Basic Algebra Flexbook Solution Key
Chapter 8
Exponents and Exponential Functions

Lesson 8.1
Exponential Properties Involving Products

1. Consider a^5.
    1.The base is *a*.
    2. The exponent is 5.
    3. The power is a^5.
    4. This power can be written using repeated multiplication as: $a*a*a*a*a$

2. negative

3. positive

4. negative

5. The difference between -5^2 and $(-5)^2$ is based on the order of operations, in the first expression you are squaring 5 (solving the exponent: 5^2) then making the answer negative. In the second expression you are squaring -5, which results in a positive answer, because the exponent is an even number.

6. 2^2

7. $(-3)^3$

8. y^5

9. $(3a)^4$

10. 4^5

11. $(3x)^3$

12. $(-2a)^4$

13. $6^3x^2y^4$

14. 1

15. 0

16. 343

17. -36

18. 625

19. $3^4 * 3^7 = 3^{(4+7)} = 3^{11} = 177147$

20. $2^6 * 2 = 2^{(6+1)} = 2^7 = 128$

21. $(4^2)^3 = 4^{(2*3)} = 4^6 = 4096$

22. 64

23. 0.0001

24. -0.216

25. $6^3 * 6^6 = 6^{(3+6)} = 6^9 = 10077696$

26. $2^2 * 2^4 * 2^6 = 2^{(2+4+6)} = 2^{12} = 4096$

27. $3^2 * 4^3 = 9 * 64 = 576$

28. $x^2 * x^4 = x^{(2+4)} = x^6$

29. $x^2 * x^7 = x^{(2+7)} = x^9$

30. $(y^3)^5 = y^{(3*5)} = y^{15}$

31. $(-2y^4)\ (-3y) = -2 * -3 * y^4 * y = 6 * y^{(4+1)} = 6y^5$

32. $(4a^2)\ (-3a)\ (-5a^4) = 4 * -3 * -5 * a^2 * a * a^4 = 60 * a^{(2+1+4)} = 60a^7$

33. $(a^3)^4 = a^{(3*4)} = a^{12}$

34. $(xy)^2 = x^2y^2$

35. $(3a^2b^3)^4 = 3^4 * a^{(2*4)} * b^{(3*4)} = 81a^8b^{12}$

36. $(-2xy^4z^2)^5 = (-2)^5 * x^5 * y^{(4*5)} * z^{(2*5)} = -32x^5y^{20}z^{10}$

37. $(3x^2y^3) * (4xy^2) = 3 * 4 * x^2 * x * y^3 * y^2 = 12 * x^{(2+1)} * y^{(3+2)} = 12x^3y^5$

38. $(4xyz) * (x^2y^3) * (2yz^4) = 4 * 2 * x * x^2 * y * y^3 * y * z * z^4 =$
$8 * x^{(1+2)} * y^{(1+3+1)} * z^{(1+4)} = 8x^3y^5z^5$

39. $(2a^3b^3)^2 = 2^2 * a^{(3*2)} * b^{(3*2)} = 4a^6b^6$

40. $(-8x)^3\ (5x)^2 = (-8)^3 * 5^2 * x^3 * x^2 = -512 * 25 * x^{(3+2)} = -12800x^5$

41. $(4a^2)\ (-2a^3)^4 = 4 * (-2)^4 * a^2 * a^{(3*4)} = 4 * 16 * a^2 * a^{(12)} = 64 * a^{(2+12)} = 64a^{14}$

42. $(12xy)\ (12xy)^2 = (12xy)^3 = 12^3 * x^3 * y^3 = 1728x^3y^3$

43. $(2xy^2)\ (-x^2y)^2\ (3x^2y^2) = 2 * -1^2 * 3 * x * x^{(2*2)} * x^2 * y^2 * y^2 * y^2 = 6 * x^{(1+4+2)} * y^{(2+2+2)}$
$= 6x^7y^6$

Mixed Review

44. *How many ways can you choose a 4-person committee from seven people?*
$${}_7C_4 = \frac{7!}{4!(7-4)!} = \frac{5040}{144} = 35$$

45. *Three canoes cross a finish line to earn medals. Is this an example of a permutation or a combination? How many ways are possible?*
This is a permutation.
The number of ways possible is 3!=6

46. *Find the slope between (-9, 11) and (18, 6)*
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11-6}{-9-18} = -\frac{5}{27}$$

47. *Name the set(s) to which $\sqrt{36}$ belongs.*
When simplified $\sqrt{36} = \pm 6$. Therefore the expression belongs to Z (the integer set of numbers.) In addition, the integer set of numbers is also part of R (the real set of numbers).

The square root of 36 belongs to sets Z and R.

48. *Simplify* $\sqrt{74x^2}$
$$\sqrt{74x^2} \approx 8.6x$$
$$\sqrt{74x^2} = \sqrt{4} * \sqrt{18.5} * \sqrt{x^2} = 2x\sqrt{18.5}$$

49.
$$\frac{78}{x} = \frac{10}{100} = \frac{1}{10}$$
$$\frac{78}{x} = \frac{1}{10}$$
$$780 = x$$

50.

$$m = \frac{3-1}{5-3} = \frac{2}{2} = 1$$

$$y = mx + b$$
$$y = x + b$$
$$(5,3)$$
$$3 = 5 + b$$
$$b = -2$$

$$y = x - 2$$

Lesson 8.2
Exponential Properties Involving Quotients

1. $\frac{5^6}{5^2} = 5^{6-2} = 5^4 = 625$

2. $\frac{6^7}{6^3} = 6^{7-3} = 6^4 = 1296$

3. $\frac{3^{10}}{3^4} = 3^{10-4} = 3^6 = 729$

4. $\left(\frac{2^2}{3^3}\right)^3 = \frac{2^{2\cdot 3}}{3^{3\cdot 3}} = \frac{2^6}{3^9} = \frac{64}{19683} = .003251536859$

5. $\frac{a^3}{a^2} = a^{3-2} = a$

6. $\frac{x^9}{x^5} = x^{9-5} = x^4$

7. $\frac{x^{10}}{x^5} = x^{10-5} = x^5$

8. $\frac{a^6}{a} = a^{6-1} = a^5$

9. $\frac{a^5b^4}{a^3b^2} = a^{5-3} \bullet b^{4-2} = a^2b^2$

10. $\frac{4^5}{4^2} = 4^{5-2} = 4^3 = 64$

11. $\frac{5^7}{5^3} = 5^{7-3} = 5^4 = 625$

12. $\left(\frac{3^4}{5^2}\right)^2 = \frac{3^{4\cdot 2}}{5^{2\cdot 2}} = \frac{3^8}{5^4} = \frac{6561}{625} = 10.4976$

13. $\left(\frac{a^3b^4}{a^2b}\right)^3 = \frac{a^{3\bullet 3}\times b^{4\bullet 3}}{a^{2\bullet 3}\times b^{1\bullet 3}} = \frac{a^9b^{12}}{a^6b^3} = a^{9-6}\times b^{12-3} = a^3b^9$

14. $\frac{x^6y^5}{x^2y^3} = x^{6-2}\times y^{5-3} = x^4y^2$

15. $\left(\frac{6x^2y^3}{2xy^2}\right) = 3 * x^{2-1} * y^{3-2} = 3xy$

16. $\left(\frac{8a^7b^3}{2a^3b}\right)^2 = \frac{8^2 * a^{7\bullet 2} * b^{3\bullet 2}}{2^2 * a^{3\bullet 2} * b^2} = \frac{64a^{14}b^6}{4a^6b^2} = 16a^{14-6}b^{6-2} = 16a^8b^4$

17. $(x^2)^2 \bullet \frac{x^6}{x^4} = x^{2\bullet 2}\bullet x^{6-4} = x^4\bullet x^2 = x^{4+2} = x^6$

18. $\left(\frac{16a^6}{4b^5}\right)^3 \bullet \frac{b^{16}}{a^{16}} = \frac{16^3a^{6\bullet 3}}{4^3b^{5\bullet 3}}\bullet\frac{b^{16}}{a^{16}} = \frac{4096a^{18}b^{16}}{64a^{16}b^{15}} = 64a^2b$

19. $\frac{6a^3}{2a^2} = 3a$

20. $\frac{15x^5}{5x} = 3x^4$

21. $\left(\frac{18a^{10}}{15a^4}\right)^4 = \frac{18^4a^{10\bullet 4}}{15^4a^{4\bullet 4}} = \frac{104976a^{40}}{50625a^{16}} = 2.0736a^{24}$

22. $\frac{25y^5x^6}{20yx^2} = 1.25y^4x^4$

23. $\left(\frac{x^6y^4}{x^4y^2}\right)^3 = \frac{x^{6\bullet 3}y^{4\bullet 3}}{x^{4\bullet 3}y^{2\bullet 3}} = \frac{x^{18}y^{12}}{x^{12}y^6} = x^{18-12}y^{12-6} = x^6y^6$

24. $\left(\frac{6a^2}{4}\right)^2 \bullet \frac{5b}{3a} = \frac{6^2a^{2\bullet 2}}{4^2}\bullet\frac{5b}{3a} = \frac{36a^4\bullet 5b}{16\bullet 3a} = \frac{180a^4b}{48a} = 3.75a^3b$

25. $\frac{(3ab)^2(4a^3b^4)^3}{(6a^2b)^4} = \frac{3^2 \bullet a^2 \bullet b^2 \bullet 4^3 \bullet a^9 \bullet b^{12}}{6^4 a^8 b^4} = \frac{576a^{11}b^{14}}{1296a^8b^4} = .4\bar{4}a^3b^{10}$

26. $\frac{(2a^2bc^2)(6abc^3)}{4ab^2c} = \frac{12a^3b^2c^5}{4ab^2c} = 3a^2c^4$

Mixed Review

27.

$x|z| - |z|$

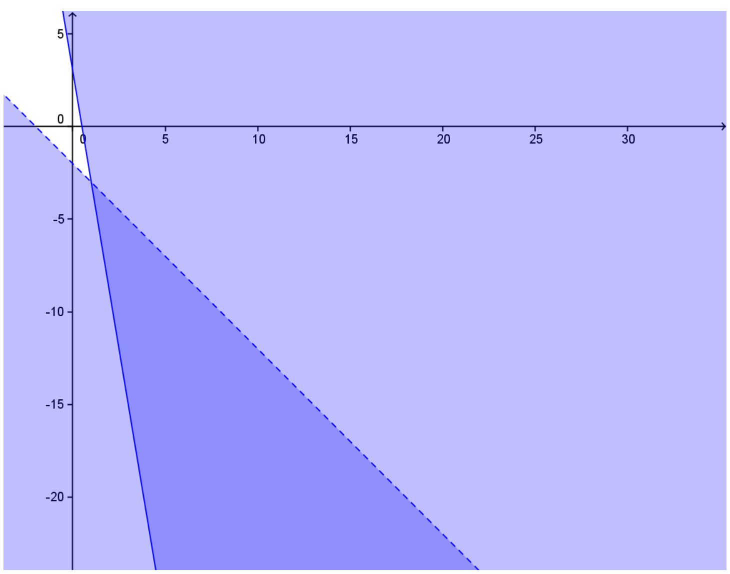
$8|-4| - |-4|$

$(8)(4) - (4)$

$32 - 4$

28

28. Graph the solution set to the system $\begin{cases} y < -x - 2 \\ y \geq -6x + 3 \end{cases}$



29. *Evaluate* $\binom{8}{4}$

$\binom{8}{4} = {}_8C_4 = \frac{8!}{4!(8-4)!} = \frac{40320}{576} = 70$

30. Answers will vary. It is correct as long as the situation involves 4 items or choices, being chosen one at a time, that can only be chosen once, so that the number of choices decreases by one with each pick.

31. $x^3 = 8$

Lesson 8.3
Zero, Negative, and Fractional Exponents

1. $x^{-1} \bullet y^2 = \frac{1}{x} \bullet y^2 = \frac{y^2}{x}$

2. $x^{-4} = \frac{1}{x^4}$

3. $\frac{x^{-3}}{x^{-7}} = x^{-3-(-7)} = x^{-3+7} = x^4$

4. $x^{-1} = \frac{1}{x}$

5. $2x^{-2} = 2 \cdot \frac{1}{x^2} = \frac{2}{x^2}$

6. $x^2 y^{-3} = x^2 \cdot \frac{1}{y^3} = \frac{x^2}{y^3}$

7. $3x^{-1}y^{-1} = 3 \cdot \frac{1}{x} \cdot \frac{1}{y} = \frac{3}{xy}$

8. $3x^{-3} = 3 \cdot \frac{1}{x^3} = \frac{3}{x^3}$

9. $a^2 b^{-3} c^{-1} = a^2 \cdot \frac{1}{b^3} \cdot \frac{1}{c} = \frac{a^2}{b^3 c}$

10. $4x^{-1}y^3 = 4 \cdot \frac{1}{x} \cdot y^3 = \frac{4y^3}{x}$

11. $\frac{2x^{-2}}{y^{-3}} = 2 \cdot \frac{1}{x^2} \cdot y^3 = \frac{2y^3}{x^2}$

12. $a^{\frac{1}{2}} \cdot a^{\frac{1}{3}} = a^{\frac{1}{2}+\frac{1}{3}} = a^{\frac{3}{6}+\frac{2}{6}} = a^{\frac{5}{6}} = \sqrt[6]{a^5}$

13. $\left(a^{\frac{1}{3}}\right)^{2} = a^{\frac{1}{3}\cdot 2} = a^{\frac{2}{3}} = \sqrt[3]{a^{2}}$

14. $\dfrac{a^{\frac{5}{2}}}{a^{\frac{1}{2}}} = a^{\frac{5}{2}-\frac{1}{2}} = a^{\frac{4}{2}} = a^{2}$

15. $\left(\dfrac{x^{2}}{y^{3}}\right)^{\frac{1}{3}} = \left(x^{2}\cdot y^{-3}\right)^{\frac{1}{3}} = x^{2\cdot\frac{1}{3}}\cdot y^{-3\cdot\frac{1}{3}} = x^{\frac{2}{3}}\cdot y^{-1} = \dfrac{x^{\frac{2}{3}}}{y} = \dfrac{\sqrt[3]{x^{2}}}{y}$

16. $\dfrac{x^{-3}y^{-5}}{z^{-7}} = \dfrac{1}{x^{3}}\cdot\dfrac{1}{y^{5}}\cdot z^{7} = \dfrac{z^{7}}{x^{3}y^{5}}$

17. $\left(x^{\frac{1}{2}}y^{-\frac{2}{3}}\right)\left(x^{2}y^{\frac{1}{3}}\right) = x^{\frac{1}{2}+2}\cdot y^{-\frac{2}{3}+\frac{1}{3}} = x^{\frac{3}{2}}\cdot y^{-\frac{1}{3}} = \dfrac{x^{\frac{3}{2}}}{y^{\frac{1}{3}}} = \dfrac{x\cdot\sqrt{x}}{\sqrt[3]{y}}$

18. $\left(\dfrac{a}{b}\right)^{-2} = \left(a^{1\cdot -2}\cdot b^{-1\cdot -2}\right) = a^{-2}b^{2} = \dfrac{1}{a^{2}}\cdot b^{2} = \dfrac{b^{2}}{a^{2}}$

19. $\left(3a^{-2}b^{2}c^{3}\right)^{3} = 3^{3}\cdot a^{-2\cdot 3}\cdot b^{2\cdot 3}\cdot c^{3\cdot 3} = 27\cdot a^{-6}\cdot b^{6}\cdot c^{9} = \dfrac{27b^{6}c^{9}}{a^{6}}$

20. $x^{-3}\cdot x^{3} = \dfrac{1}{x^{3}}\cdot x^{3} = \dfrac{x^{3}}{x^{3}} = 1$

21. $\dfrac{a^{-3}\left(a^{5}\right)}{a^{-6}} = \dfrac{a^{-3+5}}{a^{-6}} = \dfrac{a^{2}}{a^{-6}} = a^{2-(-6)} = a^{8}$

22. $\dfrac{5x^{6}y^{2}}{x^{8}y} = 5\cdot x^{6-8}\cdot y^{2-1} = 5x^{-2}y$

23. $\dfrac{\left(4ab^{6}\right)^{3}}{\left(ab\right)^{5}} = \dfrac{4^{3}\cdot a^{3}\cdot b^{6\cdot 3}}{a^{5}b^{5}} = \dfrac{64a^{3}b^{18}}{a^{5}b^{5}} = 64a^{3-5}b^{18-5} = 64a^{-2}b^{13}$

24. $\left(\frac{3x}{y^{\frac{1}{3}}}\right)^3 = \frac{3^3x^3}{y^{\frac{1}{3}\cdot 3}} = \frac{27x^3}{y} = 27x^3y^{-1}$

25. $\frac{4a^2b^3}{2a^5b} = \left(\frac{4}{2}\right)\cdot a^{2-5}\cdot b^{3-1} = 2a^{-3}b^2$

26.
$\left(\frac{x}{3y^2}\right)^3\cdot\frac{x^2y}{4} = \frac{x^3}{3^3y^{2\cdot 3}}\cdot\frac{x^2y}{4} = \frac{x^{3+2}y}{27\cdot y^6\cdot 4} = \frac{x^5y}{108y^6} = \frac{1}{108}\cdot x^5\cdot y^{1-6} = \frac{1}{108}x^5y^{-5} \approx .00925926x^5y^{-}$

27. $\left(\frac{ab^{-2}}{b^3}\right)^2 = \frac{a^2b^{-2\cdot 2}}{b^{3\cdot 2}} = \frac{a^2b^{-4}}{b^6} = a^2b^{-4-6} = a^2b^{-10}$

28. $\frac{x^{-3}y^2}{x^2y^{-2}} = x^{-3-2}y^{2-(-2)} = x^{-5}y^4$

29. $\frac{3x^2y^{\frac{3}{2}}}{xy^{\frac{1}{2}}} = 3\cdot x^{2-1}\cdot y^{\frac{3}{2}-\frac{1}{2}} = 3xy$

30. $\frac{(3x^3)(4x^4)}{(2y)^2} = \frac{12x^{3+4}}{2^2y^2} = \frac{12x^7}{4y^2} = 3x^7y^{-2}$

31. $\frac{a^{-2}b^{-3}}{c^{-1}} = a^{-2}b^{-3}c$

32. $\frac{x^{\frac{1}{2}}y^{\frac{5}{2}}}{x^{\frac{3}{2}}y^{\frac{3}{2}}} = x^{\frac{1}{2}-\frac{3}{2}}\cdot y^{\frac{5}{2}-\frac{3}{2}} = x^{-1}y$

33. $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

34. $(6.2)^0 = 1$

35. $8^{-4}\cdot 8^6 = 8^{-4+6} = 8^2 = 64$

36. $\left(16^{\frac{1}{2}}\right)^{3} = 16^{\frac{1}{2}\cdot 3} = 16^{\frac{3}{2}} = \sqrt{16^{3}} = \sqrt{4096} = \pm 64$

37. $5^{0} = 1$

38. $7^{2} = 7 \cdot 7 = 49$

39. $\left(\frac{2}{3}\right)^{3} = \frac{2^{3}}{3^{3}} = \frac{8}{27} \approx .2962963$

40. $3^{-3} = \frac{1}{3^{3}} = \frac{1}{27} \approx .03703704$

41. $16^{\frac{1}{2}} = \sqrt{16} = 4$

42. $8^{-\frac{1}{3}} = \frac{1}{8^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{8}} = \frac{1}{2} = 0.5$

43.

$$
\begin{aligned}
&2x^{2} - 3y^{3} + 4z \\
&2(2^{2}) - 3(-1^{3}) + 4(3) \\
&2(4) - 3(-1) + 12 \\
&8 + 3 + 12 \\
&11 + 12 \\
&23
\end{aligned}
$$

44.

$$
\begin{aligned}
&\left(x^{2} - y^{2}\right)^{2} \\
&\left(2^{2} - (-1^{2})\right)^{2} \\
&(4 - 1)^{2} \\
&3^{2} \\
&9
\end{aligned}
$$

45.

$$\left(\frac{3x^2y^5}{4z}\right)^{-2}$$

$$\left(\frac{3(2^2)(-1^5)}{4(3)}\right)^{-2} = \left(\frac{3(4)(-1)}{12}\right)^{-2} = \left(\frac{-12}{12}\right)^{-2} = (-1)^{-2} = -\frac{1}{1^2} = -1$$

46.

$$x^2 4x^3y^4 4y^2$$

$$(2^2)\cdot 4\cdot (2^3)\cdot(-1^4)\cdot 4\cdot(-1^2)$$

$$4\cdot 4\cdot 8\cdot 1\cdot 4\cdot 1$$

$$512$$

47.

$$a^4(b^2)^3 + 2ab$$

$$(-2^4)(1^2)^3 + 2(-2)(1)$$

$$(16)(1) + -4$$

$$16 - 4$$

$$12$$

48.

$$5x^2 - 2y^3 + 3z$$

$$5(3^2) - 2(2^3) + 3(4)$$

$$5(9) - 2(8) + 12$$

$$45 - 16 + 12$$

$$41$$

49. $\left(\frac{a^2}{b^3}\right)^{-2} = \left(\frac{5^2}{3^3}\right)^{-2} = \left(\frac{25}{27}\right)^{-2} = \frac{25^{-2}}{27^{-2}} = \frac{27^2}{25^2} = \frac{729}{625} = 1.1664$

50.

$$3 \cdot 5^5 - 10 \cdot 5 + 1$$
$$3 \cdot 3125 - 10 \cdot 5 + 1$$
$$9375 - 50 + 1$$
$$9325 + 1$$
$$9326$$

51. $\frac{2 \cdot 4^2 - 3 \cdot 5^2}{3^2} = \frac{2 \cdot 16 - 3 \cdot 25}{9} = \frac{32 - 75}{9} = \frac{-43}{9} \approx -4.7777778$

52. $\left(\frac{3^3}{2^2}\right)^{-2} \cdot \frac{3}{4} = \frac{3^{-6}}{2^{-4}} \cdot \frac{3}{4} = \frac{2^4}{3^6} \cdot \frac{3}{4} = \frac{16}{729} \cdot \frac{3}{4} = \frac{4}{243} \approx .01646091$

53.

When the test is answered each of the true/false questions can be answered in 2 ways. This allows them to be answered in 2^7 ways. The 3 multiple choice questions have 4 options and can be answered in 4^3 ways. $2^7 * 4^3$ = 128*64 = 8192. The test can be answered in 8192 ways.

54.

$$3a^4b^4 \cdot a^{-3}b^{-4}$$
$$3 \cdot a^{4+(-3)} \cdot b^{4+(-4)}$$
$$3a$$

55.

$\left(x^4 y^2 \cdot xy^0\right)^5$

$\left(x^{4+1} \cdot y^{2+0}\right)^5$

$\left(x^5 y^2\right)^5$

$x^{5\cdot 5} y^{2\cdot 5}$

$x^{25} y^{10}$

56. $\dfrac{v^2}{-vu^{-2} \cdot u^{-1} v^4} = \dfrac{v^2}{-v^{1+4} \cdot u^{-2+(-1)}} = \dfrac{v^2}{-v^5 u^{-3}} = \dfrac{v^2}{-v^5} \cdot \dfrac{1}{u^{-3}} = -v^{2-5} \cdot u^3 = -v^{-3} u^3 = \dfrac{u^3}{-v^3}$

57.

$-6(4n + 3) = n + 32$

$-24n - 18 = n + 32$

$-32 - 18 = 24n + n$

$-50 = 25n$

$n = -2$

Check

$-6(4n + 3) = n + 32$

$-6(4(-2) + 3) = -2 + 32$

$-6(-8 + 3) = 30$

$-6(-5) = 30$

$30 - 30$

Lesson 8.4
Scientific Notation

1. 310.2

2. 74000

3. 0.00175

4. 0.000029

5. 0.00000000999

6. 2.784×10^{18}

7. 1.976×10^{-22}

8. 4.59×10^{-5}

9. 3.6782×10^{-4}

10. 1.3684×10^{15}

11. 0.62963×10^{18}

12. 1.2×10^{5}

13. 1.765244×10^{6}

14. 6.3×10

15. 9.654×10^{3}

16. 6.53937×10^{8}

17. 1×10^{9}

18. 1.2×10

19. 2.81×10^{-3}
20. 2.7×10^{-8}

21. 3×10^{-3}

22. 5.6×10^{-5}

23. 5.007×10^{-5}

24. 9.54×10^{-12}

25.
$$SA = 4\pi r^2$$
$$SA = 4(3.14)(1.08x10^3)^2$$
$$SA = 4(3.14)(1.08^2)(10^6)$$
$$SA = 4(3.14)(1.1664)(10^6)$$
$$SA = 14.649984 \times 10^6$$
$$SA = 1.47 \times 10^7$$

26.
$$\left(1.60 \times 10^{-19}\right) \cdot \left(6.02 \times 10^{23}\right)$$
$$9.632 \times 10^4$$

27.
$$\left(2.5 \times 10^{13}\right) \div \left(3.7 \times 10^4\right)$$
$$0.67567568 \times 10^9$$
$$6.76 \times 10^8$$

Mixed Review

28.

$$\frac{5.6}{18} = \frac{x}{100}$$
$$560 = 18x$$
The concentration is approximately 31% sugar.
$$x \approx 31.11$$

29.

$$\begin{cases} 6x + 3y = 18 \\ -15 = 11y - 5x \end{cases}$$

$$-15 = 11y - 5x \rightarrow -5x + 11y = -15$$

$$-5x + 11y = -15 \rightarrow 6(-5x + 11y = -15) \rightarrow -30x + 66y = -90$$

$$6x + 3y = 18 \rightarrow 5(6x + 3y = 18) \rightarrow 30x + 15y = 90$$

$$\begin{array}{l} 30x + 15y = 90 \\ \underline{-30x + 66y = -90} \\ 0x + 81y = 0 \\ 81y = 0 \\ y = 0 \end{array}$$

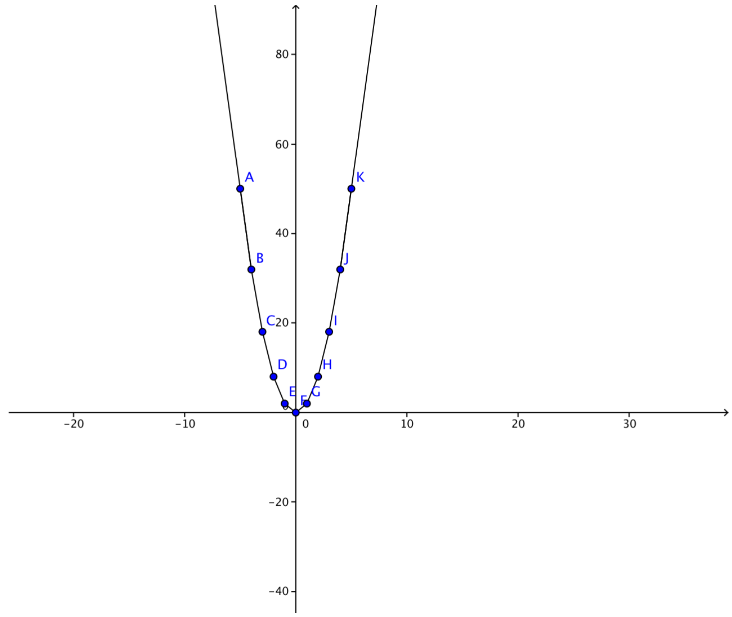
$$\begin{array}{l} 6x + 3y = 18 \\ 6x + 3(0) = 18 \\ 6x = 18 \\ x = 3 \end{array}$$

Check

$$\begin{array}{ll} 6x + 3y = 18 & -15 = 11y - 5x \\ 6(3) + 3(0) = 18 & -15 = 11(0) - 5(3) \\ 18 + 0 = 18 & -15 = 0 - 15 \\ 18 = 18 & -15 = -15 \end{array}$$

30. *Graph the function by creating a table:* $f(x) = 2x^2$. *Use the following values for x:* $-5 \le x \le 5$.

x	$2x^2$	f(x)
-5	$2(-5^2)=2(25)=50$	50
-4	$2(-4^2)=2(16)=32$	32
-3	$2(-3^2)=2(9)=18$	18
-2	$2(-2^2)=2(4)=8$	8
-1	$2(-1^2)=2(1)=2$	2
0	$2(0^2)=2(0)=0$	0
1	$2(1^2)=2(1)=2$	2
2	$2(2^2)=2(4)=8$	8
3	$2(3^2)=2(9)=18$	18
4	$2(4^2)=2(16)=32$	32
5	$2(5^2)=2(25)=50$	50



31. $\dfrac{5a^6b^2c^{-6}}{a^{11}b} = 5 \cdot a^{6-11} \cdot b^{2-1} \cdot c^{-6} = 5 \cdot a^{-5} \cdot b \cdot c^{-6} = 5 \cdot \dfrac{1}{a^5} \cdot b \cdot \dfrac{1}{c^6} = \dfrac{5b}{a^5c^6}$

32.

$\left(2.3 \times 10^8\right) \div \left(3.07 \times 10^8\right)$

$.74917 \times 10^0$

7.4917×10^{-1}

230,000,000 / 307,006,550 = 0.749169683838993

33.

$$V = l(w)(h)$$
$$312 = 12(8)(h)$$
$$312 = 96h$$
$$h = 3.25$$

The box is 3.25 inches tall.

Quick Quiz

1. $$\frac{\left(2x^{-4}y^{3}\right)^{-3}\cdot x^{-3}y^{-2}}{-2x^{0}y^{2}}$$

$$\frac{2^{-3}\cdot x^{-4\cdot -3}\cdot y^{3\cdot -3}\cdot x^{-3}\cdot y^{-2}}{-2(1)y^{2}}$$

$$\frac{8^{-1}\cdot x^{12}\cdot y^{-9}\cdot x^{-3}\cdot y^{-2}}{-2y^{2}}$$

$$\frac{8^{-1}\cdot x^{12+-3}\cdot y^{-9+(-2)}}{-2y^{2}}$$

$$\frac{x^{9}\cdot y^{-11}}{8(-2)y^{2}} = x^{9}\cdot y^{-11-2}\cdot\frac{1}{-16} = \frac{x^{9}}{-16y^{13}}$$

2.

1. Two million = 2,000,000 = 2.0 x 10^{6}
2. Five and a half million = 5,500,000 = 5.5 x 10^{6}
3. Three hundred seventy five thousand = 375,000 = 3.75 x 10^{5}

3. FALSE

$$\left(\frac{5}{4}\right)^{-3} \neq -\frac{125}{64}$$

$$\left(\frac{5}{4}\right)^{-3} = \frac{64}{125}$$

Lesson 8.5
Exponential Growth Functions

1. The general equation for an exponential equation is $y = a(b)^x$, where a is the starting value, b is the growth factor, x is the number of times the growth factor has been applied and y is the solution.

2. In a linear equation we often find a variable with an exponent, while in an exponential growth equation the variable is always the exponent.

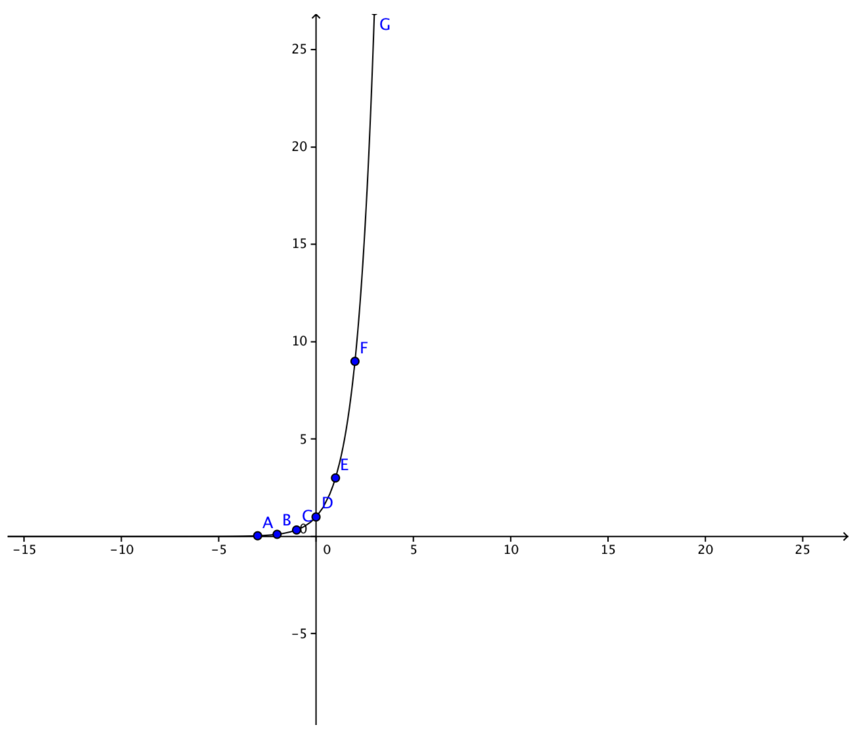
3. The growth factor of an exponential equation must always be greater than 1.

4. FALSE The form of the equation is correct, but b must be greater than 1.

5. The y-intercept (when $x = 0$) in any exponential growth function will always be equal to a or the starting value. (regardless of what b is, when it is raised to the x power and $x = 0$ then $b^x=1$, and a times 1 is always a)

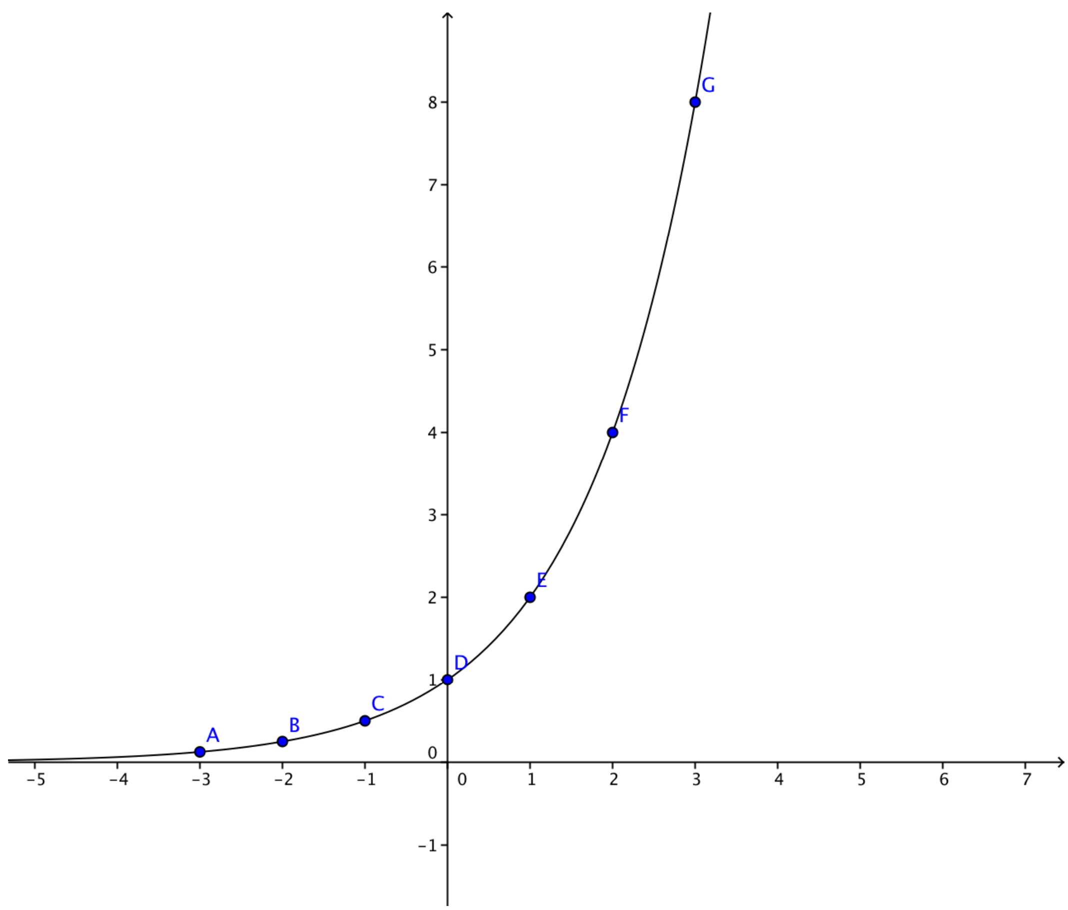
6. $y = 3^x$

x	3^x	y
-3	$3^{-3} = 1/3^3 = 1/27$	1/27
-2	$3^{-2} = 1/3^2 = 1/9$	1/9
-1	$3^{-1} = 1/3^1 = 1/3$	1/3
0	$3^0 = 1$	1
1	$3^1 = 3$	3
2	$3^2 = 9$	9
3	$3^3 = 27$	27



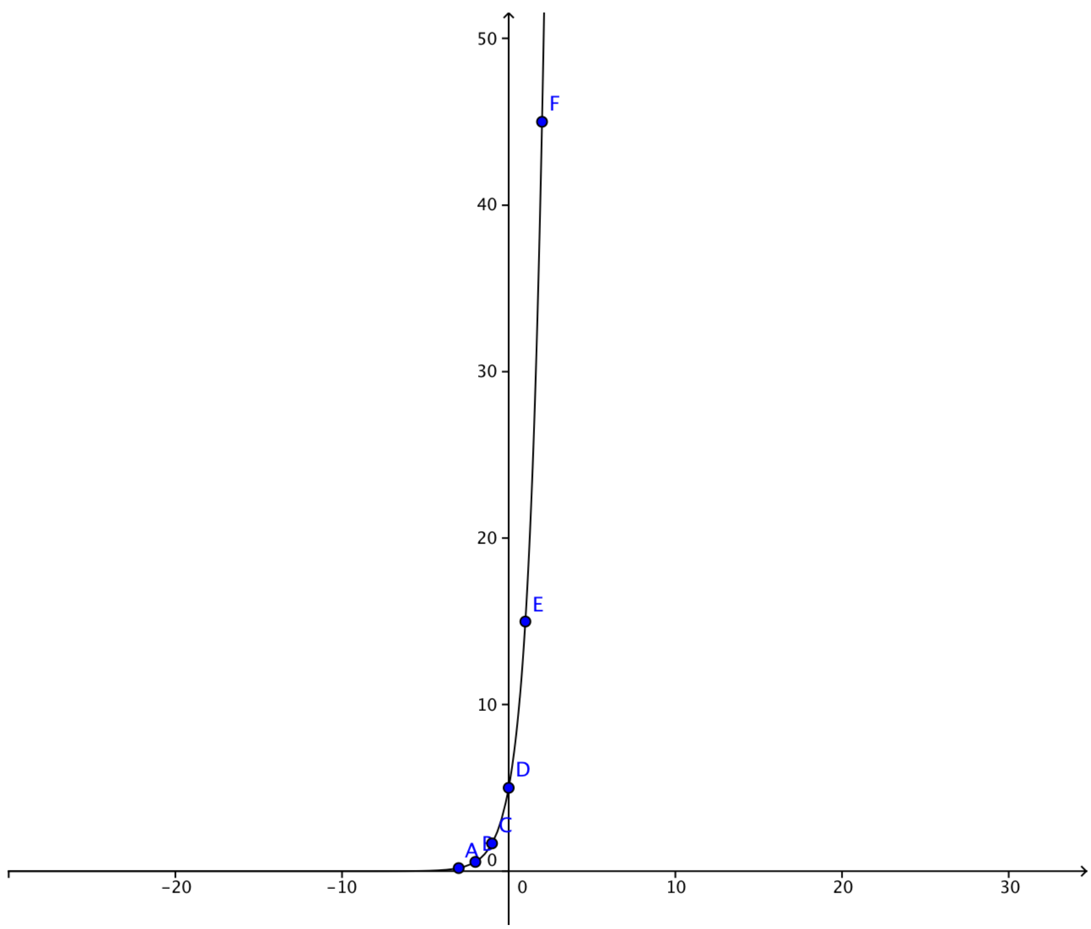
7. $y = 2^x$

x	2^x	y
-3	$2^{-3} = 1/2^3 = 1/8$	1/8=0.125
-2	$2^{-2} = 1/2^2 = 1/4$	¼=0.25
-1	$2^{-1} = 1/2^1 = 1/2$	½=0.5
0	$2^0 = 1$	1
1	$2^1 = 2$	2
2	$2^2 = 4$	4
3	$2^3 = 8$	8



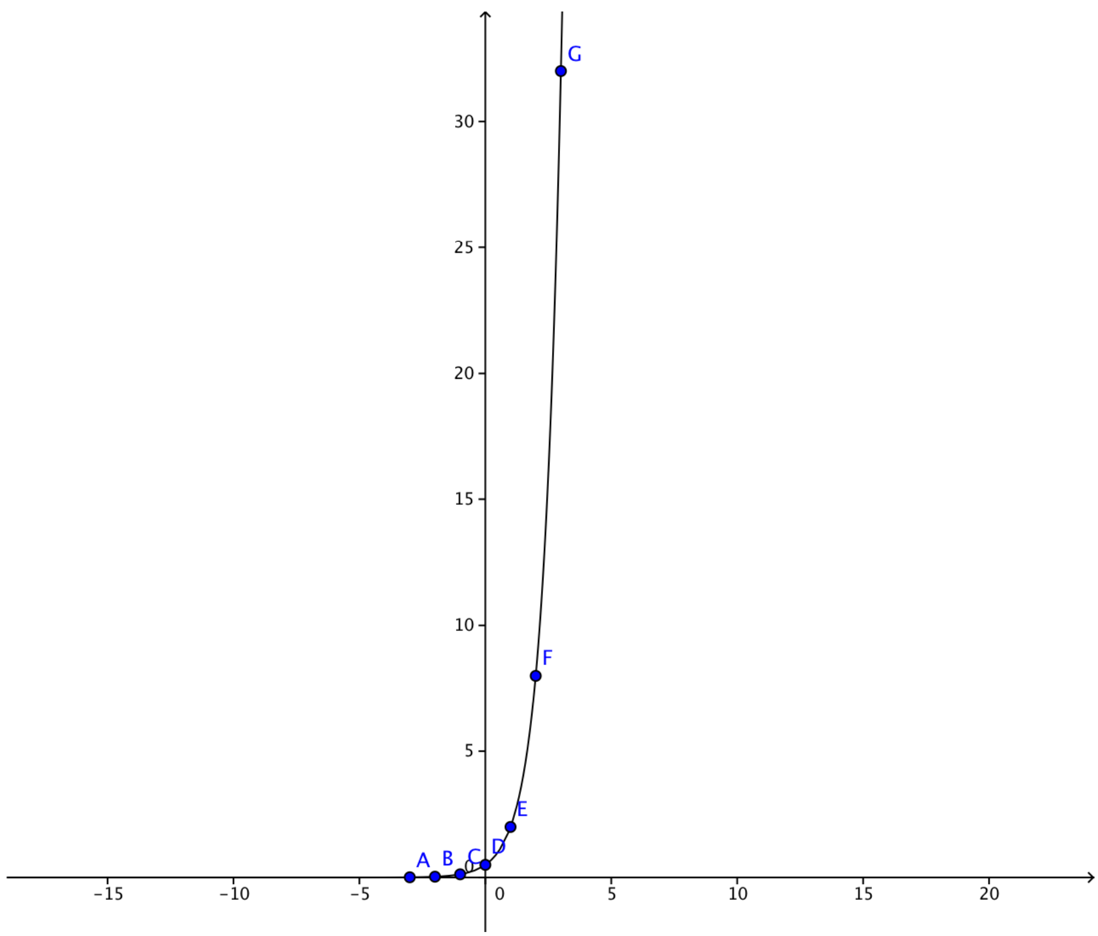
8. $y = 5 \cdot 3^x$

x	$5*3^x$	y
-3	$5*3^{-3}=5*1/3^3=5*1/27=5/27$	5/27=0.185
-2	$5*3^{-2}=5*1/3^2=5*1/9=5/9$	5/9=0.556
-1	$5*3^{-1}=5*1/3^1=5*1/3=5/3$	5/3=1.667
0	$5*3^0=5*1=5$	5
1	$5*3^1=5*3=15$	15
2	$5*3^2=5*9=45$	45
3	$5*3^3=5*27=135$	135



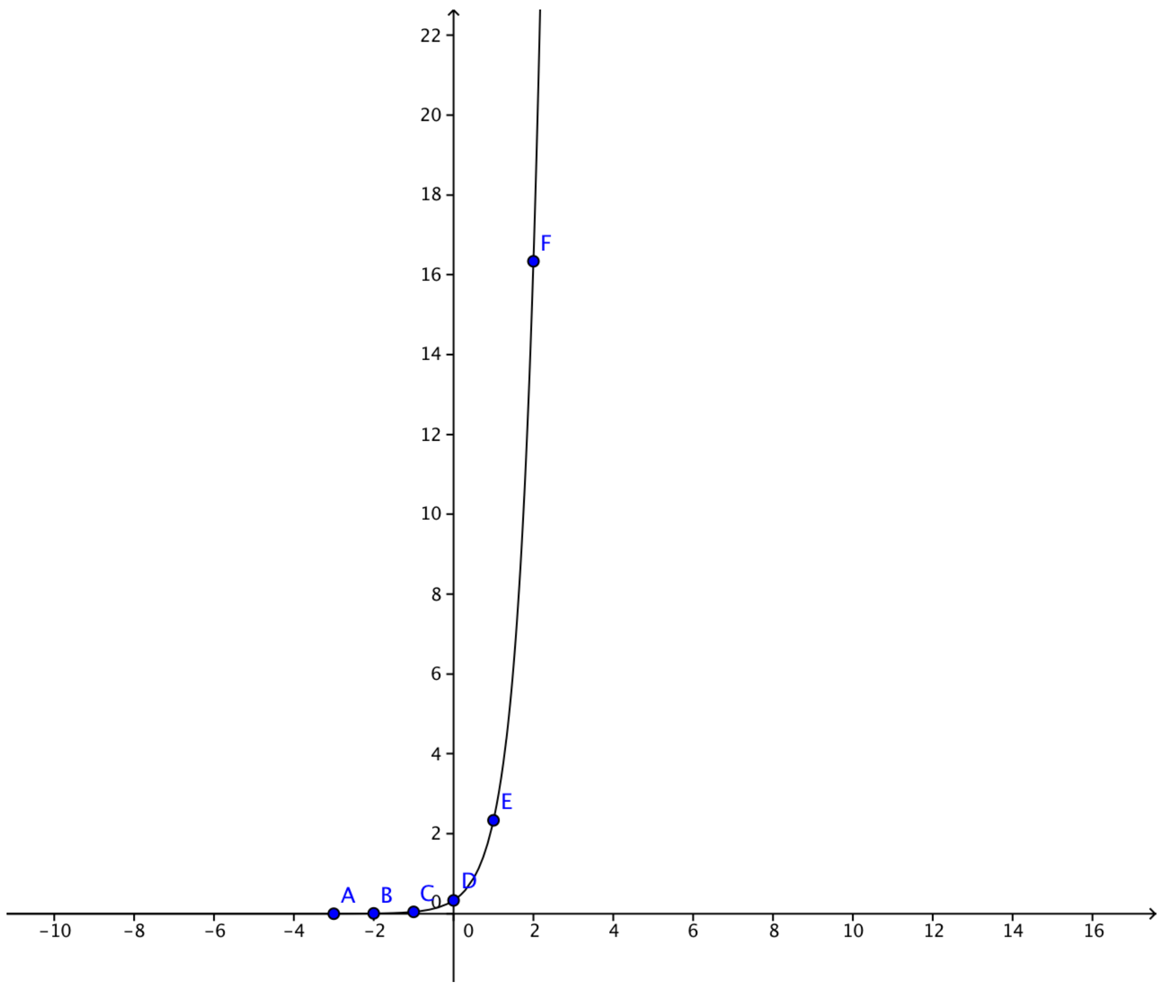
9. $y = \frac{1}{2} \cdot 4^x$

x	½*4^x	y
-3	½*4^{-3}=1/2*1/4^3=1/2*1/64=1/128	1/128=0.0078
-2	½*4^{-2}=1/2*1/4^2=1/2*1/16=1/32	1/32=0.0312
-1	½*4^{-1}=1/2*1/4^1=1/2*1/4=1/8	1/8=0.125
0	½*4^0=1/2*1=1/2	½=0.5
1	½*4^1=1/2*4=2	2
2	½*4^2=1/2*16=8	8
3	½*4^3=1/2*64=32	32



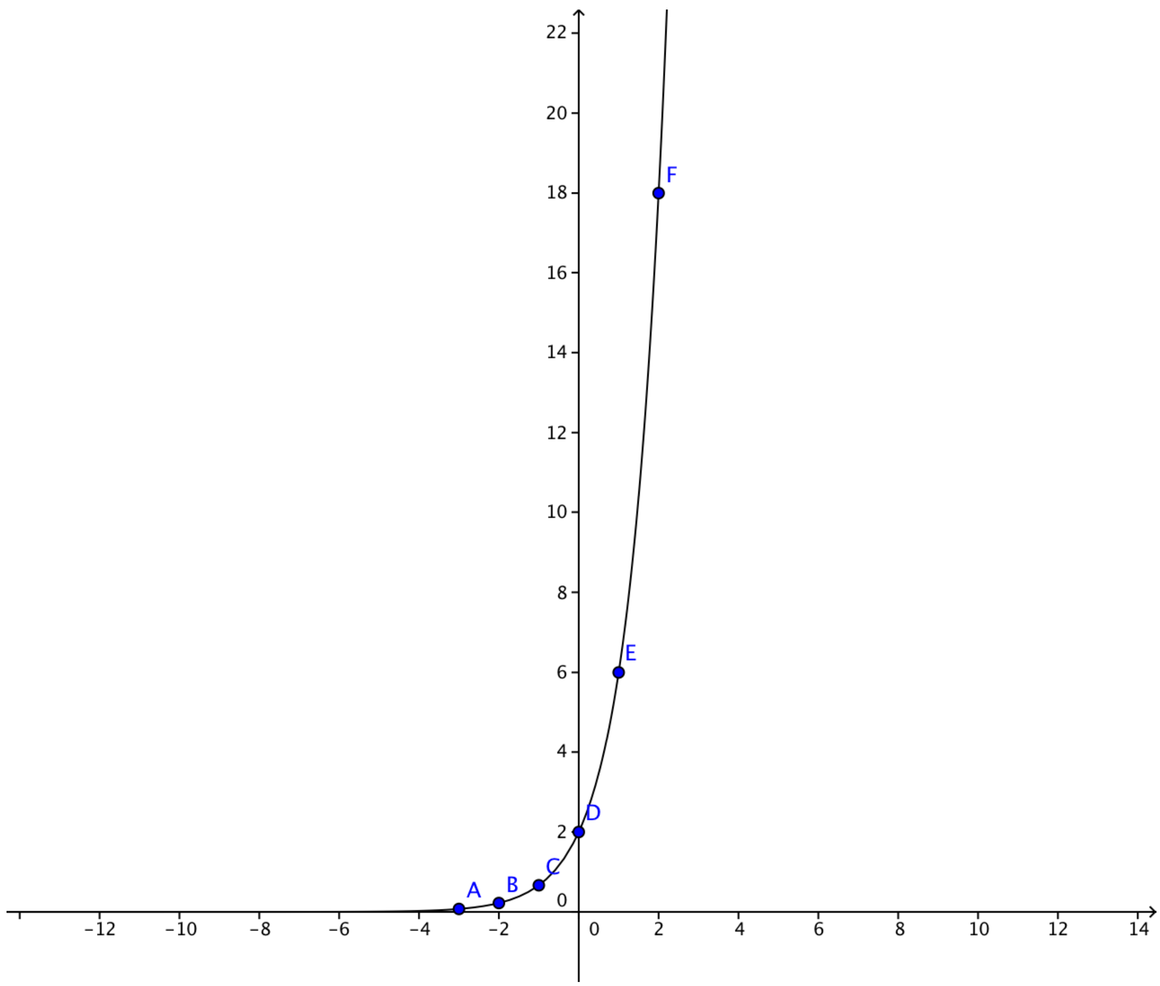
10. $f(x)=\frac{1}{3}\cdot 7^x$

x	$1/3*7^x$	f(x)
-3	$1/3*7^{-3}=1/3*1/7^3=1/3*1/343=1/1029$	1/1029
-2	$1/3*7^{-2}=1/3*1/7^2=1/2*1/49=1/98$	1/98
-1	$1/3*7^{-1}=1/3*1/7^1=1/3*1/7=1/21$	1/21
0	$1/3*7^0=1/3*1=1/3$	1/3
1	$1/3*7^1=1/3*7=7/3$	2.3334
2	$1/3*7^2=1/3*49=49/3$	16.3334
3	$1/3*7^3=1/3*343=343/3$	114.3334



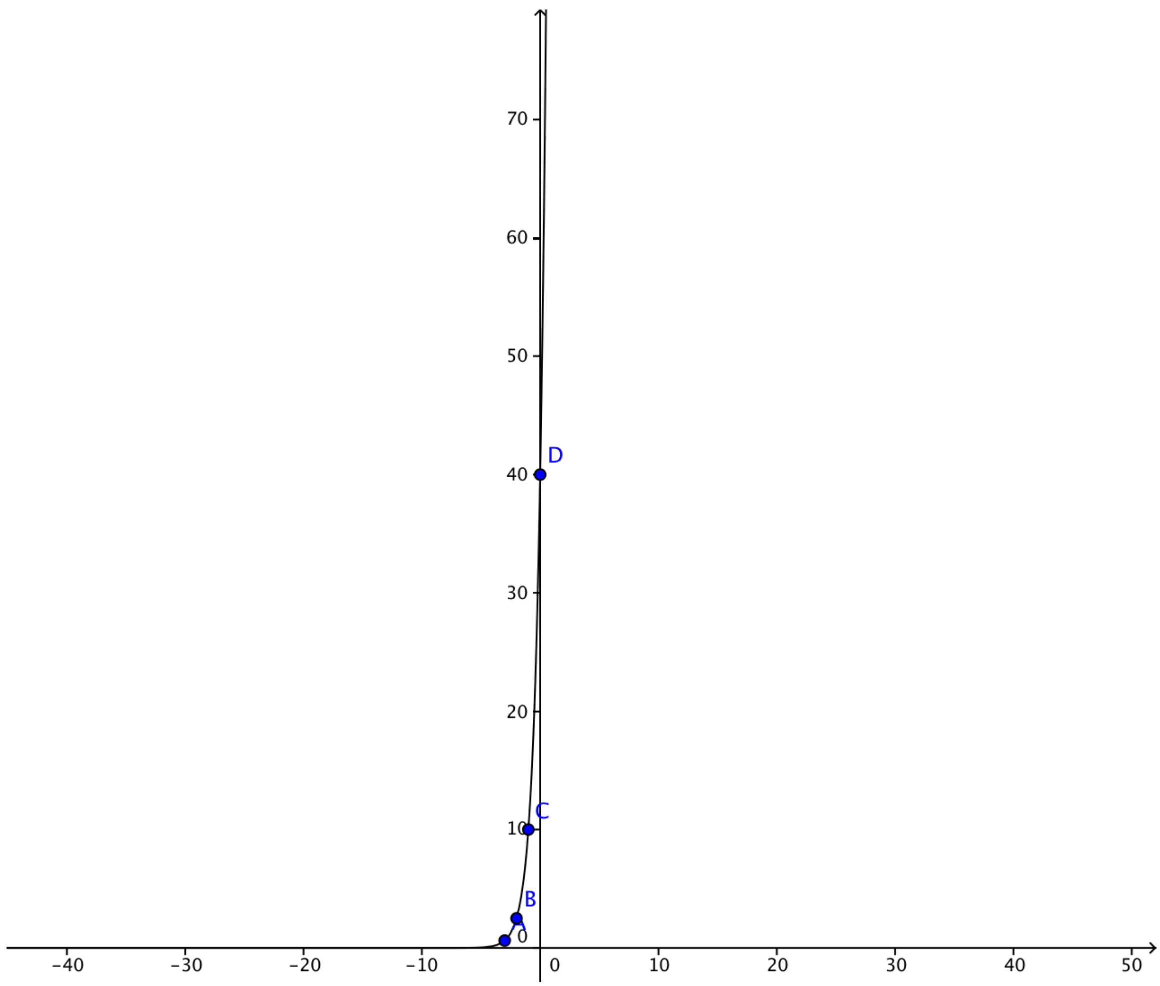
11. $f(x) = 2 \cdot 3^x$

x	$2*3^x$	f(x)
-3	$2*3^{-3}=2*1/3^3=2*1/27=2/27$	2/27=0.074
-2	$2*3^{-2}=2*1/3^2=2*1/9=2/9$	2/9=0.2223
-1	$2*3^{-1}=2*1/3^1=2*1/3=2/3$	2/3=0.6667
0	$2*3^0=2*1=2$	2
1	$2*3^1=2*3=6$	6
2	$2*3^2=2*9=18$	18
3	$2*3^3=2*27=54$	54



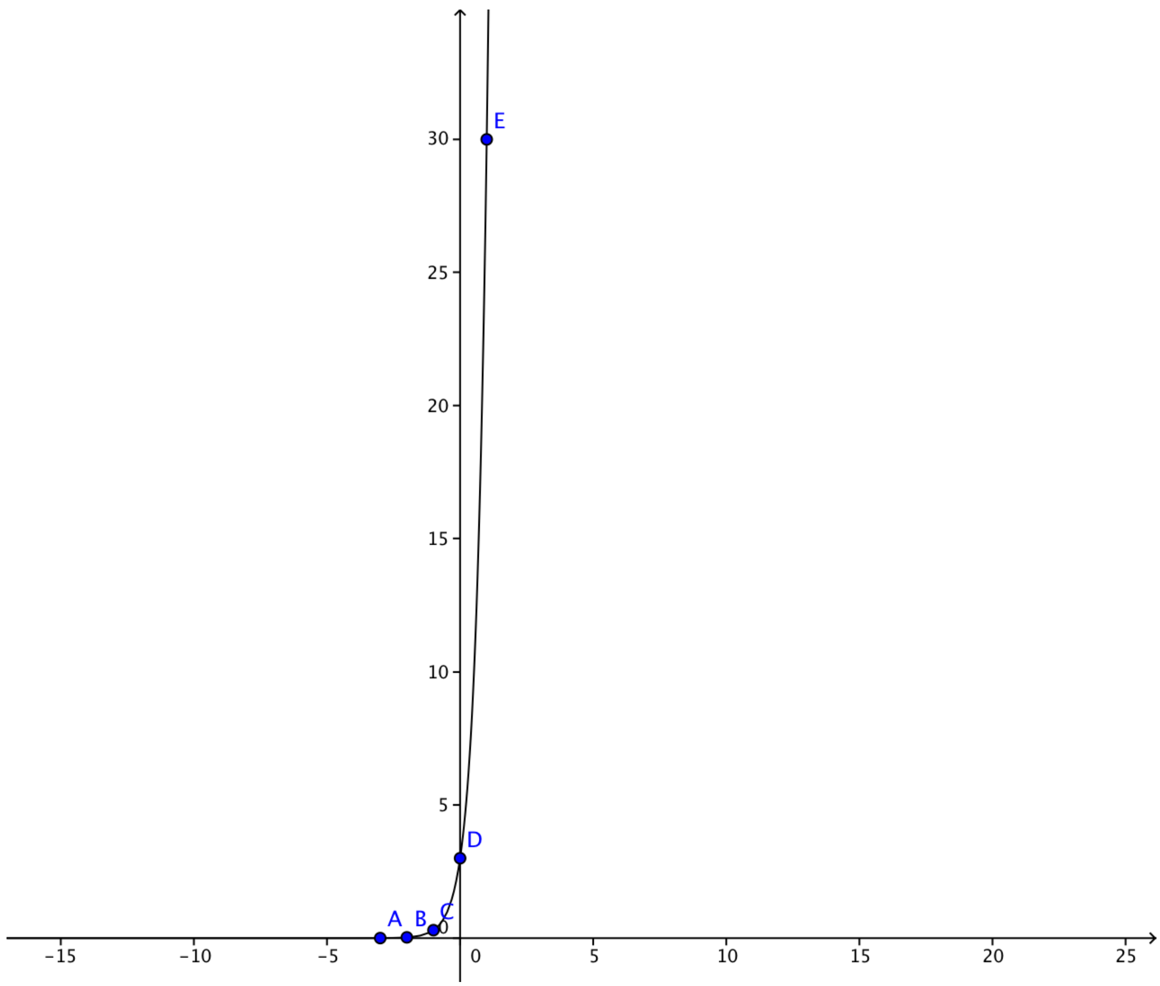
12. $y = 40 \cdot 4^x$

x	$40*4^x$	y
-3	$40*4^{-3}=40*1/4^3=40*1/64=40/64=5/8$	5/8=0.625
-2	$40*4^{-2}=40*1/4^2=40*1/16=40/16=5/2$	5/2=2.5
-1	$40*4^{-1}=40*1/4^1=40*1/4=40/4=10$	10
0	$40*4^0=40*1=40$	40
1	$40*4^1=40*4=160$	160
2	$40*4^2=40*16=640$	640
3	$40*4^3=40*64=2560$	2560



13. $y = 3 \cdot 10^x$

x	$3*10^x$	y
-3	$3*10^{-3}$=3*0.001=0.003	0.003
-2	$3*10^{-2}$=3*0.01=0.03	0.03
-1	$3*10^{-}$=3*0.1=0.3	0.3
0	$3*10^{0}$=3*1=3	3
1	$3*10^{1}$=3*10=30	30
2	$3*10^{2}$=3*100=300	300
3	$3*10^{3}$=3*1000=3000	3000



14.

The exponential growth formula is $y=a(b)^x$

a=113,505

b=100% +1.2%=1.012

x=2012-2007=5

$$y = a(b)^x$$
$$y = 113{,}505(1.012)^5$$
$$y = (113{,}505)(1.061457)$$
$$y = 120{,}480.68$$

The population in 2012 will be approximately 120,481.

15.

The exponential growth formula is $y=a(b)^x$

a=20

b=2

x=15/2=7.5

$$y = a(b)^x$$
$$y = 20(2)^{7.5}$$
$$y = 20(181.0193)$$
$$y = 3620.386$$

There will be approximately 3620 bacteria present 15 hours into the experiment.

16.

The exponential growth formula is $y=a(b)^x$

a=$12

b=100%+2.3%=1.023

x=5 years

$$y = a(b)^x$$
$$y = 12(1.023)^5$$
$$y = (12)(1.120413)$$
$$y = 13.444956$$

In five years the same item will cost approximately $13.45

17.

The exponential growth formula is $y=a(b)^x$

a=10 people

b=10

x=6 weeks

$$y = a(b)^x$$
$$y = 10(10)^6$$
$$y = 10^7$$
$$y = 10{,}000{,}000$$

If everyone maintains the chain then 10 million people will receive the letter the sixth week.

18.

The exponential growth formula is $y=a(b)^x$

a=$200

b=100%+7.5%=1.075

x=21-10=11 years

$$y = a(b)^x$$
$$y = 200(1.075)^{11}$$
$$y = 200(2.215609)$$
$$y = 443.1218$$

By her 21st birthday Nadia will have approximately $443.12 in her account.

Mixed Review

19.

The total number of elements in the set consisting of the alphabets is 26. Of these the elements of interest are M, K and L. The total number of desired elements is 3.
This allows us to calculate the probability of choosing the letters M, K or L when a letter is randomly chosen from the alphabet as 3/26. The required probability is 3/26.

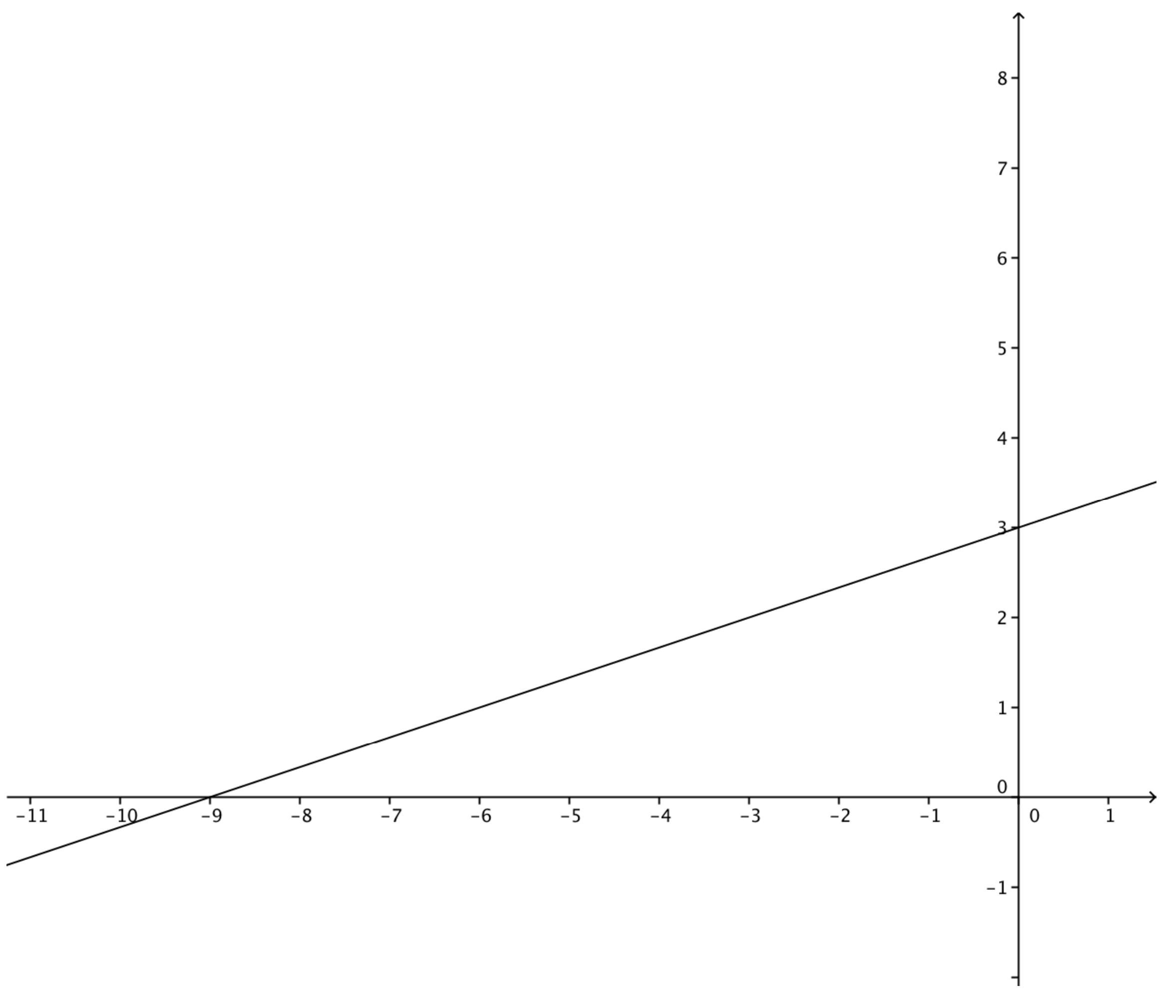
20.

$$t^4 \cdot t^{\frac{1}{2}}$$
$$9^4 \cdot 9^{\frac{1}{2}}$$
$$6561 \cdot \sqrt{9}$$
$$6561 \cdot 3$$
$$19683$$

21.

$$28-(x-16)$$
$$28-x+16$$
$$44-x$$

22. $y-1=\frac{1}{3}(x+6)$



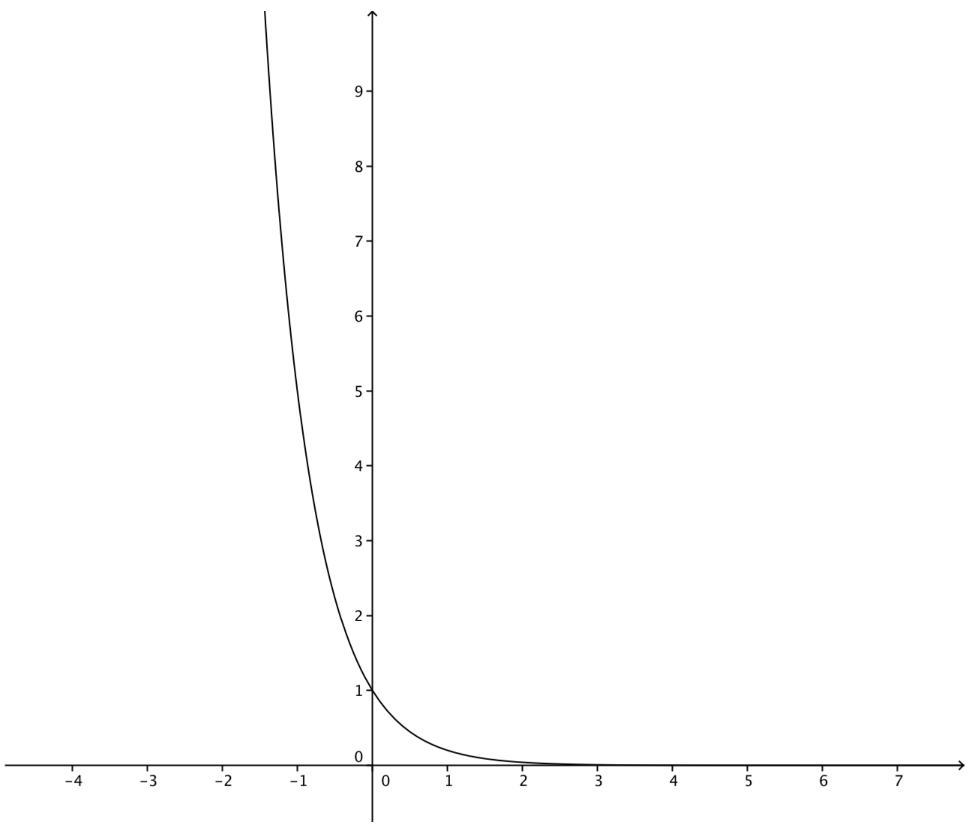
Lesson 8.6
Exponential Decay Functions

1. Exponential decay is when a function decreases from its original value regularly according to the noted decay factor.

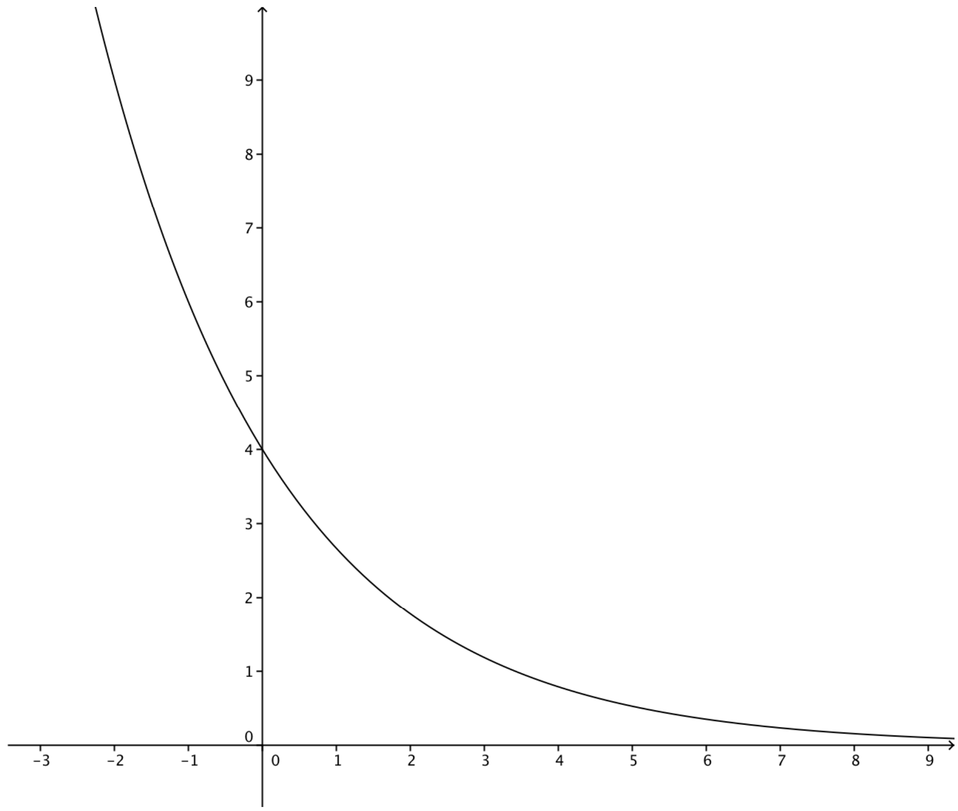
2. "b" in a exponential decay function must always be a fraction between 0 and 1.

3. If $f(x) = a(b)^x$ then *f(0)* is always equal to a. (b^0 will always equal 1 and *a*1* is *a*). This means that the y-intercept of an exponential function is always equal to *a* or the starting value.

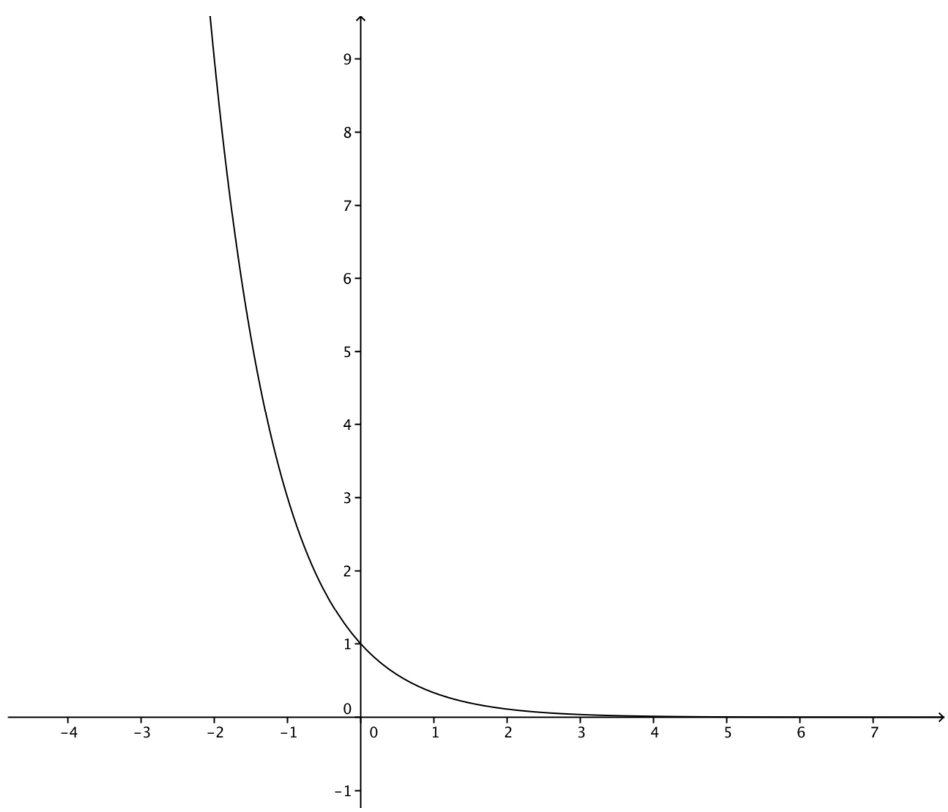
4. $y = \frac{1}{5}^x$



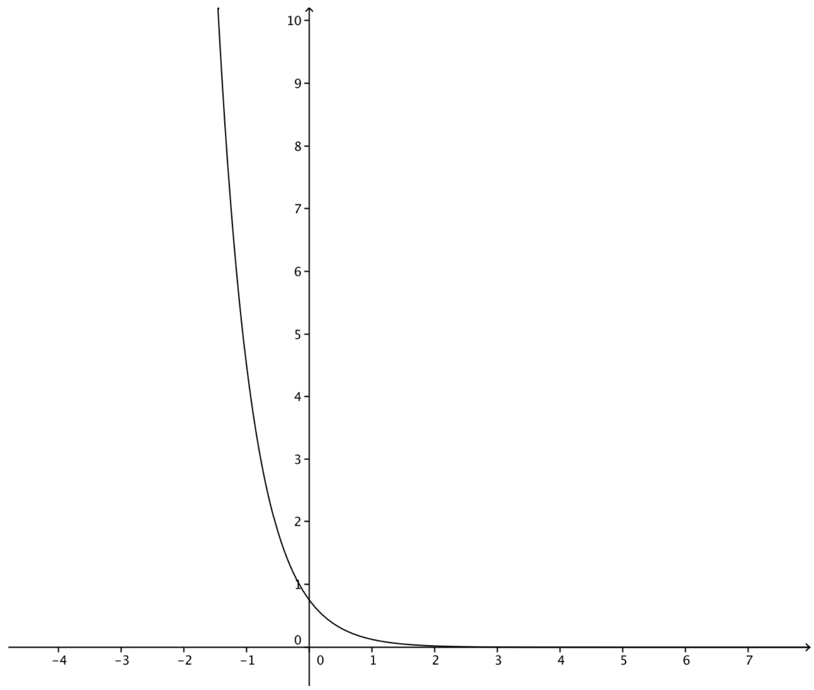
5. $y = 4 \cdot \left(\frac{2}{3}\right)^x$



6. $y = 3^{-x}$

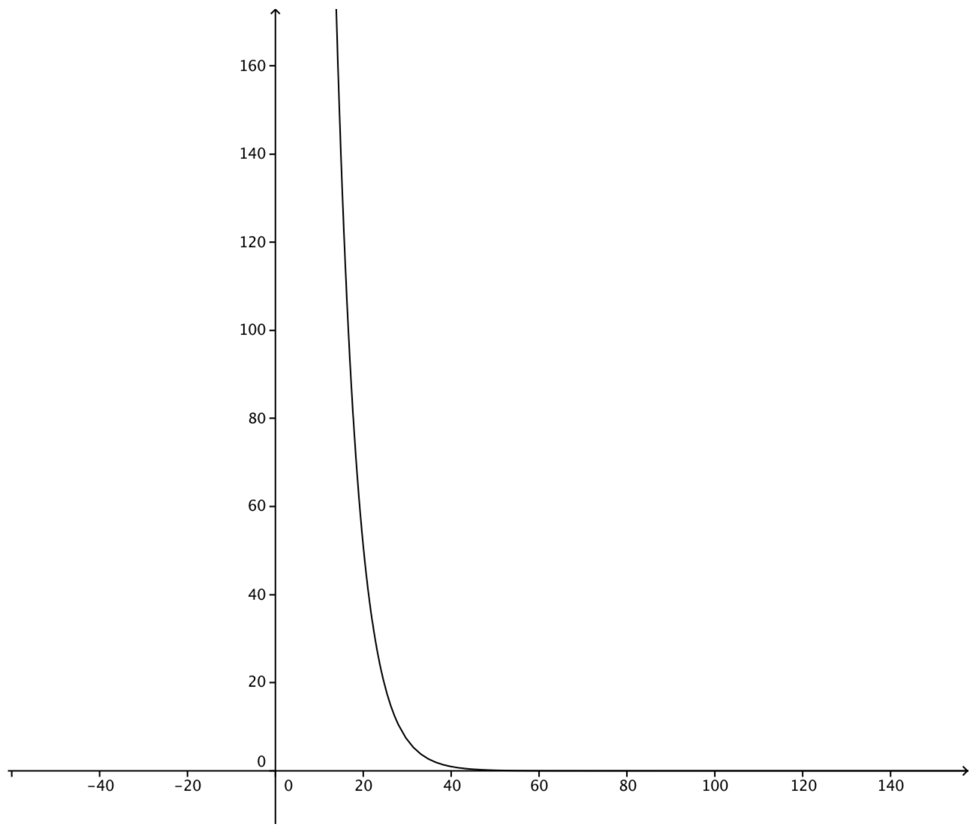


7. $y = \frac{3}{4} \cdot 6^{-x}$



8.

1.



2. $y = 2700(0.82)^x$

$y = 2700(0.82)^{10}$

3. $y = 2700(.137448)$

$y = 371.1096$

The ATV is worth approximately \$371.11 after 10 years.

9.

1. If $y = a(b)^x$ then a= 10,112,620 and b= 0.995 and x= the number of years difference from 2004.

$$y = a(b)^x$$

$$y = 10{,}112{,}620(0.995)^x$$

2. x= 2012-2004=7

$$y = 10{,}112{,}620(0.995)^8$$

$$y = 10{,}112{,}620(0.96069304)$$

$$y \approx 9{,}715{,}124$$

The population in 2012 will be approximately 9,715,124.

3. We will make a table to find when y= 9,900,000. We know that x must be greater than zero and less than 8, based on our previous calculations.

x	$10{,}112{,}620(0.995)^x$	y
~~1~~	~~$10{,}112{,}620(0.995)^1$~~	~~10,062,057~~
~~2~~	~~$10{,}112{,}620(0.995)^2$~~	~~10,011,747~~
~~3~~	~~$10{,}112{,}620(0.995)^3$~~	~~9,961,688~~
4	$10{,}112{,}620(0.995)^4$	9,911,879
5	$10{,}112{,}620(0.995)^5$	9,862,320
~~6~~	~~$10{,}112{,}620(0.995)^6$~~	

We can see that the solution must be greater than 4 and less than 5. If desired we can continue our chart using x values that fall between 4 and 5 and we would find that the answer falls between 4.2 and 4.3.

(The exact answer would be found using a logarithm)

$$\begin{pmatrix} 9{,}900{,}000 = 10{,}112{,}620(0.995)^x \\ 9{,}900{,}000 \div 10{,}112{,}620 = 0.995^x \\ 0.978974785961 = 0.995^x \\ \log(0.978974785961) = x\log(0.995) \\ -0.009228493548 = x(-0.002176919254) \\ x \approx 4.239 \end{pmatrix}$$

4. x=2000-2004= -4

$$y = 10{,}112{,}620(0.995)^{-4}$$

$$y = 10{,}112{,}620(1.0202525)$$

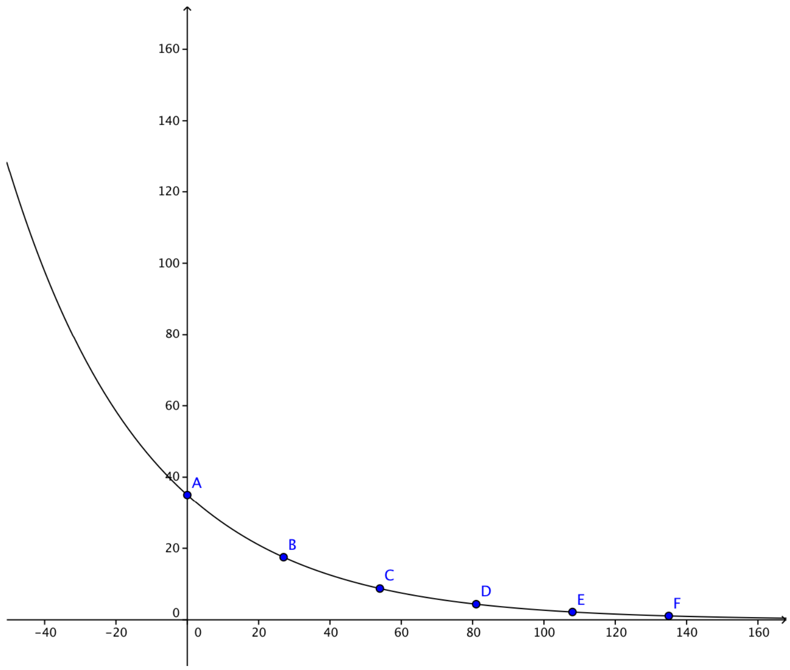
$$y \approx 10{,}317{,}426$$

The population in 2000 would have been approximately 10,317,426.

10.

1.

Year	0	27	54	81	108	135
Amount	35	17.5	8.75	4.375	2.1875	1.09375



2. If y= a(b)^x then in this case a= 35 (starting value) and b= ½ (the decay factor). However we must also take into consideration that the decay factor is only being applied at intervals of 27 years. Therefore the time is x but the exponent is x/27.
y=35(1/2)$^{x/27}$

3. y= 35(1/2)$^{x/27}$

$$y = 35\left(\frac{1}{2}\right)^{\frac{92}{27}}$$

$$y = 35\left(\frac{1}{2}\right)^{3.4}$$

$$y = (35)(0.0947323)$$

$$y \approx 3.3156$$

11.

1. The growth factor is 0.70

2. The initial value is 1

3. $V(d) = 0.70^d$

$$V(65) = 0.70^{65}$$

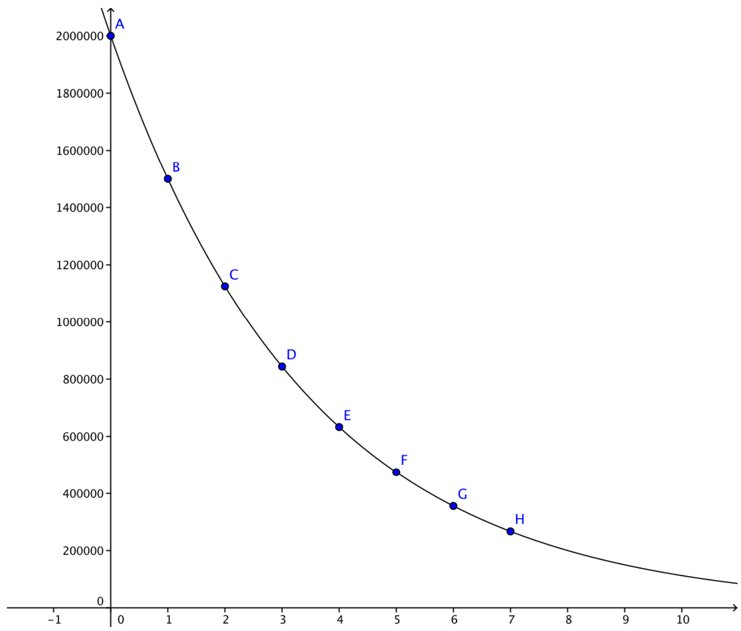
$$V(65) = .0000000000854$$

$$V(65) = 8.54 \times 10^{-11}$$

12.

1. The bacteria population is reduced by ¼ each day, therefore there are ¾ left each day.

Bacteria	2,000,000	1,500,000	1,125,000	843,750	632,812.50	474,609.38	355,957.04	266,967.78
Day	0	1	2	3	4	5	6	7



2. $y = a(b)^x$ The initial value is 2,000,000 and the growth factor is ¾ or .75, with x= time in days $y = 2{,}000{,}000(0{,}75)^x$

3. y=2000000(0.75)^x x=10

$$y = 2000000(0.75)^{10}$$

$$y = 2000000(0.0563135147095)$$

$$y = 112{,}627.029419$$

$$y \approx 112{,}627$$

4. y=2000000(0.75)^x x=14

$$y = 2000000(0.75)^{14}$$

$$y = 2000000(0.0178179480135)$$

$$y = 35635.896027$$

$$y \approx 35{,}636$$

Mixed Review

13.

1. $y=a(b)^x$
a= 14,578
b=100% + 2.14% = 1.0214
x= the number of years before or after 2010

$y= 14578(1.0214)^x$

2. x=2015-2010=5

$$y = 14578(1.0214)^5$$
$$y = 14578(1.1116786565)$$
$$y = 16206.051454457$$
$$y \approx 16{,}206$$

3. x = 2000-2010 = -10

$$y = 14578(1.0214)^{-10}$$
$$y = 14578(0.809173152302)$$
$$y = 11796.1262143$$
$$y \approx 11{,}796$$

14.

diameter = 11 inches
radius = 5.5 inches = 11/2

$$v = \frac{4}{3}\pi r^3$$
$$v = \frac{4}{3}\left(\frac{22}{7}\right)\left(\frac{11}{2}\right)^3$$
$$v = \frac{4}{3}\left(\frac{22}{7}\right)\left(\frac{1331}{8}\right)$$
$$v = \frac{14641}{21}$$
$$v \approx 697$$

15. $\frac{6x^2}{14y^3}\cdot\frac{7y}{x^8}\cdot x^0 y = \frac{6x^{2+0}7y^{1+1}}{14y^3x^8} = \frac{6x^2y^2}{14y^3x^8} = \left(\frac{6}{14}\right)x^{2-8}y^{2-3} = \left(\frac{3}{7}\right)x^{-6}y^{-1} = \frac{3}{7x^6y}$

16. $3(x^2y^3x)^2 = 3(x^{2+1}y^3)^2 = 3(x^{3\cdot 2}y^{3\cdot 2}) = 3x^6y^6$

17. $ax + by = c$

$y - 16 + x = -4x + 6y + 1$

$y - 16 + x + 4x = 6y + 1$

$y - 16 + 5x = 6y + 1$

$y - 16 + 5x - 6y = 1$

$-5y - 16 + 5x = 1$

$-5y + 5x = 1 + 16$

$5x - 5y = 17$

Lesson 8.7
Geometric Sequences and Exponential Functions

1. A *geometric sequence* is a sequence of numbers in which each number in the sequence is found by multiplying the previous number by a fixed amount called the common ratio.

2.

1. $a_4 = 1 \cdot 2^3 = 8$
2. $a_{10} = 1 \cdot 2^9 = 512$
3. $a_{25} = 1 \cdot 2^{24} = 16777216$
4. $a_{60} = 1 \cdot 2^{59} = 576460752303000000 \approx 5.76 \times 10^{17}$

3. Yes the example is a geometric sequence. If we divide any number by the number preceding it we always get the same answer: -1. Therefore it is a sequence found by multiplying the previous number by a common ratio and therefore a geometric sequence.

4.

$$a_n = a_1 r^{n-1}$$
$$a_1 = 1$$
$$r = 2$$
$$n - 1 = 19$$
$$a_{20} = 1 \cdot 2^{19} = 2^{19} = 524288 = \$5242.88$$

5.
$$6 \div 8 = .75$$
$$r = 0.75$$

6.
$$4 \div 2 = 2$$
$$r = 2$$

7.
$$3 \div 9 = \frac{3}{9} = \frac{1}{3}$$
$$r = \frac{1}{3}$$

8.
$$-8 \div 2 = -4$$
$$r = -4$$

9. $a_1 = 2, r = 3$

$2, 6, 18, 54, 162$

10. $a_1 = 90, r = -\frac{1}{3}$

$90, -30, 10, -\frac{10}{3}, \frac{10}{9}$

11. $a_1 = 6, r = -2$

$6, -12, 24, -48, 96$

12. $3, \_, 48, 192, \_$

$192 \div 48 = 4, r = 4$

$3, 12, 48, 192, 768$

13.

In a geometric progression we know that $a_n = a_1 r^{n-1}$ where a_n is any number in the progression, a_1 is the first term and r is the common ratio.

$$a_1 = 81$$

$$a_5 = 1$$

$$n - 1 = 4$$

$$1 = 81 \cdot r^4$$

$$\frac{1}{81} = r^4$$

$$\sqrt[4]{\frac{1}{81}} = r$$

$$r = \frac{\sqrt[4]{1}}{\sqrt[4]{81}} = \pm\frac{1}{3}$$

When r=1/3 then the progression is 81, 27, 9, 3, 1

When r= -1/3 then the progression is 81, -27, 9, -3, 1

14.

In a geometric progression we know that $a_n = a_1 r^{n-1}$ where a_n is any number in the progression, a_1 is the first term and r is the common ratio.

$$a_1 = \frac{9}{4}$$

$$a_4 = \frac{2}{3}$$

$$n - 1 = 3$$

$$\frac{2}{3} = \frac{9}{4} \cdot r^3$$

$$\frac{8}{27} = r^3$$

$$\sqrt[3]{\frac{8}{27}} = r$$

$$r = \frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \frac{2}{3}$$

r= 2/3, therefore the progression is 9/4, 3/2, 1. 2/3, 4/9

15. $a_6 = 4 \cdot 2^5 = 128$

16. $a_4 = -7 \cdot \left(-\frac{3}{4}\right)^3 = 2.953125$

17. $a_{10} = -10 \cdot (-3)^9 = 196830$

18.

1.

$$a_n = a_1 r^{n-1}$$

$$a_1 = 4\,ft$$

$$r = 80\% = 0.80$$

$$a_n = 4 \cdot (0.8)^{n-1}$$

2.

$$a_n = 4 \cdot (0.8)^{n-1}$$

$$a_5 = 4 \cdot (0.8)^{5-1}$$

$$a_5 = 4 \cdot (0.8)^{4}$$

$$a_5 = 4 \cdot (0.4096)$$

$$a_5 = 1.6384$$

19.

1.

$$a_n = a_1 r^{n-1}$$

$$n = 8$$

$$a_1 = 1$$

$$r = 3$$

$$n - 1 = 7$$

$$a_8 = 1 \cdot 3^7 = 2187$$

2. We know that *n* has to be greater than 3 and less than 7. Let's make a table.

n	**$1*3^{n-1}$**	**a_n**
4	$1*3^{4-1}=3^3=27$	27
5	$1*3^{5-1}=3^4=81$	81
6	$1*3^{6-1}=3^5=243$	243
7	$1*3^{7-1}=3^6=$	729

Now we know that *n* must be between 6 and 7. Let's make another table.

n	**$1*3^{n-1}$**	**a_n**
~~6.1~~	~~$1*3^{6.1-1}=3^{5.1}=271$~~	~~271~~
~~6.2~~	~~$1*3^{6.2-1}=3^{5.2}=303$~~	~~303~~
6.3	$1*3^{6.3-1}=3^{5.3}=338$	338
6.4	$1*3^{6.4-1}=3^{5.4}=$	377

We can see that *n* falls between 6.3 and 6.4.

(The exact answer is found using logarithms.)

$$\begin{pmatrix} 343 = 1 \cdot 3^{n-1} \\ 343 = 3^{n-1} \\ (n-1)\log 3 = \log(343) \\ (n-1)(0.47712125472) = 2.53529412004 \\ n-1 = 5.31373124748 \\ n = 6.31373124748 \\ n \approx 6.314 \end{pmatrix}$$

20.

1.

$$a_n = a_1 r^{n-1}$$

$$n = 3$$

$$a_1 = 20$$

$$r = 75\% = 0.75$$

$$n - 1 = 2$$

$$a_3 = 20 \cdot (0.75)^2 = 20 \cdot 0.5625 = 11.25$$

The ball will bounce 11.25 feet high after the third bounce.

2.

$$a_n = a_1 r^{n-1}$$

$$n = 17$$

$$a_1 = 20$$

$$r = 75\% = 0.75$$

$$n - 1 = 16$$

$$a_{17} = 20 \cdot (0.75)^{16} = 20 \cdot (0.010022595758) = 0.2005$$

The ball will bounce 0.2005 feet high after the seventeenth bounce.

21.

1.

$$a_n = a_1 r^{n-1}$$

$$n = 3$$

$$a_1 = 120$$

$$r = 60\% = 0.60$$

$$n - 1 = 2$$

$$a_3 = 120 \cdot (0.60)^2 = 120 \cdot 0.36 = 43.2$$

The rope will stretch 43.2 feet on the third bounce.

2.

$$a_n = a_1 r^{n-1}$$

$$n = 12$$

$$a_1 = 120$$

$$r = 60\% = 0.60$$

$$n - 1 = 11$$

$$a_{12} = 120 \cdot (0.60)^{11} = 120 \cdot (0.00362797056) = 0.4354$$

The rope will stretch 0.4354 feet on the twelfth bounce.

22.

1. The formula for exponential growth is $y = a(b)^x$, where a is the initial value (30 in this case), b is the growth factor (2 in this case) and x is the time passed (in this case the exponent will be 5 hours divided by 30 minutes, because the growth factor is applied every 30 minutes. x = 10)

$$y = a(b)^x$$

$$y = 30 \cdot 2^{10} = 30 \cdot 1024 = 30720$$

2. We know that x>10, let's make a table and find some practice values.

x	**$30*2^x$**	**y**
11	$30*2^{11}$=30*2048=61440	61,440
12	$30*2^{12}$=30*4096=122880	122,880
13	$30*2^{13}$=30*8192=245760	245,760
14	$30*2^{14}$=30*16384=491520	491,520
15	$30*2^{15}$=30*32768=983040	983,040
16	$30*2^{16}$=30*65536=1966080	1,966,080

Now we know that x>15 and x<16. We have to remember that x = the number of 30 minute intervals that have passed. So 15*30 minutes=450 minutes/60 minutes= 7.5 hours.

23.

1. The formula for exponential decay is $y = a(b)^x$, where *a* is the initial value (12mg in this case), **b** is the growth factor (1/2 in this case) and x is the time passed (this question asks for 4 half life periods so x=4)

$$y = a(b)^x$$

$$y = 12 \cdot (0.5)^4 = 12 \cdot 0.0625 = 0.75$$

2. We know that 0<x<4 half-life periods. Let's make a table.

x	**$12*0.5^x$**	**y**
1	$12*0.5^1$=12*0.5=6	6
2	$12*0.5^2$=12*0.25=3	3

We can see that she will have less than 3 mg in her system after 2 half-life periods or 50 days. Anytime after 50 days she will have less than 3mg in her system.

3. The growth factor is ½ or 0.5.

Mixed Review

24. $x^2 < 2x + 15$

25. $y = \frac{2}{3}x - 7$

(The equation is in y= mx + b form, where m is the slope and b is the y-intercept.)
Slope = 2/3
y-intercept is -7

26. 10!=3628800

27. 1 mile = 1760 yards
6 miles * 1760= 10560 yards

28. $\frac{5y^2 - 3y^2}{4y^{11}} = \frac{2y^2}{4y^{11}} = \frac{1}{2} \cdot y^{2-11} = \frac{1}{2}y^{-9} = \frac{1}{2y^9}$

29. $3x^2 \cdot x^6 + 4x^3x^5 = 3x^{2+6} + 4x^{3+5} = 3x^8 + 4x^8 = 7x^8$

30. $\left(\frac{27}{64}\right)^{-\frac{1}{3}} = \frac{\sqrt[3]{64}}{\sqrt[3]{27}} = \frac{4}{3} = 1\frac{1}{3}$

Lesson 8.8
Problem Solving Strategies

1.
The formula for exponential decay is $y = a(b)^x$, where a is the initial value (in this case simply 1), b is the growth factor (in this case the decay factor is 3.5% so the growth factor is 96.5% or 0.965) and x is the number of times the growth factor is applied (in this problem, the number of hours, which is 6.)

$$y = a(b)^x$$

$$y = 1 \cdot (0.965)^6$$

$$y = 1 \cdot (0.807539696082)$$

$$y \approx 0.808$$

After six hours there will be approximately 80.8% left.

2.
The formula for exponential decay is $y = a(b)^x$, where a is the initial value (in this case 1200), b is the growth factor (in this case the decay factor is 1.5% so the growth factor is 98.5% or 0.985) and x is the number of times the growth factor is applied (in this problem, the number of decades, which is 2020-1990=30/10=3.)

$$y = a(b)^x$$

$$y = 1200 \cdot (0.985)^3$$

$$y = 1200 \cdot (0.955671625)$$

$$y = 1146.80595$$

$$y \approx 1147$$

In the year 2020 there will be approximately 1147 bird species left.

3

The formula for exponential growth is $y = a(b)^x$, where a is the initial value (in this case 200), b is the growth factor (in this case 100%+ 8% or 1.08) and x is the number of times the growth factor is applied (in this problem, the number of years, 2007-1999=8.)

$$y = a(b)^x$$

$$y = 200 \cdot (1.08)^8$$

$$y = 200 \cdot (1.85093021028)$$

$$y = 370.186042056$$

$$y \approx 370$$

In the year 2007 there would be approximately 370 stores in Nadia's chain.

4.

The formula for exponential growth is $y = a(b)^x$, where a is the initial value (in this case \$360), b is the growth factor (in this case 100%+ 7.25% or 1.0725) and x is the number of times the growth factor is applied (in this problem, the number of years, 12)

$$y = a(b)^x$$

$$y = 360 \cdot (1.0725)^{12}$$

$$y = 360 \cdot (2.31615495052)$$

$$y = 833.815782186$$

$$y \approx 833.82$$

In 12 years there will be approximately $833.82 in the account.

Lesson 8.9
Chapter 8 Review

1. An *exponent* is the power of a number that shows how many times that number is multiplied by itself.

2. A *geometric sequence* is a sequence of numbers in which each number in the sequence is found by multiplying the previous number by a fixed amount called the common ratio.

3. 5*5*5*5= 5^4= 625

4. $(3x^2y^3)\cdot(4xy^2) = 12x^{2+1}y^{3+2} = 12x^3y^5$

5. $a^3 \cdot a^5 \cdot a^6 = a^{3+5+6} = a^{14}$

6. $(y^3)^5 = y^{3\cdot 5} = y^{15}$

7. $(x \cdot x^3 \cdot x^5)^{10} = (x^{1+3+5})^{10} = (x^9)^{10} = x^{9\cdot 10} = x^{90}$

8. $(2a^3b^3)^2 = 2^2 \cdot a^{3\cdot 2} \cdot b^{3\cdot 2} = 4a^6b^6$

9. $\dfrac{c^5}{c^3} = c^{5-3} = c^2$

10. $\dfrac{a^6}{a} = a^{6-1} = a^5$

11. $\dfrac{a^5b^4}{a^3b^2} = a^{5-3} \cdot b^{4-2} = a^2b^2$

12. $\dfrac{x^4y^5z^2}{x^3y^2z} = x^{4-3} \cdot y^{5-2} \cdot z^{2-1} = xy^3z$

13. $\dfrac{6^5}{6^5} = 1$

14. $\dfrac{y^2}{y^5} = y^{2-5} = y^{-3} = \dfrac{1}{y^3}$

15. $\frac{7^3}{7^6} = 7^{3-6} = 7^{-3} = \frac{1}{7^3} = \frac{1}{343}$

16. $\frac{2}{x^3} = 2x^{-3}$

17. $\sqrt[4]{a^3} = a^{\frac{3}{4}}$

18. $\left(a^{\frac{1}{3}}\right)^2 = a^{\frac{1}{3}\cdot 2} = a^{\frac{2}{3}} = \sqrt[3]{a^2}$

19. $\left(\frac{x^2}{y^3}\right)^{\frac{1}{3}} = \frac{x^{2\cdot\frac{1}{3}}}{y^{3\cdot\frac{1}{3}}} = \frac{x^{\frac{2}{3}}}{y} = \frac{\sqrt[3]{x^2}}{y} = \sqrt[3]{x^2}\cdot y^{-1}$

20. 557000 = 5.57 x 10^5

21. 600000 = 6 x 10^5

22. 20 = 2 x 10^1

23. 0.04 = 4 x 10^{-2}

24. 0.0417 = 4.17 x 10^{-2}

25. 0.0000301 = 3.01 x 10^{-5}

26. 384,403 km = 3.84 x 10^5 km

27. 483,780,000 miles = 4.8378 x 10^8 miles

28. 15 parts per billion = 1.5 x 10^{-8}

29. 3.53 x 10^3 = 3530

30. 8.9 x 10^6 = 8900000

31. 2.12 x 10^6 = 2,120,000

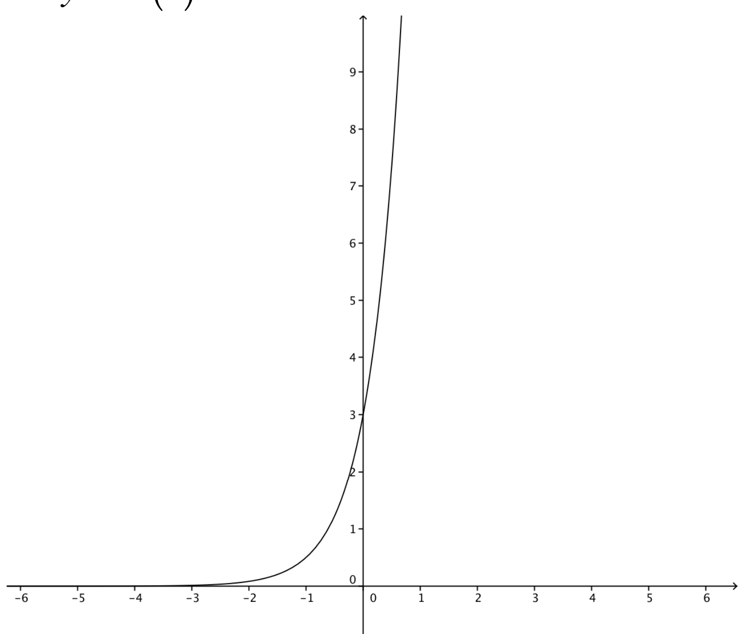
32. 5.4 x 10^1 = 54
33. 7.9 x 10^{-3} = 0.0079

34. 4.69 x 10^{-2} = 0.0469

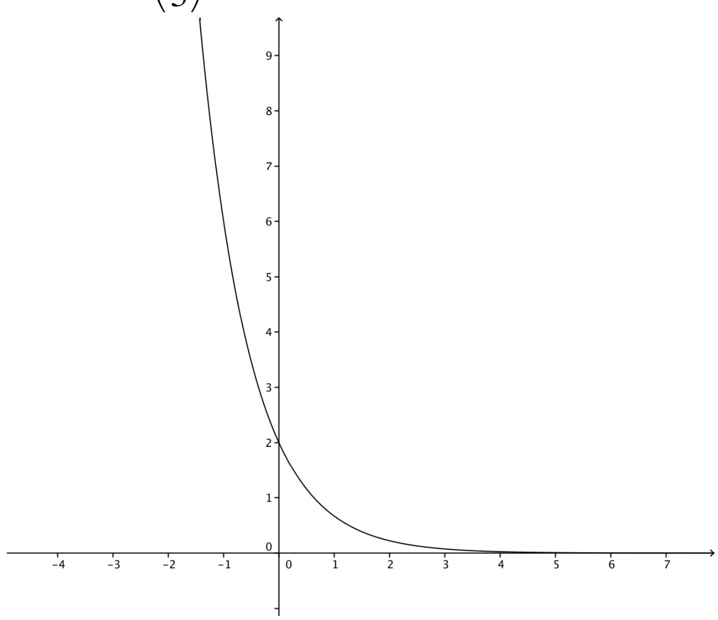
35. 1.8 x 10^{-5} = .000018

36. 8.41 x 10^{-3} = 0.00841

37. $y = 3 \cdot (6)^x$

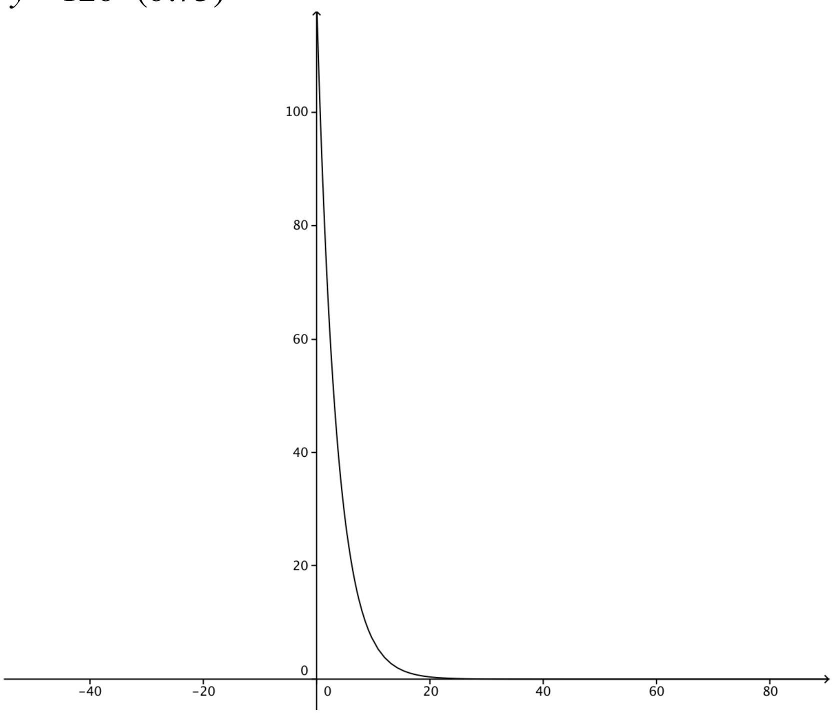


38. $y = 2 \cdot \left(\frac{1}{3}\right)^x$



39.

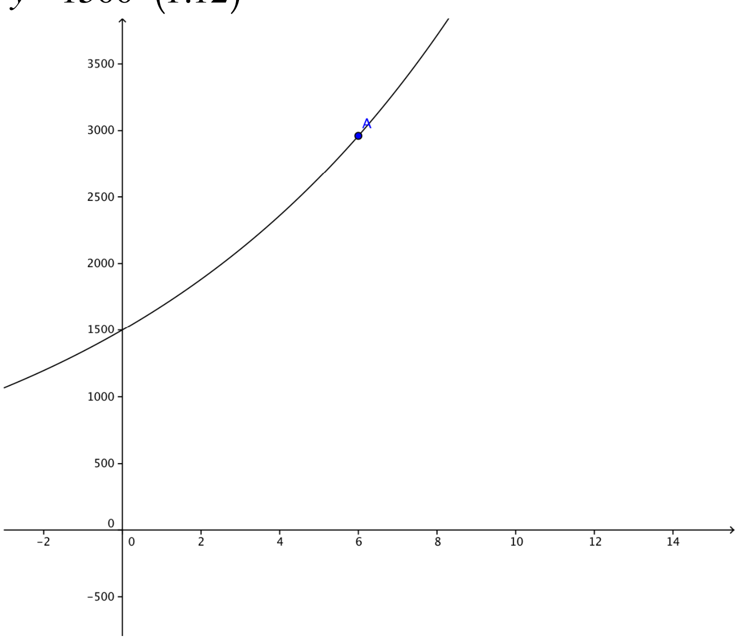
$$y = 120 \cdot (0.75)^x$$



Marissa will run out of candy in approximately 20 days.

40.

$$y = 1500 \cdot \left(1.12\right)^x$$



Jacoby will have approximately $2900 in his account after 6 years.

41.

3/1=3
r=3
1, 3, 9, 27, 81

42.

625/125=5
r=5
1, 5, 25, 125, 625

43.

2401/343=7
r=7
7, 49, 343, 2401, 16807

44.

1.5/5=0.3
r=0.3
5, 1.5, 0.45, 0.135, 0.0405

45.

$$a_n = a_1 r^{n-1}$$
$$n = 30$$
$$a_1 = 2$$
$$r = 3$$
$$n - 1 = 29$$
$$a_{30} = 2 \cdot 3^{29} = 2 \cdot (68630377364900) = 137260754730000$$

Ben will have 137,260,754,730,000 ants at the end of a month.

Lesson 8.10
Chapter 8 Test

1. $x^3 \cdot x^4 \cdot x^5 = x^{3+4+5} = x^{12}$

2. $\left(a^3\right)^7 = a^{3\cdot 7} = a^{21}$

3. $\left(y^2 z^4\right)^7 = y^{2\cdot 7} \cdot z^{4\cdot 7} = y^{14} z^{28}$

4. $\dfrac{a^3}{a^5} = a^{3-5} = a^{-2} = \dfrac{1}{a^2}$

5. $\dfrac{x^3 y^2}{x^6 y^4} = x^{3-6} \cdot y^{2-4} = x^{-3} y^{-2} = \dfrac{1}{x^3 y^2}$

6.
$$\left(\frac{3x^8 y^2}{9x^6 y^5}\right)^3 = \left(\frac{3}{9} \cdot x^{8-6} \cdot y^{2-5}\right)^3 = \left(\frac{1}{3} x^2 y^{-3}\right)^3 = \left(\frac{1}{3}\right)^3 \cdot x^{2\cdot 3} \cdot y^{-3\cdot 3} = \frac{1}{27} \cdot x^6 \cdot y^{-9} = \frac{x^6}{27y^9}$$

7. $\dfrac{3^4}{3^4} = 1$

8. $\dfrac{2}{x^3} = 2x^{-3}$

9.
$\sqrt[3]{5^6} = \sqrt[3]{15625} = 25$
$\sqrt[3]{5^6} = 5^{\frac{6}{3}} = 5^2 = 25$

10. 0.00002 = 2 x 10^{-5}

11.

Day	1	2	3	4	5	6	7	8	9	10	11
Turkeys	2	4	8	16	32	64	128	256	512	1024	2048

12.

$$y = a(b)^x$$

$$y = 89 \cdot (1.2)^{15}$$

$$y = 89 \cdot (15.4070215746)$$

$$y = 1371.22492014$$

$$y \approx 1371$$

After 15 years the population of the town will be approximately 1371 people.

13.

$$y = a(b)^x$$

$$y = 1 \cdot (0.975)^9$$

$$y = 1 \cdot (0.796235508571)$$

$$y \approx 0.7962$$

After 9 hours there will be approximately 79.6% of the substance left.

14.

$$y = a(b)^x$$

$$y = 100(0.84)^3$$

$$y = 100 \cdot 0.592704$$

$$y = 59.2704$$

There will be approximately 59 cockroaches left after 3 hours.