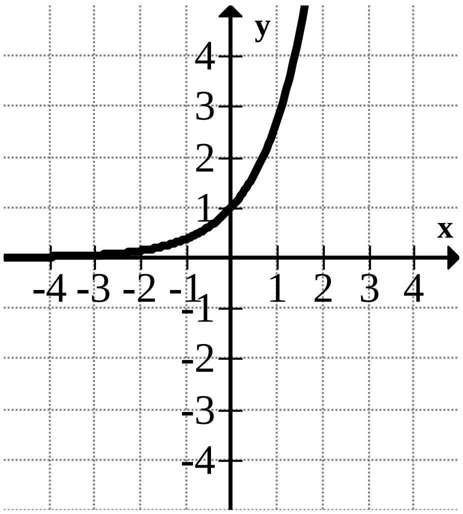
1.1 Functions Families

Answers

1.



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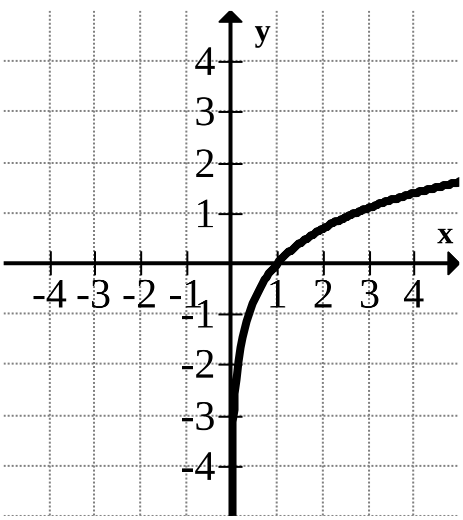

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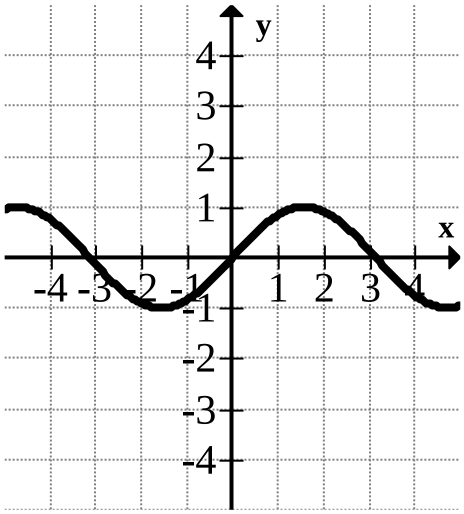
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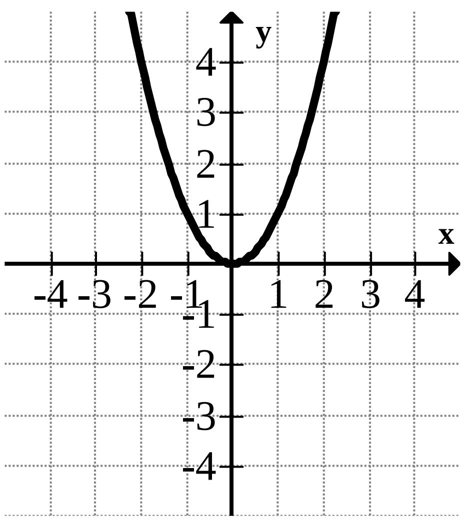
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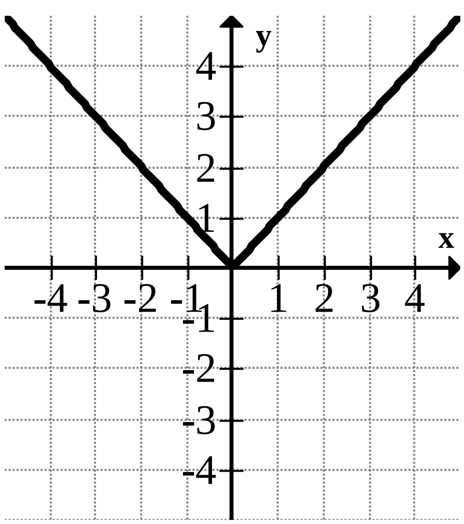
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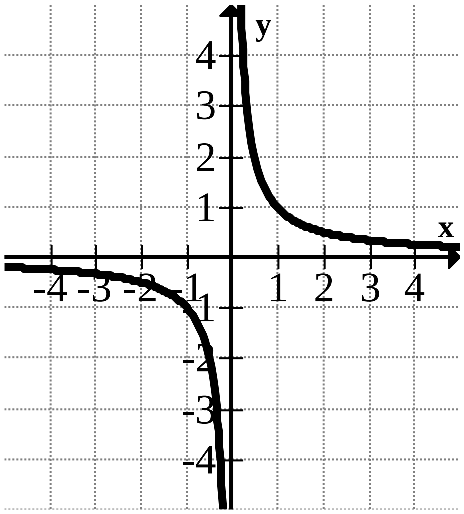
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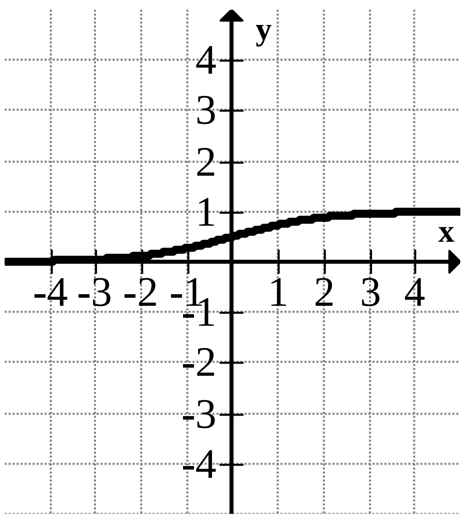
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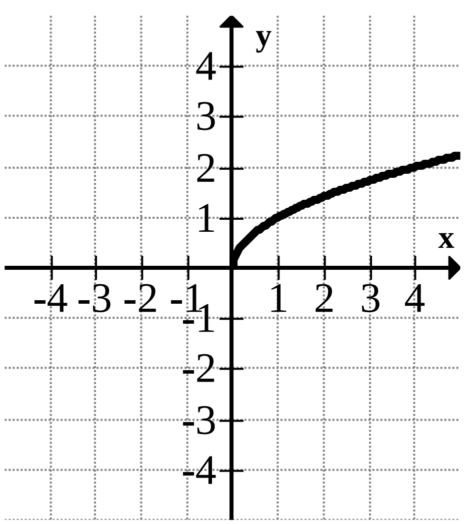
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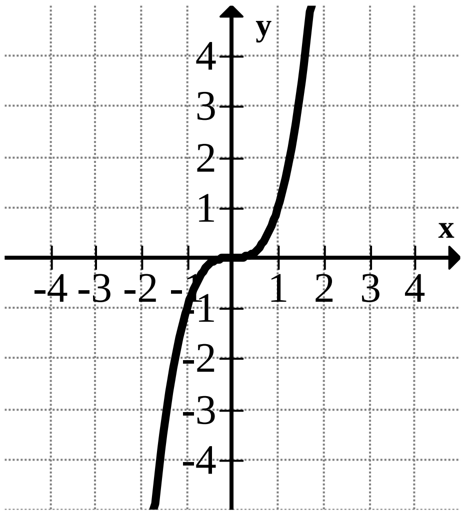

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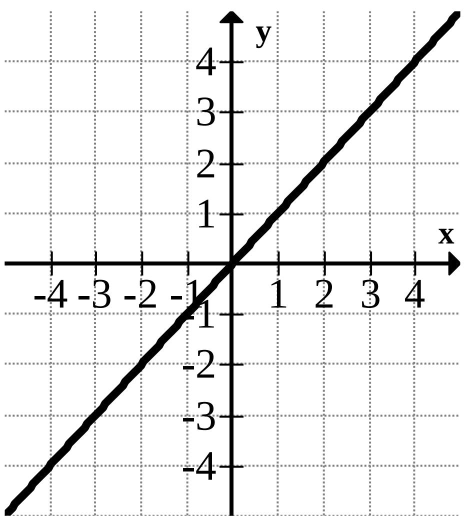


11. $y = \frac{1}{x}$, because $\frac{1}{0}$ is undefined.

12. $y = e^x,\ y = x^2,\ y = \sqrt{x},\ y = |x|$

13. One difference is $y = x^2$ has a minimum value, while $y = x^3$ doesn't.

14. The two graphs are reflections of one another across the line $y = x$.

15. $y = \sqrt{x}$ is not defined for all values of x because the square root of any negative number is not a real number.

1.2 Graphical Transformations

Answers

1. Reflection across the x axis and reflection across the y axis.

2. Reflection across the x axis and a horizontal shift left 3 units.

3. Horizontal shift left 1 unit and vertical shift down 2 units.

4. Reflection across the y axis and horizontal shift right 3 units.

5. Reflection across the x axis and horizontal compression by a factor of 2.

6. Vertical stretch by a factor of 4, horizontal stretch by a factor of 2, and horizontal shift left 2 units.

7. A reflection across the x axis, a horizontal shift right 2 units, vertical shift down 2 units, and a vertical stretch by a factor of 3.

8. Vertical stretch by a factor of 5 and a horizontal shift left 1 unit.

9. $2h(x-2)+3$

10. $-f(x+2)-1$

11. $\frac{1}{4}g(-x)$

12. $j(3(x-2))+3$

13. $k\left(\frac{1}{4}(x+1)\right)+3$

14. $\frac{1}{2}h\big(-(x-3)\big)$

15. $-5f(x)$

1.3 Point Notation and Function Notation

Answers

1. Vertical reflection across the x axis, vertical compression by a factor of 2, horizontal shift one unit left.

$(x, y) \to \left(x - 1, -\frac{1}{2}y\right)$

x	y
0	5
1	6
2	7

→

x	y
-1	-2.5
0	-3
1	-3.5

2. Vertical stretch by a factor of 2, horizontal compression by a factor of 3, and vertical shift up 2 units.

$(x, y) \to \left(\frac{1}{3}x, 2y + 2\right)$

x	y
0	5
1	6
2	7

→

x	y
0	12
1/3	14
2/3	16

3. Reflection across the x axis, horizontal shift 4units to the right, vertical shift 3 units down.

$(x, y) \to (x + 4, -y - 3)$

x	y
0	5
1	6
2	7

→

x	y
4	-8
5	-9
6	-10

4. Vertical stretch by a factor of 3, horizontal compression by a factor of 2, horizontal shift 2 units to the right, and vertical shift up 1 unit.

$(x, y) \to \left(\frac{x}{2} + 2, 3y + 1\right)$

x	y
0	5
1	6
2	7

→

x	y
2	16
2.5	19
3	22

5. Reflection across the x axis, horizontal shift right 3 units.

$(x, y) \to (x + 3, -y)$

x	y
0	5
1	6
2	7

→

x	y
3	-5
4	-6
5	-7

6. $f(x) \to f(2x - 6) - 4$

7. $f(x) \to -f\left(\frac{x}{2} - 2\right) + 1$

8. $f(x) \to 3f\left(\frac{1}{4}x\right) - 5$

9. $f(x) \to -f\left(\frac{1}{2}x\right) + 1$

10. $f(x) \to -f\left(\frac{1}{3}x\right) + 1$

11. $(x, y) \to (x + 2, 3y + 1)$

12. $(x, y) \to (x + 1, -4y + 3)$

13. $(x, y) \to \left(\frac{x}{2} - 1, \frac{1}{2}y - 5\right)$

14. $(x, y) \to (2x + 4, 5y - 1)$

15. $(x, y) \to \left(\frac{x}{2} + 2, \frac{1}{4}y\right)$

1.4 Domain and Range

Answers

1. $(0, \infty)$

2. $[-1,1)$

3. $[1,2) \cup (2,3) \cup (3,5]$

4. $(5, \infty)$

5. $(-\infty, 0) \cup (0, \infty)$

6. $(-4, 5]$

7. $(0, \infty)$

8. $(-\infty, 4] \cup (5, \infty)$

9. Domain: $x \in (-\infty, \infty)$ Range: $y \in [-1,1]$

10. Domain: $x \in (-\infty, 2) \cup [3, \infty)$ Range: $y \in [-8, \infty)$

11. Answers vary. Domain should be $x \in [0, \infty)$ and range should be $y \in (-2,2]$.

12. Answers vary. Domain should be $x \in [-4,1) \cup (1, \infty)$ and range should be $y \in (-\infty, \infty)$.

13. Domain: $x \in \left\{-2, \frac{3}{4}, \frac{\pi}{2}, 2, 3\right\}$ Range: $y \in \{1, \pi, 5, 7\}$

14. Domain: $x \in [-4, \infty)$

15. Domain: $x \in (-\infty, -6) \cup (-6, \infty)$

16. Domain: $x \in (-\infty, 1) \cup (1, \infty)$

1.5 Maximums and Minimums

Answers

1. There is a global minimum at (3,0).

2. There are no local extrema.

3. Global minimum at $(-\pi/2, -1)$ and global maximum at $(\pi/2, 1)$.

4. Local maximum at $(-\pi, 0)$ and global minimum at $(\pi, 0)$.

5. There are no global extrema.

6. There are no local extrema.

7. There are no global extrema.

8. Local minimums: (0.4, -1), (2.5, -13). Local maximums: (-1.5, 22), (1, 0). *Note: points are approximate.*

9. There are no global extrema.

10. Local minimum: (3, 0). Local maximum: (0.5, 9.5). *Note: points are approximate.*

11. A global maximum is the overall highest point on the graph, while the local maximum is the highest point within a certain neighborhood of the graph.

12. Answers vary. Graph should show a global minimum, a local maximum, no global maximum (there can be a local minimum).

13. Answers vary. Graph should have no global extrema, but both types of local extrema.

14. Local maximum: (-1.16, 36.24). Local minimum: (-4, 0). No global maximum. Global minimum: (2,16, -18.49).

15. Local maximum: (-1,0). Local minimum: (0.22, -3.23). Global maximum: (2.28, 9.91). No global minimum.

1.6 Symmetry

Answers

1. Even

2. Odd

3. Neither

4. Neither

5. Odd

6. Neither

7. Neither

8. Even

9. $f(-x) = h(x) + g(x)$

10. $f(-x) = -\frac{h(x)}{g(x)}$

11. $f(-x) = -h(x)g(x)$

12. Yes. If h(x) and g(x) are both even and f(x)=h(x)+g(x), then:

$$f(-x) = h(-x) + g(-x) = h(x) + g(x) = f(x).$$

13. Yes. If h(x) and g(x) are both odd and f(x)=h(x)+g(x), then:

$$f(-x) = h(-x) + g(-x) = -h(x) - g(x) = -[h(x) + g(x)] = -f(x).$$

14. There are some functions that do not have reflection symmetry across the y axis or rotation symmetry about the origin.

15. If a function is even then it is symmetrical across the y axis. If a function is odd then it has rotation symmetry about the origin.

1.7 Increasing and Decreasing

Answers

1. $x \in (3, \infty)$

2. $x \in (-\infty, 3)$

3. $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

4. $x \in \left(-\pi, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right)$

5. $x \in (-\infty, \infty)$

6. None

7. $x \in (-\infty, -1.4) \cup (0.3, 1) \cup (2.5, \infty)$ *Note that points are approximate*

8. $x \in (-1.4, 0.3) \cup (1, 2.5)$ *Note that points are approximate*

9. $x \in (-\infty, 0.3) \cup (3, \infty)$

10. $x \in (0.3, 3)$

11. Answers vary. Possible answer: A line with a positive slope.

12. Answers vary. Possible answer: A line with a negative slope.

13. Increasing: $x \in (-\infty, 1) \cup (3, \infty)$. Decreasing: $x \in (1, 3)$.

14. Increasing: $x \in (-\infty, 1)$. Decreasing: $x \in (1, \infty)$.

15. Increasing: $x \in (5,\ \infty)$. Decreasing: $x \in (-\infty, 5)$

1.8 Zeroes and Y-Intercepts of Functions

Answers

1. y-intercept: (0, -4); Zeroes: (-1,0) and (0, -4)

2. y-intercept: (0, -12); Roots: (-3,0), (1,0) and (2, 0)

3. y-intercept is approximately (0, 5), x-intercepts are (-2,0) and (1, 0)

4. Both x and y –intercepts are (0,0)

5. Both x and y –intercepts are (0, 0)

6. No y-intercept; x-intercept is (1, 0)

7. No x or y- intercepts

8. y-intercept is (0, 1); no x-intercept

9. Both x and y-intercepts are (0,0)

10. Yes, because there are functions that are undefined when x=0.

11. Yes, because there are functions with no real solutions when y=0.

12. The x-intercept of f(x) is called a zero because it is the solution to f(x)=0.

13. y-intercept: (0, 10); x-intercepts: (2,0), (-1,0), (5,0)

14. y-intercept: (0, -7); x-intercepts: (-1,0),(7,0)

15. y-intercept: (0, 5); x-intercepts: (5,0), (-1/2, 0), (1,0)

1.9 Asymptotes and End Behavior

Answers

1. There are no asymptotes. As x approaches positive infinity, y approaches positive infinity. As x approaches negative infinity, y approaches negative infinity.

2. There are no asymptotes. As x approaches both positive and negative infinity, y approaches positive infinity.

3. There are no asymptotes. As x approaches positive infinity, y approaches positive infinity. As x approaches negative infinity, y approaches negative infinity.

4. There are no asymptotes. As x approaches positive infinity, y approaches positive infinity.

5. There is a horizontal asymptote at y=0 and a vertical asymptote as x=0. As x approaches both positive and negative infinity, y approaches 0.

6. As x approaches negative infinity there is a horizontal asymptote at y=0. As x approaches positive infinity, y approaches positive infinity. There is no vertical asymptote.

7. There is a vertical asymptote at x=0. As x approaches positive infinity, y approaches positive infinity. There is no horizontal asymptote.

8. As x approaches negative infinity there is a horizontal asymptote at y=0. As x approaches positive infinity there is a horizontal asymptote at y=1. There is no vertical asymptote.

9. There is a vertical asymptote at x=0. As x approaches positive infinity there is a horizontal asymptote at y=0. As x approaches negative infinity there is a horizontal asymptote at y=2.

10. There is a vertical asymptote at x=1. As x approaches both positive and negative infinity there is a horizontal asymptote at y=2.

11. There is a vertical asymptote at x=4. As x approaches both positive and negative infinity there is a horizontal asymptote at y=1.

12. Because when $x = 0, y = \frac{1}{0}$ which is undefined.

13. Because when $x = -3, y = \frac{1}{0}$ which is undefined.

14. $x = 2$

15. $x = -4$

1.10 Continuity and Discontinuity

Answers

1. This function is continuous.

2. This function is continuous.

3. This function is continuous.

4. This function is continuous.

5. Infinite discontinuity at $x = 0$.

6. This function is continuous.

7. This function is continuous.

8. This function is continuous.

9. Removable discontinuity at $x = -2$, infinite discontinuity at $x = 0$, jump discontinuity at $x = 4$.

10. Removable discontinuity at $x = 2$.

11. Jump discontinuity at $x = 0.3$.

12. Answers vary, but should show $f(x)$ has a jump discontinuity at $x = 3$, a removable discontinuity at $x = 5$, and another jump discontinuity at $x = 6$.

13. Answers vary, but should show $g(x)$ has a jump discontinuity at $x = -2$, an infinite discontinuity at $x = 1$, and another jump discontinuity at $x = 3$.

14. Answers vary, but should show $h(x)$ has a removable discontinuity at $x = -4$, a jump discontinuity at $x = 1$, and another jump discontinuity at $x = 7$.

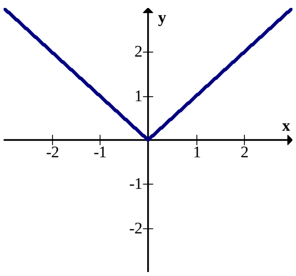
15. Answers vary, but should show $j(x)$ has an infinite discontinuity at $x = 0$, a removable discontinuity at $x = 1$, and a jump discontinuity at $x = 4$.

1.11 Function Composition

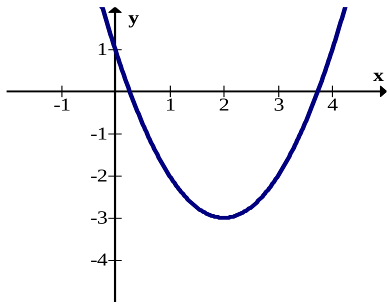
Answers

1. Here are the three graphs:

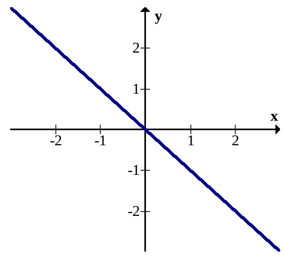
$f(x)$:



$g(x)$:

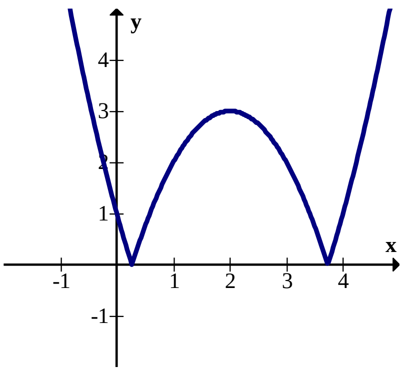


$h(x)$:



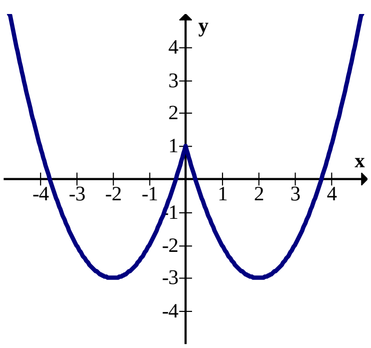
2. $f(g(x)) = |(x-2)^2 - 3|$

3. The negative y values of the original parabola have been reflected across the x-axis:



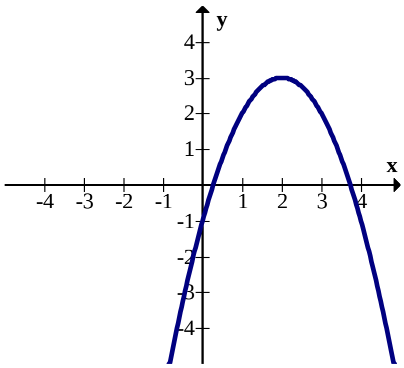
4. $g(f(x)) = (|x|-2)^2 - 3$

5. The portion of the original parabola to the right of the y-axis has been reflected across the y-axis.



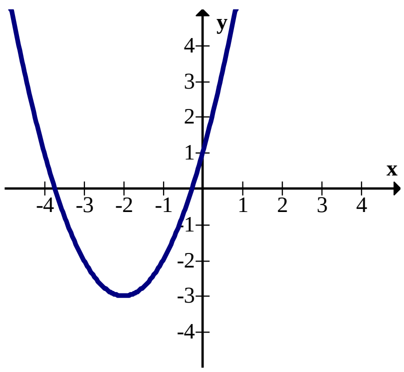
6. $h(g(x)) = -[(x-2)^2 - 3]$

7. The original parabola has been reflected across the x-axis.



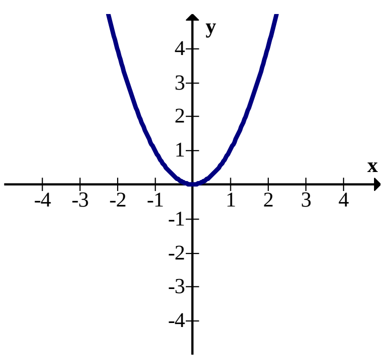
8. $g\left(h(x)\right) = (-x-2)^2 - 3$

9. The original parabola has been reflected across the y-axis.

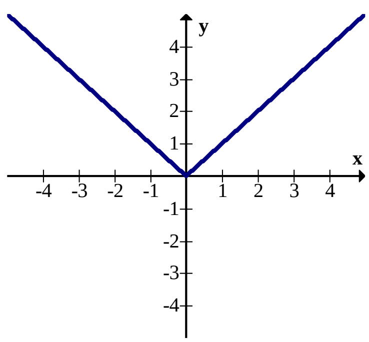


10. Here are the three graphs:

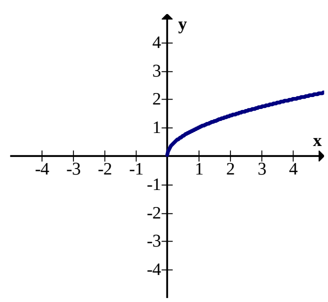
$j(x) = x^2$



$k(x) = |x|$

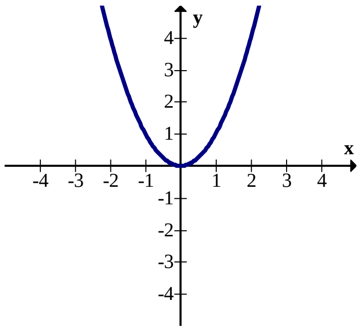


$m(x) = \sqrt{x}$



11. $j\left(k(x)\right) = (|x|)^2$

12. The graph looks the same as $j(x)$.

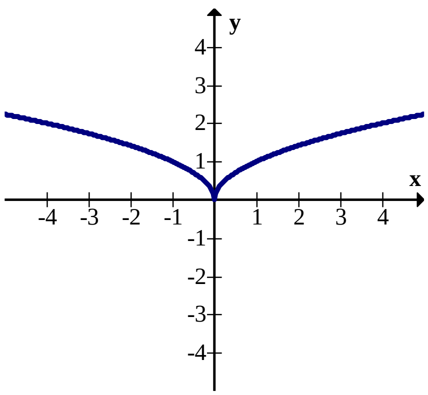


13. $k(m(x)) = |\sqrt{x}|$

14. The graph looks the same as $m(x)$.

15. $m(k(x)) = \sqrt{|x|}$

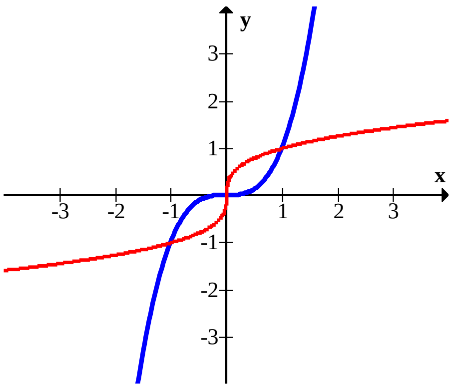
16. The original square root graph is there, as well as its reflection across the y-axis.



1.12 Inverses of Functions

Answers

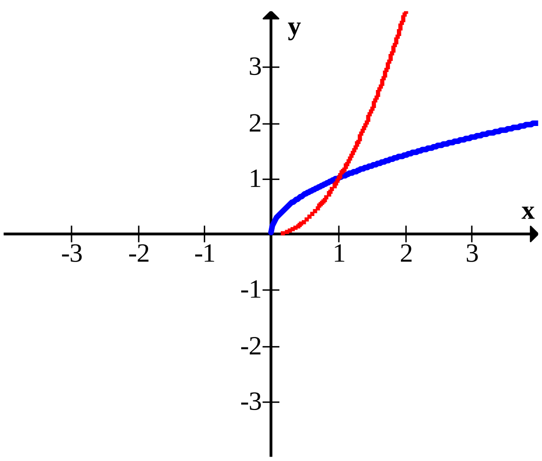
1. $f(x)$ is shown in blue while $f^{-1}(x)$ is shown in red.



2. $f^{-1}(x) = x^{\frac{1}{3}}$. It is a function.

3. $f(f^{-1}(x)) = (x^{\frac{1}{3}})^3 = x$. $f^{-1}(f(x)) = (x^3)^{\frac{1}{3}} = x$

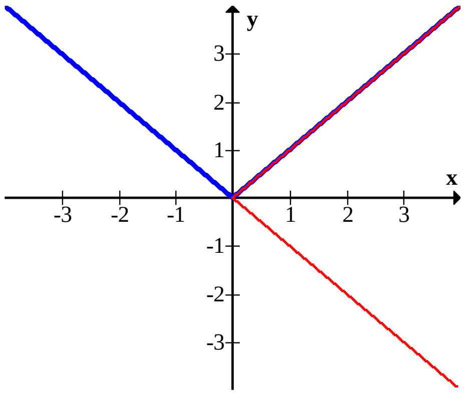
4. $g(x)$ is shown in blue while $g^{-1}(x)$ is shown in red.



5. $g^{-1}(x) = x^2$ for $x \geq 0$.

6. $g(g^{-1}(x)) = (\sqrt{x})^2 = x$. $g^{-1}(g(x)) = \sqrt{x^2} = x$.

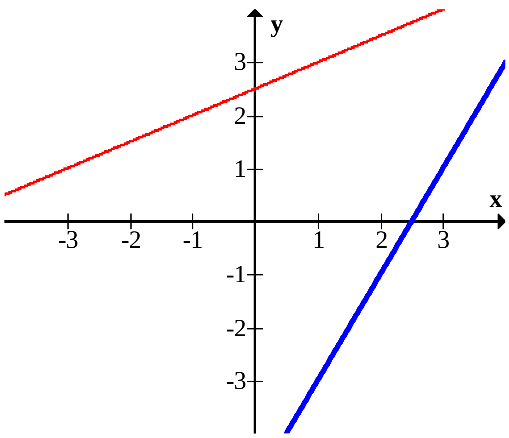
7. $h(x)$ is shown in blue while $h^{-1}(x)$ is shown in red.



8. The inverse is $x = |y|$ and is not a function.

9. You can see from the graph that they are inverses because they are symmetrical across the line y=x.

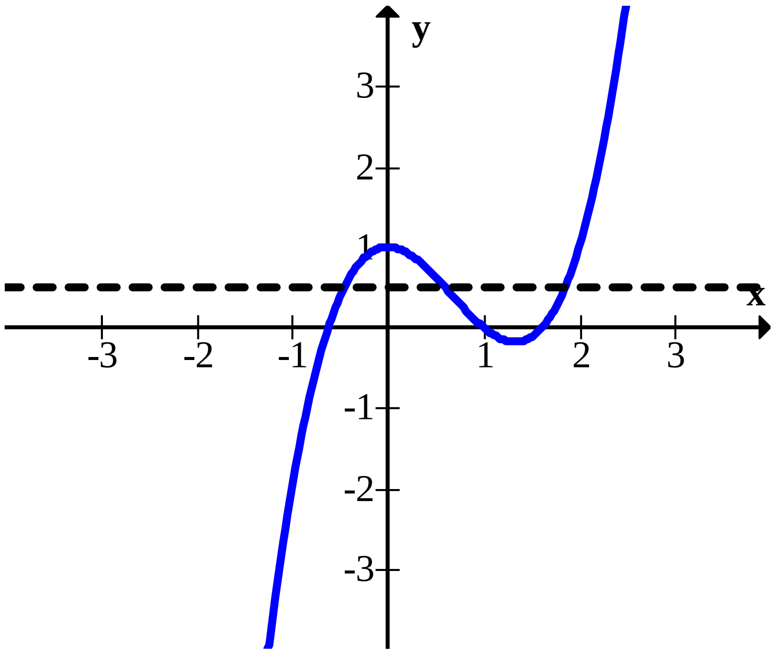
10. $j(x)$ is shown in blue while $j^{-1}(x)$ is shown in red.



11. $j^{-1}(x) = \frac{x+5}{2}$. It is a function.

12. $j(j^{-1}(x)) = 2\left(\frac{x+5}{2}\right) - 5 = x + 5 - 5 = x.$ $j^{-1}(j(x)) = \frac{(2x-5)+5}{2} = \frac{2x}{2} = x.$

13. It is not:



14. No. The inverse of $g(x)$ is $g^{-1}(x) = e^x - 1$.

15. You could switch the x and y coordinates given in the original table to make the table for the inverse.