Permutations with Repetition

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Permutations with Repetition

There is a subset of permutations that takes into account that there are double objects or repetitions in a permutation problem. In general, repetitions are taken care of by dividing the permutation by the factorial of the number of objects that are identical.

If you look at the word TOOTH, there are 2 O's in the word. Both O's are identical, and it does not matter in which order we write these 2 O's, since they are the same. In other words, if we exchange 'O' for 'O', we still spell TOOTH. The same is true for the T's, since there are 2 T's in the word TOOTH as well. In how many ways can we arrange the letters in the word TOOTH?

We must account for the fact that these 2 O's are identical and that the 2 T's are identical. We do this using the formula:

\[ \frac{nP_r}{x_1!x_2!} \]

where \( x \) is the number of times a letter is repeated.
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\[
\frac{n^P_r}{x_1!x_2!} = \frac{5^P_5}{2!2!}
\]
\[
\frac{5^P_5}{2!2!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1}
\]
\[
\frac{5^P_5}{2!2!} = 120
\]
\[
\frac{5^P_5}{2!2!} = 4
\]
\[
\frac{5^P_5}{2!2!} = 30
\]

We can arrange the letters in the word TOOTH in 30 different orders.

**Rearranging Words**

How many different 5-letter arrangements can be formed from the word APPLE?

There are 5 letters in the word APPLE, so \( n = 5 \). We want 5-letter arrangements; therefore, we are choosing 5 objects at a time. In this example, \( r = 5 \), and we are using a word with letters that repeat. In the word APPLE, there are 2 P’s, so \( x_1 = 2 \).

\[
\frac{5^P_5}{x_1!} = \frac{5^P_5}{2!}
\]

There are 5 letters, \( n = 5 \), and you are choosing all 5 digits, \( r = 5 \)

\[
\frac{5^P_5}{2!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = \frac{120}{2}
\]

\[
\frac{5^P_5}{2!} = 60 \text{ arrangements}
\]

There are 60 5-letter arrangements that can be formed from the word APPLE.

**Writing Numerals**

How many different 6-digit numerals can be written using the following 7 digits?

Assume the repeated digits are all used.

3, 3, 4, 4, 4, 5, 6

There are 7 digits, so \( n = 7 \). We want 6-digit arrangements; therefore, we are choosing 6 objects at a time. In this example, \( r = 6 \), and we are using a group of digits with numbers that repeat. In the group of 7 digits (3, 3, 4, 4, 4, 5, 6), there are two 3’s and three 4’s, so \( x_1 = 2 \) and \( x_2 = 3 \).
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There are 420 6-digit numerals that can be written using the 7 digits.

Example

Example 1

If there are 4 chocolate chip, 2 oatmeal, and 2 double chocolate cookies in a box, in how many different orders is it possible to eat all of these cookies?

There are 8 cookies, so \( n = 8 \). We want to eat all of the cookies; therefore, we are choosing 8 objects at a time. In this example, \( r = 8 \), and we are eating a group of cookies with flavors that repeat. In the group of 8 cookies, there are 4 chocolate chip, 2 oatmeal, and 2 double chocolate cookies, so \( x_1 = 4 \), \( x_2 = 2 \), and \( x_3 = 2 \).
It is possible to eat the cookies in 420 different orders.

**Review**

1. In how many ways can the letters of the word REFERENCE be arranged?
2. In how many ways can the letters of the word MISSISSIPPI be arranged?
3. In how many ways can the letters of the word MATHEMATICS be arranged?
4. A math test is made up of 15 multiple choice questions. 5 questions have the answer A, 4 have the answer B, 3 have the answer C, 2 have the answer D, and 1 has the answer E. How many answer sheets are possible?
5. How many different 5-digit numerals can be written using the following 9 digits? 2, 2, 2, 7, 7, 8, 8, 8, 9
6. How many different 4-digit numerals can be written using the following 10 digits? 1, 3, 3, 4, 4, 5, 5, 6, 6, 9
7. How many different 6-digit numerals can be written using the following 12 digits? 1, 1, 7, 7, 7, 7, 8, 8, 8, 9, 9, 9
8. If there are 4 cans of cola, 3 cans of lemonade, and 5 cans of iced tea in a cooler, in how many orders is it possible to consume these drinks?
9. A clothing store has a certain shirt in 4 sizes: small, medium, large, and extra large. If it has 2 small, 3 medium, 6 large, and 2 extra large in stock, in how many orders can it sell all the shirts?
10. In question 9, suppose the store decides not to sell the extra large shirts. In how many orders can it sell the remaining shirts?

**Review (Answers)**

To view the Review answers, open this PDF file and look for section 2.4.
<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>combination</td>
<td>Combinations are distinct arrangements of a specified number of objects without regard to order of selection from a specified set.</td>
</tr>
<tr>
<td>Permutation</td>
<td>A permutation is an arrangement of objects where order is important.</td>
</tr>
<tr>
<td>repetitions</td>
<td>Repetitions are double objects in a permutation problem.</td>
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</tbody>
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