. not factorable 14. $4(8x-13)(8x+13)$

15. When something is squared, there are two of them. Therefore $(2x-5)^2=(2x-5)(2x-5)$. Spencer distributed the "2," which is incorrect. If Spencer had FOILed (2x – 5)(2x – 5), he would have ended up with the answer the teacher showed him.

Solving Quadratics by Factoring

1. x = -9, 1 2. x = 0, -6 3. $x=-\frac{3}{2}, 4$

4. $x=\frac{2}{3}, -\frac{5}{4}$ 5. x = 3, -3 6. $x=-\frac{5}{3}$; double root

7. not factorable 8. $x=0, \frac{1}{2}$ 9. $x=-\frac{4}{3}$, -8

10. $x=-\frac{7}{6}, \frac{7}{6}$ 11. not factorable 12. $x=-2, -\frac{2}{5}$

13. $x=-\frac{5}{2}, \frac{3}{4}$ 14. $x=-\frac{1}{3}, -\frac{7}{3}$ 15. $x=\frac{1}{6}, \frac{5}{2}$

16. The length is 64 feet, the width is 39 feet.

Solving Quadratics by using Square Roots

Review Queue Answers

1. 8, -8; yes 2. 4 and 5 because 18 is between 16 and 25.

3. 4.24 4. x = 5, -5

Simplifying Square Roots

1. 7.48 2. 3.46 3. 9.59

4. $3\sqrt{2}$ 5. $5\sqrt{3}$ 6. $11\sqrt{5}$

7. $4\sqrt{3}$ 8. 10 9. $12\sqrt{7}$

10. $4\sqrt{30}$ 11. 80 12. $18\sqrt{2}$

13. $4+4\sqrt{2}$ 14. $3\sqrt{7}$ 15. $-10\sqrt{3}$

16. $\sqrt{2}$ 17. $5\sqrt{6}$ 18. $5\sqrt{10}+7\sqrt{5}$

Dividing Square Roots

1. $\frac{2}{5}$ 2. $\frac{4\sqrt{6}}{11}$ 3. $\sqrt{5}$

4. $\frac{2\sqrt{15}}{5}$ 5. $\frac{2\sqrt{21}}{7}$ 6. $\frac{8\sqrt{15}}{5}$

7. $2\sqrt{6}$ 8. $\frac{4\sqrt{11}}{11}$ 9. $\frac{3\sqrt{42}}{4}$

10. $2\sqrt{5}$ 11. $\frac{16\sqrt{10}}{5}$ 12. $2\sqrt{3}+4\sqrt{13}$

Solving Quadratics using Square Roots

1. x = 12, -12 2. x = 2, -2 3. x = $\pm\sqrt{3}$

4. x = -9, 5 5. x = $5\pm\sqrt{3}$ 6. x = $\pm 2\sqrt{15}$

7. x = 4, -4 8. x = $\pm 3\sqrt{14}$ 9. x = $-7\pm\sqrt{13}$

10. x = -3, $\frac{1}{3}$ 11. x = $10\pm 6\sqrt{2}$ 12. x = $1\pm\sqrt{37}$

13. x = 11, 5 14. x = $\pm 3\sqrt{2}$ 15. x = $-\frac{5}{2}, -\frac{4}{3}$

16. Answers will vary. 17. x = 3, -3, opinions will vary.

18. x = 2, $-\frac{2}{3}$; the answers are the same for both quadratics. This means the quadratic equations are the same (or reduce to be the same).

19. w = 36.61, h = 20.59 20. 3.54 seconds

Complex Numbers

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1. There is no real solution; you cannot take the square root of a negative number.

2. 72 3. $\frac{2\sqrt{5}}{5}$ 4. x = $6\pm\sqrt{17}$

Defining Complex Numbers

1. 3i
2. $11i\sqrt{2}$
3. $18i\sqrt{5}$
4. $-4i\sqrt{2}$
5. $12\sqrt{6}$
6. $21i\sqrt{14}$
7. 1
8. -16
9. -9i
10. i
11. -2i
12. $\frac{i\sqrt{5}}{5}$
13. 17 – 12i
14. -6 – 7i
15. 5 + 10i
16. -1 – 6i
17. 0.8 + 0.5i
18. 22 – 13i
19. 11 + 35i
20. 31 + 9i

Multiplying and Dividing Complex Numbers

1. 2i + 7
2. -18 + 48i
3. -8 – 22i
4. 84 – 100i
5. -68 + 79i
6. -2 – 6i
7. 60-112i
8. 89
9. $3-\frac{4}{3}i$
10. -1 – 6i
11. $\frac{7}{5}-\frac{12}{5}i$
12. $\frac{1}{2}+\frac{1}{6}i$
13. $\frac{3}{5}-\frac{4}{5}i$
14. $\frac{13}{5}-\frac{6}{5}i$
15. $-\frac{82}{449}+\frac{343}{449}i$

Solving Quadratic Equations with Complex Number Solutions

1. $x=-1\pm 11i$
2. $x=\pm 3i$
3. $x=\pm i\sqrt{6}$
4. $x=9\pm 4i\sqrt{5}$
5. $x=-6\pm 2i\sqrt{3}$
6. $x=5\pm i\sqrt{14}$
7. $x=\frac{1}{2}\pm i\sqrt{7}$
8. $x=-\frac{5}{6}\pm i\sqrt{2}$
9. $x=\frac{3}{4}\pm\frac{\sqrt{371}}{28}i$
10. 4 + i
11. 6 – 2i
12. x^2– 6x +13

Completing the Square

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1. x = 16, 2
2. $x = 4 \pm 3i\sqrt{3}$
3. x = 2, $-\frac{3}{4}$
4. x = $\pm 9\sqrt{2}$

Completing the Square When the First Coefficient Equals 1

1. 4
2. 1
3. 64
4. $(x+3)^2$
5. $\left(x-\frac{7}{2}\right)^2$
6. $\left(x-\frac{1}{4}\right)^2$
7. $x = -3 \pm 2\sqrt{6}$
8. $x = -5 \pm 2i$
9. $x = 7 \pm 2i\sqrt{5}$
10. $x = \frac{3}{2} \pm \frac{\sqrt{71}}{2}i$
11. $x = \frac{9}{2} \pm \frac{\sqrt{193}}{2}$
12. $x = 10 \pm 4\sqrt{10}$
13. x = -6, 5
14. $x = 9 \pm 3i$
15. x = -7, -8
16. $x = -\frac{3}{2} \pm \frac{\sqrt{153}}{2}$
17. $x = 2 \pm 5i$
18. x = -21, -3
19. x = 11, -4; opinions will vary.
20. $x = -\frac{17}{16} \pm \frac{3\sqrt{167}}{16}i$

Completing the Square When the First Coefficient Doesn't Equal 1

1. $x = 1 \pm \frac{\sqrt{78}}{6}$
2. $x = 3 \pm \sqrt{34}$
3. $x = 3 \pm i\sqrt{2}$
4. $x = 2, -\frac{3}{2}$
5. $x = -7 \pm i\sqrt{89}$
6. $x = 3, -\frac{5}{3}$
7. x = 2, -1
8. $x = -\frac{7}{2}, -1$
9. $x = -4 \pm 3i$
10. $x = -5 \pm \frac{2\sqrt{15}}{3}$
11. $x = 2, -\frac{1}{3}$
12. $x = -2 \pm \frac{2\sqrt{30}}{5}i$
13. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
14. $x = \frac{1}{2}, \frac{5}{4}$
15. yes, (4x – 5)(2x + 1)

16. In the third step, they should have added 144 to the right side, $4 \cdot 36$, no just 36. The correct answer is $x = 6 \pm \frac{\sqrt{155}}{2}$.

The Quadratic Formula

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1. $x = 2 \pm 4i$
2. $x = \frac{11}{2}, -\frac{3}{2}$
3. $x = -\frac{5}{4}, -\frac{4}{3}$
4. $x = 3, \frac{9}{2}$

Deriving and Using the Quadratic Formula

1. $x = -4 \pm \sqrt{7}$
2. $x = 4, -\frac{3}{4}$
3. $x = \frac{1 \pm \sqrt{41}}{4}$
4. $x = \frac{11}{14} \pm \frac{\sqrt{215}}{14} i$
5. $x = -\frac{1}{3} \pm \frac{\sqrt{11}}{6} i$
6. x = 7; double root
7. x = 10, -15
8. $x = \frac{3}{4}, -\frac{1}{2}$
9. $x = \frac{9}{5} \pm \frac{\sqrt{39}}{5} i$
10. $x = -\frac{1}{2}, \frac{2}{5}$
11. $x = 8 \pm 2\sqrt{15}$
12. $x = \pm \frac{14}{3}$
13. $x = -\frac{5}{2}$; double root
14. x = 21, -3
15. Answers will vary. In general, students should try to factor an equation first. If it cannot be factored, then they should either complete the square or use the Quadratic Formula. Both of the later options work if the solutions are imaginary or irrational.

Using the Discriminant

1. 1 real solution
2. 2 real solutions
3. 2 imaginary solutions
4. 2 real solutions
5. 2 real solutions
5. 2 imaginary solutions
7. x = 20, -3
8. $x = \frac{\sqrt{30}}{3}$
9. $x = -\frac{5}{4} \pm \frac{3\sqrt{7}}{4} i$
10. c < 1, c = 1, c > 1
11. c < 9, c = 9, c > 9
12. c < 36, c = 36, c > 36
13. $4k^2 - 16$
14. k > 2 and k < -2, k = 2 and -2, -2 < k < 2

Analyzing the Graph of a Quadratic Function

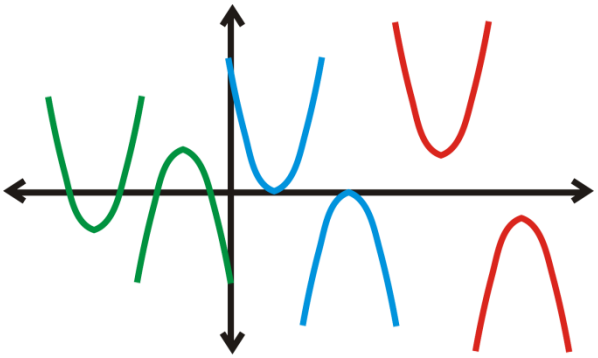
Review Queue Answers

1. x = -9, 3
2. x = $5 \pm 2i$
3. x = 4
4. Answers will vary.

Finding the Parts of a Parabola

1. (6, -25); minimum
2. (-5, -43), minimum
3. $\left(\frac{2}{3}, 18\frac{1}{3}\right)$; maximum
4. $\left(2\frac{1}{4}, -21\frac{1}{8}\right)$; minimum
5. (3, 0); maximum
6. (16, 31); maximum
7. (6, -25), (11, 0), (1, 0), (0, 11), x = 6
8. $\left(-1\frac{1}{4}, 24\frac{1}{2}\right)$, (-4, 0,), $\left(\frac{3}{2}, 0\right)$, (0, 12), x = $-\frac{5}{4}$
9. (-6, -27), (-15, 0), (3, 0), (0, -15), x = -6
10. $\left(-4\frac{1}{3}, -65\frac{1}{3}\right)$, (-9, 0), $\left(\frac{1}{3}, 0\right)$, (0, -9), x = $-\frac{13}{3}$
11. (5, 0), (5, 0), (0, -25), x = 5
12. $\left(5, 40\frac{1}{2}\right)$, (14, 0), (-4, 0), (0, 28), x = 5
13. Quadratic Formula or completing the square.
14. (4, 7), $\left(4+\sqrt{7}, 0\right)$, $\left(4-\sqrt{7}, 0\right)$
15. $\left(\frac{1}{4}, -8\frac{1}{8}\right)$, $\left(\frac{1+\sqrt{65}}{4}, 0\right)$, $\left(\frac{1-\sqrt{65}}{4}, 0\right)$
16. 17, 5, 1, 5, 17
17. 45, 13, 9, 13, 45
18. These two parabolas do not have any x-intercepts, which means they do not have real solutions. The solutions for these quadratic equations are imaginary. For #16, x = $1 \pm i$.
19. A parabola can intersect the x-axis three different ways; 2, 1, or 0. If it intersects it twice, there are two real solutions. If it intersects the x-axis once, then the vertex is the only solution (repeated root;

#11). If it does not intersect the x-axis at all, then there are no real solutions or two imaginary solutions (#16 and #17).



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20. $y = \dfrac{-b^2 + 4ac}{4a}$

Vertex, Intercept, and Standard Form

1.

	Equation	**Vertex**	**Intercepts** (or how to find the intercepts)
Standard Form	$y = ax^2 + bx + c$	$\left(\dfrac{-b}{2a}, f\left(\dfrac{-b}{2a}\right)\right)$	Factor or use the Quadratic Formula to find the intercepts.
Intercept Form	$y = a(x-p)(x-q)$	$\left(\dfrac{p+q}{2}, f\left(\dfrac{p+q}{2}\right)\right)$	(p, 0) and (q, 0)
Vertex Form	$y = a(x-h)^2 + k$	(h, k)	Continue to solve equation using square roots.

2. (4, -9), (7, 0), (1, 0)

3. (1, -49), (-6, 0), (8, 0)

4. (-1, -9), (2, 0), (-4, 0)

5. (-1, 36), (5, 0), (-7, 0)

6. (-1, -3), $\left(\dfrac{-2+\sqrt{6}}{2}, 0\right)$, $\left(\dfrac{-2-\sqrt{6}}{2}, 0\right)$

7. (2, 4), other points: (0, 16), (4, 16)

8. (3, -12), (9, 0), (-3, 0)

9. (-2, 7), $\left(-2+\sqrt{7}, 0\right)$, $\left(-2-\sqrt{7}, 0\right)$

10. $\left(1\dfrac{5}{8}, -22\dfrac{9}{16}\right)$, (4, 0), $\left(-\dfrac{3}{4}, 0\right)$

11. y = (x – 2)(x – 1)

12. y = -(x + 12)(x – 2)

13. $y = 4\left(x + \dfrac{1}{2}\right)(x+4)$

14. $y=(x+6)^2-8$

15. $y=-(x+5)^2-1$

16. $y=2(x-2)^2+7$

17. $y=x^2-6x+17$

18. $y=2x^2-11x+12$

19. $y=-\frac{1}{2}x^2-6x-29$

Using the Graphing Calculator to Graph Quadratic Equations

1. (0.5, -6.25), (-2, 0), (3, 0)

2. (1.5, 30.25), (-4, 0), (7, 0)

3. (-2.75, -55.125), (-8, 0), (2.5, 0)

4. (3, -2), (1.59, 0), (4.41, 0)

5. (-4, -3), (-5.73, 0), (-2.27, 0)

6. (-3, 25), x = $-3\pm5i$

7. (0.65, -7.225), (-0.2, 0), (1.5, 0)

8. (1.5, 6), (0.28, 0), (2.72, 0)

9. (4, 12), x = $4\pm6i$

10. a) *a* effects the width (or breadth) of the parabola. If |a| > 1, then the parabola is wider than $y=x^2$. If 0 < |a| < 1, then the parabola will be narrower.

    b) If *a* is negative, then if flips the parabola upside-down.

    c) *h* shifts the parabola to the right or left. If *h* is negative in the equation, it will shift the parabola to the right. If it is positive, it will shift the parabola to the left.

    d) *k* shifts the parabola up and down. If *k* is negative in the equation, it will shift the parabola down. If it is positive, it will shift the parabola up.

11. The maximum height is the *y*-coordinate of the vertex or 144.8 feet. The ball travelled a total distance of 436.6 feet.

Modeling with Quadratic Functions

1. $y=-2(x+1)^2+1$

2. $y=\frac{1}{4}(x+2)(x-2)$

3. $y=(x-9)^2-4$

4. $y=-\frac{1}{2}(x-8)(x+5)$

5. $y=\frac{3}{2}(x+9)(x+7)$

6. $y=-3(x-6)^2+10$

7. $y=\frac{4}{9}(x+4)^2-15$

8. $y=-\frac{7}{8}x^2+2$

9. $y=-\frac{2}{3}(x-3)(x-16)$

10. $y=-2x^2+x-15$

11. $y=6x^2+2x-9$

12. $y=\frac{3}{4}x^2+x+8$

13. See Complete Solution Key

14. $y=-0.07x^2+1.25x$

15. $y=-0.01x^2+0.98x+5.25$

16. a) $y=-0.18x^2+8.52x-3.95$

    b) Answers will vary.