2.1 Factoring Review

Answers

1. $(x+2)(x+3)$
2. $(x^2+2)(x^2+3)$
3. $(x-2)(x+2)(x^2+4)$
4. $2(x-\sqrt{10})(x+\sqrt{10})$
5. $3(x+2)(x+1)$
6. $\frac{1}{2}(x-3)(x+3)(x-1)(x+1)$
7. $\frac{2}{3}(x-4)(x+4)(x-1)(x+1)$
8. $\left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right)$
9. $\frac{1}{4}(x-3)(x+3)(2x-1)(2x+1)$ or $\left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right)(x-3)(x+3)$
10. $\frac{3}{4}(x-5)(x+5)(x-2)(x+2)$
11. $\frac{1}{2}(x-5)(x+5)(x-2)(x+2)$
12. $\frac{1}{2}\left(x-\frac{1}{3}\right)\left(x+\frac{1}{3}\right)(x-1)(x+1)$
13. $\left(x-\frac{1}{3}\right)\left(x+\frac{1}{3}\right)\left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right)$
14. The degree is the maximum number of linear factors.
15. Some of the roots are complex.

2.2 Advanced Factoring

Answers

1. $(2x+3)(x-4)$

2. $(3x-1)(4x+3)$

3. $(5x-1)(2x+3)$

4. $(6x-1)(3x+2)$

5. $(2x+1)(3x+2)$

6. $(4x+7)(2x+5)$

7. $(5x+3)(x+4)$

8. $(4x-1)(3x-2)$

9. a^9+b^9

10. a^7+b^7

11. The binomial will have a subtraction sign. The second factor will have all addition signs.

12. The binomial will have an addition sign. The second factor will have alternating addition and subtraction signs, ending with an addition sign.

13. $(3x-4)(9x^2+12x+12)$

14. $(x-y)(x^4+x^3y+x^2y^2+xy^3+y^4)$

15. $(2a-b)(16a^4+8a^3b+4a^2b^2+2ab^3+b^4)$

16. $(2x+y)(16x^4-8x^3y+4x^2y^2-2xy^3+y^4)$

17. $(2x+3)(4x^2-6x+9)$

18. $(2x+1)(x+y)$

19. $(4x^2+1)(2x+3)$

20. $(x+y)(3x-4)$

2.3 Polynomial Expansion and Pascal's Triangle

Answers

1. $(x+y)^2$
2. $(x+1)^3$
3. $(x+1)^4$
4. $(3x-1)^3$
5. $(x+4)^3$
6. $8x^3-36x^2+54x-27$
7. $81x^4+432x^3+864x^2+768x+256$
8. $x^7-7x^6y+21x^5y^2-35x^4y^3-21x^2y^5+7xy^6-y^7$
9. $a^{10}+10a^9b+45a^8b^2+120a^7b^3+210a^6b^4+252a^5b^5+210a^4b^6+120\,a^3b^7+45a^2b^8+10ab^9+b^{10}$
10. $32x^5+400x^4+2000x^3+5000x^2+6250x+3125$
11. $256x^4-256x^3+96x^2-16x+1$
12. $125x^3+150x^2+60x+8$
13. $x^6+6x^5y+15x^4y^2+20x^3y^3+15x^2y^4+6xy^5+y^6$
14. $27x^3+54x^2y+36xy^2+8y^3$
15. $625x^4-1000x^3y^2-160xy^3+16y^4$

2.4 Rational Expressions

Answers

1. $5x$

2. $\frac{7}{x+2}$

3. $\frac{4}{5}$

4. $\frac{2x+2}{x-4}$

5. $\frac{x^2+4}{x-1}$

6. $\frac{1}{x-3}$

7. $-\frac{(x-3)}{(x+1)}$

8. $\frac{2x-7}{x^2-49}$

9. $\frac{84}{x^2-49}$

10. $\frac{5}{x-5}$

11. $\frac{4}{x-6}$

12. $-\frac{1}{x+6}$

13. $-\frac{3}{x+5}$

14. $\frac{x+2}{x-2}$

15. $x=-2$

2.5 Polynomial Long Division and Synthetic Division

Answers

1. $\pm\frac{p}{q}=\frac{1}{1},\frac{1}{5},\frac{3}{1},\frac{3}{5},\frac{5}{1},\frac{15}{1}$

2. $\pm\frac{p}{q}=\frac{1}{1},\frac{1}{7},\frac{2}{1},\frac{2}{7},\frac{3}{1},\frac{3}{7},\frac{6}{1},\frac{6}{7},\frac{9}{1},\frac{9}{7},\frac{18}{1},\frac{18}{7}$

3. $\pm\frac{p}{q}=\frac{1}{1},\frac{1}{11},\frac{2}{1},\frac{2}{11},\frac{3}{1},\frac{3}{11},\frac{4}{1},\frac{4}{11},\frac{6}{1},\frac{6}{11},\frac{12}{1},\frac{12}{11}$

4. $\pm\frac{p}{q}=\frac{1}{1},\frac{1}{3},\frac{2}{1},\frac{2}{3},\frac{7}{1},\frac{7}{3},\frac{14}{1},\frac{14}{3}$

5. $\pm\frac{p}{q}=\frac{1}{1},\frac{1}{2},\frac{3}{1},\frac{3}{2},\frac{9}{1},\frac{9}{2}$

6. $(x+1)(x-4)(x+2)(2x+1)$

7. $(x+3)(x-2)(x+1)(x+5)$

8. $(x+4)(x-2)(x+2)(x-1)$

9. $(x+1)(x+6)(x-3)(4x+3)$

10. $(x-2)(x+5)(2x-3)(x+7)$

11. $x^3+3x^2-7x-3-\frac{18}{x+4}$

12. $x^3+5x^2-5x-21+\frac{12}{x+2}$

13. $x^3-8x+12-\frac{20}{x+3}$

14. $(2x+1)$

15. $x-2$

2.6 Solving Rational Equations

Answers

1. $x = 10$

2. $x = \frac{2}{3}$

3. $x = 0$ or $x = 7$

4. $x = \frac{(-1 \pm \sqrt{769})}{6}$

5. $x = \frac{(-8 \pm \sqrt{22})}{3}$

6. No solution. $x = 4$ is extraneous.

7. $x = \frac{2}{3}$

8. $x = -3, x = -2$

9. $x = -\frac{3}{2}, x = -1$

10. $x = -\frac{3}{4}, x = 2$

11. $x = \frac{(-5 \pm \sqrt{13})}{2}$

12. $x = 5 \pm \sqrt{17}$

13. $x = -4$. $x = -6$ is extraneous.

14. $x = -2$. $x = -5$ is extraneous.

15. An extraneous solution is like a "fake" solution that you get when you solve the equation. It is not actually a solution because it causes one of the original denominators to be equal to zero.

2.7 Holes in Rational Functions

Answers

1. You can find holes in a rational function by setting the denominator in the rational part of the function equal to 0 and solving for the variable in the rational part of the function

2. A hole is an undefined point of the function evaluated on that precise x value. A removable discontinuity is a hole in a function which could be filled with singular points.

3. Seeing a hollow circle means that the point is not evaluated at that precise x value.

4. There is a hole at $x = 1$.

5. There is a hole at $x = -3$.

6. There are holes at $x = 1$ and $x = -2$.

7. There is a hole at $x = 1$.

8. There is a hole at $x = -3$.

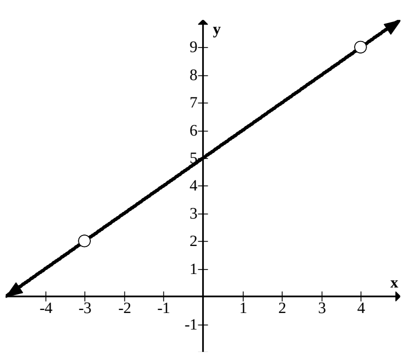
9. There is a hole at $x = 4$.

10. There is a hole at $x = 7$

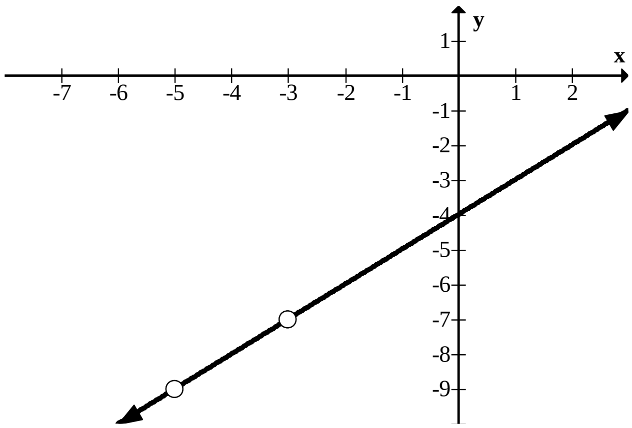
11. $y = (2x - 1) \cdot \frac{x+1}{x+1}$ or $y = \frac{2x^2+x-1}{x+1}$

12. $y = (x^2 - 4) \cdot \frac{x+1}{x+1}$ or $y = \frac{x^3+x^2-4x-4}{x+1}$

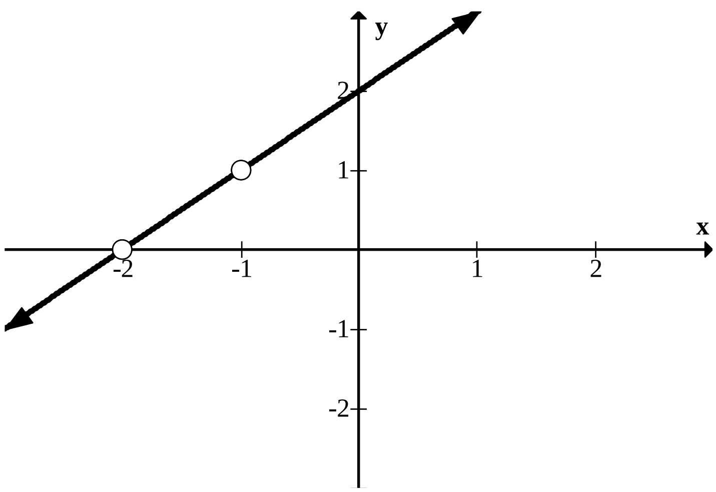
13.



14.



15.



2.8 Zeroes of Rational Functions

Answers

1. $y-intercept: (0,-4); x-intercepts: (-4,0), (1,0); hole: (2,6)$

2. $y-intercept: (0,-2); x-intercepts: \left(-\frac{2}{3},0\right), \left(\frac{1}{2},0\right); hole: (3,55)$

3. $y-intercept: (0,-2); x-intercept: (-2,0); hole: (1,3)$

4. $y-intercept: (0,-8); x-intercept: (4,0); hole: (-2,0)$

5. $y-intercept: (0,0); x-intercepts: (0,0), (4,0), (-2,0); holes: (3,-105), (-4,0)$

6. $y-intercept: (0,0); x-intercept: (0,0); holes: (3,-105), (-1,0)$

7. $y-intercept: (0,-1); x-intercept: none; hole: (1,0), (-1,-2)$

8. $y-intercept: (0,2); x-intercept: (-2,0); hole: (2,-4)$

9. Answers vary. Possible answer: $f(x)=\frac{(x-1)(x-2)(x-3)(x-5)(x-9)}{(x-3)(x-5)(x-9)}$

10. Answers vary. Possible answer: $f(x)=\frac{(x-1)(x+1)(x-4)}{(x+1)(x-4)}$

11. Answers vary. Possible answer: $f(x)=\frac{(x-2)(x-3)(x)(x-5)}{x(x-5)}$

12. Answers vary. Possible answer: $f(x)=\frac{(x-4)(x+3)(x-5)}{(x+3)(x-5)}$

13. Answers vary. Possible answer: $f(x)=\frac{(x)(x-3)(x+2)(x-6)}{(x+2)(x-6)}$

14. Answers vary. Possible answer: $f(x)=\frac{(x)(x-6)(x-1)(x-5)}{(x-1)(x-5)}$

15. Answers vary. Possible answer: $f(x)=\frac{(x-3)(x-2)(x-7)}{(x-2)(x-7)}$

2.9 Vertical Asymptotes

Answers

1. $f(x) = \frac{10(x-3)(x-5)(x-6)}{(x-1)(x-4)(x-6)}$

2. $f(x) = \frac{5(x-3)(x-1)(x-5)}{(x-3)(x+2)(x-2)}$

3. $f(x) = \frac{20(x-8)(x-1)(x-2)}{(x-8)(x)(x-3)}$

4. $f(x) = \frac{4(x-4)(x-5)}{(x-4)(x-2)(x-6)}$

5. $f(x) = \frac{(x-5)(x)(x-3)}{(x-5)(x-4)}$

6. $x = 2, x = 4$

7. $x = -1, x = 3$

8. $x = -1, x = 5$

9. $x = 3, x = -2$

10. $x = -3, x = 4$

11. Holes: $(-4, -1.875), (4, -0.375), (-1, 1.5)$. Vertical asymptote at $x = -2$.

12. Holes: $(3, -0.1875), (8, -.1693)$. Vertical asymptote at $x = -1$.

13. $(-\infty, 0) \cup (0, 1) \cup (1, 3) \cup (3, \infty)$

14. $(-\infty, -2) \cup (-2, 8) \cup (8, \infty)$

15. $(-\infty, -3) \cup (-3, -1) \cup (-1, 0) \cup (0, \infty)$

2.10 Horizontal Asymptotes

Answers

1. $y = 5$

2. $none$

3. $y = 1$

4. $y = 0$

5. $y = -\frac{2}{7}$

6. $y = 0$

7. $y = 1$ as x gets infinitely large and $y = -1$ as x gets infinitely small.

8. $f(x) = \frac{2(x-3)(x-5)(x-6)}{3(x-6)(x-1)(x-4)}$

9. $f(x) = \frac{(x-1)(x-5)(x-3)}{(x-3)(x+2)(x-2)}$

10. $f(x) = \frac{2(x-1)(x-2)(x-8)}{(x)(x-3)(x-8)}$

11. Possible answer: $f(x) = \frac{(x-4)(x-5)}{(x-4)(x-2)(x-6)}$

12. Possible answer: $f(x) = \frac{(x-5)(x)(x-3)}{(x-5)(x-4)}$

13. Vertical asymptotes: $x = -\frac{5}{4}, x = 6$. Horizontal asymptote at $y = \frac{1}{2}$.

14. Vertical asymptote: $x = -\frac{3}{4}$. No horizontal asymptote.

15. Vertical asymptotes: $x = -2, x = 4$. Horizontal asymptote at $y = 0$.

2.11 Oblique Asymptotes

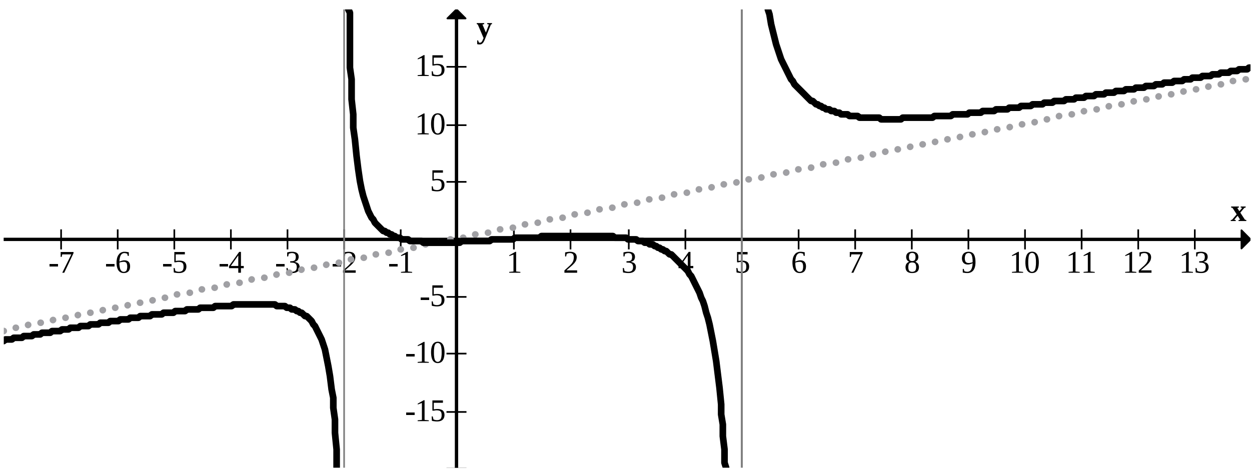
Answers

1. An oblique asymptote is a slanted line that the graph tends towards as x values get very big and very small.

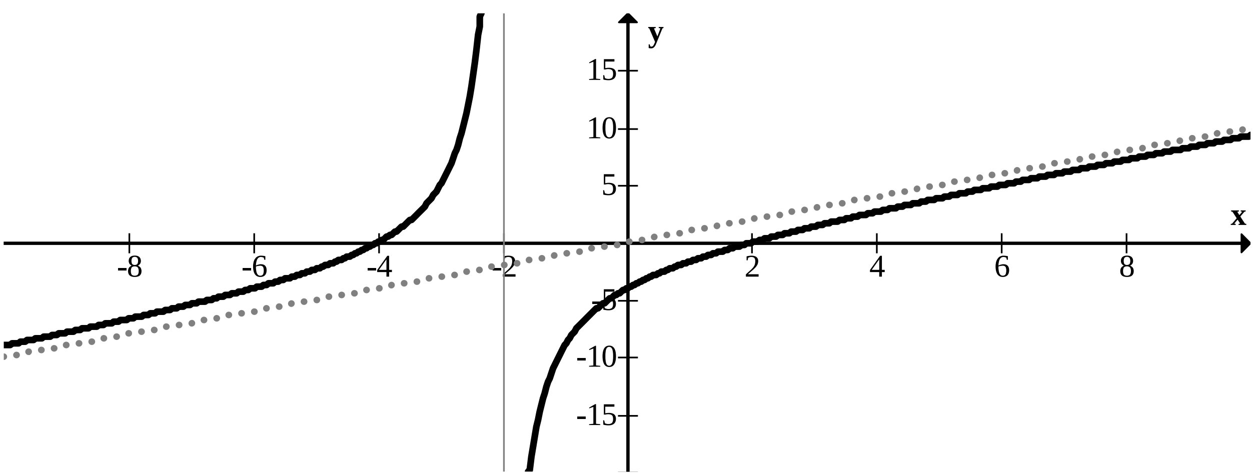
2. If the degree of the numerator is exactly one more than the degree of the denominator.

3. No, because if the degree of the numerator is exactly one more than the degree of the denominator then the function will not a have a horizontal asymptote.

4. Here is a sketch of the graph with the oblique asymptote.

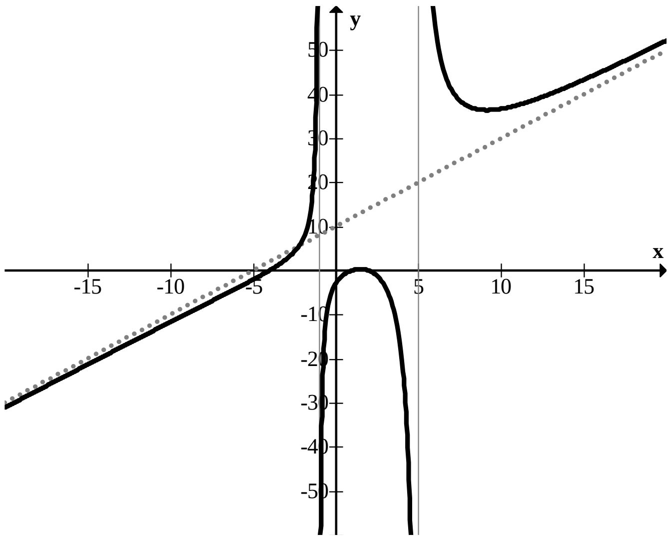


5. Here is a sketch of the graph with the oblique asymptote.



6. No asymptote. Parabolas don't have asymptotes.

7. Here is a sketch of the graph with the oblique asymptote.



8. No oblique asymptote. Only horizontal and vertical asymptotes.

9. $y = x + 2$

10. No oblique asymptote because the degree of the numerator is smaller than the degree of the denominator.

11. No oblique asymptote because the degree of the numerator is the same as the degree of the denominator.

12. The backbone is the parabola $y = x^2 + 7x + 6$. There is no oblique asymptote because the degree of the numerator is two more than the degree of the denominator.

13. While there are an infinite number of functions that match these criteria, one example is: $f(x) = 2x - 1 + \frac{(x-7)}{(x-3)(x-7)}$

14. While there are an infinite number of functions that match these criteria, one example is:

$$f(x) = x + \frac{1}{(x-1)(x+3)}$$

15. Parabolas and cubics do not have oblique asymptotes. Only rational functions can have oblique asymptotes.

2.12 Sign Test for Rational Function Graphs

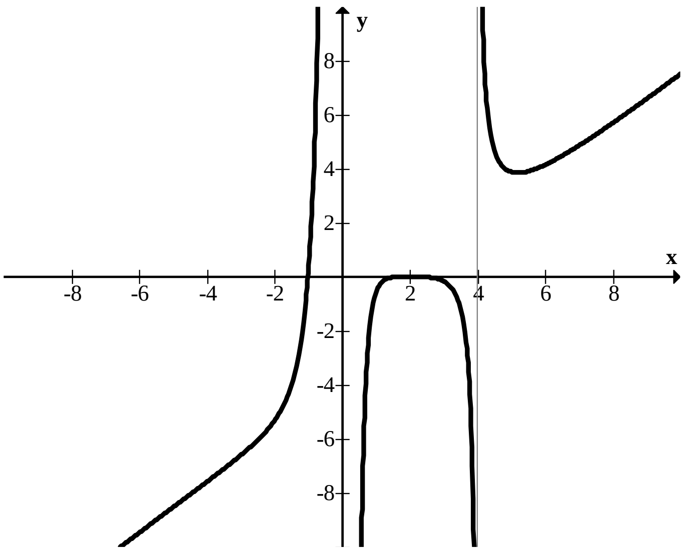
Answers

1. Vertical asymptotes at $x = 0, x = 4$.

2. It will have an oblique asymptote at $y = x - 3$.

3. Test $x = 4.01, x = 3.99, x = 0.01, x = -0.01$

4. Here is the graph. Students are only required to sketch the graph near the asymptotes.

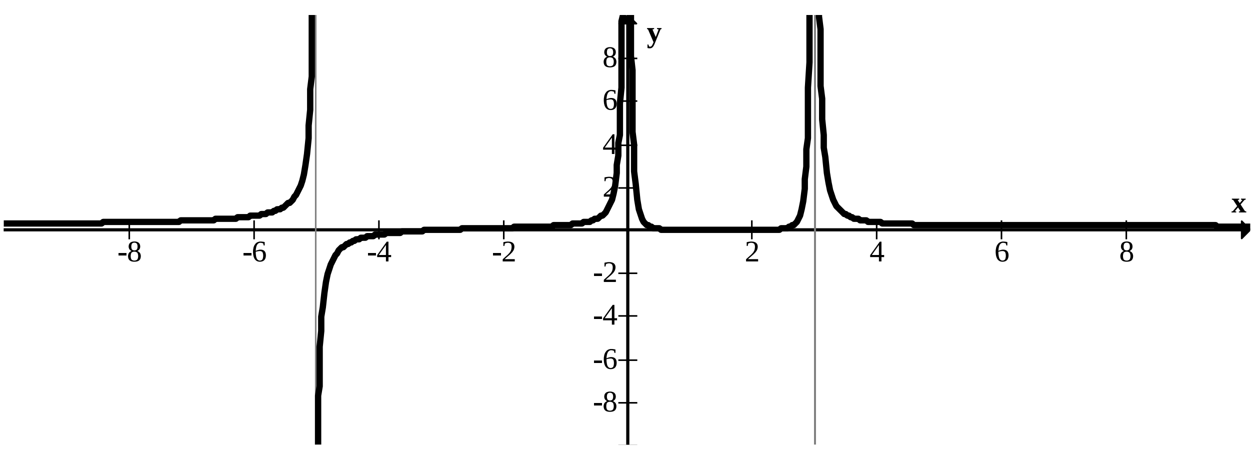


5. Vertical asymptotes at $x = 0, x = -5, x = 3$.

6. Horizontal asymptote at $y = \frac{1}{5}$.

7. Sign test at $x = 0.01, x = -0.01, x = -5.01, x = -4.99, x = 3.01, x = 2.99$.

8. Here is the graph. Students are only required to sketch the graph near the asymptotes.

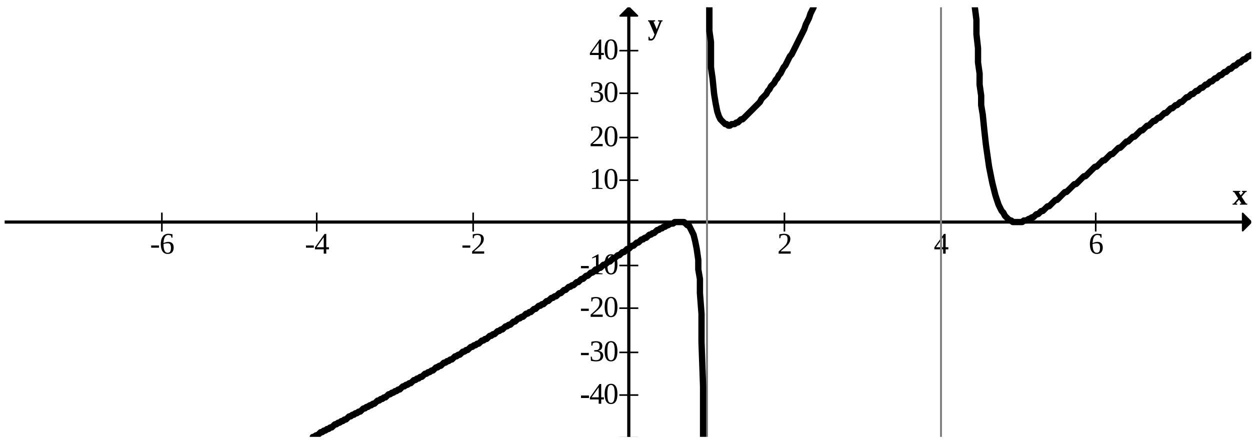


9. Vertical asymptotes at $x = 4, x = 1$.

10. Oblique asymptote at $y = 9x - 21$.

11. Sign test at $x = 4.01, x = 3.99, x = 1.01, x = 0.99$.

12. Here is the graph. Students are only required to sketch the graph near the asymptotes.

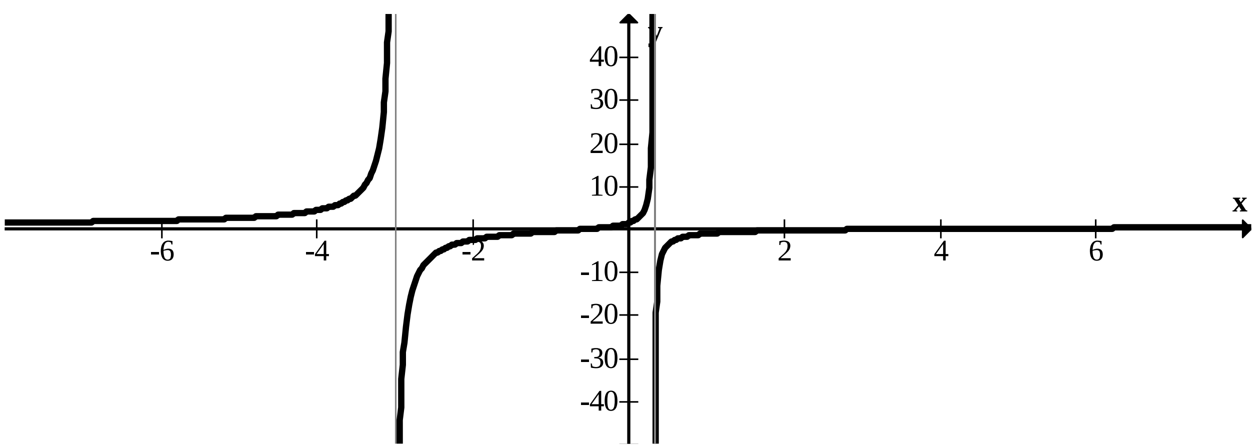


13. Vertical asymptotes at $x = -3, x = \frac{1}{3}$.

14. Horizontal asymptote at $y = \frac{2}{3}$.

15. Sign test at $x = -3.01, x = -2.99, x = 0.35, x = 0.32$.

16. Here is the graph. Students are only required to sketch the graph near the asymptotes.



2.13 Graphs of Rational Functions by Hand

Answers

1. No holes. Vertical asymptotes at $x = 1$ and $x = -2$.

2. Sign test at $x = 1.01\ (-), x = 0.99(-), x = -2.01\ (-), x = -1.99\ (+)$

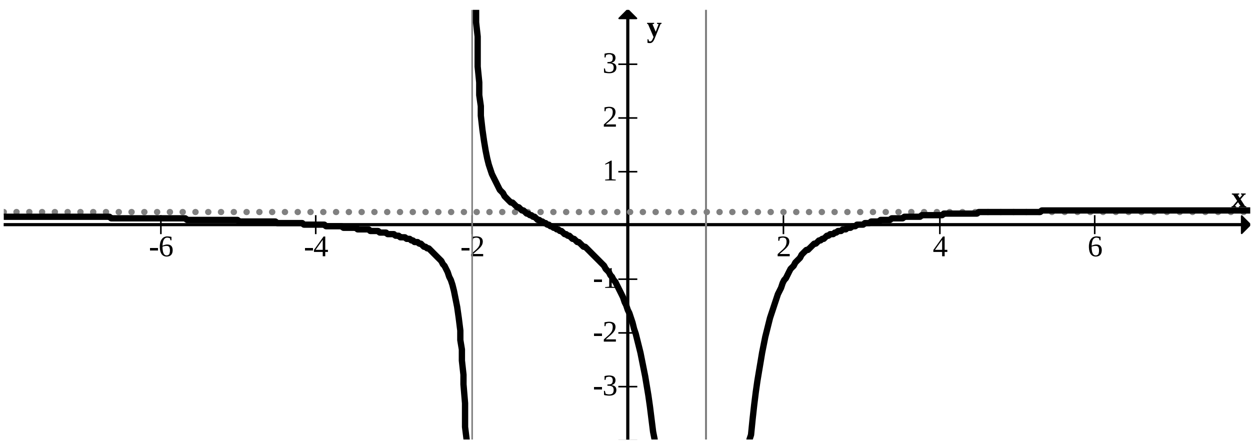
3. Horizontal asymptote at $y = \frac{1}{4}$.

4. Function approaches positive infinity as x values get large and as x values get small.

5. Zeroes at $x = -4, x = 3, x = -1$.

6. y-intercept at -1.5.

7.



8. No holes. Vertical asymptote at $x = 2, x = -1$.

9. Sign test at $x = 2.01(-), x = 1.99(+), x = -1.01(+), x = -0.99(+)$.

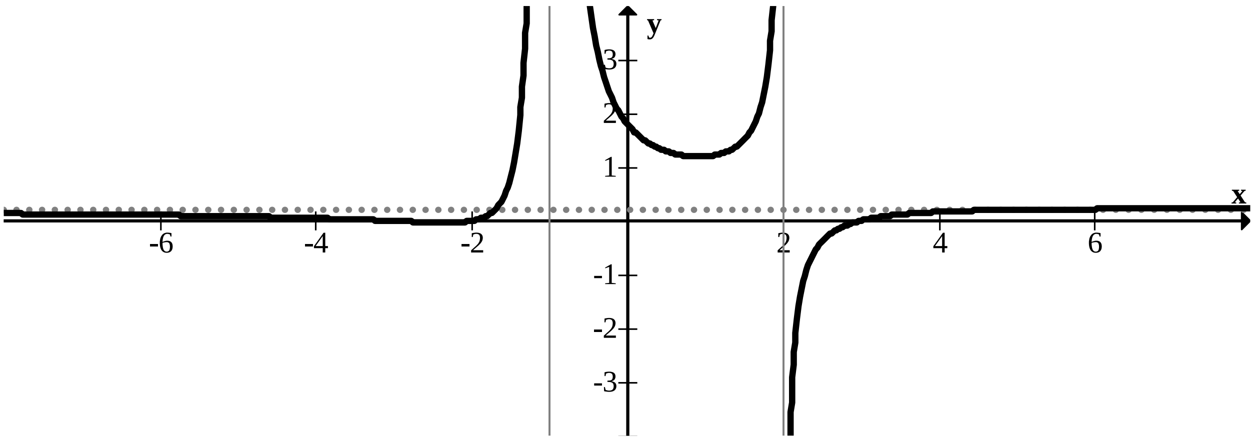
10. Horizontal asymptote at $y = \frac{1}{5}$.

11. Approaches positive infinity in both directions.

12. Zeroes at $x = 3, x = -3, x = -2$.

13. y-intercept at $y = 1.8$.

14.



15. Here is the complete graph: