Chapter 2 – Rigid Transformations

2.1 Transformations in the Plane

Answers

1. True
2. True
3. False
4. False
5. True
6. False
7. True
8. The triangle is rotated around point D to create a new triangle. This looks like a rigid transformation.
10. The triangle is moved to the right and stretched vertically to create a new triangle. This does not look like a rigid transformation.
11. Answers vary.
12. The triangle is translated along vector \( \vec{u} \) to create a new triangle. This is a rigid transformation.
14. The triangle is reflected across the line and then moved to the right. This is a rigid transformation.
15. Answers vary.
2.2 Translations

Answers

1. Yes

2. A vector tells you how far and in what direction to translate your shape.

3. The lines that connect corresponding points are all parallel to each other and the vector that defines the translation.

4. If the vector that connects each pair of corresponding points is the same.

5. The vector moved each point one unit to the left and three units up.

6. The vector moved each point two units to the right.

7. The vector moved each point three units to the right and two units down.

8. The vector moved each point three units to the left and one unit down.

9.

10.
11. Yes, each point moves 6 units to the right and three units down.

12. No.

13. No. Though distances and angles are preserved, the lines that connect corresponding points are not parallel.
2.3 Geometry Software for Translations

Answers

1-7: Answers vary.

8. You can check that you did it correctly by verifying that both images are in the exact same place.

9-14: Answers vary.

15. One way to do this is to define a point on the circle. Construct lines parallel to the vector through the center of the circle and the point on the circle. Copy and paste the vector onto those lines at the center of the circle and the point on the circle. Construct a new circle by using the "circle with center though point" button and the endpoints of the two copied vectors.

16. Answers vary
2.4 Reflections

Answers

1. The x-axis.

2. The line $x = 7$.

3. The line $AC$ (or $A'C'$).

4. The line $AB$ (or $A'B'$).

5.

6.
7.

8. The image looks like same although B and C have switched places.

9. Yes. The line of reflection is the line segment connecting C and E.

10. No. This is a rotation.

11. \((x, y) \rightarrow (-x, y)\)

12. \((-7,2)\)

13. \((x, y) \rightarrow (y, x)\)

14. \((2,7)\)

15. \((x, y) \rightarrow (-y, -x)\)

16. \((-2, -7)\)
2.5 Geometry Software for Reflections

Answers

1-5: Answers vary.

6. You can check that you did it correctly by verifying that both images are in the exact same place.

7-11: Answers vary.

12. Lines that pass through the center of the circle.

13. Yes. Because of the nature of circles, when "flipped", they will not appear to change orientation. Therefore, unless specific points have been defined on the circles, either a translation or a reflection could have occurred.


15. One way to do this is to define a point on the circle. Construct lines perpendicular to the line of reflection that pass through the center of the circle and the point on the circle. Copy and paste the segments connecting those points with the line of reflection to the other side of the line of reflection. Construct a new circle by using the "circle with center though point" button and the endpoints of the two copied segments.
2.6 Reflection Symmetry

Answers

1. A shape has symmetry if it can be transformed and look exactly the same and be in the same location.

2. A shape has reflection symmetry if it can be reflected across some line and still look the same and be in the same location.

3. A shape would have translation symmetry if it could be translated and look the same and be in the same location. No shapes have translation symmetry, but lines do because they go on forever.

4. Yes, it has reflection symmetry. 3 lines of symmetry through each midpoint/opposite vertex pair.

5. Yes, it has reflection symmetry. 1 line of symmetry through the midpoint of the base and the opposite vertex.

6. No reflection symmetry.

7. No reflection symmetry.

8. Yes, it has reflection symmetry. 2 lines of symmetry through opposite vertices.

9. Yes, it has reflection symmetry. 5 lines of symmetry through each midpoint/opposite vertex pair.

10. Yes, it has reflection symmetry. 6 lines of symmetry through opposite vertices and opposite midpoints.

11. Yes, it has reflection symmetry. 12 lines of symmetry through opposite vertices and opposite midpoints.

12. Yes, it has reflection symmetry. n lines of symmetry through opposite vertices and opposite midpoints.

13. Yes, it has reflection symmetry. There are an infinite number of lines of symmetry. Any diameter of the circle is a line of symmetry.

14. Yes, it has reflection symmetry. 1 line of symmetry through vertices that connect the sides that are the same length.

15. Yes, in fact it must have two halves that are exactly the same.

16. There are many examples. One example is a person. It is thought that the more symmetrical a person's face, the more they are perceived as being beautiful.
2.7 Rotations

Answers

1. 120° counterclockwise rotation about point O or 240° clockwise rotation about point O.

2. 180° clockwise or counterclockwise rotation about point O.

3. 250° counterclockwise rotation about point O or 110° clockwise rotation about point O.

4. The image and the original shape will be indistinguishable because they will be in the same place.

5. 

6. \((x, y) \rightarrow (y, -x)\)

7. \((2, -3)\)

8. \((-3, -2)\)

9. \((-2, 3)\)

10. The center of rotation is the center of a circle that the shape is rotating around.

11. O is the center of the circle containing points B and B’. \(\angle B O B' = 130°\).
12.

13.

14.

15. No, that rule only works for rotations about the origin.
2.8 Geometry Software for Rotations

Answers

1-2: Answers vary.

3. The polygon should end up in the same place because 100° counterclockwise is the same as 260° clockwise due to the fact that 100 + 260 = 360, a full circle.

4-5: Answers vary. Your rotation is correct if the images for #4 and #5 end up in the same place.

6. The center of the decagon.

8. \( \frac{360}{10} = 36° \).
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9-10. Answers vary.

11.

12. The center of the circle.

13. Yes, assuming there are no distinguishing points on the circle.

14.

15. You need to rotate both the center of the circle and a point on the circle.
2.9 Rotation Symmetry

Answers

1. A shape has symmetry if it can be transformed and its transformed image is indistinguishable from the original shape.

2. A shape has rotation symmetry if it can be rotated less than $360^\circ$ and be carried onto itself.

3. When any shape is rotated $360^\circ$ it will be carried onto itself, so this would not be a unique property.

4. Yes, about its center $120^\circ, 240^\circ$

5. No

6. No

7. Yes, about its center $180^\circ$

8. Yes, about its center $180^\circ$

9. Yes, about its center $72^\circ, 144^\circ, 216^\circ, 288^\circ$

10. Yes, about its center $60^\circ, 120^\circ, 180^\circ, 240^\circ, 300^\circ$

11. Yes, about its center $30^\circ, 60^\circ, 90^\circ, 120^\circ, 150^\circ, 180^\circ, 210^\circ, 240^\circ, 270^\circ, 300^\circ, 330^\circ$

12. Yes, about its center $k \cdot \frac{360}{n}$ for integer values of $k$ less than $n$.

13. Yes, about its center any number of degrees.

14. No

15. The center of rotation will be in the center of the shape.
2.10 Composite Transformations

Answers

1. A composite transformation is two or more transformations performed one after another.

2. Yes, in general the order matters. For some composite transformations it will not matter, but these are special cases.

3. Answers vary. Possible answer: Rotation of 180° about the origin followed by a translation.

4. Answers vary. Possible answer: A reflection across the x-axis followed by a reflection across the y-axis followed by a translation.


9. Answers vary

10. Translation

11. Translation

12. Translation

13. Rotation

14. Rotation

15. Rotation