Polynomial Functions

Properties of Exponents

Review Queue Answers

1. 25
2. 27
3. 32
4. -12
5. 36
6. -9

Product and Quotient Properties

1. 64
2. 1,000
3. -243
4. 0.00390625
5. 262,144
6. 46,656
7. 64
8. 216
9. b^9
10. $25x^{13}$
11. y^7
12. a^4b^5
13. $81x^3$
14. $d^{14}f^{10}$
15. $8m^7n^{10}$
16. $81p^4q^6$
17. a) -2 b) 4 c) -8 d) 16 e) -32 f) 64
18. Even powers will always generate positive answers, odd powers will give negative answers.
19. Yes, $(-2)^4 = 16$ and $-2^4 = -16$. The negative was not included the power.

Negative and Zero Exponents

1. $\frac{1}{64}$
2. $\frac{1}{x^9}$
3. $\frac{1}{7}$
4. $\frac{1}{y^{19}}$
5. $\frac{1}{xy^2}$
6. $\frac{b}{a^6}$
7. $\frac{2c^4}{3d}$
8. $\frac{4g^9}{15h}$
9. $\frac{x^6y^8}{2}$
10. $\frac{3}{h^4}$
11. $\frac{3b^{19}}{20}$
12. $\frac{3g^{31}}{2h}$
13. $\frac{1}{5^4}, \frac{1}{5^3}, \frac{1}{5^2}, \frac{1}{5^1}, 5^0, 5^1, 5^2, 5^3, 5^4$
14. $\frac{1}{625}, \frac{1}{125}, \frac{1}{25}, \frac{1}{5}, 1, 5, 25, 125, 625$
15. multiply, divide

Power Properties

1. $2^{15} = 32{,}768$
2. $81x^4$
3. $\frac{16}{25}$
4. $216x^9$
5. $\frac{128a^{21}}{b^{35}}$
6. $\frac{1}{16x^{16}}$
7. $49h^9$
8. $\frac{8x^{21}}{125y^9}$
9. $\frac{81}{m^4n^{48}}$
10. $\frac{4}{125x^4y^{14}}$
11. $\frac{5r^{14}}{243}$
12. $8s^9$
13. $\frac{a^{21}b^{14}}{75}$
14. $\frac{1}{2xz^{11}}$
15. $\frac{243g^{51}j^{21}}{h^2}$
16. $\frac{12b^{66}}{a^{18}}$
17. $(2^2)^3 = 2^6$
18. $(3^2)^2 = 3^4$
19. x = 6
20. x = 4

Adding, Subtracting and Multiplying Polynomials

Review Queue Answers

1. $3x^2 + 11x - 4$
2. $(x - 3)^2$
3. $4x^2 - 11x - 55$
4. 4x + 7

Adding and Subtracting Polynomials

1. No, x is in the denominator of a fraction.
2. Yes, degree is 4, leading coefficient is 8.
3. Yes, degree is 3, leading coefficient is 1.
4. No, x has negative exponents.
5. Yes, degree is 2, leading coefficient is $\sqrt{2}$.
6. Yes, degree is 4, leading coefficient is $\frac{1}{3}$.
7. No, x is in the denominator of a fraction.
8. Yes, degree is 6, leading coefficient is -10.
9. $4x^3 + 3x^2 - 19x + 20$
10. $-6x^4 + 8x^3 - 2x^2 - 12x + 7$
11. $x^4 + 7x^3 + 7x^2 - 3x + 19$
12. $-6x^4 + 17x^3 - 3x^2 - 4$
13. $3x^3 + 15x^2 - 11x - 11$
14. $2x^5 - 4x^4 + 3x^3 + 7x^2 - 5x + 17$
15. $7x^3 - 4x^2 + 20x - 12$

Multiplying Polynomials

1. $5x^3-30x^2+40x$
2. $-8x^5+11x^3-20x^2$
3. $21x^6-7x^5+112x^4+70x^3$
4. $x^3-5x^2+4x-20$
5. $6x^3-21x^2-8x+28$
6. $-x^3-2x^2+9x+18$
7. x^4-2x^3-2x-1
8. $5x^4-x^3+40x^2-68x+12$
9. $3x^4-25x^3+36x^2-139x-105$
10. $2x^3+9x^2-64x-64$
11. $2x^5+8x^4+3x^3+12x^2-10x-40$
12. $25x^2-120x+144$
13. $-6x^7-33x^6+2x^5+11x^4$
14. $16x^2+72x+81$
15. $8x^5-6x^4+25x^3-12x^2+3x-18$
16. $10x^5-26x^4-15x^3+61x^2+10x-28$
17. $x^5-4x^4-15x^3+29x^2-51x-90$

Factoring and Solving Polynomial Equations

Review Queue Answers

1. (x – 11)(x +2)
2. (2x – 5)(2x + 5)
3. (2x – 1)(3x + 5)
4. x = $\frac{1}{2}$ or $-\frac{3}{5}$

Sum and Difference of Cubes

1. $(x-3)(x^2+3x+9)$
2. $(4+x)(16-4x+x^2)$
3. $4(2x-1)(4x^2+2x+1)$
4. $(4x+7)(16x^2-28x+49)$
5. $(8x-9)(64x^2+72x+81)$
6. $x(5x+2)(25x^2-10x+4)$
7. $81(2x+1)(4x^2-2x+1)$
8. $5x^3(x-3)(x^2+3x+9)$
9. $2x^4(7x-8)(49x^2+56x+64)$
10. $x=-\frac{1}{5}$
11. $x=\frac{4}{9}$
12. $x=0$ and $\frac{7}{2}$
13. $x=0,\ -5,\ \frac{5}{2}+\frac{5\sqrt{3}}{2}i,\ \frac{5}{2}-\frac{5\sqrt{3}}{2}i$
14. $x=-\frac{10}{7},\ \frac{35}{49}+\frac{35\sqrt{3}}{49}i,\ \frac{35}{49}-\frac{35\sqrt{3}}{49}i$
15. $V=4x(18-x)(21-x)$, 1360, 3240, 4160

Factoring by Grouping

1. $(x^2+3)(x-4)$
2. $(x-3)(x+3)(x+6)$
3. $(x^2+5)(3x-4)$
4. $(2x-3)(x-2)(x^2+2x+4)$
5. $(2x-5)(2x+5)(x+1)$
6. $(2x^2-5)(2x+9)$
7. $(3x-5)(2x+3)(4x^2-6x+9)$
8. $(3x^2-2)(5x+2)$
9. $(x-5)(x+5)(4x+5)$
10. $x=-\frac{2}{3}, \frac{2}{3}, 6$
11. $x=3$ and -3
12. $x=2,\ 2,\ -2$
13. $x=-3,\ -1,\ 1,\ 3,\ i,\ -i$
14. $x=-3,\ 4i,\ -4i$

Factoring Polynomials in Quadratic Form

1. $(x-2)(x+2)(x^2-2)$
2. $(x-3)(x+3)(x^2+5)$
3. $(x^2-3)(4x^2+1)$
4. $(3x^2+8)(2x^2+1)$
5. $(x-3)(x+3)(x^2+9)$
6. $(2x-1)(2x+1)(4x^2+1)$
7. $2x(3x^2-2)(x^2+5)$
8. $4x^2(x-3)(x+3)(x^2+9)$
9. $(5-3x)(5+3x)(25+9x^2)$
10. x = 2, -2
11. x = $\pm\frac{\sqrt{7}}{2}$
12. x = 0

Dividing Polynomials

Review Queue Answers

1. 15
2. 84
3. 165
4. 423

Long Division

1. $2x^2+3x-10+\frac{4}{x+1}$
2. $x^3-5x^2-10x-35+\frac{145}{x-5}$
3. $2x^2-8x+6-\frac{19x-5}{x^2-1}$
4. $3x^2-6x+9-\frac{8}{3x+2}$
5. $3x^3+7x^2+7x-2$
6. $x^3-4x+3-\frac{3x-11}{2x^2+3}$
7. 1, -3, and -5
8. 6, $\frac{1}{2}$, and $-\frac{1}{3}$

9. $2x^3 - 7x^2 - 10x + 24$

10. Any polynomial with the form $a(x^4 - 13x^3 + 39x^2 + 13x - 40)$, such that *a* is an integer.

Synthetic Division

1. $x^2 + 4x - 1 + \frac{12}{x+2}$

2. $4x^2 + 17x + 16$

3. $4x - 2 - \frac{4}{2x+1}$

4. $2x^3 - 3x^2 - 3x + 7 + \frac{21}{x+9}$

5. $x^2 + 2x - 1$

6. $3x^4 + 3x^3 + 7x^2 + 7x + 6 + \frac{4}{x-1}$

7. #2 and #5. k is a zero because the remainder is zero.

8. k is a zero, (x – k) is a factor of f(x). f(k) = 0 if and only if k is a zero.

9. f(-2) = -14

10. The remainder is -14, which is the same as f(-2).

11. -4, $\frac{1}{6}$, and $-\frac{5}{2}$

12. 5, $\pm\sqrt{2}$

Finding all Solutions of Polynomial Functions

Review Queue Answers

1. $(2x - 3)(4x^2 + 6x + 9)$

2. $(x + 7)(3x + 1)$

3. $(3x + 7)(2x + 1)$

4. $(x + 2)(x - 2)(x - 9)$

5. $(x + 2)(x + 1)(x^2 - x + 1)$

6. $2x(3x + 4)(3x - 4)(9x^2 + 16)$

Finding Rational and Real Zeros

1. $\pm1, \pm2, \pm4, \pm5, \pm10, \pm20$

2. $\pm\frac{1}{4}, \pm\frac{1}{2}, \pm\frac{3}{4}, \pm1, \pm\frac{5}{4}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm3, \pm\frac{15}{4}, \pm5, \pm\frac{15}{2}, \pm15$

3. $\pm\frac{1}{2}, \pm1, \pm2, \pm4, \pm8$

4. $\pm1, \pm3, \pm9$

5. $\frac{1}{2}, \frac{1}{3}, 2$

6. $1, -2, -4, -4$

7. $\frac{1}{4}, \frac{1}{4}, -3$

8. $\frac{3}{2}, 3 \pm \sqrt{3}$

9. $5, \frac{-8 \pm \sqrt{5}}{2}$

10. $1, -\frac{5}{2}, \frac{2 \pm \sqrt{7}}{3}$

11. $2, 2, -3, -4 \pm \sqrt{3}$

12. $\pm\frac{\sqrt{21}}{3}$, the other two solutions are imaginary.

Finding Imaginary Solutions

1. x = 1, -2, $\pm\sqrt{10}$

2. x = $5, \pm\frac{\sqrt{3}}{2}$

3. x = $\pm\sqrt{6}, \pm\frac{i\sqrt{10}}{2}$

4. x = -3, $\pm i\sqrt{5}$

5. x = -2, $\frac{3 \pm i}{2}$

6. x = $\pm\frac{\sqrt{15}}{3}, \pm i\sqrt{3}$

7. x = 3, $\pm\frac{3i\sqrt{2}}{2}$

8. x = -5, $-\frac{7}{3}, \frac{1}{2}$, 8

9. x = 1, 1, $\frac{-4 \pm \sqrt{5}}{3}$

10. x = 1, 3, $-\frac{5}{2}$, $4 \pm \sqrt{6}$

11. $x^3 - 4x^2 + x - 4$

12. $x^3 + 3x^2 + 4x + 12$

13. $x^4 + 2x^3 - 3x^2 - 10x - 10$

14. $3x^4 - 29x^3 + 80x^2 - 54x - 28$

15. Answers will vary.

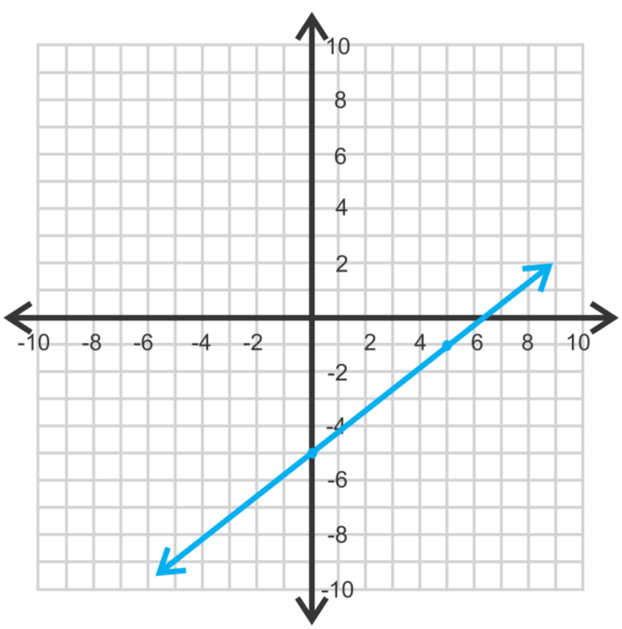
16. Answers will vary.

17. $x = -2, \pm i, 1 \pm \sqrt{3}$

Analyzing the Graph of Polynomial Functions

Review Queue Answers

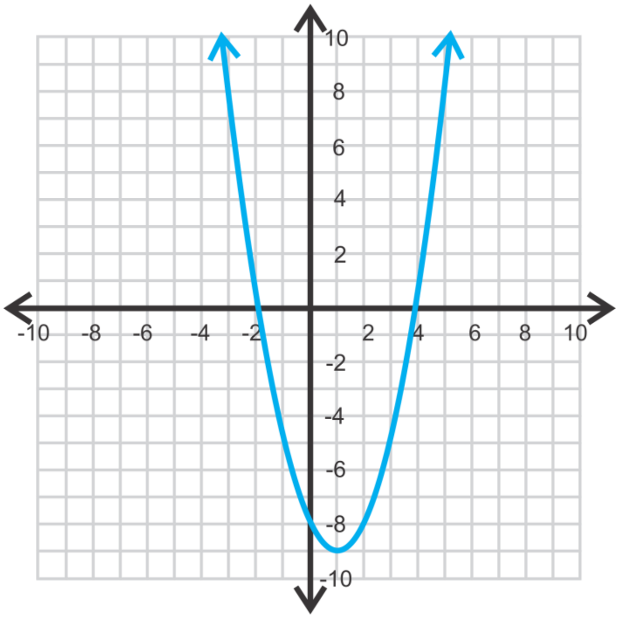
1.



m = $\frac{4}{5}$, y-intercept: (0, -5), x-intercept: $\left(\frac{25}{4}, 0\right)$

ans-0607-01

2.



x-intercepts: (-2, 0) and (4, 0)

vertex: (1, -9)

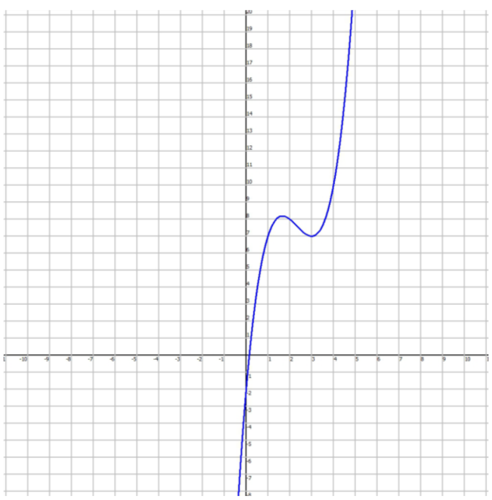
y-intercept: (0, 8)

ans-0607-02

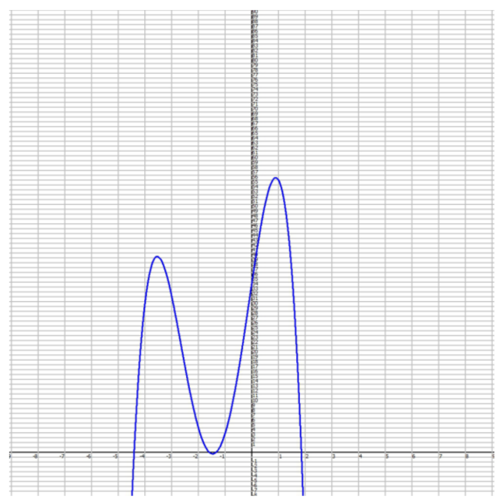
3. The vertex is a maximum and (3, 41).

Finding and Defining the Parts of a Polynomial Function Graph

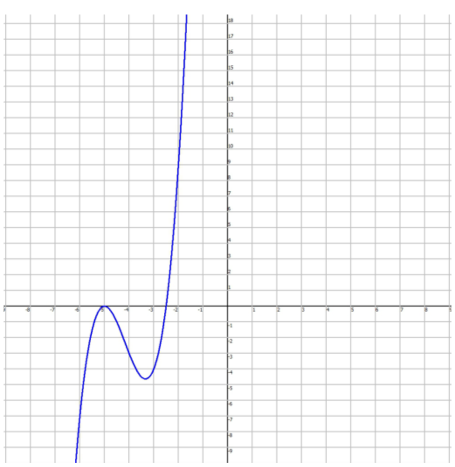
1. ans-0607-04



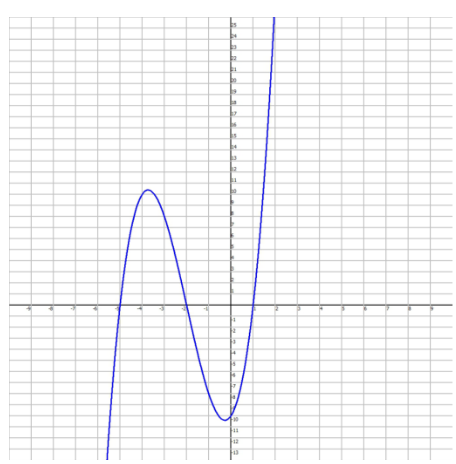
2. ans-0607-05



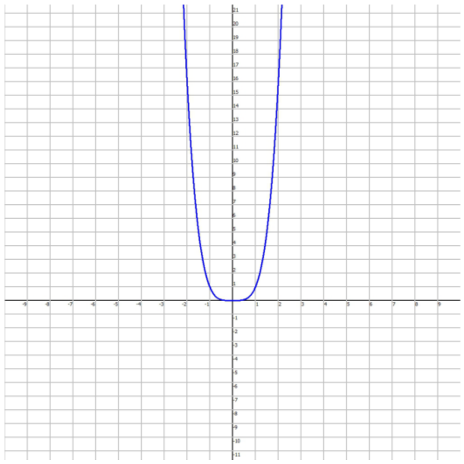
3. ans-0607-06



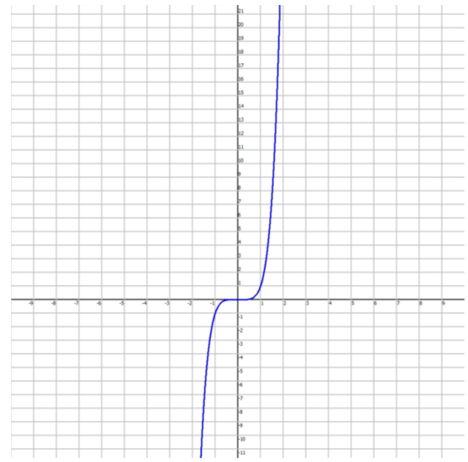
4. ans-0607-07



5. ans-0607-08



6. ans-0607-09



7. All even functions have the same end behavior. For example, x^2, x^4, and x^6 all open up. If there is a negative sign in front of the function, they will all open downward. All odd functions have opposite end behavior. They will be mostly increasing (positive leading coefficient) or mostly decreasing (negative leading coefficient).

8. A function can have as many repeated roots as the degree. For example, a parent graph will have a repeated root of zero the same as the degree number.

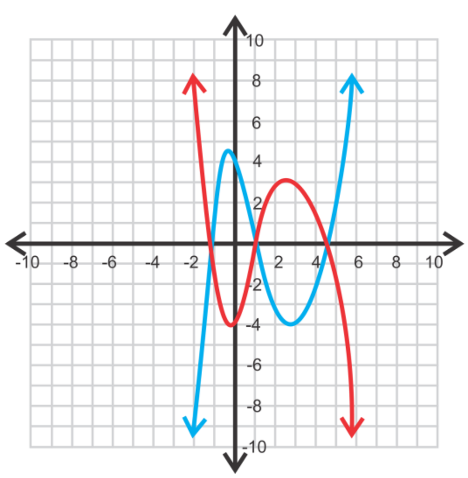
9. Zeros: (-6, 0) and (2, 0) both repeated twice, y-intercept: (0, -5), maximums at the zeros, minimum: (-2, -8), negative end behavior, domain and range are both all reals.

10. Zeros: (4, 0) and (7, 0) and two imaginary solutions, y-intercept: (0, 7.5), maximum: (-1, 8.5), local minimum: (-5.5, 1.5), absolute minimum: (5.5, -2.5), positive end behavior, domain: all reals, range: [-2.5, ∞).

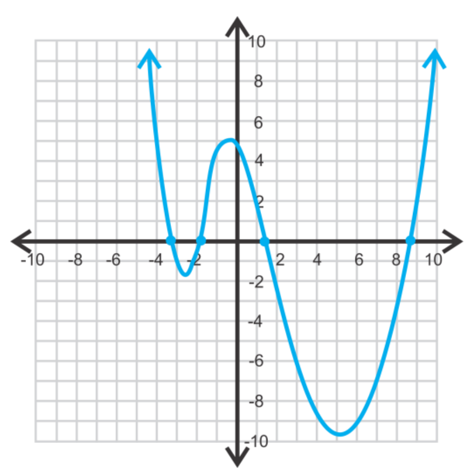
11. Zeros: (-4, 0), (-1.5, 0), and (4, 0), y-intercept: (0, 6), maximum: (1, 7.5), minimum: (-2.8, -1.5), negative end behavior, domain and range are both all reals.

12. Zeros: (-6.2, 0), (-2.8, 0), (-1.8), (0, 0) and (3.5, 0), y-intercept: (0, 0), absolute maximum: (-4.5, 24), local maximum (-0.8, 9), local minimum: (-2, -4), absolute minimum: (2, -45), positive end behavior, domain and range are both all reals.

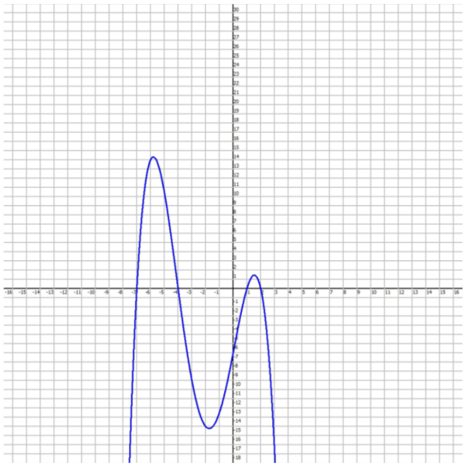
13. ans-0607-10



14. ans-0607-11



15. ans-0607-03



16. $y = -\frac{1}{8}(x+7)(x+4)(x-1)(x-2)$

Graphing Polynomial Functions with a Graphing Calculator

1. zeros: (-2, 0), $\left(\frac{3}{2}, 0\right)$

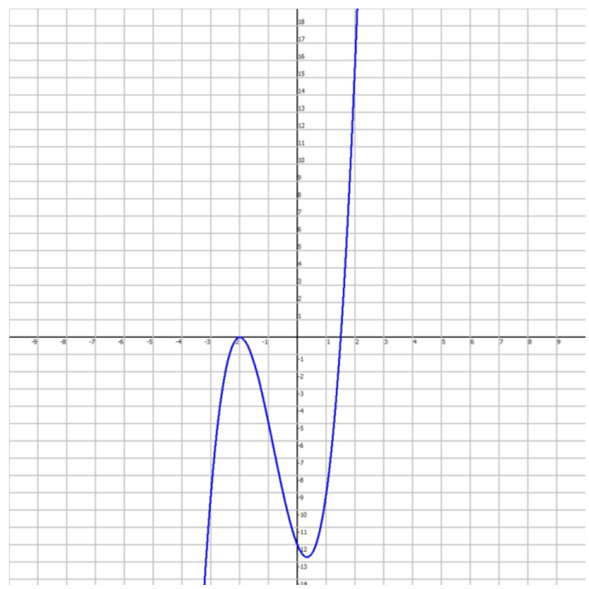
y-intercept: (0, -12)

max: (-2, 0), min: $\left(\frac{1}{3}, -12.7\right)$

generally increasing/positive end behavior

domain and range all reals

ans-0607-12



2. zeros: (-6.65, 0), (-1.35, 0)

y-intercept: (0, -9)

max: (-5.02, 43.51), no min

negative end behavior/opens down

domain: all reals

range: (-∞, 43.51]

ans-0607-13

3. zero: (2, 0)

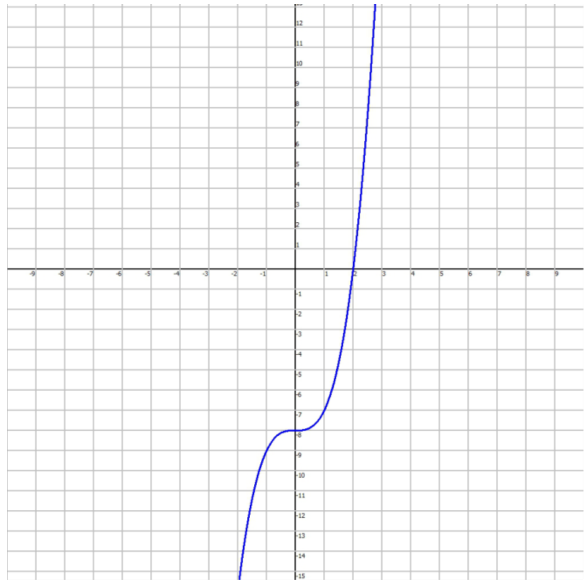
y-intercept: (0, -8)

no max or min

generally increasing/positive end behavior

domain and range all reals

ans-0607-14



4. zero: $\left(\frac{1}{2}, 0\right)$

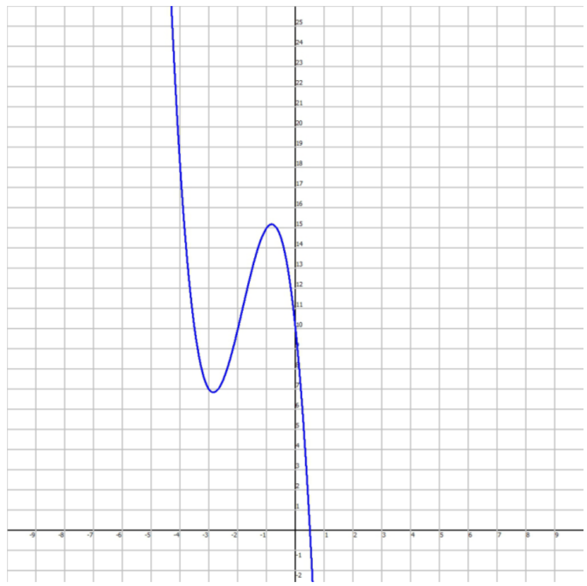
y-intercept: (0, 10)

max: (-0.82, 15.19), min: (-2.85, 6.85)

generally decreasing/negative end behavior

domain and range all reals

ans-0607-15



5. zeros: (-4.162, 0), (-1.5, 0), (2, 0), (2.162, 0)

y-intercept: (0, 54)

max: (-0.06, 54.09)

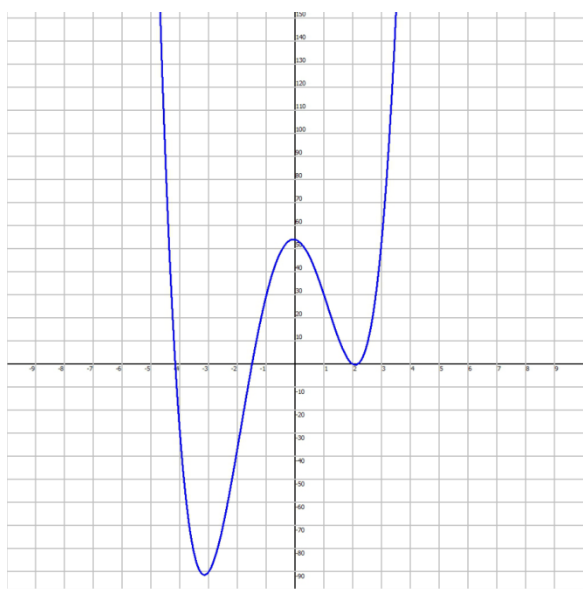
local min: (2.08, -0.29)

absolute min: (-3.15, -91.39)

positive end behavior/opens up

domain: all reals, range: [-91.39, ∞)

ans-0607-16



6. zeros: (2.45, 0), (-2.45, 0), (-1, 0)

y-intercept: (0, -6)

max: (-1, 0)

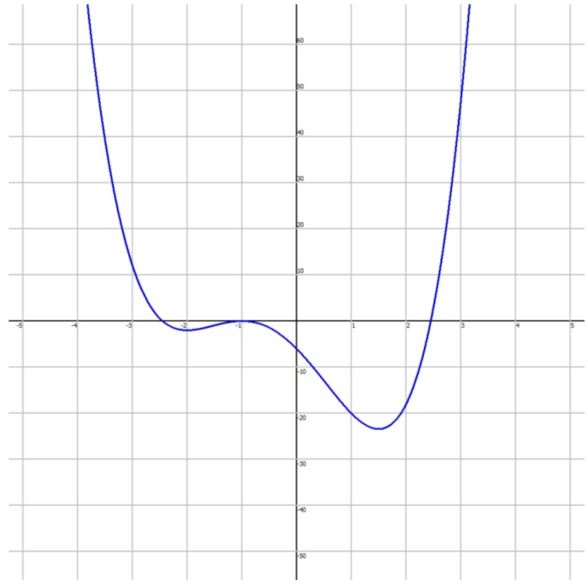
local min: (-2, -2)

absolute min: (1.5, -23.44)

positive end behavior/opens up

domain: all reals, range: [-23.44, ∞)

ans-0607-17



7. 2 irrational, 2 imaginary

8. $x = -1 \pm i\sqrt{3}$

9. $-1 \pm \sqrt{10}$

10. Sometimes, when the leading coefficient is negative.

11. Always

12. Never, imaginary solutions always come in pairs.

13. Sometimes, a function can have imaginary solutions.

14. $-4 - \sqrt{7}$, $x = \pm 2i$