

11.1 F-Distribution

Answers

1. Variance
2. Answers will vary but could include that the t-test compares the means of independent samples whereas the F-distribution compares the variance of independent sample or that the t-test is a symmetrical representation whereas the F-distribution is not.
3. F-ratio
4. larger; smaller

5. a)
$$H_o : \sigma_1^2 = \sigma_2^2 \text{ or } \frac{\sigma_1^2}{\sigma_2^2} = 1$$

b)
$$H_a : \sigma_1^2 \neq \sigma_2^2 \text{ or } \frac{\sigma_1^2}{\sigma_2^2} \neq 1$$

b) 2.04

c) 2.25

d) Reject the null hypothesis because the F-ratio is greater than the critical value.

e) The variance of the student achievement scores in course #2 is less than the variance for the students in course #1. This would help guidance counselors help select an appropriate program.

6. True

7. a)
$$H_o : \sigma_1^2 = \sigma_2^2 \text{ or } \frac{\sigma_1^2}{\sigma_2^2} = 1$$

b) 0.157908545

c) Accept the null hypothesis as the F-ratio does not exceed the critical value and conclude that there is no difference between the two variables.

8. Answers may vary but could include that the F-distributions are not symmetric and only span non-negative numbers. The F-distribution also has varying shapes depending on the degrees of freedom.

9. Answers may vary but could include that the chi-squares test is a measure of the squared differences between observed and expected frequencies to determine if there is a difference and it is also a test for single variance. The F-test is used for testing the equality of two population variances and is used for small sample sizes. The chi-squared can be used for both large and small sample sizes.
10. a) True
b) False, two parameters
c) False, they can be difference sizes
11. B
12. A
13. A
14. A
15. A and C
16. C
17. $F = 4$. The critical value of F at 0.10 level of significance is 1.822. Therefore reject the null hypothesis that the variances are equal.
18. $F = 1.73$. The critical value of F at 0.05 level of significance is 2.11. Therefore accept the null hypothesis that the variances are equal.
19. $F = 1.59$
20. $F = 1.73$. The critical value of F at 0.01 level of significance is 1.97. Therefore accept the null hypothesis that the variances are equal.
21. C
22. B
23. $F = 1.50$. The critical value of F at 0.05 level of significance is 1.95. Therefore accept the null hypothesis that the variances are equal.

11.2 One-Way ANOVA Tests

Answers

1. Analysis of Variance
2. Increase or increase exponentially
3. C
4. E, A, D, B, C
5. = a) $H_o : \mu_1 = \mu_2 = \mu_3$

b)

	Ms. Jones	Mr. White	Mrs. White	Totals
Number (n_k)	10	9	8	27
Total (T_k)	109	131	137	377
Mean (\bar{x})	10.9	14.6	17.1	42.6
Sum of Squared observations ($\sum_{i=1}^{n_k} x_{ik}^2$)	1339	2113	2529	5981
Sum of Obs. Squares/Number of obs ($\frac{T_k^2}{n_k}$)	1188	1907	2346	5441

- c) 26.35
 - d) 4.03
 - e) 6.54
 - f) 3.40
 - g) The test statistic exceeds the critical value so the null hypothesis is rejected. Thus the population means are not all equal.
6. A one-way ANOVA is a test to determine if three or more means are equal. The ANOVA analyses the variation of the scores within groups and between groups.

- 7. a) Yes as there are four groups of women being compared.
b) Yes, the ratings for each song are being compared.
- 8. a) $H_o : \mu_1 = \mu_2 = \mu_3$. There is no connection between the body mass index and age.
b) The small p-value indicates that the variances are different.
- 9. The small p-value indicates that the variances are different and the choice of seating (front, middle, or back) is not connected to student height.
- 10. a) $F = 6.27$. The critical value of F at 0.05 level of significance is 3.89. Therefore accept the null hypothesis that the variances are equal. The critical value of F at 0.01 level of significance is 6.93. Therefore reject the null hypothesis that the variances are equal.
b) $F = 3.27$. The critical value of F at 0.05 level of significance is 3.11. Therefore accept the null hypothesis that the variances are equal. The critical value of F at 0.01 level of significance is 4.94. Therefore reject the null hypothesis that the variances are equal.
c) This is a repeat of question b.

11.

	ANOVA			
Source	<i>Df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>
Between	5	40	8	1.33
Error	10	60	6	xxxxx
Total	15	100	xxxxx	xxxxx

12.

	ANOVA			
Source	<i>Df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>
Between	2	10	5	0.4827586207
Error	28	290	10.35714286	xxxxx
Total	30	300	xxxxx	xxxxx

13. The purpose of the ANOVA test is test if the value of a single variable differs significantly among three or more levels of a factor.

14. 2.282

15. 2.8

16. 0.9929

17. df (within) = $k - 1$

df (between) = $n - k$

18. Answers will vary but should include statements like the F-distribution starts at zero and goes to infinity and that the F-distribution is skewed to the right.

19. $F = 6.089$

20. a) $H_o : \mu_1 = \mu_2 = \mu_3$. There is no difference among the three treatments for depression.

b) $H_o : \mu_1 \neq \mu_2 \neq \mu_3$. There is a difference among the three treatments for depression.

c) ANOVA test yields $F = 11.167$.

d) Significance level test = 0.05 as it is normal test of significance.

e) Critical value of $F = 3.89$

f) The critical value of F at 0.05 level of significance is 3.89 and is less than the F statistic, the conclusion is to accept the null hypothesis that there is no difference in the treatments.

21.

	ANOVA			
Source	<i>Df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>
Between	2	116.133	58.067	4.697
Error	12	151.587	12.632	xxxxx
Total	14	267.720	xxxxx	xxxxx

a) $H_o : \mu_1 = \mu_2 = \mu_3$. There is no connection between house color and people’s stay (in years).

b) $H_o : \mu_1 \neq \mu_2 \neq \mu_3$. There is a connection between house color and people’s stay (in years).

c) Critical value of $F = 3.89$ at 0.05 level of significance

d) The critical value of F at 0.05 level of significance is 3.89 and is less than the F statistic. The conclusion is to accept the null hypothesis that there is no connection between house color and people’s stay (in years).

22. $F = 4.166667$. The critical value of F at 0.05 level of significance is 4.26 and is slightly more than the F statistic, the conclusion is to reject the null hypothesis that there is no difference in the four groups.

11.3 Two-Way ANOVA Tests

Answers

1. Interaction

2. d

3. D

$$H_o : \mu_M = \mu_F$$

4. a) $H_o : \mu_1 = \mu_2 = \mu_3$

$$H_o : \text{all effects} = 0$$

b) Answers will vary but should be similar to statements such as the independent variable in the rows are equal to each other, the means of the independent variables of the rows and columns are equal to each other or there is no interaction between variables.

c) The three critical values are 4.07, 3.23, and 3.23. If the calculated exceeds these values we would reject the null hypothesis.

d) The first two null hypotheses would be rejected and the third would be accepted.

e) Answers will vary but students should conclude that not all means in the populations are equal with respect to gender and drug dosage. The conclusion should be made that there is no difference in performance of the males and females across levels of drug dosage.

5. a) response variable: weight loss

Two factors A and B: drink alcohol or not

The number of categories in each factor: 3

Number of levels for Factors A and B: 6

b) response variable: results on a reading test

Two factors A and B: attend preschool and have older siblings

The number of categories in each factor: Three levels for preschool, two levels for siblings

Number of levels for Factors A and B: 6

6.

a) ANOVA table completed in EXCEL

9							
10	Anova: Two-Factor Without Replication						
11							
12	<i>SUMMARY</i>	<i>Count</i>	<i>Sum</i>	<i>Average</i>	<i>Variance</i>		
13	Row 1	3	97.1	32.36667	2.943333		
14	Row 2	3	73.3	24.43333	2.743333		
15	Row 3	3	105.6	35.2	9.03		
16							
17	Column 1	3	95.5	31.83333	67.40333		
18	Column 2	3	89.5	29.83333	23.26333		
19	Column 3	3	91	30.33333	14.24333		
20							
21							
22	ANOVA						
23	<i>Source of Variation</i>	<i>SS</i>	<i>df</i>	<i>MS</i>	<i>F</i>	<i>P-value</i>	<i>F crit</i>
24	Rows	186.8867	2	93.44333	16.29826	0.011946	6.944272
25	Columns	6.5	2	3.25	0.56686	0.607093	6.944272
26	Error	22.93333	4	5.733333			
27							
28	Total	216.32	8				
29							
30							

- b) (i) The null hypothesis is that there is no connection between the route that the commuter takes to work (Factor A) and the speed of getting to work during different parts of the week. The alternative hypothesis is that there is a connection between the route that the commuter takes to work (Factor A) and the speed of getting to work during different parts of the week.
- (ii) The null hypothesis is that there is no connection between the day of the week the commuter takes to work (Factor B) and the speed of getting to work. The alternative hypothesis is that there is a connection between the day of the week the commuter takes to work (Factor B) and the speed of getting to work.
- (iii) The null hypothesis is that there is no connection between the route that the commuter takes to work (Factor A), the day of the week the commuter takes that route (Factor B) and the speed of getting to work. The alternative hypothesis is that there is a connection between the route that the commuter takes to work (Factor A), the day of the week the commuter takes that route (Factor B) and the speed of getting to work.

- c) (i) $F = 15.21$. The critical value of F at 0.05 level of significance is 4.26 and is less than the F statistic; the conclusion is to reject the null hypothesis that there is a connection in the route taken and the speed of going to work at different parts of the week.
- (ii) $F = 0.529$. The critical value of F at 0.05 level of significance is 3.63 and is more than the F statistic, the conclusion is to accept the null hypothesis that there is no connection in the day of the week going to work and the speed of getting to work.
- (iii) $F = 0.933$. The critical value of F at 0.05 level of significance is 3.63 and is more than the F statistic, the conclusion is to accept the null hypothesis that there is no connection in the route the commuter takes to work and the day of the week going to work.

7.

	ANOVA			
Source	<i>Df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>
Factor A	2	450	225	10.285
Factor B	1	840	840	38.398
Interaction	2	130	65	2.971
Error	48	1050	21.875	xxxxx
Total	53	2470	xxxxx	xxxxx

- 8. (a) $F = 10.285$. The critical value of F at 0.05 level of significance is 3.191 and is less than the F statistic, the conclusion is to accept the null hypothesis that there is no connection in the between the levels in Factor A.
 - (b) $F = 38.398$. The critical value of F at 0.05 level of significance is 4.043 and is less than the F statistic, the conclusion is to accept the null hypothesis that there is no connection in the between the levels in Factor B.
 - (c) $F = 2.971$. The critical value of F at 0.05 level of significance is 3.191 and is less than the F statistic, the conclusion is to accept the null hypothesis that there is no interaction between the levels in Factor A and the levels in Factor B.
9. There was no table in problem 3??
- 10. a) You would do a two-way ANOVA where Factor A would be the three levels of smokers and Factor B would be the different physical activities. The two-way ANOVA would yield test statistics that would allow you to accept or reject the null hypothesis.
 - b) The null hypothesis would be that there is no connection between the levels of smokers and the levels of physical activity.
11. You would do a two-way ANOVA where Factor A would be the test scores from the three universities and Factor B would be whether they went through the agency or did not. The two-way ANOVA would yield test statistics that would allow you to accept or reject the null hypothesis that there was no difference in the test scores.