Basic Algebra Flexbook Solution Key
Chapter 11
Radicals and Geometry Connections; Data Analysis

Lesson 11.1
Graphs of Square Root Functions

1. In the definition of a square root function, (x-h) must always be greater than or equal to zero because a negative number under a radical gives you an imaginary number. If the number under the radical were negative there would be no real solution and it would be impossible to graph or solve.

2. $f(x) = \sqrt{x}$
Domain: $x \geq 0$
Range: $y \geq 0$

3.
$f(x) = \sqrt{x-2}$
$h = 2$
$k = 0$
$(h,k) = (2,0)$

4.
$g(x) = \sqrt{x+4} + 6$
$h = -4$
$k = 6$
$(h,k) = (-4,6)$

5.
$h(x) = \sqrt{x-1} - 1$
$h = 1$
$k = -1$
$(h,k) = (1,-1)$

6.

$y = \sqrt{x} + 3$

$h = 0$

$k = 3$

$(h, k) = (0, 3)$

7.

$f(x) = \sqrt{2x} + 4$

$h = 0$

$k = 4$

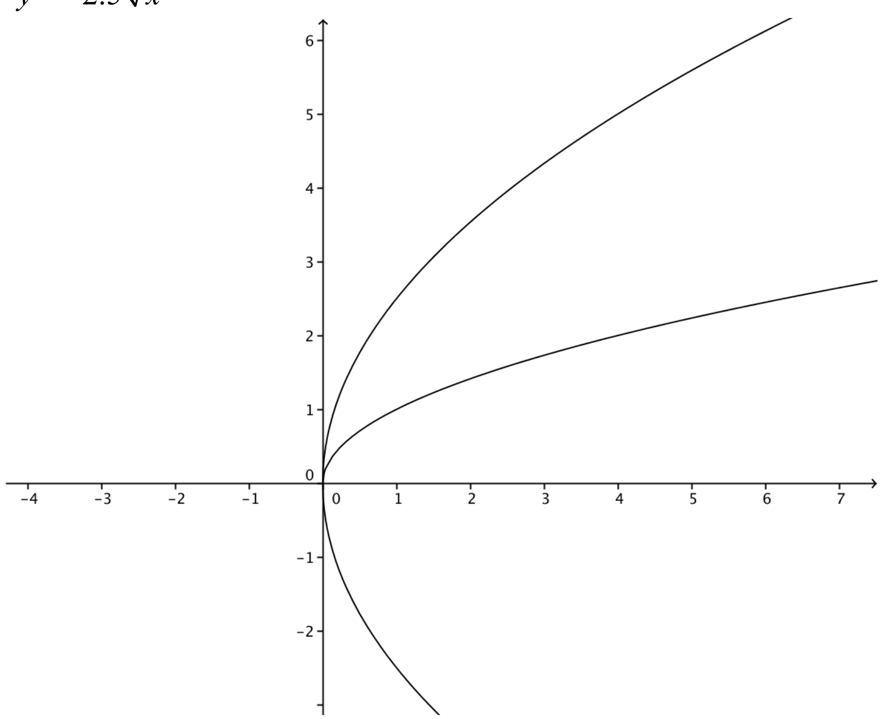
$(h, k) = (0, 4)$

8.

$y = \sqrt{x}$

$y = 2.5\sqrt{x}$

$y = -2.5\sqrt{x}$

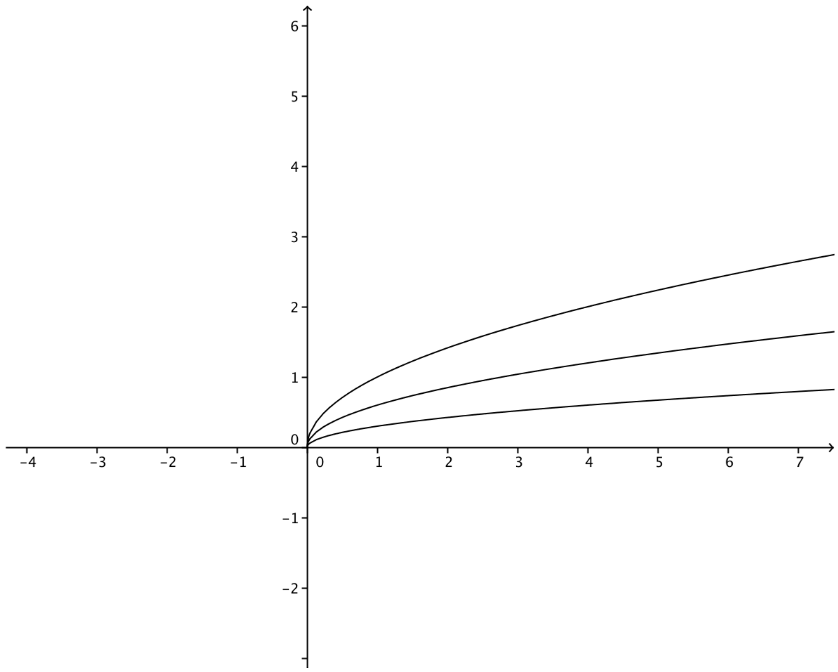


9.

$y=\sqrt{x}$

$y=0.3\sqrt{x}$

$y=0.6\sqrt{x}$

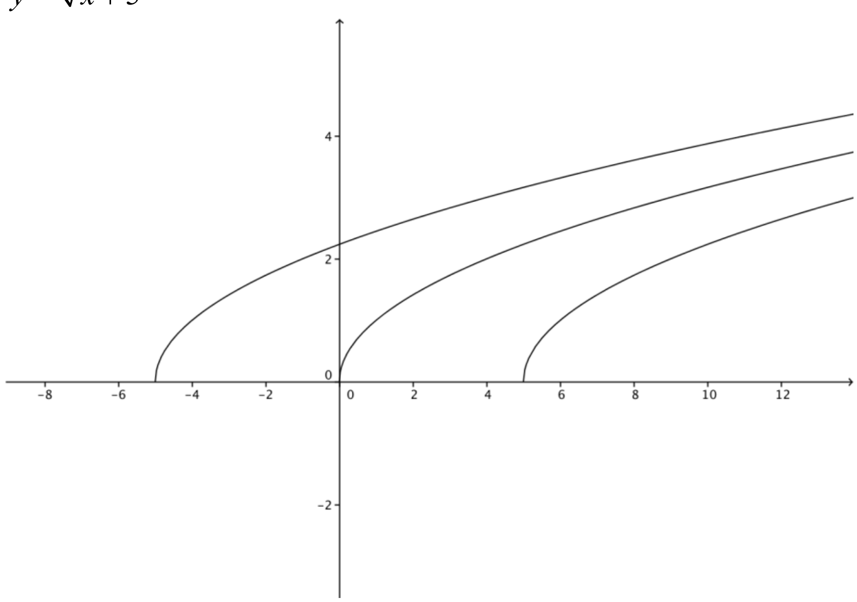


10.

$y=\sqrt{x}$

$y=\sqrt{x-5}$

$y=\sqrt{x+5}$

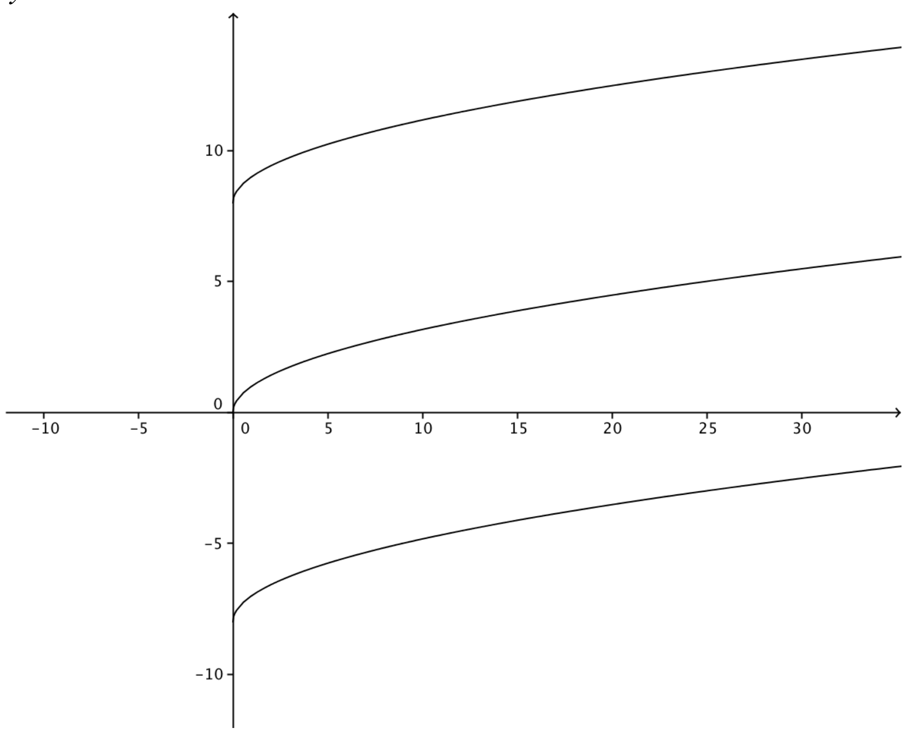


11.

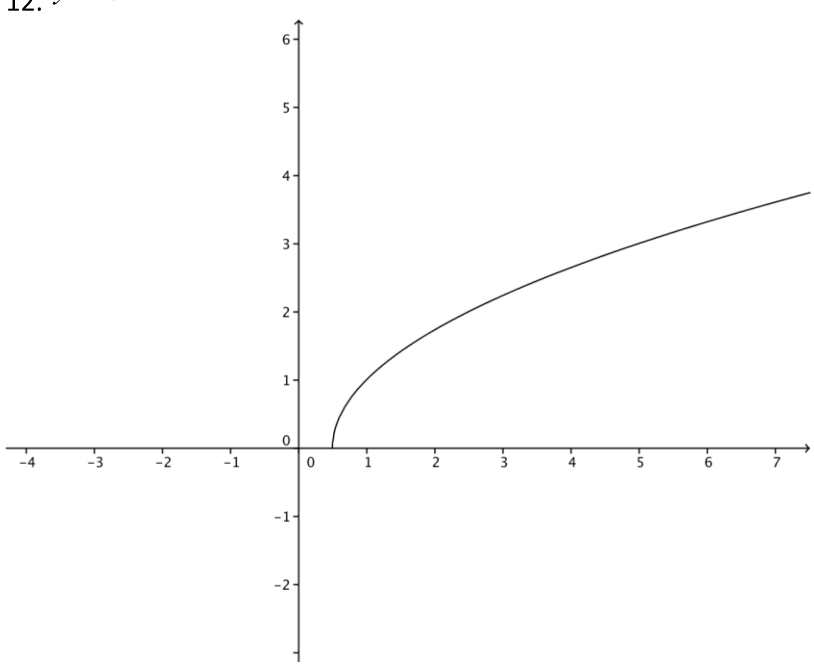
$y = \sqrt{x}$

$y = \sqrt{x} + 8$

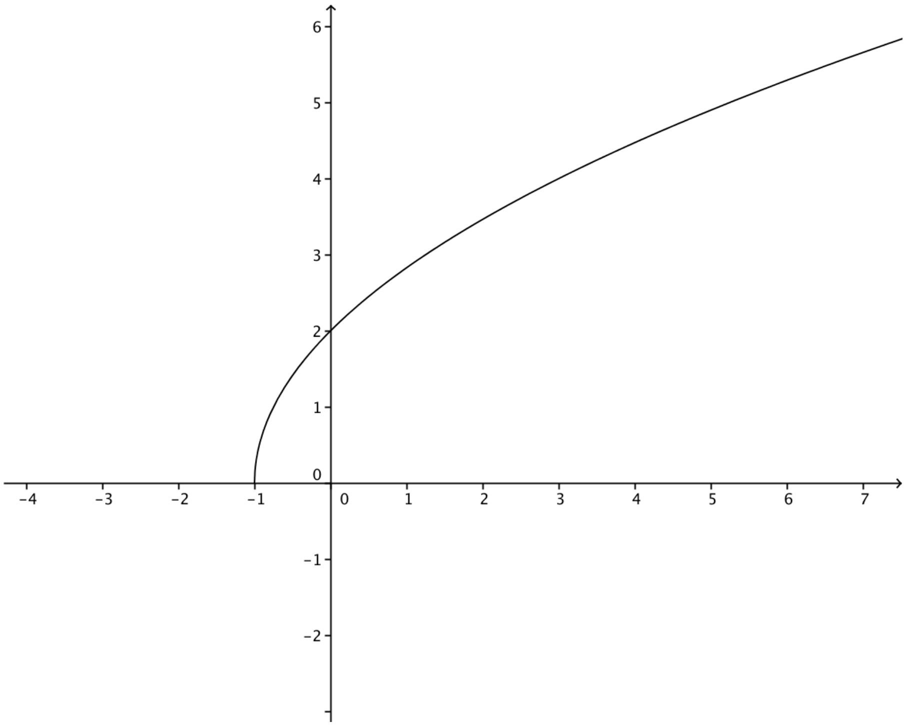
$y = \sqrt{x} - 8$



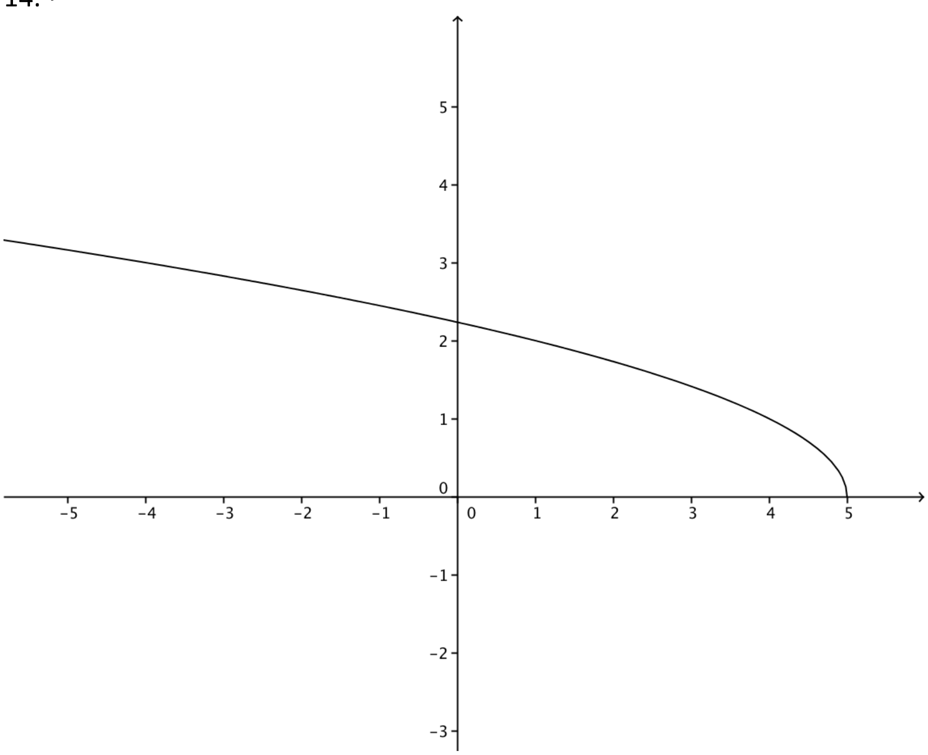
12. $y = \sqrt{2x - 1}$



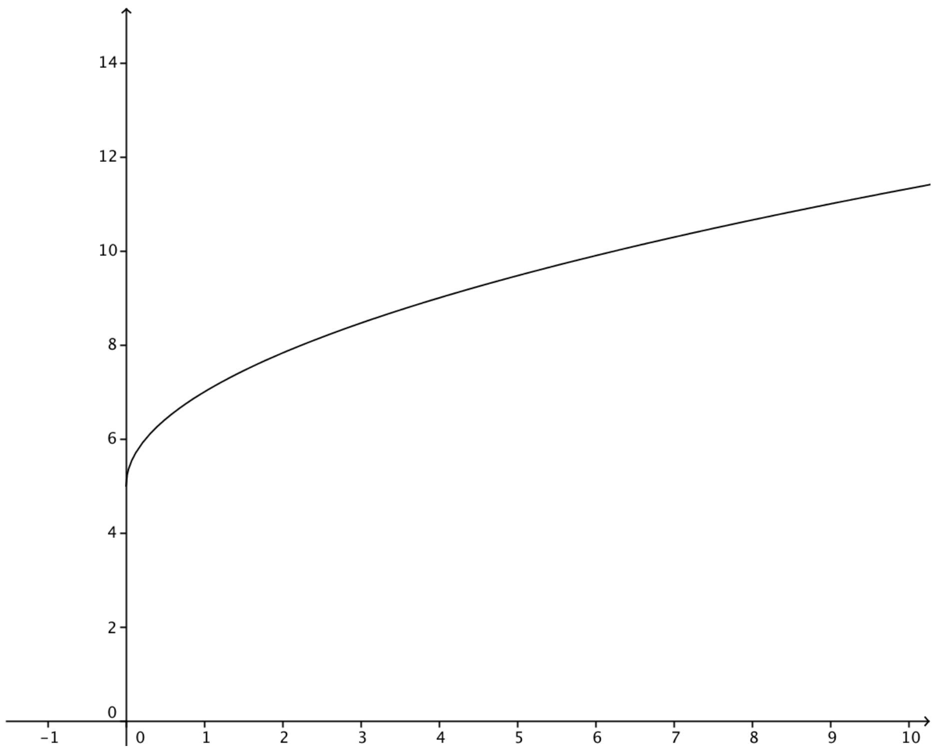
13. $y=\sqrt{4x+4}$



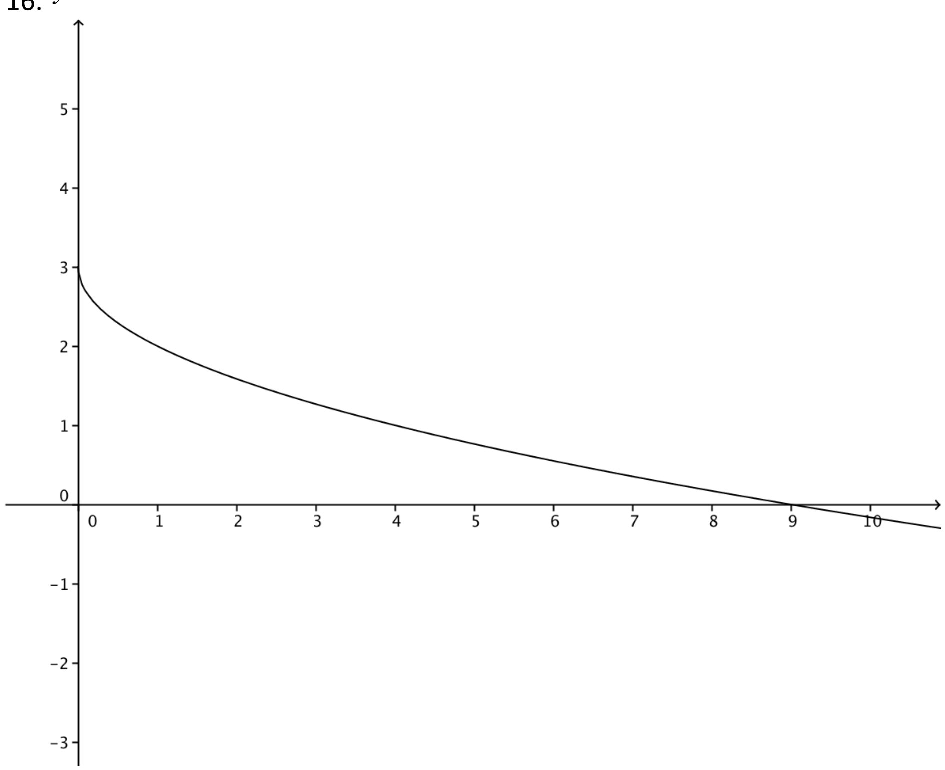
14. $y=\sqrt{5-x}$



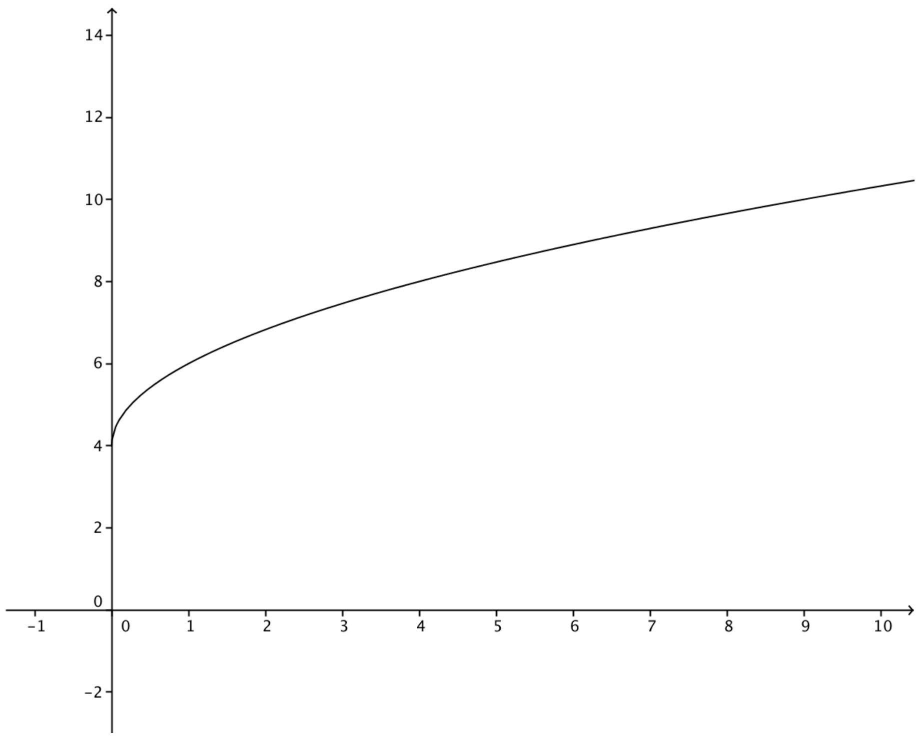
15. $y = 2\sqrt{x} + 5$



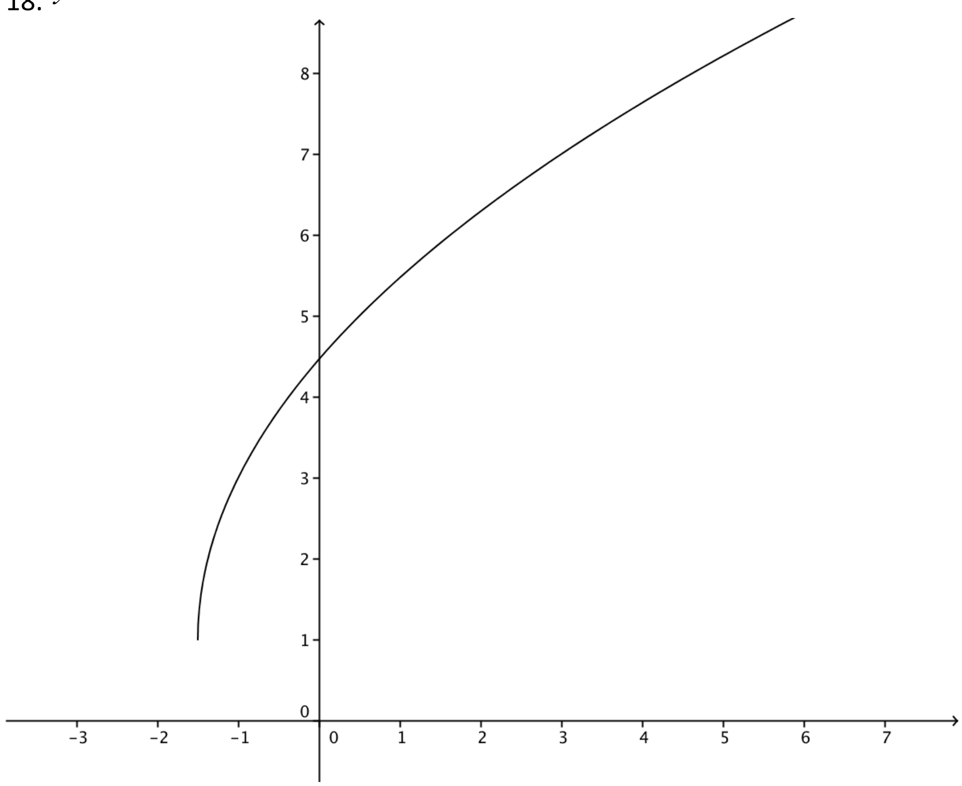
16. $y = 3 - \sqrt{x}$



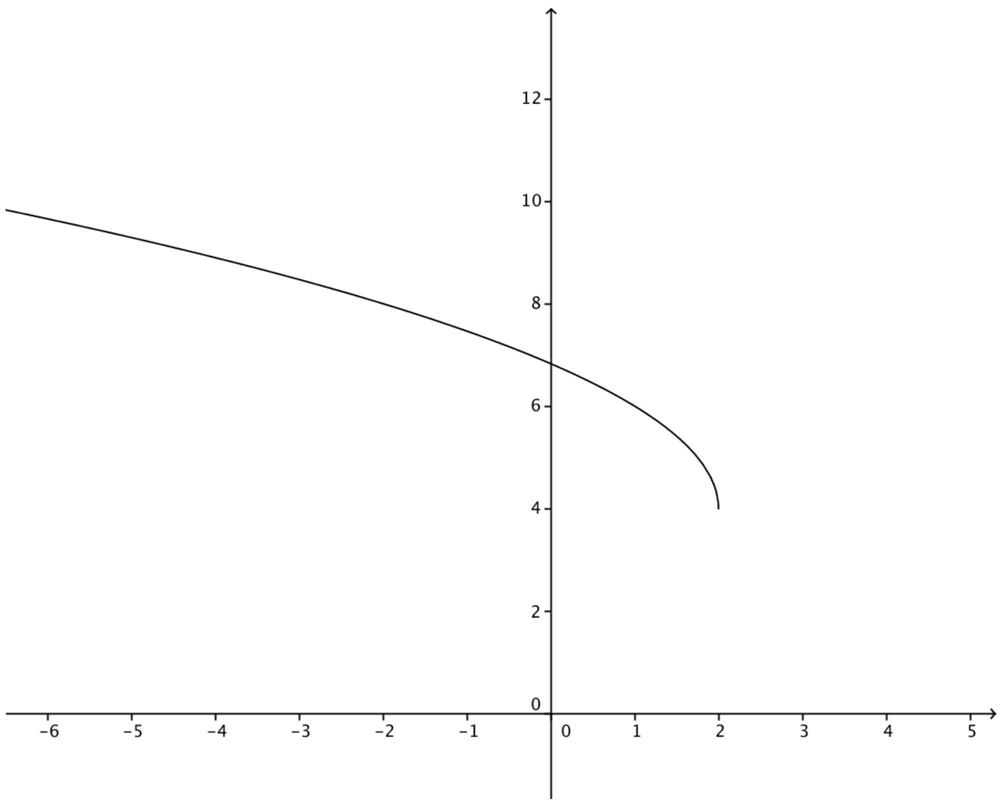
17. $y = 4 + 2\sqrt{x}$



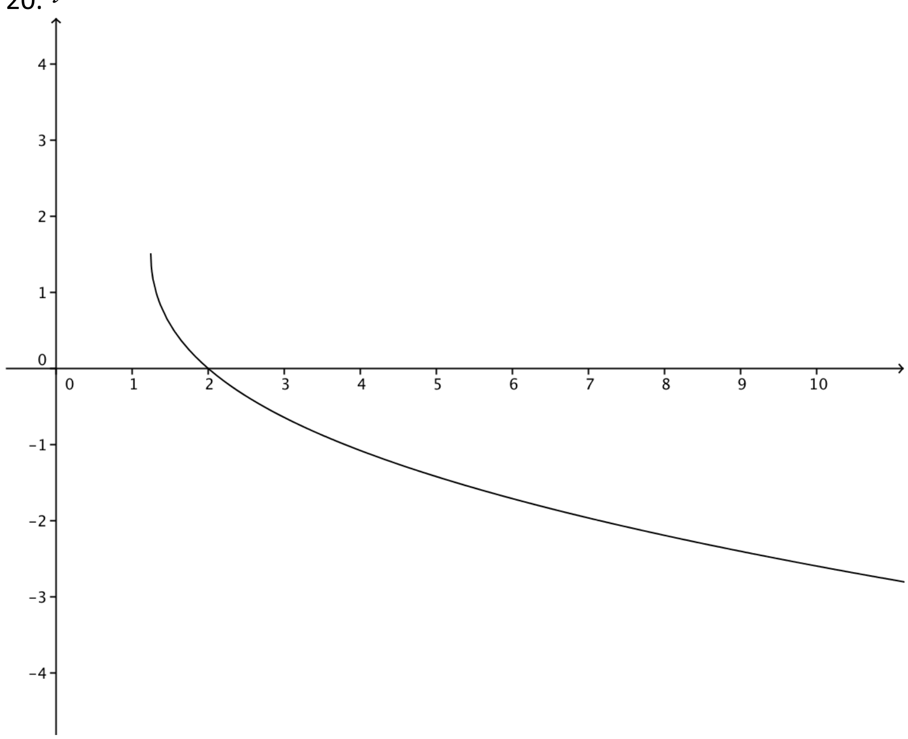
18. $y = 2\sqrt{2x+3} + 1$



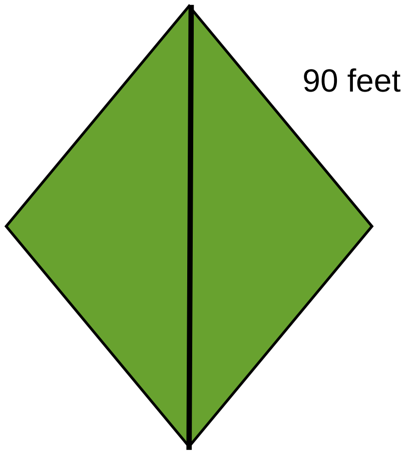
19. $y=4+2\sqrt{2-x}$

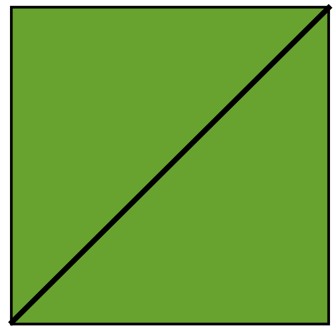


20. $y=\sqrt{x+1}-\sqrt{4x-5}$



21.





The diagonal walked by the catcher is the hypotenuse of a right triangle, so we can use pythagorean theorem.

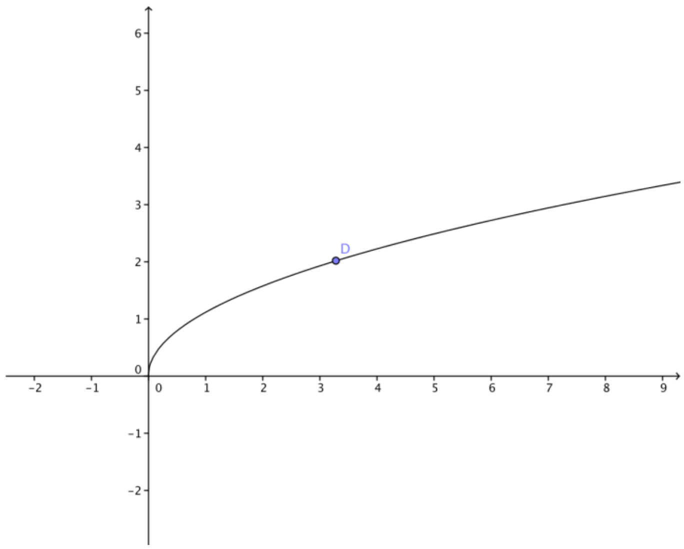
$$a^2 + b^2 = c^2$$
$$90^2 + 90^2 = c^2$$
$$8100 + 8100 = c^2$$
$$16200 = c^2$$
$$\sqrt{16200} = \sqrt{c^2}$$
$$c = \sqrt{16200}$$
$$c \approx 127.2792$$

The distance run by the runner is 90 +90 = 180 feet.
The distance covered by the catcher is approximately 127.2792 feet.
The catcher goes (180-127.2792=52.7208) approximately 53 feet less.

22.

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$T = 2\pi\sqrt{\frac{L}{32}}$$

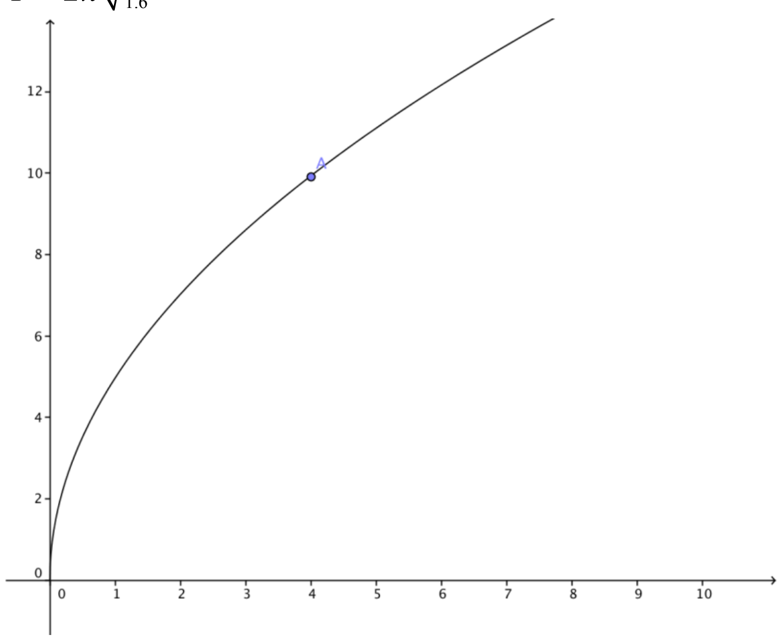


When T=2 then the length of the pendulum is 3.24ft

23.

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$T = 2\pi\sqrt{\frac{L}{1.6}}$$

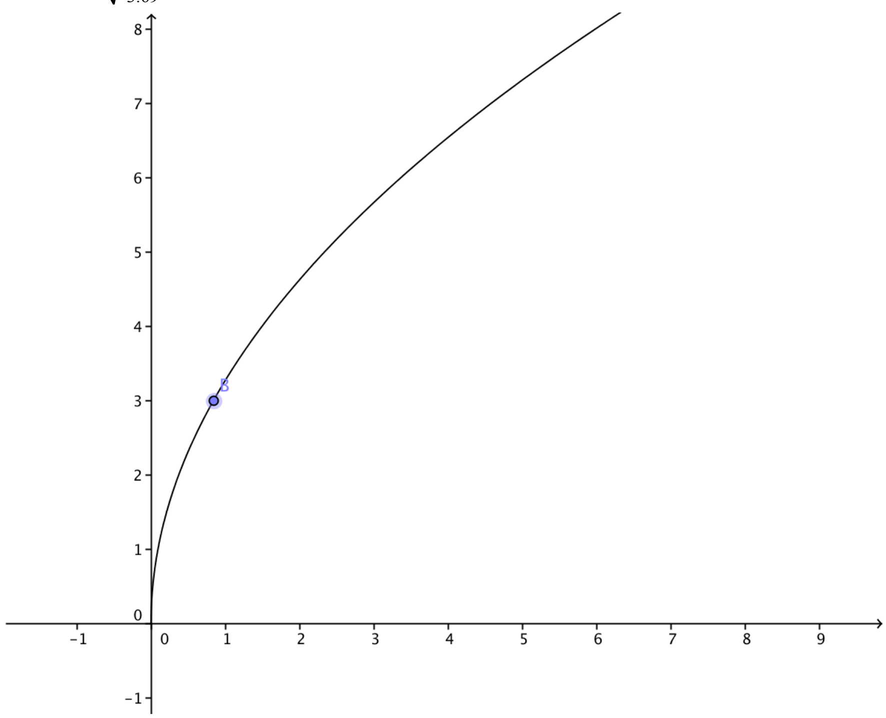


When T=10 then the length of the pendulum is 4.052 m

24.

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$T = 2\pi\sqrt{\frac{L}{3.69}}$$



When T=3 then the length of the pendulum is 0.8412 m

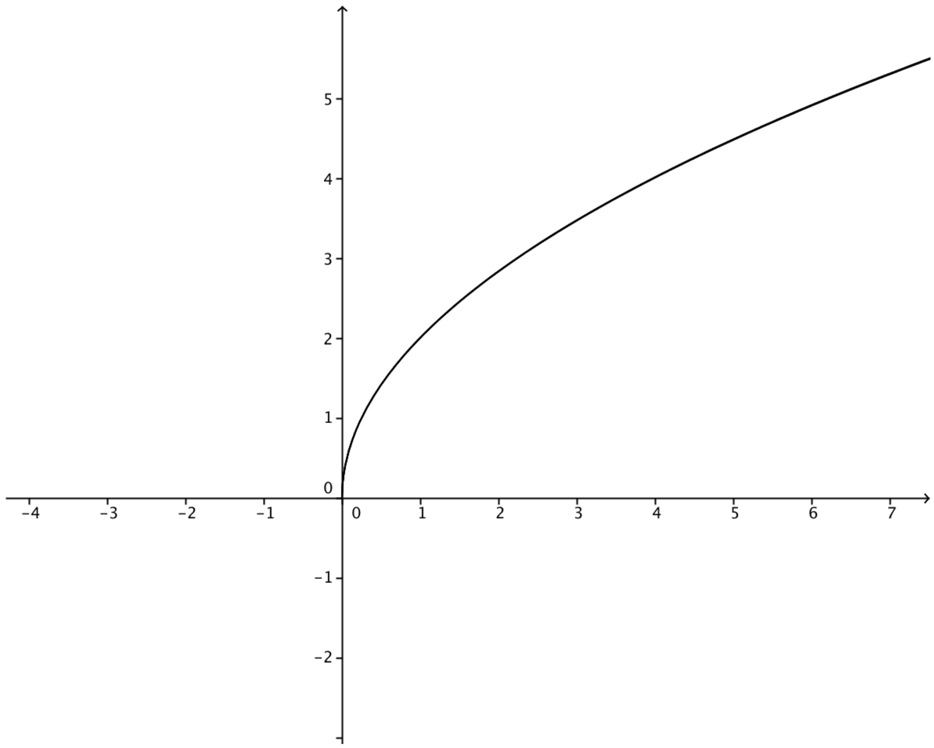
25.

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$T = 2\pi\sqrt{\frac{L}{9.819}}(Hel\sin ki)$$

$$T = 2\pi\sqrt{\frac{L}{9.796}}(LosAngeles)$$

$$T = 2\pi\sqrt{\frac{L}{9.779}}(MexicoCity)$$



(The functions are so close they graph on top of on another.)

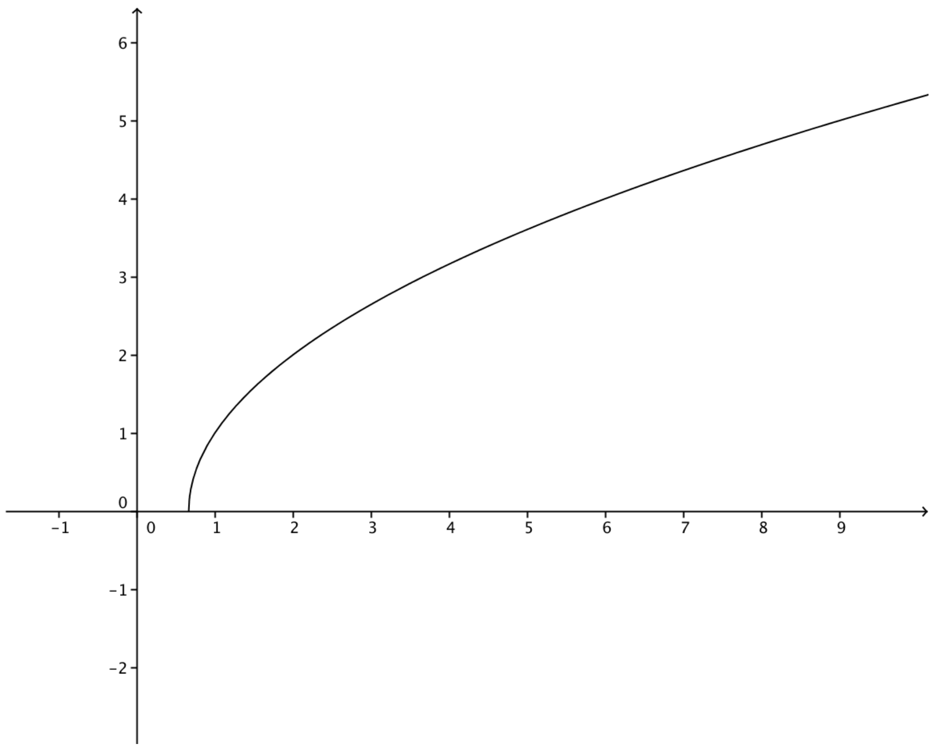
$$T = 2\pi\sqrt{\frac{L}{9.819}}(Hel\sin ki)$$
$$8 = 2(3.14)\sqrt{\frac{L}{9.819}}$$
$$8 = 6.28\sqrt{\frac{L}{9.819}}$$
$$1.2739 = \sqrt{\frac{L}{9.819}}$$
$$1.2739 = \sqrt{L} \div \sqrt{9.819}$$
$$1.2739 = \sqrt{L} \div 3.1335$$
$$3.9918 = \sqrt{L}$$
$$3.9918^2 = \left(\sqrt{L}\right)^2$$
$$L = 15.9345$$

$$T = 2\pi\sqrt{\frac{L}{9.796}}(LosAngeles)$$
$$8 = 2(3.14)\sqrt{\frac{L}{9.796}}$$
$$8 = 6.28\sqrt{\frac{L}{9.796}}$$
$$1.2739 = \sqrt{\frac{L}{9.796}}$$
$$1.2739 = \sqrt{L} \div \sqrt{9.796}$$
$$1.2739 = \sqrt{L} \div 3.1299$$
$$3.9872 = \sqrt{L}$$
$$3.9872^2 = \left(\sqrt{L}\right)^2$$
$$L = 15.8978$$

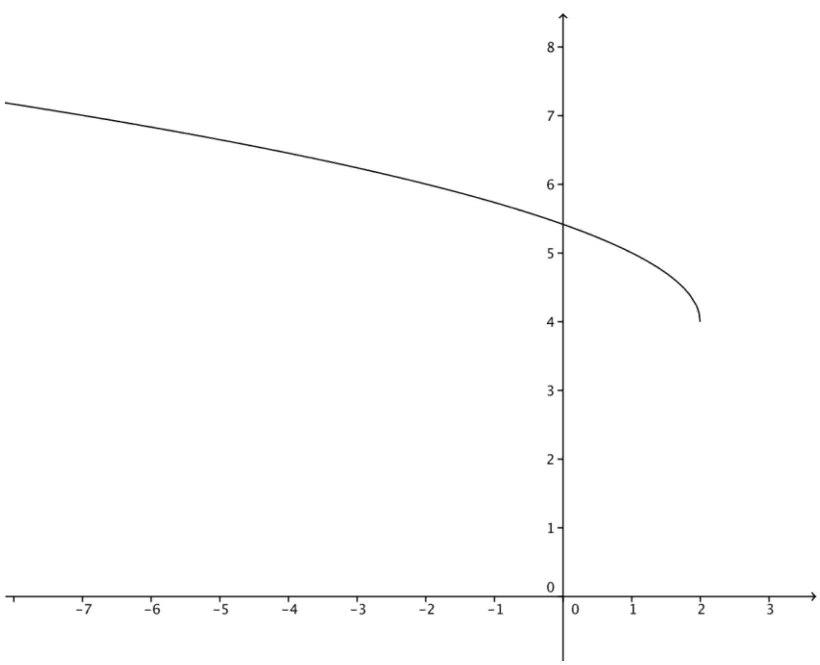
$$T = 2\pi\sqrt{\frac{L}{9.779}}(MexicoCity)$$
$$8 = 2(3.14)\sqrt{\frac{L}{9.779}}$$
$$8 = 6.28\sqrt{\frac{L}{9.779}}$$
$$1.2739 = \sqrt{\frac{L}{9.779}}$$
$$1.2739 = \sqrt{L} \div \sqrt{9.779}$$
$$1.2739 = \sqrt{L} \div 3.1271$$
$$3.9836 = \sqrt{L}$$
$$3.9836^2 = \left(\sqrt{L}\right)^2$$
$$L = 15.8691$$

When the period is 8 seconds the length of the pendulum in Helsinki is 15.9345 meters.
When the period is 8 seconds the length of the pendulum in Los Angeles is 15.8978 meters.
When the period is 8 seconds the length of the pendulum in Mexico City is 15.8691 meters

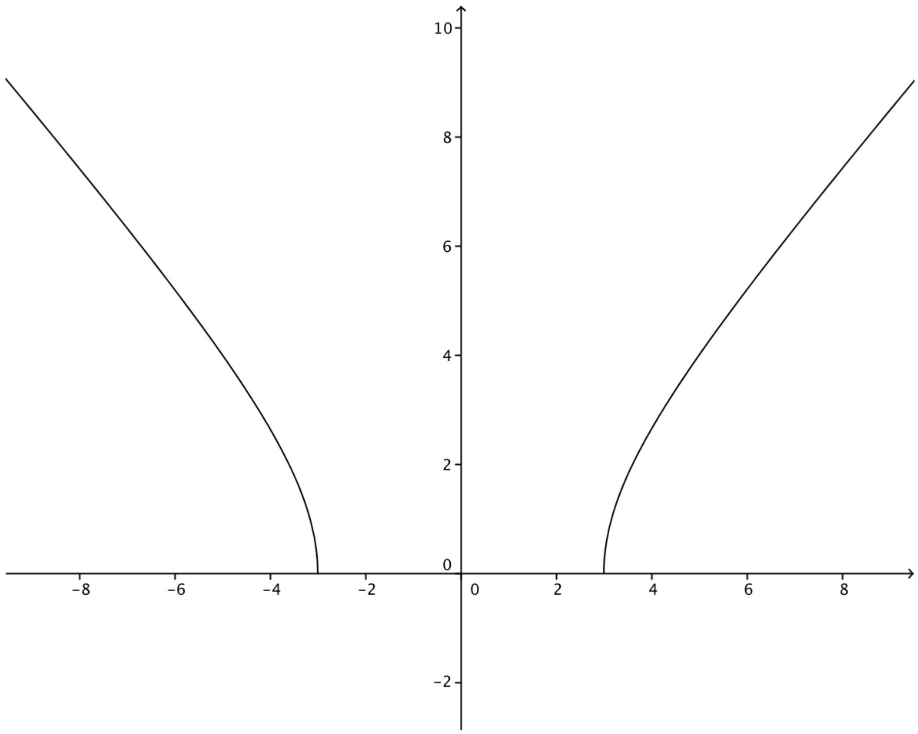
26. $y=\sqrt{3x-2}$



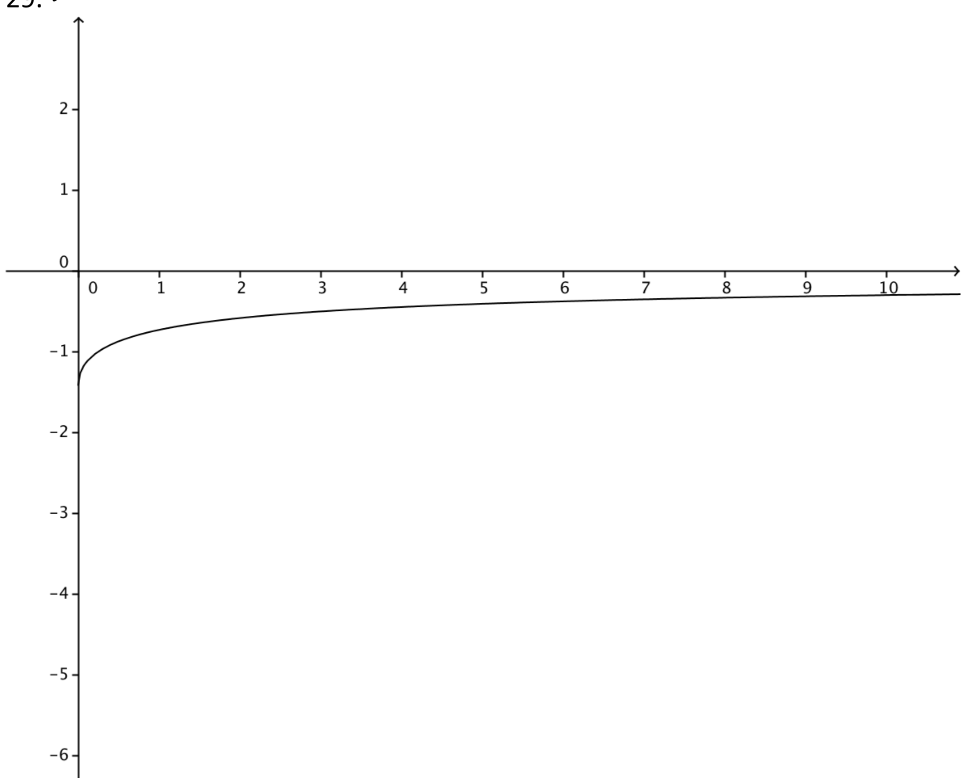
27. $y=4+\sqrt{2-x}$



28. $y=\sqrt{x^2-9}$



29. $y=\sqrt{x}-\sqrt{x+2}$



Mixed Review

30.

$$16 = 2x^2 - 3x + 4$$

$$0 = 2x^2 - 3x - 12$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-12)}}{2(2)}$$

$$y = \frac{3 \pm \sqrt{9 + 96}}{4}$$

$$y = \frac{3 \pm \sqrt{105}}{4}$$

$$y = \frac{3 + \sqrt{105}}{4} \approx \frac{3 + 10.247}{4} \approx \frac{13.247}{4} \approx 3.3118$$

$$y = \frac{3 - \sqrt{105}}{4} \approx \frac{3 - 10.247}{4} \approx \frac{-7.247}{4} \approx -1.8118$$

$$y \approx 3.3118, -1.8118$$

31.

$$y - y_1 = m(x - x_1)$$

$$y - 10 = 0.2(x - 1)$$

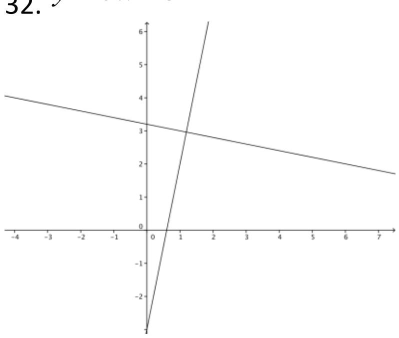
$$y - 10 = 0.2x - 0.2$$

$$y = 0.2x + 9.8$$

32.

$$x + 5y = 16$$

$$y = 5x - 3$$



The lines are perpendicular.

33.

$$20x + 32y$$

$$(50, 0)$$

$$20(50) + 32(0) = 1000$$

$$(0, 60)$$

$$20(0) + 32(60) = 1920$$

$$(15, 30)$$

$$20(15) + 32(30) = 1260$$

The vertex (50,0) minimizes the expression 20x + 32y

34. No the graph is not a function. Many of the x-values have more than one corresponding y-value.

35. The square root of 205 is between 14 and 15.

Lesson 11.2
Radical Expressions

1. $\sqrt[n]{-16}$ is undefined when n is even.

2. $\sqrt{169} = 13$

3. $\sqrt[4]{81} = 3$

4. $\sqrt[3]{-125} = -5$

5. $\sqrt[5]{1024} = 4$

6. $\sqrt[3]{14} = 14^{\frac{1}{3}}$

7. $\sqrt[4]{zw} = (zw)^{\frac{1}{4}}$

8. $\sqrt{a} = a^{\frac{1}{2}}$

9. $\sqrt[9]{y^3} = y^{\frac{3}{9}} = y^{\frac{1}{3}}$

10. $\sqrt{24} = \sqrt{4} * \sqrt{6} = 2\sqrt{6}$

11. $\sqrt{300} = \sqrt{3} * \sqrt{100} = 10\sqrt{3}$

12. $\sqrt[5]{96} = \sqrt[5]{32} * \sqrt[5]{3} = 2\sqrt[5]{3}$

13.
$\sqrt{\frac{240}{567}} =$
$\sqrt{240} \div \sqrt{567} =$
$\left(\sqrt{16} * \sqrt{15}\right) \div \left(\sqrt{4} * \sqrt{141.75}\right) =$
$4\sqrt{15} \div 2\sqrt{141.75} =$
$\frac{2\sqrt{15}}{\sqrt{141.75}}$

14. $\sqrt[3]{500} = \sqrt[3]{125} * \sqrt[3]{4} = 5\sqrt[3]{4}$

15. $\sqrt[6]{64x^8} = \left(\sqrt[6]{64} * \sqrt[6]{x^6} * \sqrt[6]{x^2}\right) = 2x\sqrt[6]{x^2} = 2x\sqrt[3]{x}$

16. $\sqrt[3]{48a^3b^7} = \sqrt[3]{8} * \sqrt[3]{6} * a^{\frac{3}{3}} * b^{\frac{7}{3}} = 2\sqrt[3]{6} * a * b^2 * b^{\frac{1}{3}} = 2ab^2\sqrt[3]{6b}$

17.

$$\sqrt[3]{\frac{16x^5}{135y^4}} =$$

$$\sqrt[3]{16x^5} \div \sqrt[3]{135y^4} =$$

$$\left(\sqrt[3]{8} * \sqrt[3]{2} * x^{\frac{5}{3}}\right) \div \left(\sqrt[3]{27} * \sqrt[3]{5} * y^{\frac{4}{3}}\right) =$$

$$\left(2 * \sqrt[3]{2} * x * x^{\frac{2}{3}}\right) \div \left(3 * \sqrt[3]{5} * y * y^{\frac{1}{3}}\right) =$$

$$2x\sqrt[3]{2x^2} \div 3y\sqrt[3]{5y} = \frac{2x\sqrt[3]{2x^2}}{3y\sqrt[3]{5y}}$$

18. FALSE

$$\left(\sqrt[7]{5} \cdot \sqrt[6]{6} = \sqrt[42]{30}\right)$$

$$5^{\frac{1}{7}} \cdot 6^{\frac{1}{6}} = 5^{\frac{6}{42}} \cdot 6^{\frac{7}{42}} = 30^{\frac{13}{42}} = \sqrt[42]{30^{13}}$$

19.

$$3\sqrt{8} - 6\sqrt{32} =$$

$$\left(3 * \sqrt{4} * \sqrt{2}\right) - \left(6 * \sqrt{16} * \sqrt{2}\right) =$$

$$\left(3 * 2 * \sqrt{2}\right) - \left(6 * 4 * \sqrt{2}\right) =$$

$$6\sqrt{2} - 24\sqrt{2} =$$

$$-18\sqrt{2}$$

20.

$$\sqrt{180} + 6\sqrt{405} =$$

$$\left(\sqrt{36} * \sqrt{5}\right) + \left(6 * \sqrt{81} * \sqrt{5}\right) =$$

$$\left(6 * \sqrt{5}\right) + \left(6 * 9 * \sqrt{5}\right) =$$

$$6\sqrt{5} + 54\sqrt{5} =$$

$$60\sqrt{5}$$

21.

$$\sqrt{6}-\sqrt{27}+2\sqrt{54}+3\sqrt{48}=$$

$$\left(\sqrt{6}\right)-\left(\sqrt{9}*\sqrt{3}\right)+\left(2*\sqrt{9}*\sqrt{6}\right)+\left(3*\sqrt{4}*\sqrt{4}*\sqrt{3}\right)=$$

$$\left(\sqrt{6}\right)-\left(3*\sqrt{3}\right)+\left(2*3*\sqrt{6}\right)+\left(3*2*2*\sqrt{3}\right)=$$

$$\sqrt{6}-3\sqrt{3}+6\sqrt{6}+12\sqrt{3}=$$

$$7\sqrt{6}+9\sqrt{3}$$

22.

$$\sqrt{8x^3}-4x\sqrt{98x}=$$

$$\left(\sqrt{4}*\sqrt{2}*\sqrt{x^2}*\sqrt{x}\right)-\left(4x*\sqrt{49}*\sqrt{2}*\sqrt{x}\right)=$$

$$\left(2*\sqrt{2}*x*\sqrt{x}\right)-\left(4x*7*\sqrt{2}*\sqrt{x}\right)=$$

$$2x\sqrt{2x}-28x\sqrt{2x}=$$

$$-26x\sqrt{2x}$$

23.

$$\sqrt{48a}+\sqrt{27a}=$$

$$\left(\sqrt{16}*\sqrt{3}*\sqrt{a}\right)+\left(\sqrt{9}*\sqrt{3}*\sqrt{a}\right)=$$

$$4\sqrt{3a}+3\sqrt{3a}=$$

$$7\sqrt{3a}$$

24.

$$\sqrt[3]{4x^3}+x\sqrt[3]{256}=$$

$$\left(\sqrt[3]{4}*\sqrt[3]{x^3}\right)+\left(x*\sqrt[3]{64}*\sqrt[3]{4}\right)=$$

$$x\sqrt[3]{4}+4x\sqrt[3]{4}=$$

$$5x\sqrt[3]{4}$$

25. $\sqrt{6}\left(\sqrt{10}+\sqrt{8}\right)=\sqrt{60}+\sqrt{48}=\left(\sqrt{4}*\sqrt{15}\right)+\left(\sqrt{16}*\sqrt{3}\right)=2\sqrt{15}+4\sqrt{3}$

26. $\left(\sqrt{a}-\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)=\left(\sqrt{a^2}+\sqrt{ab}-\sqrt{ab}-\sqrt{b^2}\right)=a-b$

27. $\left(2\sqrt{x}+5\right)\left(2\sqrt{x}+5\right)=\left(4\sqrt{x^2}+10\sqrt{x}+10\sqrt{x}+25\right)=4x+20\sqrt{x}+25$

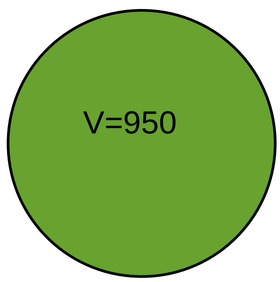
28. $\frac{7}{\sqrt{15}}=\frac{7*\sqrt{15}}{\sqrt{15}*\sqrt{15}}=\frac{7\sqrt{15}}{15}$

29. $\frac{9}{\sqrt{10}}=\frac{9*\sqrt{10}}{\sqrt{10}*\sqrt{10}}=\frac{9\sqrt{10}}{10}$

30. $\frac{2x}{\sqrt{5x}}=\frac{2x*\sqrt{5x}}{\sqrt{5x}*\sqrt{5x}}=\frac{2x\sqrt{5x}}{5x}$

31. $\frac{\sqrt{5}}{\sqrt{3y}}=\frac{\sqrt{5}*\sqrt{3y}}{\sqrt{3y}*\sqrt{3y}}=\frac{\sqrt{15y}}{3y}$

32.



$V=\frac{4}{3}\pi R^3$

$950=\frac{4}{3}\pi R^3$

$950=\left(\frac{4}{3}\right)(3.14)R^3$

$950=4.1867R^3$

$227.8888=R^3$

$\sqrt[3]{227.8888}=\sqrt[3]{R^3}$

$R\approx 6.1081$

The radius of the balloon is approximately 6.1081.

33.

$$A=(9+2W)(12+2W)$$

$$180=(9+2W)(12+2W)$$

$$180=108+18W+24W+4W^2$$

$$72=42W+4W^2$$

$$0=4W^2+42W-72$$

$$y=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$y=\frac{-42\pm\sqrt{42^2-4(4)(-72)}}{2(4)}$$

$$y=\frac{-42\pm\sqrt{1764+1152}}{8}$$

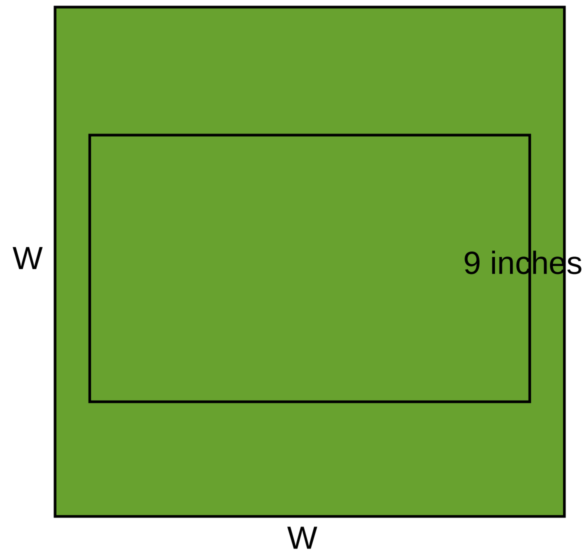
$$y=\frac{-42\pm\sqrt{2916}}{8}$$

$$y=\frac{-42+\sqrt{2916}}{8}=\frac{-42+54}{8}=\frac{12}{8}=1.5$$

$$y=\frac{-42-\sqrt{2916}}{8}=\frac{-42-54}{8}=\frac{-96}{8}=-12$$

$$y=1.5$$

The width of the frame is 1.5 inches.



34.

$$V=\pi r^2 h$$

$$h=4r^2$$

$$V=355$$

$$355=\pi r^2\left(4r^2\right)$$

$$355=3.14r^2\left(4r^2\right)$$

$$355=12.56r^4$$

$$28.2643=r^4$$

$$\sqrt[4]{28.2643}=\sqrt[4]{r^4}$$

$$r=2.3057$$

The radius of the base of the cylinder is approximately 2.3057 cm.

Mixed Review

35.

$$c - 15\% = 612.99$$
$$c - 0.15c = 612.99$$
$$.85c = 812.99$$
$$c = 956.4588$$

The original price of the item was $956.46

36.

$$\frac{x+3}{6} = \frac{21}{x}$$
$$x(x+3) = 21 * 6$$
$$x^2 + 3x = 126$$
$$x^2 + 3x + 2.25 = 126 + 2.25$$
$$(x+1.5)^2 = 128.25$$
$$\sqrt{(x+1.5)^2} = \sqrt{128.25}$$
$$x + 1.5 = \sqrt{128.25}$$
$$x = \sqrt{128.25} - 1.5$$

$$x \approx \pm 11.3248 - 1.5$$
$$x \approx 11.3248 - 1.5$$
$$x \approx 9.8248$$
$$x \approx -11.3248 - 1.5$$
$$x \approx -12.8248$$
$$x \approx 9.8248, -12.8248$$

37.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{7.25 - 3.35}{2009 - 1989}$$
$$m = \frac{3.9}{20} = 0.195$$

The average rate of change is approximately $0.20 per year.

38.

$$y = a(x-h)^2 + k$$

$$y = 2(x+1)^2 + 4$$

$$a = 2$$

$$h = -1$$

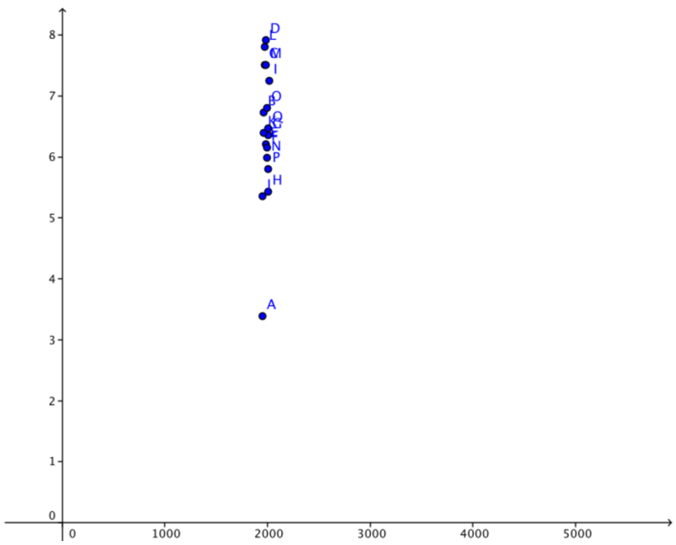
$$k = 4$$

Vertex = (-1,4)

The vertex is a minimum.

39.

1.



2. The best model for this data is linear.

$$y = mx + b$$

$$m = 0.0105$$

$$b = 5.5606$$

$$y = 0.0105x + 5.5606$$

$$x = 1999 = 99$$

$$y = 0.0105(99) + 5.5606$$

$$y = 1.0395 + 5.5606$$

3. $y = 6.6001$

Using the model of best fit the minimum wage (adj. for inflation) in 1999 would be $6.60

4. My prediction was only $0.2 off the EPI data.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{7.52 - 6.40}{1965 - 1960} = \frac{1.12}{5} = 0.224$$

$$y - y_1 = m(x - x_1)$$

$$y - 6.40 = 0.224(x - 1960)$$

$$y - 6.40 = 0.224x - 439.04$$

$$y = 0.224x - 432.64$$

$$x = 1962$$

$$y = 0.224(1962) - 432.64$$

5. $y = 6.848$

Using interpolation, the minimum wage in 1962 would be approximately $6.85

Lesson 11.3
Radical Equations

1.

$$\sqrt{x+2}-2=0$$
$$\sqrt{x+2}=2$$
$$\left(\sqrt{x+2}\right)^2=2^2$$
$$x+2=4$$
$$x=2$$

$$\sqrt{2+2}-2=0$$
$$\sqrt{4}-2=0$$
$$2-2=0$$
$$0=0$$

2.

$$\sqrt{3x-1}=5$$
$$\left(\sqrt{3x-1}\right)^2=5^2$$
$$3x-1=25$$
$$3x=26$$
$$x=\tfrac{26}{3}$$
$$x\approx 8.6667$$

$$\sqrt{3\left(\tfrac{26}{3}\right)-1}=5$$
$$\sqrt{26-1}=5$$
$$\sqrt{25}=5$$
$$5=5$$

3.

$$2\sqrt{4-3x}+3=0$$
$$2\sqrt{4-3x}=-3$$
$$\sqrt{4-3x}=-1.5$$
$$\left(\sqrt{4-3x}\right)^2=(-1.5)^2$$
$$4-3x=2.25$$
$$-3x=-1.75$$
$$x=0.5833$$

$$2\sqrt{4-3(0.5833)}+3=0$$
$$2\sqrt{4-1.7499}+3=0$$
$$2\sqrt{2.2501}+3=0$$
$$2(1.5)+3=0$$
$$3+3=0$$
$$6\neq 0$$

This is an extraneous solution. The equation has no real roots.

4.

$$\sqrt[3]{x-3}=1$$
$$\left(\sqrt[3]{x-3}\right)^3=1^3$$
$$x-3=1$$
$$x=4$$

$$\sqrt[3]{4-3}=1$$
$$\sqrt[3]{1}=1$$
$$1=1$$

5.

$$\sqrt[4]{x^2-9}=2$$

$$\left(\sqrt[4]{x^2-9}\right)^4=2^4$$

$$x^2-9=16$$

$$x^2=25$$

$$x=5$$

$$\sqrt[4]{5^2-9}=2$$

$$\sqrt[4]{25-9}=2$$

$$\sqrt[4]{16}=2$$

$$2=2$$

6.

$$\sqrt[3]{-2-5x}+3=0$$

$$\sqrt[3]{-2-5x}=-3$$

$$\left(\sqrt[3]{-2-5x}\right)^3=(-3)^3$$

$$-2-5x=-27$$

$$-5x=-25$$

$$x=5$$

$$\sqrt[3]{-2-5(5)}+3=0$$

$$\sqrt[3]{-2-25}+3=0$$

$$\sqrt[3]{-27}+3=0$$

$$-3+3=0$$

$$0=0$$

7.

$$\sqrt{x} = x - 6$$

$$\left(\sqrt{x}\right)^2 = (x-6)^2$$

$$x = x^2 - 12x + 36$$

$$0 = x^2 - 13x + 36$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(1)(36)}}{2(1)}$$

$$y = \frac{13 \pm \sqrt{169 - 144}}{2}$$

$$y = \frac{13 \pm \sqrt{25}}{2}$$

$$y = \frac{13 + \sqrt{25}}{2} = \frac{13+5}{2} = \frac{18}{2} = 9$$

$$y = \frac{13 - \sqrt{25}}{2} = \frac{13-5}{2} = \frac{8}{2} = 4$$

$$\sqrt{9} = 9 - 6$$

$$3 = 3$$

$$\sqrt{4} = 9 - 4$$

$$2 \neq 5$$

There is one real solution (x=9) and one extraneous solution (x=4).

8.

$\sqrt{x^2-5x}-6=0$

$\sqrt{x^2-5x}=6$

$\left(\sqrt{x^2-5x}\right)^2=6^2$

$x^2-5x=36$

$x^2-5x+6.25=36+6.25$

$(x-2.5)^2=42.25$

$\sqrt{(x-2.5)^2}=\sqrt{42.25}$

$x-2.5=\pm6.5$

$x=\pm6.5+2.5$

$x=6.5+2.5$

$x=9$

$x=-6.5+2.5$

$x=-4$

$\sqrt{9^2-5(9)}-6=0$

$\sqrt{81-45}-6=0$

$\sqrt{36}-6=0$

$6-6=0$

$\sqrt{(-4)^2-5(-4)}-6=0$

$\sqrt{16+20}-6=0$

$\sqrt{36}-6=0$

$6-6=0$

$0=0$

There are two real solutions, x=9 and x=-4

9.

$$\sqrt{(x+1)(x-3)}=x$$
$$\left(\sqrt{(x+1)(x-3)}\right)^2=x^2$$
$$(x+1)(x-3)=x^2$$
$$x^2-2x-3=x^2$$
$$-2x-3=0$$
$$-2x=3$$
$$x=-1.5$$

$$\sqrt{(-1.5+1)(-1.5-3)}=-1.5$$
$$\sqrt{(-0.5)(-4.5)}=-1.5$$
$$\sqrt{2.25}=-1.5$$
$$-1.5=-1.5$$

10.

$$\sqrt{x+6}=x+4$$
$$\left(\sqrt{x+6}\right)^2=(x+4)^2$$
$$x+6=x^2+8x+16$$
$$6=x^2+7x+16$$
$$0=x^2+7x+10$$
$$(x+5)(x+2)=0$$
$$x+5=0$$
$$x=-5$$
$$x+2=0$$
$$x=-2$$

$$\sqrt{-5+6}=-5+4$$
$$\sqrt{1}=-1$$
$$-1=-1$$
$$\sqrt{-2+6}=-2+4$$
$$\sqrt{4}=2$$
$$2=2$$

There are two real solutions, x=-5 and x=-2

11.

$\sqrt{x} = \sqrt{x-9} + 1$

$\sqrt{x} - 1 = \sqrt{x-9}$

$\left(\sqrt{x} - 1\right)^2 = \left(\sqrt{x-9}\right)^2$

$\left(\sqrt{x}\right)^2 - 1\sqrt{x} - 1\sqrt{x} + 1 = x - 9$

$x - 2\sqrt{x} + 1 = x - 9$

$-2\sqrt{x} = -10$

$\sqrt{x} = 5$

$x = 25$

$\sqrt{25} = \sqrt{25-9} + 1$

$5 = \sqrt{16} + 1$

$5 = 4 + 1$

$5 = 5$

12.

$\sqrt{3x+4} = -6$

$\left(\sqrt{3x+4}\right)^2 = (-6)^2$

$3x + 4 = 36$

$3x = 32$

$x = \frac{32}{3}$

$x \approx 10.6667$

$\sqrt{3\left(\frac{32}{3}\right) + 4} = -6$

$\sqrt{32+4} = -6$

$\sqrt{36} = -6$

$-6 = -6$

13.

$$\sqrt{10-5x}+\sqrt{1-x}=7$$

$$\sqrt{10-5x}=-\sqrt{1-x}+7$$

$$\left(\sqrt{10-5x}\right)^2=\left(-\sqrt{1-x}+7\right)^2$$

$$10-5x=\left(-\sqrt{1-x}\right)^2-7\sqrt{1-x}-7\sqrt{1-x}+49$$

$$10-5x=1-x-14\sqrt{1-x}+49$$

$$10-5x=50-x-14\sqrt{1-x}$$

$$-4x=40-14\sqrt{1-x}$$

$$-4x-40=-14\sqrt{1-x}$$

$$\frac{-4x-40}{-14}=\sqrt{1-x}$$

$$\left(\frac{-4x-40}{-14}\right)^2=\left(\sqrt{1-x}\right)^2$$

$$\frac{16x^2+1600}{196}=1-x$$

$$16x^2+1600=196(1-x)$$

$$16x^2+1600=-196x+196$$

$$16x^2-196x+1404=0$$

$$16x^2-196x=-1404$$

$$16(x^2-12.25x)=-1404$$

$$16(x^2-12.25x+37.5156)=-1404+37.5156(16)$$

$$16(x-6.125)^2=-803.7504$$

$$(x-6.125)^2=-50.2344$$

$$\sqrt{(x-6.125)^2}=\sqrt{-50.2344}$$

$$x-6.125=\sqrt{-50.2344}$$

$$x=\sqrt{-50.2344}+6.125$$

The equation has no real roots.

14.

$$\sqrt{2x-2}-2\sqrt{x}+2=0$$

$$\sqrt{2x-2}=2\sqrt{x}-2$$

$$\left(\sqrt{2x-2}\right)^2=\left(2\sqrt{x}-2\right)^2$$

$$2x-2=4x-4\sqrt{x}-4\sqrt{x}+4$$

$$2x-2=4x-8\sqrt{x}+4$$

$$-6=2x-8\sqrt{x}$$

$$-2x-6=-8\sqrt{x}$$

$$\frac{-2x-6}{-8}=\sqrt{x}$$

$$\left(\frac{-2x-6}{-8}\right)^2=\left(\sqrt{x}\right)^2$$

$$\frac{4x^2+36}{64}=x$$

$$4x^2+36=36x$$

$$4x^2-36x+36=0$$

$$y=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$y=\frac{-(-36)\pm\sqrt{(-36)^2-4(4)(36)}}{2(4)}$$

$$y=\frac{36\pm\sqrt{1296-576}}{8}$$

$$y=\frac{36\pm\sqrt{720}}{8}$$

$$y=\frac{36+\sqrt{720}}{8}\approx\frac{36+26.8328}{8}\approx\frac{62.8328}{8}\approx 7.8541$$

$$y=\frac{36-\sqrt{720}}{8}\approx\frac{36-26.8328}{8}\approx\frac{9.1672}{8}\approx 1.1459$$

$$\sqrt{2\left(7.8541\right)-2}-2\sqrt{7.8541}+2\approx 0$$

$$3.7025-5.605+2\approx 0$$

$$0=0$$

$$\sqrt{2\left(1.1459\right)-2}-2\sqrt{1.1459}+2\approx 0$$

$$0.5402-2.1409+2\approx 0$$

$$.3992\neq 0$$

This equation has one real root (x is approximately 7.8541) and one extraneous solution.

15.

$\sqrt{2x+5}-3\sqrt{2x-3}=\sqrt{2-x}$

$\left(\sqrt{2x+5}-3\sqrt{2x-3}\right)^2=\left(\sqrt{2-x}\right)^2$

$(2x+5)-3^2(2x-3)=2-x$

$2x+5+9(2x-3)=2-x$

$2x+5+18x-27=2-x$

$20x-22=2-x$

$21x=24$

$x=\frac{24}{21}$

$x\approx 1.1429$

$\sqrt{2(1.1429)+5}-3\sqrt{2(1.1429)-3}=\sqrt{2-(1.1429)}$

$\sqrt{2.2858+5}-3\sqrt{2.2858-3}=\sqrt{0.8571}$

$\sqrt{7.2858}-3\sqrt{-0.7142}=\sqrt{0.8571}$

This is an extraneous solution.

16.

$$3\sqrt{x}-9=\sqrt{2x-14}$$

$$\left(3\sqrt{x}-9\right)^2=\left(\sqrt{2x-14}\right)^2$$

$$9x-27\sqrt{x}-27\sqrt{x}+81=2x-14$$

$$7x-54\sqrt{x}+95=0$$

$$7x+95=54\sqrt{x}$$

$$\frac{7x+95}{54}=\sqrt{x}$$

$$\left(\frac{7x+95}{54}\right)^2=\left(\sqrt{x}\right)^2$$

$$\frac{49x^2+9025}{2916}=x$$

$$49x^2+9025=2916x$$

$$49x^2-2916x+9025=0$$

$$y=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$y=\frac{-(-2916)\pm\sqrt{(-2916)^2-4(49)(9025)}}{2(9025)}$$

$$y=\frac{2916\pm\sqrt{8503056-1768900}}{18050}$$

$$y=\frac{2916\pm\sqrt{6734156}}{18050}$$

$$y=\frac{2916+\sqrt{6734156}}{18050}\approx\frac{2916+2595.0252}{18050}\approx\frac{5511.0252}{18050}\approx 0.3053$$

$$y=\frac{2916-\sqrt{6734156}}{18050}\approx\frac{2916-2595.0252}{18050}\approx\frac{320.9748}{18050}\approx 0.01778$$

$$3\sqrt{0.3053}-9=\sqrt{2\left(0.3053\right)-14}$$

$$-7.3424\neq\sqrt{-13.3894}$$

$$3\sqrt{0.01778}-9=\sqrt{2\left(0.01778\right)-14}$$

$$-8.6\neq\sqrt{-13.9644}$$

These are extraneous solutions. The equation has no real roots.

17.

$A = \frac{1}{2}bh$

$A = \frac{1}{2}b(2b)$

$24 = \frac{1}{2}b(2b)$

$24 = \frac{b}{2}(2b)$

$24 = \dfrac{2b^2}{2}$

$24 = b^2$

$b = \sqrt{24}$

$b = 4.899$

$24 = \frac{1}{2}(4.899)(2 * 4.899)$

$24 = 2.4495(9.798)$

$24 = 24$

The base of the triangle is 4.899 in and the height of the triangle is 9.798 in

18.

$V = \dfrac{A(h)}{3}$

$1600 = \dfrac{A(10)}{3}$

$4800 = A(10)$

$480 = A$

The area of the base is 480 meters squared.

19.

$V = \pi r^2 h$

$245 = \left(\frac{22}{7}\right)\left(\frac{1}{2}d\right)^2\left(\frac{1}{3}d\right)$

$245 = \left(\frac{22}{7}\right)\left(\frac{1}{4}d^2\right)\left(\frac{1}{3}d\right)$

$245 = \left(\frac{22}{7}\right)\left(\frac{1}{12}d^3\right)$

$245 = \frac{22}{84}d^3$

$\frac{20580}{22} = d^3$

$\sqrt[3]{\frac{20580}{22}} = \sqrt[3]{d^3}$

$d = \sqrt[3]{\frac{20580}{22}}$

$d \approx 9.78$

$h \approx 3.26$

$h = 3.26 + 2 = 5.26$

$r = 9.78 \div 2 = 4.89$

$V = \pi r^2 h$

$V = (3.14)(4.89^2)(5.26)$

$V = (3.14)(21.0681)(5.26)$

$V = 347.9692$

The volume of the new cylinder is approximately 347.9692 cm cubed

20.

$h = -16t^2 + 256$

$120 = -16t^2 + 256$

$-136 = -16t^2$

$8.5 = t^2$

$\sqrt{8.5} = \sqrt{t^2}$

$t = \sqrt{8.5}$

$t \approx 2.9155$

The golf ball will reach a height of 120 feet at approximately 2.9155 seconds.

21.

s= the number of synthetic skeins and w= the number of wool skeins

$$12w + 9s = 432$$

$$12w + 9(16) = 432$$

$$12w + 144 = 432$$

$$12w = 288$$

$$w = 24$$

Joy sold 24 skeins of wool yarn.

22.

$$16 \geq |x - 4|$$

$$|x - 4| \leq 16$$

$$x - 4 \leq 16$$

$$x \leq 20$$

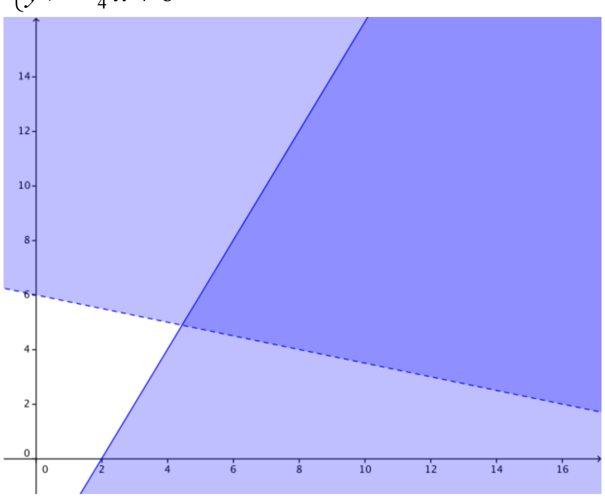
$$x - 4 \geq -16$$

$$x \geq -12$$

$$20 \geq x \geq -12$$

23.

$$\begin{cases} y \leq 2x - 4 \\ y > -\frac{1}{4}x + 6 \end{cases}$$



24.
February 2011 had 28 and 4 Mondays. The probability of landing on a Monday is 4/28 or 1/7.

25.

$$y = a(b)^x$$

$$a = initialamount$$

$$b = decayfactor = \tfrac{1}{2}$$

$$x = time \div 5730$$

$$y = a(\tfrac{1}{2})^{\frac{x}{5730}}$$

26.
Inconsistent systems have no solutions.

Lesson 11.4
The Pythagorean Theorem and its Converse

1.
$$a^2 + b^2 = c^2$$
$$12^2 + 9^2 = 15^2$$
$$144 + 81 = 225$$
$$225 = 225$$
Yes this is a right triangle.

2.
$$a^2 + b^2 = c^2$$
$$6^2 + 6^2 = \left(6\sqrt{2}\right)^2$$
$$36 + 36 = 36 * 2$$
$$72 = 72$$
Yes this is a right triangle.

3.
$$a^2 + b^2 = c^2$$
$$8^2 + \left(8\sqrt{3}\right)^2 = 16^2$$
$$64 + (64 * 3) = 256$$
$$256 = 256$$
Yes this is a right triangle.

4.
$$a^2 + b^2 = c^2$$
$$12^2 + 16^2 = c^2$$
$$144 + 256 = c^2$$
$$400 = c^2$$
$$\sqrt{400} = \sqrt{c^2}$$
$$c = 20$$

5.

$$a^2 + b^2 = c^2$$
$$a^2 + 20^2 = 30^2$$
$$a^2 + 400 = 900$$
$$a^2 = 500$$
$$\sqrt{a^2} = \sqrt{500}$$
$$a = \sqrt{100} * \sqrt{5}$$
$$a = 10\sqrt{5}$$

6.

$$a^2 + b^2 = c^2$$
$$4^2 + b^2 = 11^2$$
$$16 + b^2 = 121$$
$$b^2 = 105$$
$$\sqrt{b^2} = \sqrt{105}$$
$$b = \sqrt{100} * \sqrt{1.05}$$
$$b = 10\sqrt{1.05}$$

7.

$$a^2 + b^2 = c^2$$
$$9^2 + 7^2 = c^2$$
$$81 + 49 = c^2$$
$$130 = c^2$$
$$\sqrt{130} = \sqrt{c^2}$$
$$c = \sqrt{100} * \sqrt{1.3}$$
$$c = 10\sqrt{1.3}$$

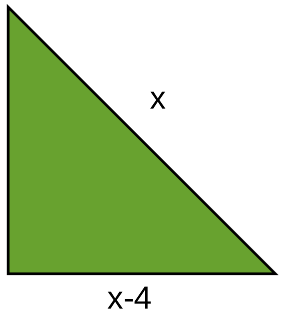
8.

$$a^2 + b^2 = c^2$$
$$a^2 + 21^2 = 35^2$$
$$a^2 + 441 = 1225$$
$$a^2 = 784$$
$$\sqrt{a^2} = \sqrt{784}$$
$$a = 28$$

9.

$$a^2 + b^2 = c^2$$
$$12^2 + b^2 = 24^2$$
$$144 + b^2 = 576$$
$$b^2 = 432$$
$$\sqrt{b^2} = \sqrt{432}$$
$$b = \sqrt{144} * \sqrt{3}$$
$$b = 12\sqrt{3}$$

10.



$a^2 + b^2 = c^2$

$12^2 + (x-4)^2 = x^2$

$144 + (x^2 - 8x + 16) = x^2$

$144 + x^2 - 8x + 16 - x^2 = 0$

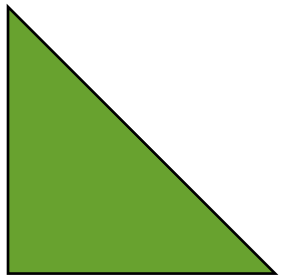
$-8x + 160 = 0$

$-8x = -160$

$x = 20$

The hypotenuse = 20
leg one =12
leg two= 16

11.



$$a^2 + b^2 = c^2$$

$$x^2 + (2x+3)^2 = (3x)^2$$

$$x^2 + 4x^2 + 12x + 9 = 9x^2$$

$$0 = 4x^2 - 12x - 9$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(-9)}}{2(4)}$$

$$y = \frac{12 \pm \sqrt{144 + 144}}{8}$$

$$y = \frac{12 \pm \sqrt{288}}{8}$$

$$y = \frac{12 + \sqrt{288}}{8} \approx \frac{12 + 16.9706}{8} \approx \frac{28.9706}{8} \approx 3.6213$$

$$y = \frac{12 - \sqrt{288}}{8} \approx \frac{12 - 16.9706}{8} \approx \frac{-4.9706}{8} \approx -0.6213$$

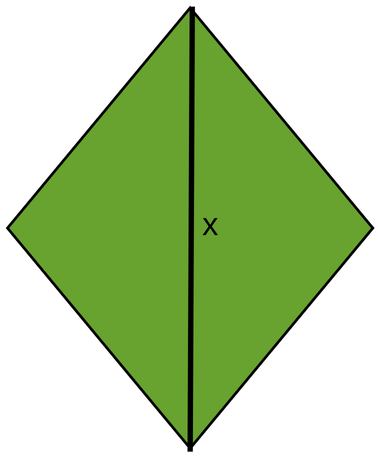
The sides of a triangle cannot be negative, so the solution is:

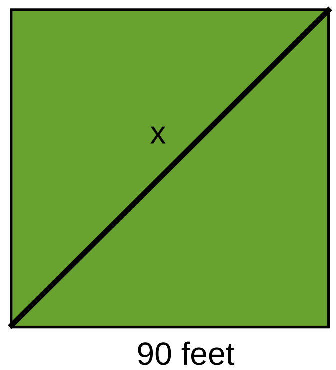
$$x = 3.6213$$

$$2x + 3 = 2(3.6213) + 3 = 10.2426$$

$$3x = 3(3.6213) = 10.8639$$

12.



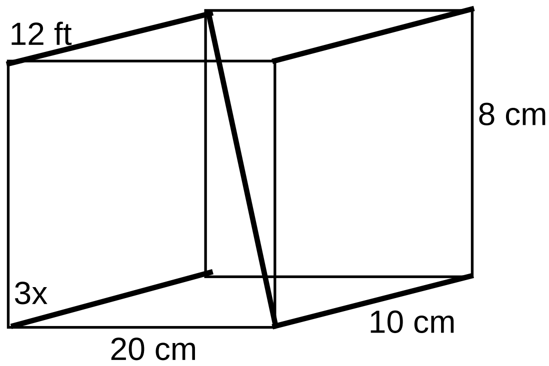


The diagonal created by home plate and second base is the hypotenuse of a right triangle.

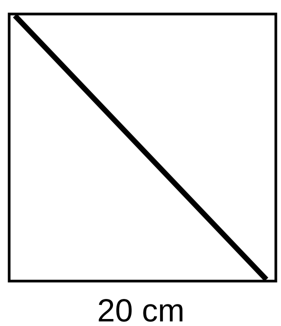
$$a^2 + b^2 = c^2$$
$$90^2 + 90^2 = c^2$$
$$8100 + 8100 = c^2$$
$$16200 = c^2$$
$$\sqrt{16200} = \sqrt{c^2}$$
$$c = \sqrt{16200}$$
$$c \approx 127.2792$$

Second base is approximately 127.2792 feet from home plate.

13.



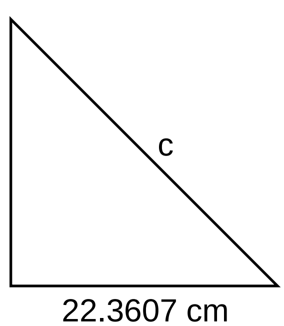
The diagonal we have to find is the hypotenuse of a right triangle whose legs are the height and the diagonal of the bottom of the box. Let's find that diagonal first.



$$a^2 + b^2 = c^2$$
$$20^2 + 10^2 = c^2$$
$$400 + 100 = c^2$$
$$500 = c^2$$
$$\sqrt{500} = \sqrt{c^2}$$
$$c \approx 22.3607$$

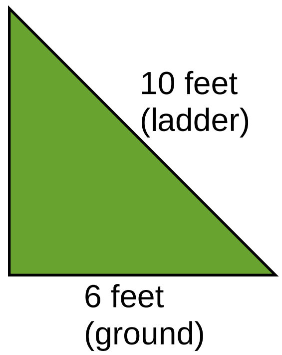
Now we have two legs of the triangle and can find the hypotenuse (the diagonal of the box).

$$a^2 + b^2 = c^2$$
$$8^2 + 22.3607^2 = c^2$$
$$64 + 500 = c^2$$
$$564 = c^2$$
$$\sqrt{564} = \sqrt{c^2}$$
$$c \approx 23.7487$$



The diagonal is approximately 23.75 cm.

14.



$a^2 + b^2 = c^2$

$x^2 + 6^2 = 10^2$

$x^2 + 36 = 100$

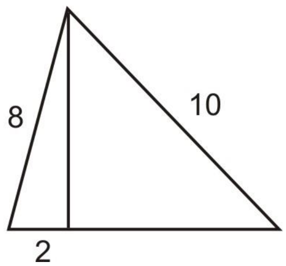
$x^2 = 64$

$\sqrt{x^2} = \sqrt{64}$

$x = 8$

The ladder touches the wall of the house 8 feet above the ground.

15.



We can find the height by using Pythagorean Theorem on the smaller triangle.

$a^2 + b^2 = c^2$

$a^2 + 2^2 = 8^2$

$a^2 + 4 = 64$

$a^2 = 60$

$\sqrt{a^2} = \sqrt{60}$

$a = \sqrt{4} * \sqrt{15}$

$a = 2\sqrt{15}$

We can find the base by again using Pythagorean Theorem on the bigger triangle and adding 2.

$a^2 + b^2 = c^2$

$\left(2\sqrt{15}\right)^2 + b^2 = 10^2$

$(4 * 15) + b^2 = 100$

$60 + b^2 = 100$

$b^2 = 40$

$\sqrt{b^2} = \sqrt{40}$

$b = \sqrt{4} * \sqrt{10}$

$a = 2\sqrt{10}$

Now we use the formula to find the area.

$A = \frac{1}{2}bh$

$A = \frac{1}{2}\left(2\sqrt{10} + 2\right)\left(2\sqrt{15}\right)$

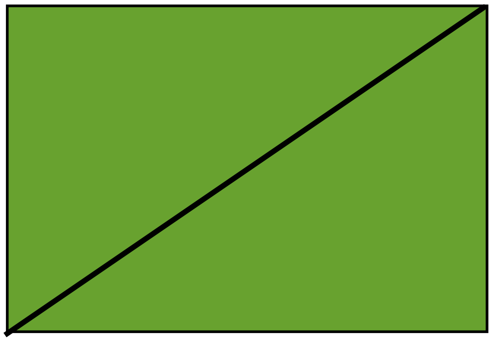
$A = \frac{1}{2}\left(4\sqrt{150} + 4\sqrt{15}\right)$

$A = 2\sqrt{150} + 2\sqrt{15}$

$A \approx 32.2409$

The area of the triangle is approximately 32. 2409

16.



We know the short side is 123 feet. We know the diagonal (hypotenuse) saves Mario 1/2 the length of the long side. If we call the long side x then the hypotenuse is 123 + 1/2x. Now we solve using the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$

$$x^2 + 123^2 = (0.5x + 123)^2$$

$$x^2 + 15129 = 0.25x^2 + 123x + 15129$$

$$x^2 + 15129 - 0.25x^2 - 123x - 15129 = 0$$

$$0.75x^2 - 123x = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-123) \pm \sqrt{(-123)^2 - 4(0.75)(0)}}{2(0.75)}$$

$$x = \frac{123 \pm \sqrt{15129 - 0}}{1.5}$$

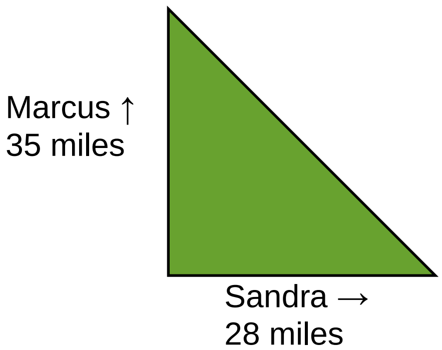
$$x = \frac{123 \pm \sqrt{15129}}{1.5}$$

$$x = \frac{123 \pm \sqrt{15129}}{1.5} = \frac{123 + 123}{1.5} = \frac{246}{1.5} = 164$$

$$x = \frac{123 \pm \sqrt{15129}}{1.5} = \frac{123 - 123}{1.5} = \frac{0}{1.5} = 0$$

The long side is 164 feet.

17.



$a^2 + b^2 = c^2$

$28^2 + 35^2 = c^2$

$784 + 1225 = c^2$

$2009 = c^2$

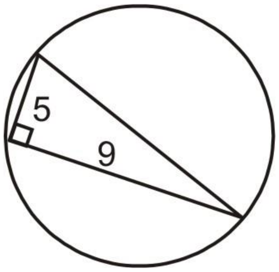
$\sqrt{2009} = \sqrt{c^2}$

$c = \sqrt{2009}$

$c \approx 44.8219$

The two boats are approximately 44.8219 miles apart after two hours.

18.



The hypotenuse of the triangle is the diameter of the circle. We will find the length of the hypotenuse using Pythagorean Theorem.

$a^2 + b^2 = c^2$

$5^2 + 9^2 = c^2$

$25 + 81 = c^2$

$106 = c^2$

$\sqrt{106} = \sqrt{c^2}$

$c = \sqrt{106}$

$c \approx 10.2956$

The diameter of the circle is approximately 10.2956. To find the area we need the radius, which is half of the diameter. 10.2956/2=5.1478

$A = \pi r^2$

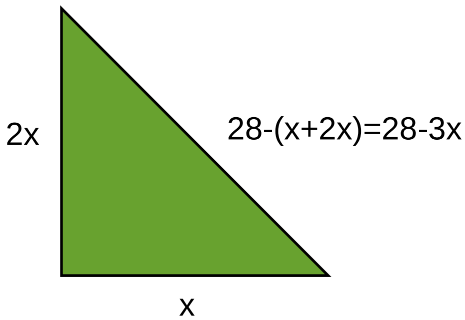
$A = (3.14)(5.1478^2)$

$A = (3.14)(26.4998)$

$A = 83.2094$

The area of the circle is approximately 83.2094

19.



$$a^2+b^2=c^2$$
$$x^2+(2x)^2=(28-3x)^2$$
$$x^2+4x^2=784-84x-84x+9x^2$$
$$5x^2=9x^2-168x+784$$
$$0=4x^2-168x+784$$
$$y=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$y=\frac{-(-168)\pm\sqrt{(-168)^2-4(4)(784)}}{2(4)}$$
$$y=\frac{168\pm\sqrt{28224-12544}}{8}$$
$$y=\frac{168\pm\sqrt{15680}}{8}$$
$$y=\frac{168+\sqrt{15680}}{8}\approx\frac{168+125.2198}{8}\approx\frac{293.2198}{8}\approx 36.6525$$
$$y=\frac{168-\sqrt{15680}}{8}\approx\frac{168-125.2198}{8}\approx\frac{42.7802}{8}\approx 5.3475$$

There are two possible sets of solutions.

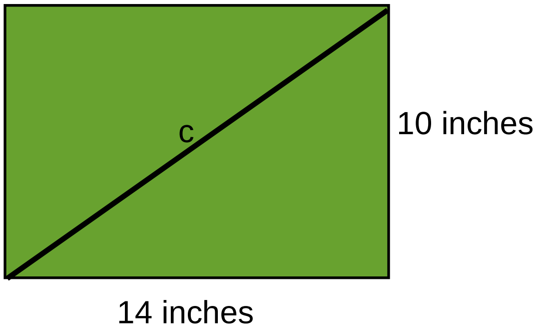
$$x=36.6525$$
$$2x=73.305$$
$$28-3x=-81.9575$$

This is an extraneous solution. (The length of the hypotenuse cannot be negative.)

$$x=5.3475$$
$$2x=10.695$$
$$28-3x=11.9575$$

The sides of the triangle are approximately 5.3475, 10.695, and 11.9575

20.



$a^2 + b^2 = c^2$

$10^2 + 14^2 = c^2$

$100 + 196 = c^2$

$296 = c^2$

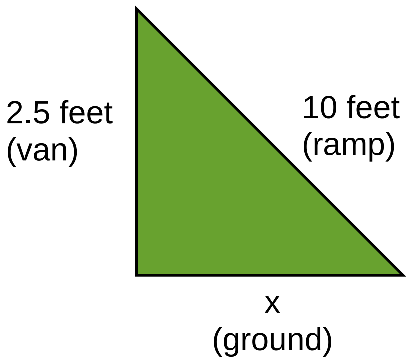
$\sqrt{296} = \sqrt{c^2}$

$c = \sqrt{296}$

$c \approx 17.2047$

The length of the diagonal is approximately 17.2047 inches

21.



$$a^2 + b^2 = c^2$$

$$2.5^2 + x^2 = 10^2$$

$$6.25 + x^2 = 100$$

$$x^2 = 93.75$$

$$\sqrt{x^2} = \sqrt{93.75}$$

$$x = \sqrt{93.75}$$

$$x \approx 9.6825$$

The ramp extends approximately 9.6825 feet past the back of the van.

Mixed Review

22.

1.

$y = a(b)^x$

$y = 121000(100\% + 1.2\%)^x$

$y = 121000(1.012)^x$

$y = 121000(1.012)^{13}$

$y = 121000(1.1677)$

$y = 141291.7$

The population in 13 years will be approximately 141,292

2.

$y = a(b)^x$

$y = 121000(100\% - 1.2\%)^x$

$y = 121000(0.988)^x$

$y = 121000(0.988)^x$

$y = 121000(0.988)^5$

$y = 121000(0.9414)$

$y = 113909.4$

The population 5 years ago was approximately 113,909

23. $1.29651843 \cdot 10^5 = 129651.843$

24.

$4, 2, 1, \frac{1}{2}, \frac{1}{6}, \frac{1}{8} \ldots$

$2 \div 4 = \frac{1}{2}$

$2 \div 1 = \frac{1}{2}$

$1 \div \frac{1}{2} = 2$

$\frac{1}{2} \div \frac{1}{6} = 3$

$\frac{1}{6} \div \frac{1}{8} = 1\frac{1}{3}$

No, this is not a geometric sequence. In a geometric sequence each successive number is found by multiplying the number before by a common ratio. Since these numbers do not have a common ratio, it is not a geometric sequence.

25.

$$6x^3\left(4xy^2+y^3z\right)=24x^4y^2+6x^3y^3z$$

26.

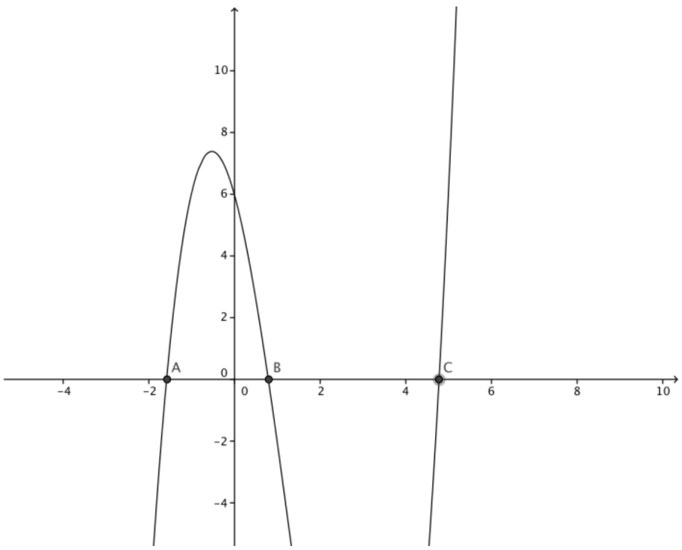
$$0=(x-2)(x+1)(x-3)$$

$$0=\left(x^2+x-2x-2\right)(x-3)$$

$$0=\left(x^2-x-2\right)(x-3)$$

$$0=x^3-x^2-2x-3x^2-3x+6$$

$$0=x^3-4x^2-5x+6$$



x-intercepts= (-1.58,0) (0.79,0) (4.78,0)

27. $\sqrt{300}=\sqrt{100}*\sqrt{3}=10\sqrt{3}\approx 17.3205$

Quick Quiz

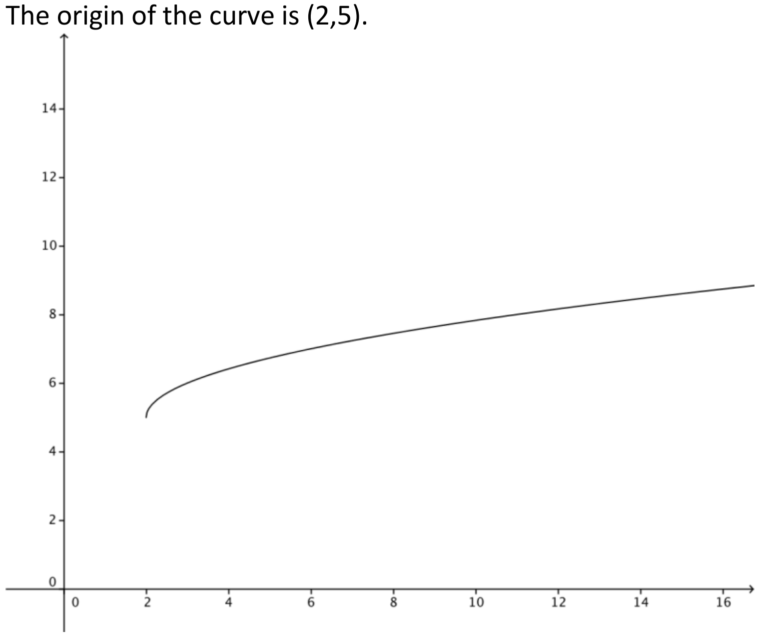
1.

$$f(x)=\sqrt{x-h}+k$$

$$h(x)=\sqrt{x-2}+5$$

$$(h,k)=(2,5)$$

The origin of the curve is (2,5).



2.

$$\frac{6}{\sqrt[3]{2}}=\frac{6}{\sqrt[3]{2}}*\frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}}=\frac{6\sqrt[3]{4}}{2}=3\sqrt[3]{4}$$

3. $\sqrt[4]{-32}$ This problem has no real solution. It is impossible to take an even root (4) of a negative number (-32).

4. An extraneous solution is a solution that occurs when a radical equation is solved, but the solution is not a possible solution for the equation. These solutions occur because you cannot have an even root of a negative number and sometimes the solution that you find results in this situation.

5.

$$a^2+b^2=c^2$$

$$3^2+4^2=6^2$$

$$9+16=36$$

$$25\neq 36$$

No 3,4,6 cannot form a right triangle.

6.

$$5 = \sqrt[3]{y+6}$$

$$5^3 = \left(\sqrt[3]{y+6}\right)^3$$

$$125 = y+6$$

$$119 = y$$

Lesson 11.5
The Distance and Midpoint Formulas

1. $d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

2.
$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$
$$d = \sqrt{(7-7)^2 + (-7-7)^2}$$
$$d = \sqrt{(0)^2 + (-14)^2}$$
$$d = \sqrt{0 + 196}$$
$$d = \sqrt{196}$$
$$d = 14$$

3.
$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$
$$d = \sqrt{(-6-6)^2 + (3-(-3))^2}$$
$$d = \sqrt{(-12)^2 + (6)^2}$$
$$d = \sqrt{144 + 36}$$
$$d = \sqrt{180}$$
$$d = \sqrt{36} * \sqrt{5} = 6\sqrt{5}$$
$$d \approx 13.4164$$

4.
$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$
$$d = \sqrt{(-8-(-1))^2 + (-5-(-3))^2}$$
$$d = \sqrt{(-7)^2 + (-2)^2}$$
$$d = \sqrt{49 + 4}$$
$$d = \sqrt{53}$$
$$d \approx 7.2801$$

5.

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$
$$d = \sqrt{(0 - (-4))^2 + (6 - 3)^2}$$
$$d = \sqrt{(4)^2 + (3)^2}$$
$$d = \sqrt{16 + 9}$$
$$d = \sqrt{25}$$
$$d = 5$$

6.

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$
$$d = \sqrt{(2 - 0)^2 + (4 - (-1))^2}$$
$$d = \sqrt{(2)^2 + (5)^2}$$
$$d = \sqrt{4 + 25}$$
$$d = \sqrt{29}$$
$$d \approx 5.3852$$

7.

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$
$$d = \sqrt{(2 - 2)^2 + (6 - (-3))^2}$$
$$d = \sqrt{(0)^2 + (9)^2}$$
$$d = \sqrt{0 + 81}$$
$$d = \sqrt{81}$$
$$d = 9$$

8.

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$
$$d = \sqrt{(-4 - (-2.5))^2 + (4 - 0.5)^2}$$
$$d = \sqrt{(-1.5)^2 + (3.5)^2}$$
$$d = \sqrt{2.25 + 12.25}$$
$$d = \sqrt{14.5}$$
$$d \approx 3.8079$$

9.

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$d = \sqrt{(-6 - (-10))^2 + (0 - 12)^2}$$

$$d = \sqrt{(4)^2 + (12)^2}$$

$$d = \sqrt{16 + 144}$$

$$d = \sqrt{160} = \sqrt{16} * \sqrt{10} = 4\sqrt{10}$$

$$d \approx 12.6491$$

10.

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$d = \sqrt{(-5.2 - 4.5)^2 + (-3.4 - 2.3)^2}$$

$$d = \sqrt{(-9.7)^2 + (-5.7)^2}$$

$$d = \sqrt{94.09 + 32.49}$$

$$d = \sqrt{126.58}$$

$$d \approx 11.2508$$

11.

$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

$10 = \sqrt{(2 - y)^2 + (4 - (-4))^2}$

$10 = \sqrt{(2 - y)^2 + (8)^2}$

$10 = \sqrt{(2 - y)^2 + 64}$

$10^2 = \left(\sqrt{(2 - y)^2 + 64}\right)^2$

$100 = (2 - y)^2 + 64$

$36 = 4 - 2y - 2y + y^2$

$0 = y^2 - 4y + 4 - 36$

$0 = y^2 - 4y - 32$

$32 = y^2 - 4y$

$32 + 4 = y^2 - 4y + 4$

$36 = (y - 2)^2$

$\sqrt{36} = \sqrt{(y - 2)^2}$

$6 = \pm(y - 2)$

$6 = (y - 2)$

$8 = y$

$6 = -(y - 2)$

$6 = -y + 2$

$4 = -y$

$-4 = y$

The points are (-4,8) and (-4,-4)

12.

$d=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$

$8=\sqrt{(5-3)^2+(-2-x)^2}$

$8=\sqrt{(2)^2+(-2-x)^2}$

$8=\sqrt{4+(-2-x)^2}$

$8^2=\left(\sqrt{4+(-2-x)^2}\right)^2$

$64=4+(-2-x)^2$

$60=(-2-x)^2$

$60=4+2x+2x+x^2$

$0=x^2+4x+4-60$

$0=x^2+4x-56$

$56=x^2+4x$

$56+4=x^2+4x+4$

$60=(x+2)^2$

$\sqrt{60}=\sqrt{(x+2)^2}$

$\sqrt{4}*\sqrt{15}=\pm(x+2)$

$7.746=x+2$

$5.746=x$

$-7.746=x+2$

$-9.746=x$

The points are (5.746, 3) and (-9.746, 3)

13. $(x_m,y_m)=\frac{(x_1+x_2)}{2},\frac{(y_1+y_2)}{2}$

14.

$(x_m,y_m)=\frac{(x_1+x_2)}{2},\frac{(y_1+y_2)}{2}$

$(x_m,y_m)=\frac{(7+(-7))}{2},\frac{(7+7)}{2}$

$(x_m,y_m)=\frac{(0)}{2},\frac{(14)}{2}=(0,7)$

15.

$$\left(x_m, y_m\right) = \frac{\left(x_1 + x_2\right)}{2}, \frac{\left(y_1 + y_2\right)}{2}$$

$$\left(x_m, y_m\right) = \frac{\left(-3 + 3\right)}{2}, \frac{\left(6 + \left(-6\right)\right)}{2}$$

$$\left(x_m, y_m\right) = \frac{(0)}{2}, \frac{(0)}{2} = (0, 0)$$

16.

$$\left(x_m, y_m\right) = \frac{\left(x_1 + x_2\right)}{2}, \frac{\left(y_1 + y_2\right)}{2}$$

$$\left(x_m, y_m\right) = \frac{\left(-3 + \left(-5\right)\right)}{2}, \frac{\left(-1 + \left(-8\right)\right)}{2}$$

$$\left(x_m, y_m\right) = \frac{(-8)}{2}, \frac{(-9)}{2} = (-4, -4.5)$$

17.

$$\left(x_m, y_m\right) = \frac{\left(x_1 + x_2\right)}{2}, \frac{\left(y_1 + y_2\right)}{2}$$

$$\left(x_m, y_m\right) = \frac{\left(3 + 6\right)}{2}, \frac{\left(-4 + 1\right)}{2}$$

$$\left(x_m, y_m\right) = \frac{(9)}{2}, \frac{(-3)}{2} = (4.5, -1.5)$$

18.

$$\left(x_m, y_m\right) = \frac{\left(x_1 + x_2\right)}{2}, \frac{\left(y_1 + y_2\right)}{2}$$

$$\left(x_m, y_m\right) = \frac{\left(2 + 2\right)}{2}, \frac{\left(-3 + 4\right)}{2}$$

$$\left(x_m, y_m\right) = \frac{(4)}{2}, \frac{(1)}{2} = (2, 0.5)$$

19.

$$\left(x_m, y_m\right) = \frac{\left(x_1 + x_2\right)}{2}, \frac{\left(y_1 + y_2\right)}{2}$$

$$\left(x_m, y_m\right) = \frac{\left(4 + 8\right)}{2}, \frac{\left(-5 + 2\right)}{2}$$

$$\left(x_m, y_m\right) = \frac{\left(12\right)}{2}, \frac{\left(-3\right)}{2} = (6, -1.5)$$

20.

$$\left(x_m, y_m\right) = \frac{\left(x_1 + x_2\right)}{2}, \frac{\left(y_1 + y_2\right)}{2}$$

$$\left(x_m, y_m\right) = \frac{\left(1.8 + \left(-0.4\right)\right)}{2}, \frac{\left(-3.4 + 1.4\right)}{2}$$

$$\left(x_m, y_m\right) = \frac{\left(1.4\right)}{2}, \frac{\left(-2\right)}{2} = (0.7, -1)$$

21.

$$\left(x_m, y_m\right) = \frac{\left(x_1 + x_2\right)}{2}, \frac{\left(y_1 + y_2\right)}{2}$$

$$\left(x_m, y_m\right) = \frac{\left(5 + \left(-4\right)\right)}{2}, \frac{\left(-1 + 0\right)}{2}$$

$$\left(x_m, y_m\right) = \frac{\left(1\right)}{2}, \frac{\left(-1\right)}{2} = (0.5, -0.5)$$

22.

$$\left(x_m, y_m\right) = \frac{\left(x_1 + x_2\right)}{2}, \frac{\left(y_1 + y_2\right)}{2}$$

$$\left(x_m, y_m\right) = \frac{\left(10 + 2\right)}{2}, \frac{\left(2 + \left(-4\right)\right)}{2}$$

$$\left(x_m, y_m\right) = \frac{\left(12\right)}{2}, \frac{\left(-2\right)}{2} = (6, -1)$$

23.

$$x_m = \frac{(x_1 + x_2)}{2}$$

$$3 = \frac{(4 + x)}{2}$$

$$6 = 4 + x$$

$$2 = x$$

$$y_m = \frac{(y_1 + y_2)}{2}$$

$$-2 = \frac{(5 + y)}{2}$$

$$-4 = 5 + y$$

$$-9 = y$$

$$(x, y) = (2, -9)$$

24.

$$x_m = \frac{(x_1 + x_2)}{2}$$

$$0 = \frac{(-10 + x)}{2}$$

$$0 = -10 + x$$

$$10 = x$$

$$y_m = \frac{(y_1 + y_2)}{2}$$

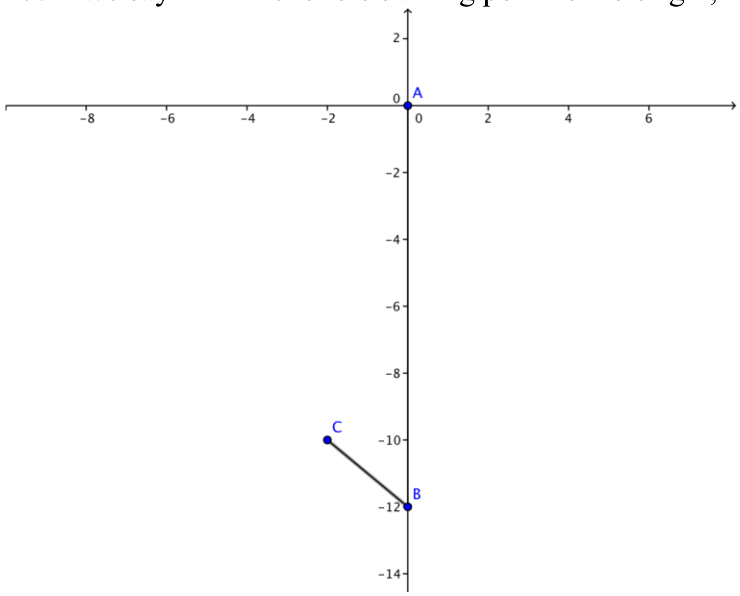
$$4 = \frac{(-2 + y)}{2}$$

$$8 = -2 + y$$

$$10 = y$$

$$(x, y) = (10, 10)$$

25. If we say that Michelle's starting point is the origin, the following is the graph of her trip.



We know the distance from A to B is 12 miles, so we need to use the distance formula to find the distance between B and C.

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$d = \sqrt{(-10 - (-12))^2 + (-2 - 0)^2}$$

$$d = \sqrt{(2)^2 + (-2)^2}$$

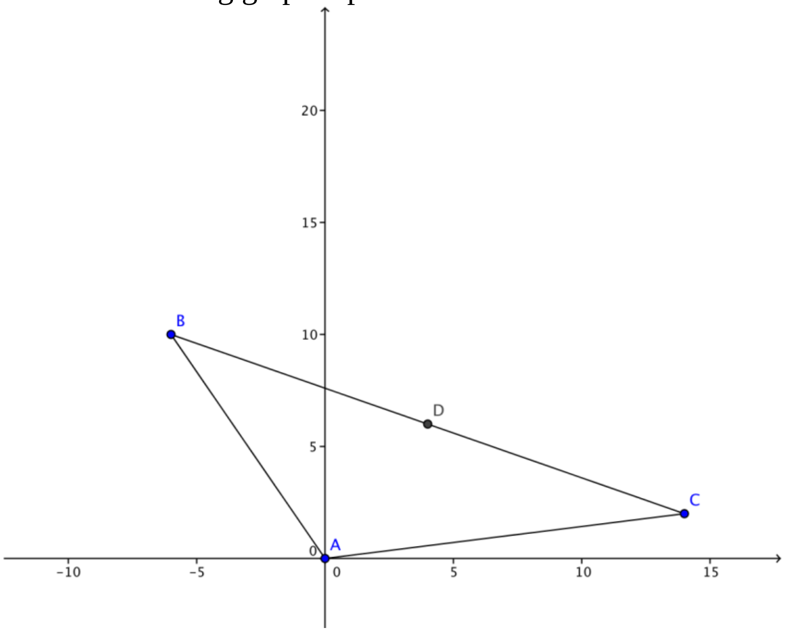
$$d = \sqrt{4 + 4}$$

$$d = \sqrt{8}$$

$$d \approx 2.8284$$

The total distance is approximately 2.8284 miles + 12 miles = 14.8284 miles

26. The following graph represents the town.



Point A (0,0) is the center of town.
Point B (-6,10) is Shawn's house.
Point C (14,2) is Kenya's house.

1.

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$
$$d = \sqrt{(2-10)^2 + (14-(-6))^2}$$
$$d = \sqrt{(-8)^2 + (20)^2}$$
$$d = \sqrt{64+400}$$
$$d = \sqrt{464} = \sqrt{16} * \sqrt{29} = 4\sqrt{29}$$
$$d \approx 21.5407$$

The two girls' houses are approximately 21.5407 blocks apart "as the crow flies".

2.

$$(x_m, y_m) = \frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}$$
$$(x_m, y_m) = \frac{(-6+14)}{2}, \frac{(10+2)}{2}$$
$$(x_m, y_m) = \frac{(8)}{2}, \frac{(12)}{2} = (4,6)$$

The halfway point between their houses is (4,6) or 4 blocks north and 6 blocks east of the center of town.

Mixed Review

27.

$(x-4)^2=121$

$x^2-4x-4x+16=121$

$x^2-8x+16=121$

$x^2-8x+16-121=0$

$x^2-8x-105=0$

$x^2-8x=105$

$x^2-8x+16=105+16$

$(x-4)^2=121$

$\sqrt{(x-4)^2}=\sqrt{121}$

$x-4=\pm 11$

$x-4=11$

$x=15$

$x-4=-11$

$x=-7$

$x=15,-7$

28.

$$\left.\begin{matrix}21ab^4\\15a^7b^2\end{matrix}\right\rangle 3ab^2$$

29.
$_{10}C_7$ is the symbol for a combination. A combination is an arrangement of objects in no particular order. 10 is the number of possible objects to choose from and 7 is the number of objects chosen at a time.

$${}_nC_k = \frac{n!}{k!(n-k)!}$$

$${}_{10}C_7 = \frac{10!}{7!(10-7)!}$$

$${}_{10}C_7 = \frac{3628800}{5040 * 6}$$

$${}_{10}C_7 = \frac{3628800}{30240}$$

$${}_{10}C_7 = 120$$

30.

$$6x^2 + 17x + 5$$

$$6x^2 + 2x + 15x + 5$$

$$2x(3x+1) + 5(3x+1)$$

$$(2x+5)(3x+1)$$

31.

$A = l * w$

$l = 16 + 2m$

$w = 12 + 2m$

$A = (16 + 2m) * (12 + 2m)$

$A = 192 + 32m + 24m + 4m^2$

$A = 4m^2 + 56m + 192$

$A = 4(m^2 + 14m + 48)$

$A = 4(m + 6)(m + 8)$

$m + 6 = 0$

$m = -6$

$m + 8 = 0$

$m = -8$

$A = (16 + 2m) * (12 + 2m)$

$A = (16 + 2(-6)) * (12 + 2(-6))$

$A = (16 - 12)(12 - 12)$

$A = 4 * 0$

$A = 4$

$A = (16 + 2m) * (12 + 2m)$

$A = (16 + 2(-8)) * (12 + 2(-8))$

$A = (16 - 16)(12 - 16)$

$A = 0 * -4$

$A \neq -4$

The area of the rectangle is 4.

32.

$x^2 - 81$

$(x + 9)(x - 9)$

Lesson 11.6
Measures of Central Tendency and Dispersion

1. Measures of Central Tendency are the central values of a data set. This lesson describes mean, median, and mode.

2. The Median is the the middle of the data set when put in ascending order. It may not be a number in the data set if the number of items in the set is even. If so, it will be the number exactly halfway between the two middle numbers of the set.
The mean is the traditional average. You find it by adding all of the numbers of the set and dividing that sum by the number of pieces of data in the set.
When data has a high range the median is a more accurate measure. The very high and very low numbers will skew the mean.

3. Bimodal is when a data set has two modes. (Answers to example will vary, any is correct as long as there are two numbers in the set that appear most often.)

4. The three measures of dispersion described in the lesson are range, variance, and standard deviation. Range is the easiest to compute.

5. $$\sigma^2 = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + ...(x_n - \bar{x})^2}{n}$$

x_1= the first number in the data set
x_2= the second number in the data set
x_n= the last or nth number in the data set
n= the number of items in the data set
$\bar{x} =$ is the mean of the data set
$\sigma^2 =$ is the symbol for variance

6. Because the variance is squared, the number is often much larger than is useful. In the housing example the variance is in the billions, while the costs of the houses are all less than a million.

7. The standard deviation measures how closely the data clusters around the mean of the set. It is found by taking the square root of the variance.

8. Logically the standard deviation of a set where all the values are the same is zero because the standard deviation is a measure of the dispersion of the numbers. If all the values are the same, there is no dispersion.
Mathematically the standard deviation of a set where all the values are the same is zero because each set of parentheses in the formula will result in a zero (the value - the mean, the mean of a set of numbers that is all the same is the number) If each pair of parentheses results in zero, then the only possible answer, even after squaring, adding, dividing, and finding the square root, is 0.

9.

$$Mean = \overline{x} = average$$

$$\overline{x} = \frac{19630 + 28920 + 56330 + 49930 + 69030 + 39130 + 35460 + 2476590}{8}$$

$$\overline{x} = \$346,877.50$$

$$Median = middle\#$$

$$Med = 19630, 28920, 35460, (39130, 49930), 56330, 69030, 2476590$$

$$Med = \frac{(39130 + 49930)}{2}$$

$$Med = \frac{89060}{2}$$

$$Med = \$44,530$$

$$Range = highest - lowest$$

$$R = 2476590 - 19630$$

$$R = \$2,456,960$$

10.

$$Mean = \overline{x} = average$$

$$\overline{x} = \frac{11+16+9+15+5+18}{6}$$

$$\overline{x} = \frac{74}{6}$$

$$\overline{x} \approx 12.3333$$

$$Median = middle\#$$

$$Med = 5,9,(11,15),16,18$$

$$Med = \frac{(11+15)}{2}$$

$$Med = \frac{26}{2}$$

$$Med = 13$$

$$Mode = \#\, appears > times$$

$$Mode = 5,9,11,15,16,18$$

$$Mode = NONE$$

$$Range = highest - lowest$$

$$R = 18-5$$

$$R = 13$$

11.

$$Mean = \overline{x} = average$$

$$\overline{x} = \frac{53+32+49+24+62}{5}$$

$$\overline{x} = \frac{220}{5}$$

$$\overline{x} = 44$$

$$Median = middle\#$$

$$Med = 24, 32, (49), 53, 62$$

$$Med = 49$$

$$Mode = \# appears > times$$

$$Mode = 24, 32, 49, 53, 62$$

$$Mode = NONE$$

$$Range = highest - lowest$$

$$R = 62 - 24$$

$$R = 38$$

12.

$$Mean = \overline{x} = average$$

$$\overline{x} = \frac{11+9+19+9+19+9+13+11}{8}$$

$$\overline{x} = \frac{100}{8}$$

$$\overline{x} = 12.5$$

$$Median = middle\#$$

$$Med = 9,9,9,(11,11),13,19,19$$

$$Med = 11$$

$$Mode = \#\,appears > times$$

$$Mode = 9,9,9,11,11,13,19,19$$

$$Mode = 9$$

$$Range = highest - lowest$$

$$R = 19 - 9$$

$$R = 10$$

13.

$Mean = \overline{x} = average$

$$\overline{x} = \frac{3+2+6+9+0+1+6+6+3+2+3+5}{12}$$

$$\overline{x} = \frac{46}{12}$$

$\overline{x} \approx 3.8333$

$Median = middle\#$

$Med = 0,1,2,2,3,(3,3),5,6,6,6,9$

$Med = 3$

$Mode = \#\, appears > times$

$Mode = 0,1,2,2,3,3,3,5,6,6,6,9$

$Mode = 3,6$

$Range = highest - lowest$

$R = 9 - 0$

$R = 9$

14.

$Mean = \overline{x} = average$

$\overline{x} = \dfrac{2+17+1+-3+12+8+12+16}{8}$

$\overline{x} = \dfrac{65}{8}$

$\overline{x} = 8.125$

$Median = middle\#$

$Med = -3,1,2,(8,12),12,16,17$

$Med = \dfrac{(8+12)}{2}$

$Med = \dfrac{20}{2}$

$Med = 10$

$Mode = \#\,appears > times$

$Mode = -3,1,2,8,12,12,16,17$

$Mode = 12$

$Range = highest - lowest$

$R = 17-(-3)$

$R = 14$

15.

$Mean = \overline{x} = average$

$$\overline{x} = \frac{11+21+6+17+9}{5}$$

$$\overline{x} = \frac{64}{5}$$

$\overline{x} = 12.8$

$Median = middle\#$

$Med = 6,9,(11),17,21$

$Med = 11$

$Mode = \#\, appears > times$

$Mode = 6,9,11,17,21$

$Mode = NONE$

$Range = highest - lowest$

$R = 21-6$

$R = 15$

16.

$$Mean = \overline{x} = average$$

$$\overline{x} = \frac{223+121+227+433+122+193+397+276+303+199+197+265+366+401+222}{15}$$

$$\overline{x} = \frac{3945}{15}$$

$$\overline{x} = 263$$

$$Median = middle\#$$

$$Med = 121,122,193,197,199,222,223,(227),265,276,303,366,397,401,433$$

$$Med = 227$$

$$Mode = \#\,appears > times$$

$$Mode = 121,122,193,197,199,222,223,227,265,276,303,366,397,401,433$$

$$Mode = NONE$$

$$Range = highest - lowest$$

$$R = 433 - 121$$

$$R = 312$$

17.

$$Mean = \bar{x} = average$$

$$\bar{x} = \frac{15+19+15+16+11+11+18+21+165+9+11+20+16+8+17+10+12+11+16+14}{20}$$

$$\bar{x} = \frac{435}{20}$$

$$\bar{x} = 21.75$$

$$Median = middle\#$$

$$Med = 8,9,10,11,11,11,11,12,14,(15,15),16,16,16,17,18,19,20,21,165$$

$$Med = 15$$

$$\sigma = \sqrt{\frac{(x_1-\bar{x})^2+(x_2-\bar{x})^2+\ldots(x_n-\bar{x})^2}{n}}$$

$$\sigma = \sqrt{\frac{\begin{array}{l}(8-21.75)^2+(9-21.75)^2+(10-21.75)^2+(11-21.75)^2+(11-21.75)^2+\\(11-21.75)^2+(11-21.75)^2+(12-21.75)^2+(14-21.75)^2+(15-21.75)^2+\\(15-21.75)^2+(16-21.75)^2+(16-21.75)^2+(16-21.75)^2+(17-21.75)^2+\\(18-21.75)^2+(19-21.75)^2+(20-21.75)^2+(21-21.75)^2+(165-21.75)^2\end{array}}{20}}$$

$$\sigma = \sqrt{\frac{21865.75}{20}}$$

$$\sigma = \sqrt{1093.2875}$$

$$\sigma \approx 33.0649$$

The median gives the best average.

18.

$$Mean = \overline{x} = average$$

$$\overline{x} = \frac{11+12+14+14+14+14+19}{7}$$

$$\overline{x} = \frac{98}{7}$$

$$\overline{x} = 14$$

$$Median = middle\#$$

$$Med = 11,12,14,(14),14,14,19$$

$$Med = 14$$

$$\sigma = \sqrt{\frac{(x_1-\overline{x})^2+(x_2-\overline{x})^2+\ldots(x_n-\overline{x})^2}{n}}$$

$$\sigma = \sqrt{\frac{(11-14)^2+(12-14)^2+(14-14)^2+(14-14)^2+(14-14)^2+(14-14)^2+(19-14)^2}{7}}$$

$$\sigma = \sqrt{\frac{38}{7}}$$

$$\sigma \approx \sqrt{5.4286}$$

$$\sigma \approx 2.3299$$

The mean and the median are the same.

19.

$$Mean = \overline{x} = average$$

$$\overline{x} = \frac{11+12+14+16+17+17+18}{7}$$

$$\overline{x} = \frac{105}{7}$$

$$\overline{x} = 15$$

$$Median = middle\#$$

$$Med = 11,12,14,(16),17,17,18$$

$$Med = 16$$

$$\sigma = \sqrt{\frac{(x_1-\overline{x})^2+(x_2-\overline{x})^2+\ldots(x_n-\overline{x})^2}{n}}$$

$$\sigma = \sqrt{\frac{(11-15)^2+(12-15)^2+(14-15)^2+(16-15)^2+(17-15)^2+(17-15)^2+(18-15)^2}{7}}$$

$$\sigma = \sqrt{\frac{44}{7}}$$

$$\sigma \approx \sqrt{6.2857}$$

$$\sigma \approx 2.5071$$

The mean and the median are so close that either makes an acceptable average.

20.

$$Mean = \bar{x} = average$$

$$\bar{x} = \frac{6+7+9+10+13}{5}$$

$$\bar{x} = \frac{45}{5}$$

$$\bar{x} = 9$$

$$Median = middle\#$$

$$Med = 6,7,(9),10,13$$

$$Med = 9$$

$$\sigma = \sqrt{\frac{(x_1-\bar{x})^2+(x_2-\bar{x})^2+\ldots(x_n-\bar{x})^2}{n}}$$

$$\sigma = \sqrt{\frac{(6-9)^2+(7-9)^2+(9-9)^2+(10-9)^2+(13-9)^2}{5}}$$

$$\sigma = \sqrt{\frac{30}{5}}$$

$$\sigma = \sqrt{6}$$

$$\sigma \approx 2.4495$$

The mean and the median are the same.

21.

$$Mean = \bar{x} = average$$

$$\bar{x} = \frac{121+122+193+197+199+222+223+227+265+276+303+366+397+401+433}{15}$$

$$\bar{x} = \frac{3945}{15}$$

$$\bar{x} = 263$$

$$Median = middle\#$$

$$Med = 121,122,193,197,199,222,223,(227),265,276,303,366,397,401,433$$

$$Med = 227$$

$$\sigma = \sqrt{\frac{(x_1-\bar{x})^2+(x_2-\bar{x})^2+\ldots(x_n-\bar{x})^2}{n}}$$

$$\sigma = \sqrt{\frac{\begin{array}{l}(121-263)^2+(122-263)^2+(193-263)^2+(197-263)^2+(199-263)^2+(222-263)^2+(223-263)^2+\\(227-263)^2+(265-263)^2+(276-263)^2+(303-263)^2+(366-263)^2+(397-263)^2+(401-263)^2+(433-263)^2\end{array}}{15}}$$

$$\sigma = \sqrt{\frac{136256}{15}}$$

$$\sigma \approx \sqrt{9083.7333}$$

$$\sigma \approx 95.3086$$

The mean is probably the best average.

22.

a. The mean will also increase by 7 points.
b. The median will also increase by 7 points.
c. The mode will also increase by 7 points. If there is no mode, there still will be none.
d. The range will stay the same.
e. The standard deviation will remain the same.

23.

a. The mean is doubled.
b. The median is doubled.
c. The mode is doubled. If there is no mode, there still will be none.
d. The range will be doubled.

24.

$$86 = \frac{88 + 76 + 97 + 84 + x}{5}$$

$$430 = 88 + 76 + 97 + 84 + x$$

$$342 = 76 + 97 + 84 + x$$

$$266 = 97 + 84 + x$$

$$169 = 84 + x$$

$$85 = x$$

Henry must score an 85 on his fifth test to have an average of 86.

25.

$$93 = \frac{88 + 76 + 97 + 84 + x}{5}$$

$$465 = 88 + 76 + 97 + 84 + x$$

$$377 = 76 + 97 + 84 + x$$

$$301 = 97 + 84 + x$$

$$204 = 84 + x$$

$$120 = x$$

It is not possible for Henry to have an average of 93 after his fifth test because it would require scoring 120 on what we can assume is a 100 point test.

26.

$$105 = \frac{x}{9}$$

$$945 = x$$

The sum of the 9 numbers is 945

27.

$$\overline{x} = \frac{163+187+194+188+205+196}{6}$$

$$\overline{x} = \frac{1133}{6}$$

$$\overline{x} = 188.8333$$

The bowler's average is approximately 189

28.

a.

$$Mean = \overline{x} = average$$

$$\overline{x} = \frac{38+45+58+38+36}{5}$$

$$\overline{x} = \frac{215}{5}$$

$$\overline{x} = 43$$

b.

$$\sigma = \sqrt{\frac{(x_1-\overline{x})^2+(x_2-\overline{x})^2+...(x_n-\overline{x})^2}{n}}$$

$$\sigma = \sqrt{\frac{(38-43)^2+(45-43)^2+(58-43)^2+(38-43)^2+(36-43)^2}{5}}$$

$$\sigma = \sqrt{\frac{328}{5}}$$

$$\sigma = \sqrt{65.6}$$

$$\sigma \approx 8.0994$$

c. The mean is not necessarily the most accurate measure of central tendency for this example because the one score of 58 is so much larger than the others. This skews the average when finding the mean. Both the median and mode are 38, a more accurate number.

29.

$$Mean = \bar{x} = average$$

$$\bar{x} = \left(\frac{1137+879+950+875+1499+875+4000+975+796.793+2100}{10}\right)(1000)$$

$$\bar{x} = \left(\frac{14086.793}{10}\right)(1000)$$

$$\bar{x} = (1408.6793)(1000)$$

$$\bar{x} = \$1408679.30$$

$$Median = middle\#$$

$$Med = 796793, 875000, 875000, 879000, (950000, 975000), 1137000, 1499000, 2100000, 4000000$$

$$Med = \frac{950000+975000}{2}$$

$$Med = \frac{1925000}{2}$$

$$Med = \$962,500$$

$$\sigma = \sqrt{\frac{(x_1-\bar{x})^2+(x_2-\bar{x})^2+\ldots(x_n-\bar{x})^2}{n}}$$

$$\sigma = \sqrt{\frac{(1137000-1408679.3)^2+(879000-1408679.3)^2+(950000-1408679.3)^2+(875000-1408679.30)^2+(1499000-1408679.30)^2+ (875000-1408679.30)^2+(4000000-1408679.30)^2+(975000-1408679.30)^2+(796793-1408679.30)^2+(2100000-1408679.30)^2}{10}}$$

$$\sigma = \sqrt{\frac{8897891382360}{10}}$$

$$\sigma = \sqrt{889789138236}$$

$$\sigma \approx 943286.35$$

The median is most often used as a measure of house prices in an area because it is not skewed by numbers at the high and low end of the range, which may be much higher or lower than most of the prices. The mean is skewed by these numbers at the far end of the range and the mode is simply unreliable (as there may be no mode or multiple modes.)

30.

(a) median
(b) median
(c) mean

31.

James- James' argument for planting by hand is valid because, as shown in the table, the mean diameter of the cabbages is larger. This means in general the cabbages planted by hand are bigger.

John- Johns' argument for planting by machine is valid because, as shown in the table, the standard deviation for the machine planted cabbages is smaller. This means that the cabbages are closer in size. John will be selling a more uniform product.

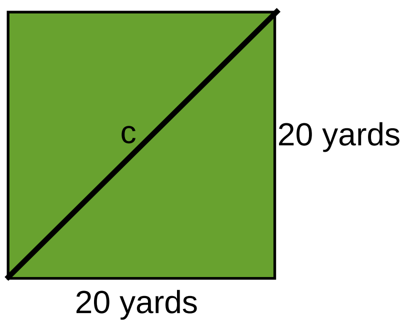
32.

(a) Samantha should take Fast-dog travel if she is trying to catch a plane. Their mean time is less and so on any given day it is more likely that she will get there in less time.

(b) Samantha should take the Inter-Cal express is she is making weekly visits and wants to reduce her overall time. Their standard deviation is less so she is more likely to make the mean time on each of her trips and spend less time overall.

Mixed Review

33.



$$a^2 + b^2 = c^2$$

$$20^2 + 20^2 = c^2$$

$$400 + 400 = c^2$$

$$800 = c^2$$

$$\sqrt{800} = \sqrt{c^2}$$

$$c \approx 28.2843$$

The distance to walk around the two joining sides is 20yds + 20yds = 40yds. The distance to walk the diagonal is 28.2843yds. The difference is 40-28.2843 = 11.7157.

It is approximately 11.7157 yards less to walk the diagonal.

34.

$$y = \frac{1}{6}x - 5$$

$$-\frac{1}{6}x + y = -5$$

35.

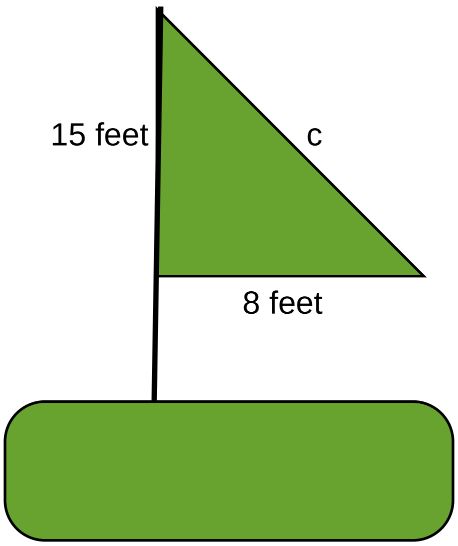
$-2 = \sqrt[4]{x+7}$

$(-2)^4 = \left(\sqrt[4]{x+7}\right)^4$

$16 = x + 7$

$9 = x$

36.



The diagonal of the sail is 17 feet.

$a^2 + b^2 = c^2$

$15^2 + 8^2 = c^2$

$225 + 64 = c^2$

$289 = c^2$

$\sqrt{289} = \sqrt{c^2}$

$c = 17$

37.

$\frac{2}{\sqrt{2}} * \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

Lesson 11.7
Stem and Leaf Plots and Histograms

1. The stem in a stem and leaf plot is the left hand column and lists the digits in the largest place. The leaf in a stem and leaf plot is the right hand column and lists the digits in the smallest place. The advantage to using a stem and leaf plot is that the chart quickly provides information from and about the data. It is also an easy method for sorting numbers manually.

2. A histogram is a bar chart that describes frequency distribution (the number of times a particular category appears in a data set). A histogram allows you to quickly determine information about the data set, such as total number of entries, highest frequency, and lowest frequency.

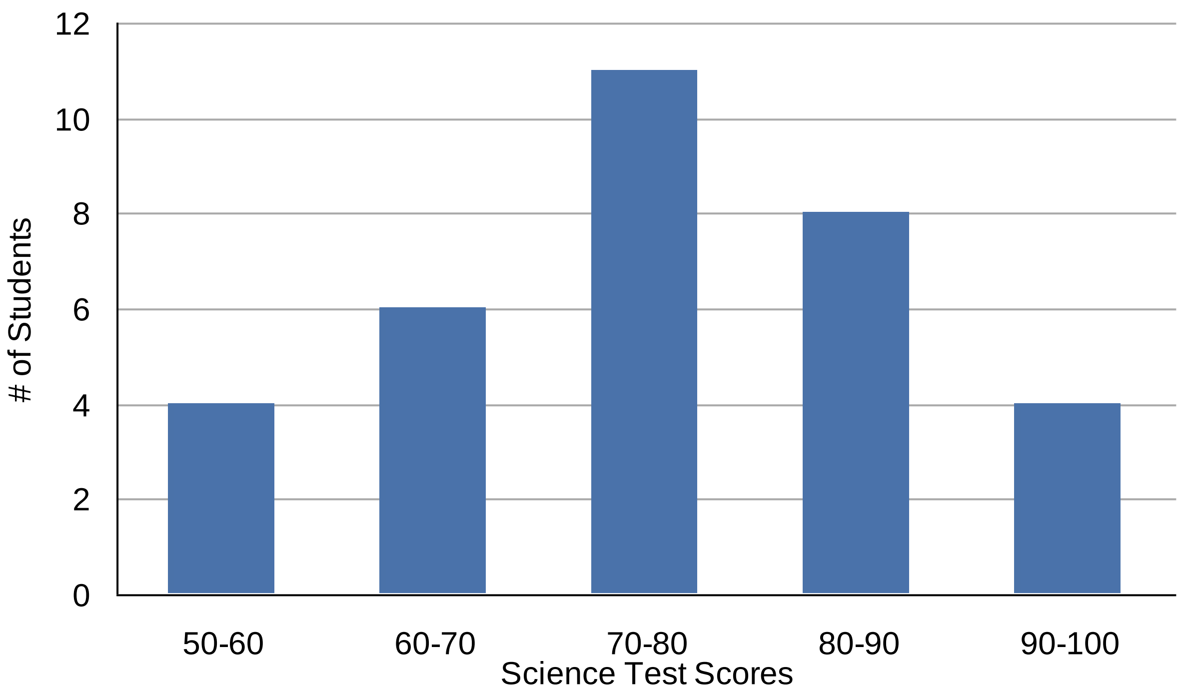
3.

1. A histogram would allow you take the large number of entries (100) and compare them quickly. A histogram would tell you in a look what the most popular drinks were (important for future ordering or stocking enough) and if any soft drinks were not chosen at all (so money would not be wasted buying these again.)

2. A histogram of this information would help the teacher determine what would be the appropriate amount of time to allot for a similar assignment. You would be able to quickly tell what amount of time it took most of the students (largest bar and any close to it). If many students were on the longer end of time the teacher knows this assignment needs more time, and if many students are on the very short end of the time scale the teacher knows that this is a quick assignment. In addition it would allow the teacher to many students take longer and might need extra help.

3. A histogram would allow the planners and builders of the mall to determine how far is too far (if few or no people parked in the spaces that distance away) and what is the ideal distance.

4.



Possible conclusions would be:

The total # of students is 33.
The greatest number of students scored between 70 and 80.
The least number of students scored between 90 and 110 or between 50 and 60.
The scores almost form a typical bell curve.
The range is 50 points.
The mean score is between 70 and 80.
The median score is between 70 and 80.
The mode is between 70 and 80.
(as well as any other data that can be observed or inferred from the histogram)

5.

1.

Stem	Leaf
8	7,9
9	4
10	0
11	8
12	2,3
13	1,7,8
14	1,3,4
15	9
16	3,4,4,5,8
17	5
18	---
19	3
20	4
21	7,9
22	0,6
23	---
24	8,8
25	0
26	6

2. There were 30 chain stores involved in selling the streak-free glass cleaner.
3. The number 11 represents 110 and the number 8 represents 80.
4. 19 out of the 33 stores sold less than 175 bottles. Approximately 58% of the stores sold less than 175 bottles.

6.

A.

Stem	Leaf
6	1,7
7	2,4,4,6,6,7,7,8,9
8	1,1,2,2,2,2,4,5,5,5,6,7,7,8,8,9,9
9	0,0,1,4,4,5,5,5,6,7,9,9

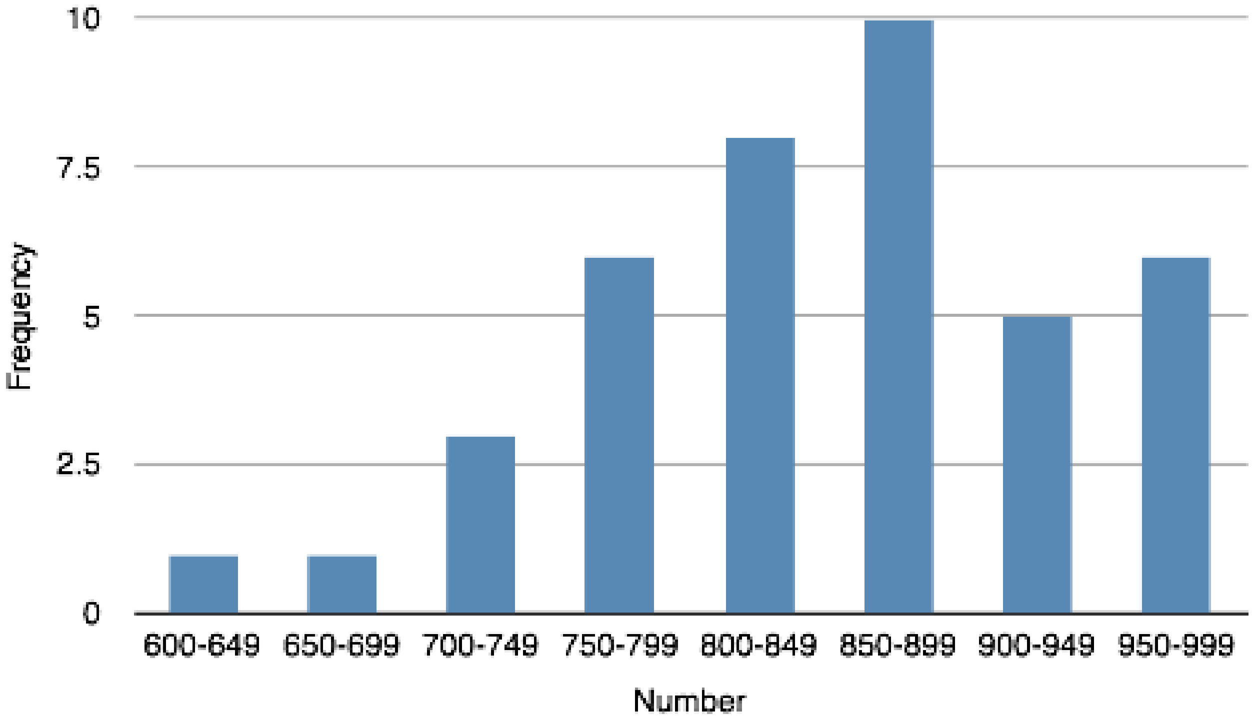
a. Mean = 84.725
b. Mode = 82
c. Median = 85

B.

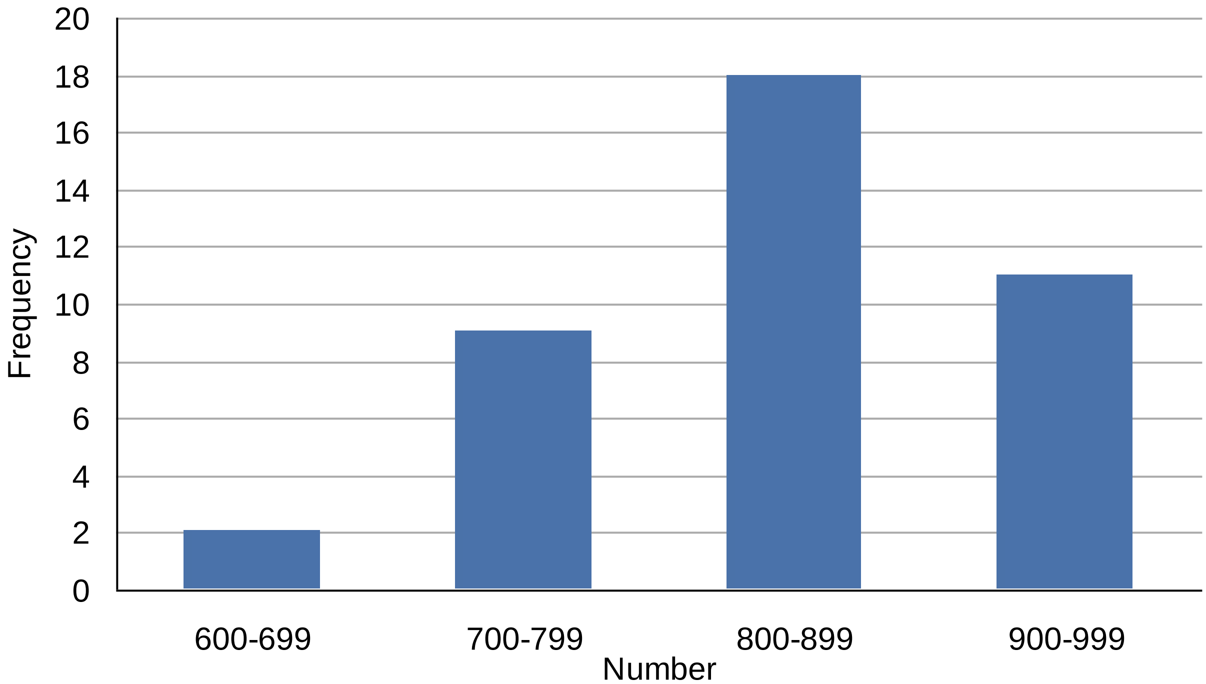
Number	Tally	Frequency
600-649	I	1
650-699	I	1
700-749	III	3
750-799	IIIIII	6
800-849	IIIIIIII	8
850-899	IIIIIIIIII	10
900-949	IIIII	5
950-999	IIIIII	6

C.

a. bin width of 50



b. bin width of 100



7.

Stem	Leaf
5	0,7,9
6	1,1,2,2,4,5,5,5,7,8,9
7	0,0,1,3,6,7,7,9,9
8	0,0,0,2,2,3,7

8.

Stem	Leaf
1	2, 3
2	1,7
3	3,4,5,7
4	0,0,1

Possible Conclusions:

The little boy is likely getting more confident (or more afraid).
The little swimmer does best at just over half an hour in the war.
They went to the pool 11 days.
The range is 29 minutes
The mean time is about 30 minutes.
The median time is 34 minutes.
The mode is 40 minutes.

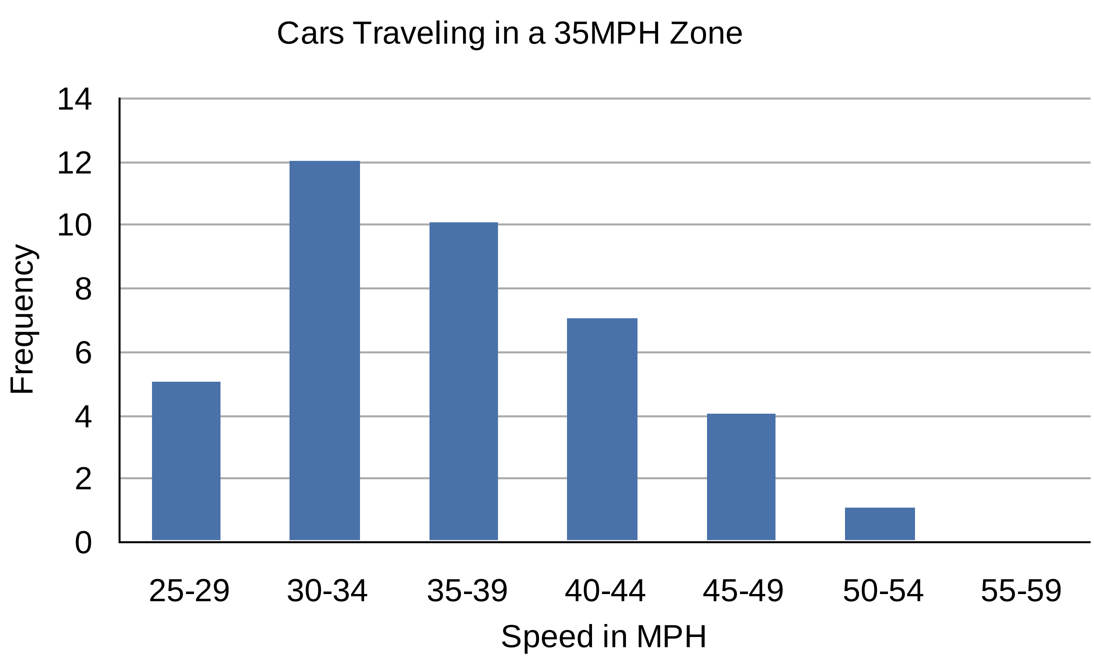
9.

1. Mean= 36.9
Median= 35.5
Mode= 32

2.

Speed in MPH	Frequency
25-29	5
30-34	12
35-39	10
40-44	7
45-49	4
50-54	1
55-59	1

3.



10.

1. Median = 2
2. Mean = 2.5679
3. Mode = 2
4. Odd # of Siblings = 38
5. (# of people with ≥ 4 siblings = 20)   % of people with ≥ 4 siblings = 24.69%

11. Answers will vary. Any situation that adequately fits the data (has 4 entries of the correct numbers) is correct.

12.

Regular Gasoline	
Stem	**Leaf**
50	
51	
52	
53	
54	0
55	0,5
56	
57	0,0
58	0,5,7,8
59	0,1
60	
61	0,5
62	
63	
64	0
65	
66	0
67	
68	
69	
70	

Premium Gasoline	
Stem	**Leaf**
50	0
51	
52	
53	
54	
55	
56	
57	
58	9
59	
60	
61	8,9
62	9
63	3,5,7,8,9
64	
65	9
66	4
67	
68	9
69	4
70	9

Mixed Review

13.

The soccer team consists of nine players. If the goalie has to be in the center there are 8 places left for the other 8 players to stand in.

As the order in which the players are standing is relevant when the photograph is being taken, the problem involves the use of permutation.

The required number of ways is

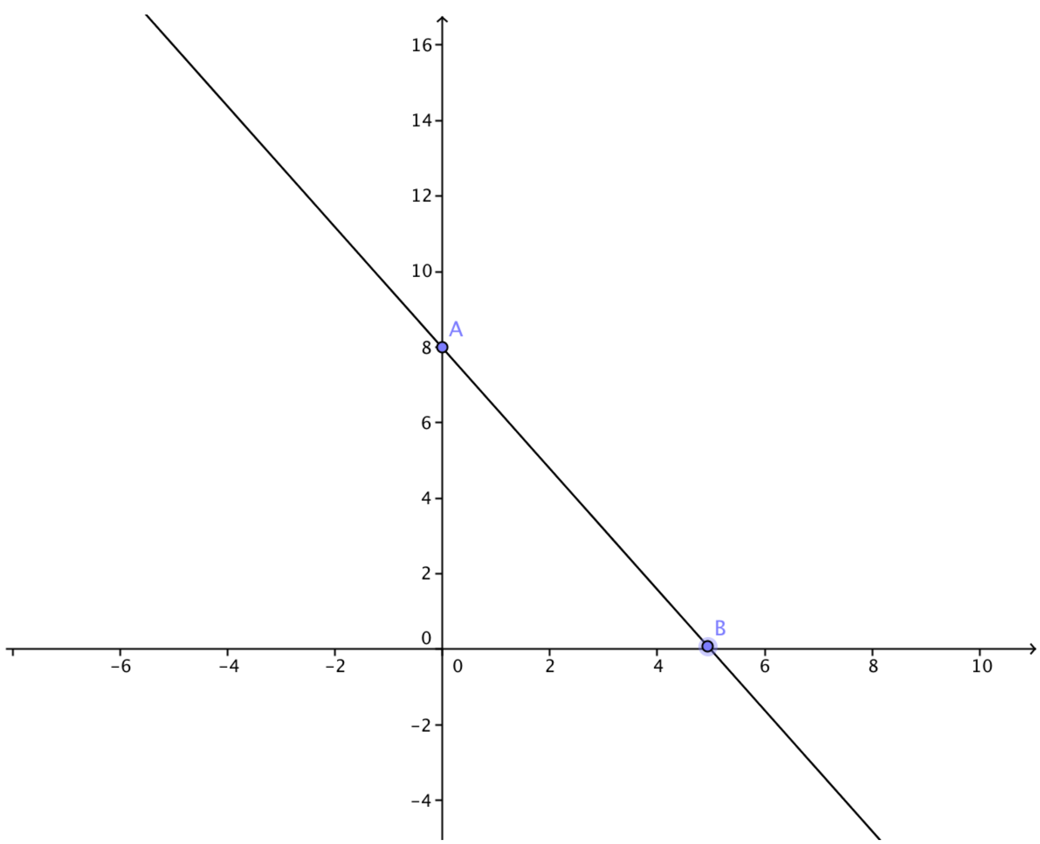
$$P(8,8) = \frac{8!}{(8-8)!} = \frac{8!}{1} = 8! = 40320$$

A nine member soccer team can line up in 40320 ways for a photograph if the goalie is in the center.

14.

x-intercept = (5,0)

y-intercept = (0,8)



15.

$$\left(\frac{7y^{-3}z^{2}}{4y^{4}z^{-6}}\right)^{-2} = \left(\frac{7}{4} * \frac{y^{-3}}{y^{4}} * \frac{z^{2}}{z^{-6}}\right)^{-2} =$$

$$\left(\frac{7}{4} * y^{(-3)-4} * z^{2-(-6)}\right)^{-2} = \left(\frac{7}{4} * y^{-7} * z^{8}\right)^{-2} =$$

$$\left(\frac{7z^{8}}{4y^{7}}\right)^{-2} = \left(\frac{7^{1(-2)} * z^{8(-2)}}{4^{1(-2)} * y^{7(-2)}}\right) = \left(\frac{7^{-2} * z^{-16}}{4^{-2} * y^{-14}}\right) = \frac{16y^{14}}{49z^{16}}$$

16.

$$y = -3(x-1)^{2} + 4$$

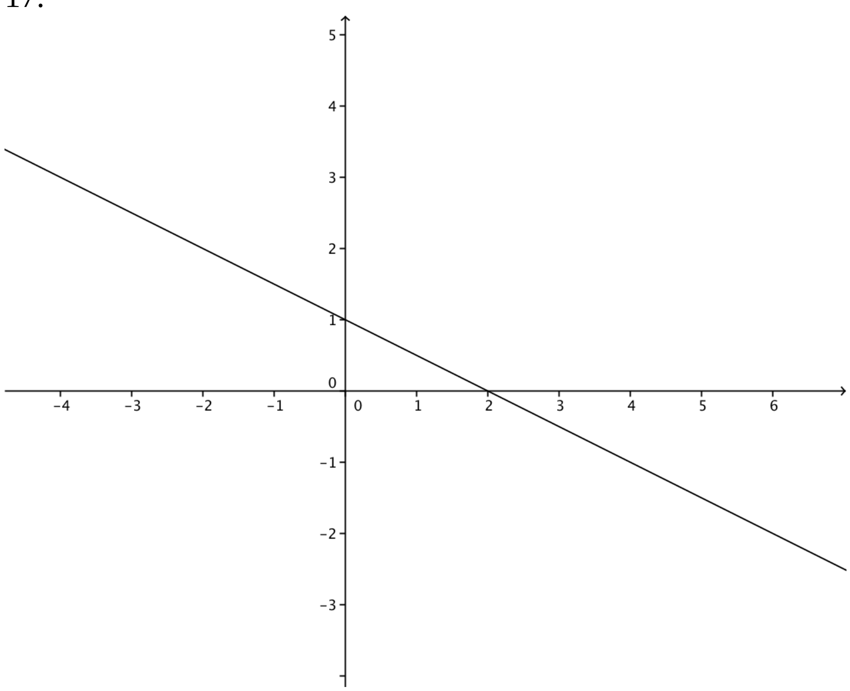
$$y = -3\left(x^{2} - x - x + 1\right) + 4$$

$$y = -3\left(x^{2} - 2x + 1\right) + 4$$

$$y = -3x^{2} + 6x - 3 + 4$$

$$y = -3x^{2} + 6x + 1$$

17. $f(x) = \dfrac{-x+2}{2}$



18.

$$h(t) = -\tfrac{1}{2}(g)t^2 + v_0 t + h_0$$

$$h(t) = -\tfrac{1}{2}(9.8)t^2 + (0)t + 10$$

$$h(t) = -4.9t^2 + 10$$

$$0 = -4.9t^2 + 10$$

$$-10 = -4.9t^2$$

$$2.0408 \approx t^2$$

$$\sqrt{2.0408} \approx \sqrt{t^2}$$

$$t \approx 1.4286$$

The ball will reach the ground at approximately 1.4286 seconds.

19.

$$\begin{cases} 3x + 4y = 9 \\ 9x + 12y = 27 \end{cases}$$

$$-3(3x + 4y = 9) \rightarrow -9x - 12y = -27$$

$$\begin{array}{l} 9x + 12y = 27 \\ \underline{-9x - 12y = -27} \\ 0 = 0 \end{array}$$

This system is consistent-dependent and has an infinite number of solutions.

Lesson 11.8
Box and Whisker Plots

1. The five-number summary are the numbers needed to construct a box-and-whisker plot: the minimum value, the lower quartile median (Q_1), the median, the upper quartile median (Q_2), and the maximum value.

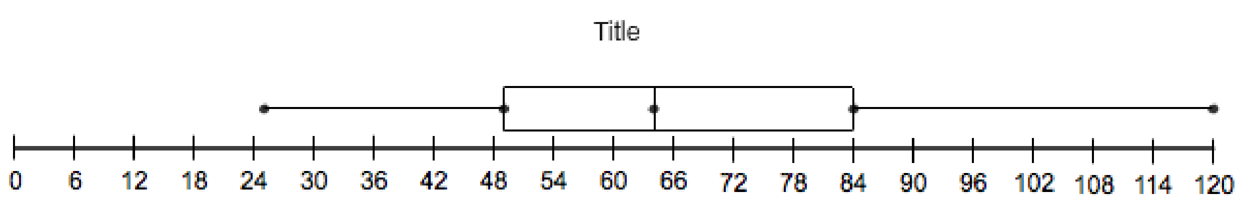
2. The purpose of a box-and-whisker plot is to display data. It shows how the data are dispersed around a median, without showing specific values. It is often used when the number of data values is large or when two or more data sets are being compared.

3. A disadvantage of representing data with a box-and-whisker plot is that it does not show distribution in as much detail as a stem-and-leaf plot or a histogram. It does not show any specific data values.

4.

$$25, 28, 34, 35, 35, 35, 40, 40, 42, 47,$$
$$49, 50, 50, 50, 50, 55, 55, 58, 60, 60,$$
$$64, 65, 70, 70, 70, 75, 75, 80, 80, 80,$$
$$84, 85, 90, 90, 93, 95, 95, 100, 110, 120$$

$$Minimun = 25$$
$$Q_1 = 48$$
$$Median = 62$$
$$Q_3 = 82$$
$$Maximum = 120$$



5.

(a) If the pass mark is 65% then 20 students would be allow in the class.
(b) If the pass mark is 60% then 10 students would be allow in the class.

6.

5, 6, 6, 7, 7, 7, 8, 8, 8, 8,
8, 9, 9, 9, 9, 9, 10, 10, 10, 10,
10, 10, 10, 10, 10, 11, 11, 11, 11, 12,
12, 12, 12, 12, 12, 12, 12, 12, 13, 13,
13, 13, 14, 14, 14, 15, 15, 15, 17, 17

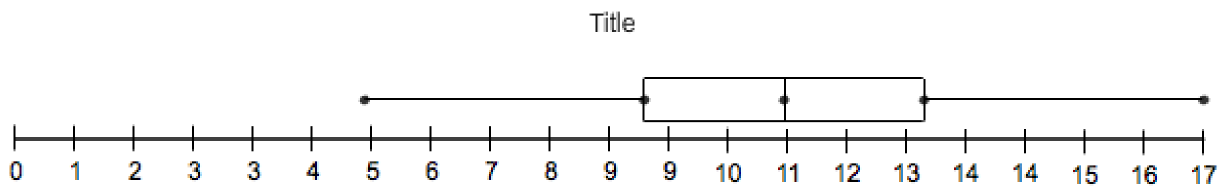
$Minimun = 5$

$Q_1 = 9$

$Median = 10.5$

$Q_3 = 12$

$Maximum = 17$



Range = 12
Interquartile Range = 4

7.

Girls

- Minimum = 1.7
- Q_1 = 2.5
- Median = 2.9
- Q_3 = 3.3
- Maximum = 4.2

The girls have a higher median and their lower 50% is higher than the boys' lower 50%.

Boys

- Minimum = 1.5
- Q_1 = 2
- Median = 2.5
- Q_3 = 3.5
- Maximum = 5.2

The boys have a higher maximum and their higher 50% is greater than the girls' higher 50%.

8.

$49, 49, 49, 50, 50, 51, 52, 53, 53, 54,$

$56, 57, 57, 57, 58, 59, 67$

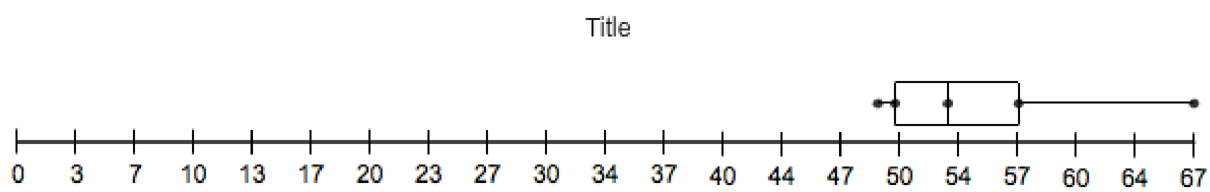
$Minimum = 49$

$Q_1 = 50$

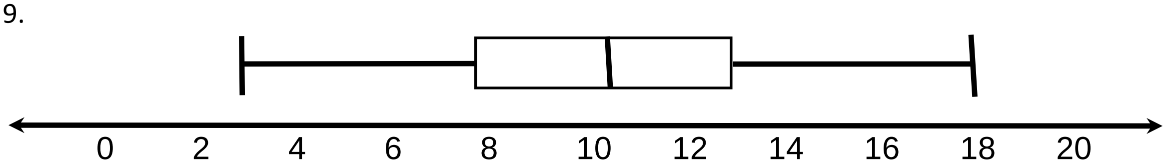
$Median = 53$

$Q_3 = 57$

$Maximum = 67$



9.



10.

Texas

Minimum = 5
Q_1 = 13
Median = 16
Q_3 = 19.5
Maximum = 35
This data has a large number of outliers. It is harder to make conclusions when this is true. The interquartile range is higher than in California.

California

Minimum = 6
Q_1 = 9.5
Median = 12
Q_3 = 15.5
Maximum = 22
This data is more centered. The distribution is smaller. The interquartile range and the maximum are lower than Texas, although the minimum is higher.

11. First we must compute the interquartile range (IQR) and determine what the minimums for outliers are

$IQR = Q_3 - Q_1$

$Q_3 = 68$

$Q_1 = 39$

$68 - 39 = 29$

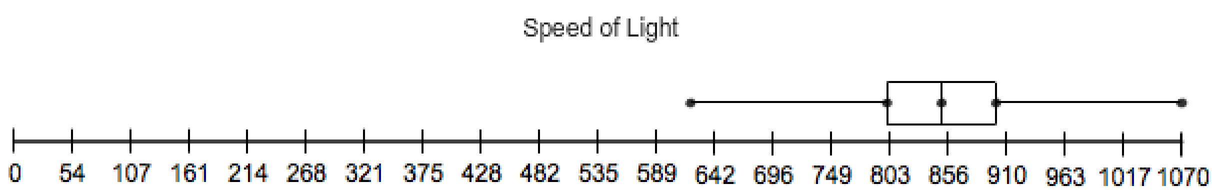
$Outlier = 1.5 * IQR$

$IQR = 29$

$1.5 * 29 = 43.5$

Based on this we know that any point 43.5 units less than 39 (Q_1) or 43.5 units more than 68 (Q_3) is an outlier. Looking at the choices:

(a) 78 is only 10 degrees more than 68: NOT an outlier
(b) -8 is 47 degrees less than 39: OUTLIER
(c) 94 is only 26 degrees more than 68: NOT an outlier
(d) 106 is only 3 degrees more than 68: NOT an outlier.
The answer is b.



12.

13.
Possible Answers:
The median of the data is 134.
The minimum is 100.
The maximum is 195.
25% of the values fall between 100 and 121.
25% of the values fall between 121 and 134.
25% of the values fall between 134 and 152.
25% of the values fall between 152 and 195.

14.

(a) Males

$Q_1 = 28$

$Q_3 = 68$ Between $28 and $68.

Females

$Q_1 = 22$

$Q_3 = 58$ Between $22 and $58

(b) $42 for males and $46 for females are the median of each data set.

(c) The middle 50% of males spent more on school lunches than the middle 50% of females. The males spent more overall on lunch than the females.

15. Choice 2

Mixed Review

16.

$Mean = average = \overline{x}$

$$\overline{x} = \frac{\begin{array}{l}63450 + 45502 + 63450 + 51769 + 63450 + \\ 35120 + 45502 + 63450 + 31100 + 42216 + \\ 49108 + 63450 + 37904\end{array}}{13}$$

$$\overline{x} = \frac{655471}{13}$$

$$\overline{x} = 50420.85$$

$Median = middle\#$

$Median = 31100, 35120, 37904, 42216, 45502, 45502, (49108), 51769, 63450, 63450, 63450, 63450, 63450$

$Median = 49108$

$Mode = \#\, appears >$

$Mode = 31100, 35120, 37904, 42216, 45502, 45502, 49108, 51769, 63450, 63450, 63450, 63450, 63450$

$Mode = 63450$

$Range = greatest - least$

$Range = 63450 - 31100$

$Range = 32350$

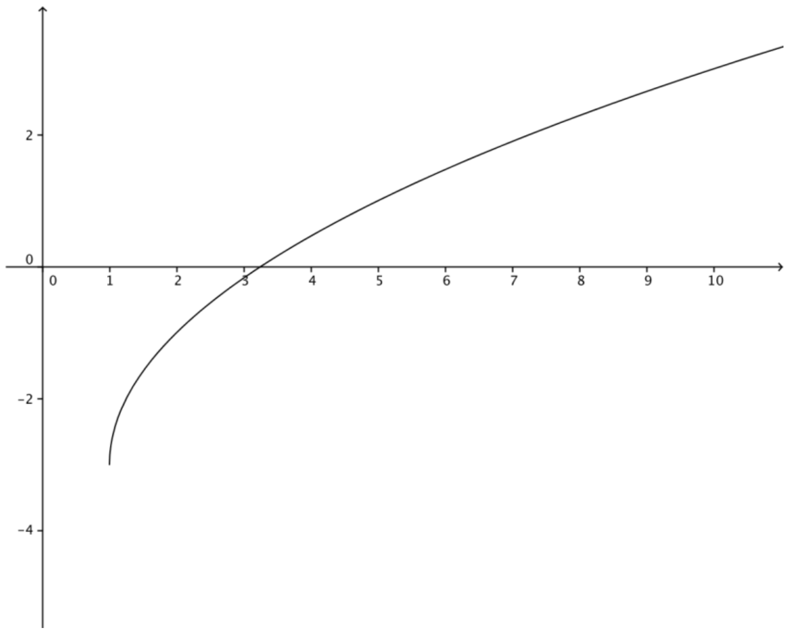
The mean is $50,420.85

The median is $49,108

The mode is $63,450

The range is $32,350

17. $g(x) = 2\sqrt{x-1} - 3$



18. $\sqrt{x+6} > 18$

19.

$$6(y-11)+9 = \tfrac{1}{3}(27+3y)-16$$
$$6y-66+9 = 9+y-16$$
$$6y-57 = y-7$$
$$5y = 50$$
$$y = 10$$

Check:

$$6(y-11)+9 = \tfrac{1}{3}(27+3y)-16$$
$$6(10-11)+9 = \tfrac{1}{3}(27+3(10))-16$$
$$6(-1)+9 = \tfrac{1}{3}(57)-16$$
$$-6+9 = 19-16$$
$$3 = 3$$

20.

(a)

$pizza = p$

$cookiedough = c$

$5p + 4c > 550$

(b)

5p and 4Cc must each be set equal to values that add up to some number greater than 550, and these values must be evenly divisible by 5 (for p) and 4 (for c).

For example, set 5p=300 and 4c=300, so that p=60 and c=75. 5p + 4c will then equal 600 (which is >550).

Select other values for 5P and 4C that meet the requirements (evenly divisible by the respective coefficients and having a sum > 550).

21.

$x + 2y = 10$

$2y = -x + 10$

$y = -\frac{1}{2}x + 5$

$m = -\frac{1}{2}$

$point = (2,1)$

$y - 1 = -\frac{1}{2}(x - 2)$

Lesson 11.9
Chapter 11 Review

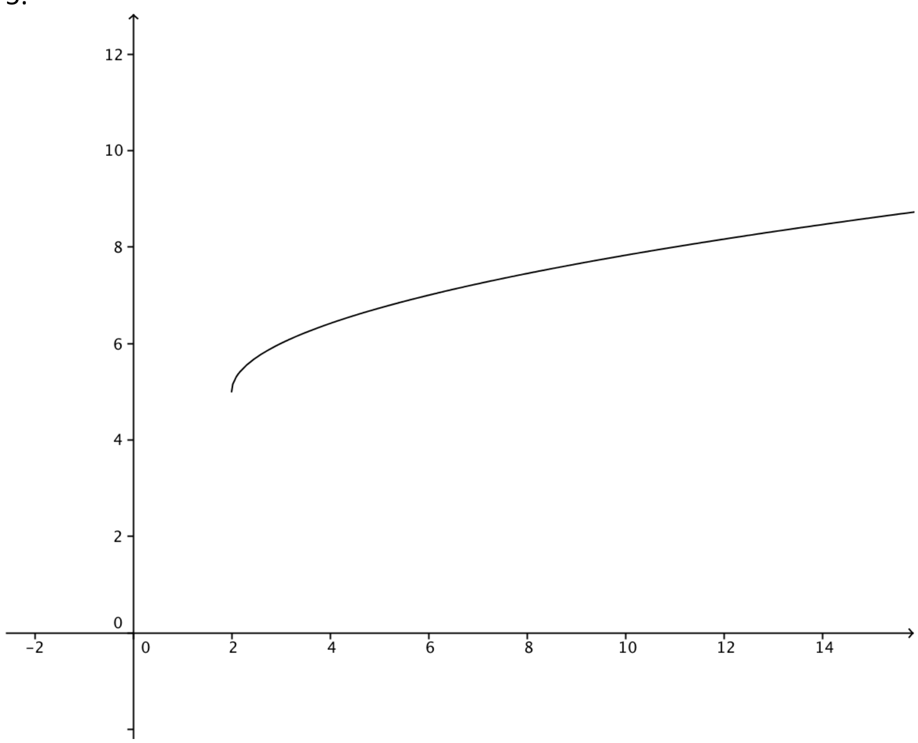
1. This function originates at (0,7) instead of (0,0) and all the y-values are 7 units greater than the values of the parent function.

2. This function originates at (-3,0) instead of (0,0) and all the x-values are 3 units less than the values of the parent function.

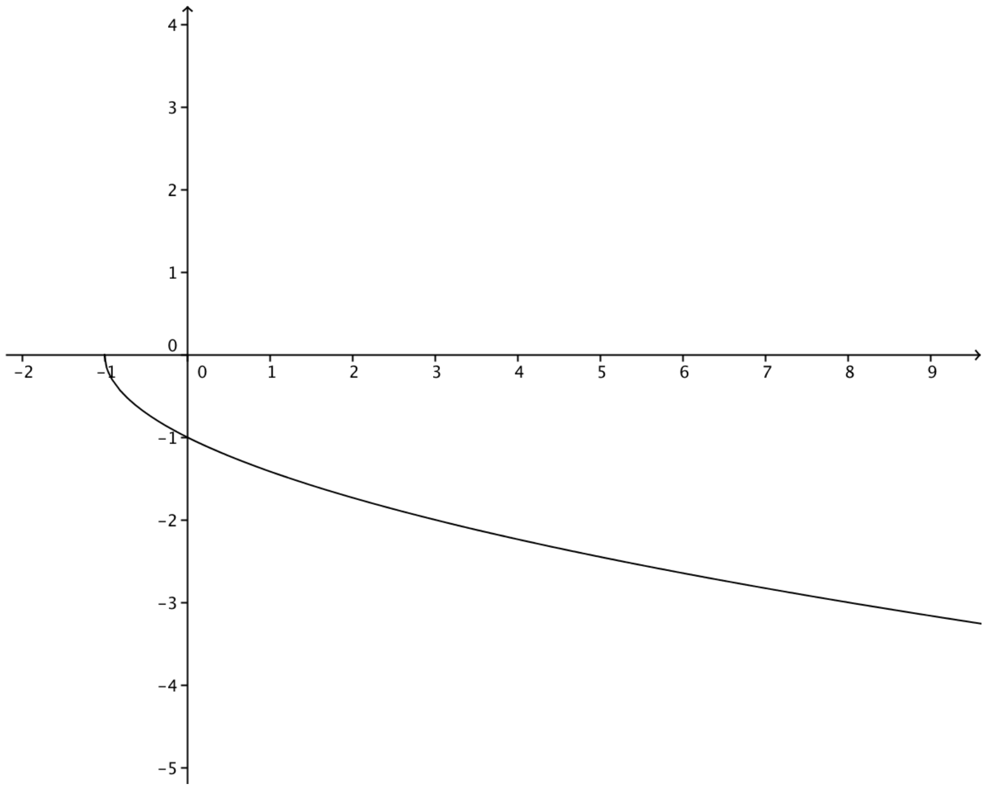
3. This function is the reflection of the parent function around the x-axis. All the y-values will be the negative of the values of the parent function.

4. This function originates at (1,3) instead of (0,0). All the x-values are 1 unit greater than the values of the parent function and all the y-values are 3 units greater than the values of the parent function.

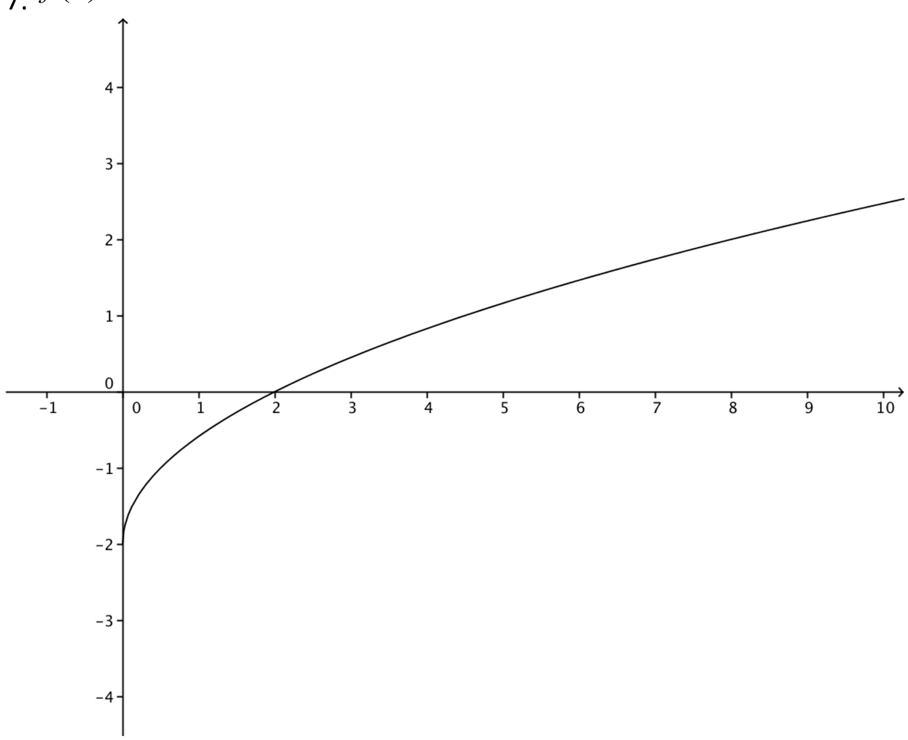
5. $f(x)=\sqrt{x-2}+5$



6. $g(x) = -\sqrt{x+1}$



7. $f(x) = \sqrt{2x} - 2$



8.

$$\sqrt{\frac{3}{7}} \times \sqrt{\frac{14}{27}} = \sqrt{\frac{3}{7} \times \frac{14}{27}} = \sqrt{\frac{2}{9}}$$

9.

$$\sqrt{5} \cdot \sqrt{7} = \sqrt{35}$$

10.

$$\sqrt{11} \times \sqrt[3]{11} = 11^{\frac{1}{2}} \times 11^{\frac{1}{3}} = 11^{\frac{5}{6}} = \sqrt[6]{11^5} = \sqrt[6]{161051}$$

11.

$$\frac{\sqrt{18}}{\sqrt{2}} = \frac{\sqrt{18}}{\sqrt{2}} * \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{36}}{2} = \frac{\pm 6}{2} = \pm 3$$

12.

$$8\sqrt[3]{4} + 11\sqrt[3]{4} = (8 + 11)\sqrt[3]{4} = 19\sqrt[3]{4}$$

13.

$$5\sqrt{80} - 12\sqrt{5}$$

$$5 \cdot \sqrt{16} \cdot \sqrt{5} - 12\sqrt{5}$$

$$\pm 20\sqrt{5} - 12\sqrt{5}$$

$$20\sqrt{5} - 12\sqrt{5} = 8\sqrt{5}$$

$$-20\sqrt{5} - 12\sqrt{5} = -32\sqrt{5}$$

14.

$$\sqrt{10} + \sqrt{2} = \sqrt{5} \cdot \sqrt{2} + \sqrt{2} = \left(1 + \sqrt{5}\right)\sqrt{2}$$

15.

$$\sqrt{24} - \sqrt{6} = \sqrt{4} \cdot \sqrt{6} - \sqrt{6} = 2\sqrt{6} - \sqrt{6} = \sqrt{6}$$

16.

$$\sqrt[3]{27} + \sqrt[4]{81} = 3 + 3 = 6$$

17.

$$4\sqrt{3}\cdot 2\sqrt{6} = 8\sqrt{18}$$

18.

$$\sqrt[3]{3}\times\sqrt{7}$$

Cannot be simplified

19.

$$6\sqrt{72} = 6\cdot\sqrt{36}\cdot\sqrt{2} = 6\cdot\pm 6\cdot\sqrt{2} = \pm 36\sqrt{2}$$

20.

$$7\sqrt{\left(\frac{40}{49}\right)} = 7\cdot\frac{\sqrt{40}}{\sqrt{49}} = 7\cdot\frac{\sqrt{40}}{\pm 7} = \pm\sqrt{40} = \pm\left(\sqrt{4}\cdot\sqrt{10}\right) = \pm 2\sqrt{10}$$

21.

$$\frac{5}{\sqrt{75}} = \frac{5}{\sqrt{75}} * \frac{\sqrt{75}}{\sqrt{75}} = \frac{5\sqrt{75}}{75} = \frac{\sqrt{75}}{15} = \frac{\sqrt{3}\cdot\sqrt{25}}{15} = \frac{\pm 5\sqrt{3}}{15} = \frac{\pm\sqrt{3}}{3}$$

22.

$$\frac{\sqrt{45}}{\sqrt{5}} = \frac{\sqrt{45}}{\sqrt{5}} * \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{225}}{5} = \frac{\pm 15}{5} = \pm 3$$

23.

$$\frac{3}{\sqrt[3]{3}} = \frac{3}{\sqrt[3]{3}} * \frac{\sqrt[3]{3^2}}{\sqrt[3]{3^2}} = \frac{3\cdot\sqrt[3]{3^2}}{3} = \sqrt[3]{3^2} = \sqrt[3]{9}$$

24.

$$8\sqrt{10} - 3\sqrt{40}$$

$$8\sqrt{10} - \left(3\cdot\sqrt{4}\cdot\sqrt{10}\right)$$

$$8\sqrt{10} - \left(3\cdot\pm 2\cdot\sqrt{10}\right)$$

$$8\sqrt{10} - \pm 6\sqrt{10}$$

$$8\sqrt{10} - 6\sqrt{10} = (8-6)\sqrt{10} = 2\sqrt{10}$$

$$8\sqrt{10} - (-6)\sqrt{10} = (8+6)\sqrt{10} = 14\sqrt{10}$$

25.

$$\sqrt{27}+\sqrt{3}$$

$$\left(\sqrt{9}\cdot\sqrt{3}\right)+\sqrt{3}$$

$$\pm 3\sqrt{3}+\sqrt{3}$$

$$3\sqrt{3}+\sqrt{3}=4\sqrt{3}$$

$$-3\sqrt{3}+\sqrt{3}=-2\sqrt{3}$$

26.

$$8=\sqrt[3]{2k}$$

$$8^3=\left(\sqrt[3]{2k}\right)^3$$

$$512=2k$$

$$k=256$$

Check

$$8=\sqrt[3]{2k}$$

$$8=\sqrt[3]{2(256)}$$

$$8=\sqrt[3]{512}$$

$$8=8$$

27.

$x=\sqrt{7x}$

$x^2=\left(\sqrt{7x}\right)^2$

$x^2=7x$

$x^2-7x=0$

$x^2-7x+12.25=12.25$

$(x-3.5)^2=12.25$

$\sqrt{(x-3.5)^2}=\sqrt{12.25}$

$x-3.5=\pm 3.5$

$x=7$

$x=0$

Check

$x=\sqrt{7x}$

$(7)=\sqrt{7(7)}$

$7=\sqrt{49}$

$7=7$

$x=\sqrt{7x}$

$0=\sqrt{7(0)}$

$0=0$

28.

$\sqrt{2+2m}=\sqrt{4-m}$

$\left(\sqrt{2+2m}\right)^2=\left(\sqrt{4-m}\right)^2$

$2+2m=4-m$

$3m=2$

$m=\frac{2}{3}$

Check

$\sqrt{2+2m}=\sqrt{4-m}$

$\sqrt{2+2\left(\frac{2}{3}\right)}=\sqrt{4-\left(\frac{2}{3}\right)}$

$\sqrt{2+\frac{4}{3}}=\sqrt{4-\left(\frac{2}{3}\right)}$

$\sqrt{\frac{10}{3}}=\sqrt{\frac{10}{3}}$

29.

$\sqrt[4]{35-2x}=-1$

$\left(\sqrt[4]{35-2x}\right)^4=\left(-1\right)^4$

$35-2x=1$

$-2x=-34$

$2x=34$

$x=17$

Check

$\sqrt[4]{35-2x}=-1$

$\sqrt[4]{35-2\left(17\right)}=-1$

$\sqrt[4]{35-34}=-1$

$\sqrt[4]{1}=-1$

$-1=-1$

30.

$14 = 6 + \sqrt{10 - 6x}$

$8 = \sqrt{10 - 6x}$

$8^2 = \left(\sqrt{10 - 6x}\right)^2$

$64 = 10 - 6x$

$54 = -6x$

$x = -9$

Check

$14 = 6 + \sqrt{10 - 6x}$

$14 = 6 + \sqrt{10 - 6(-9)}$

$14 = 6 + \sqrt{10 + 54}$

$14 = 6 + \sqrt{64}$

$14 = 6 + 8$

$14 = 14$

31.

$4 + \sqrt{\tfrac{n}{3}} = 5$

$\sqrt{\tfrac{n}{3}} = 1$

$\left(\sqrt{\tfrac{n}{3}}\right)^2 = 1^2$

$\tfrac{n}{3} = 1$

$n = 3$

Check

$4 + \sqrt{\tfrac{n}{3}} = 5$

$4 + \sqrt{\tfrac{3}{3}} = 5$

$4 + \sqrt{1} = 5$

$4 + 1 = 5$

$5 = 5$

32.

$$\sqrt{-9-2x}=\sqrt{-1-x}$$
$$\left(\sqrt{-9-2x}\right)^2=\left(\sqrt{-1-x}\right)^2$$
$$-9-2x=-1-x$$
$$-x=8$$
$$x=-8$$

Check

$$\sqrt{-9-2x}=\sqrt{-1-x}$$
$$\sqrt{-9-2(-8)}=\sqrt{-1-(-8)}$$
$$\sqrt{-9+16}=\sqrt{7}$$
$$\sqrt{7}=\sqrt{7}$$

33.

$$-2=\sqrt[3]{t-6}$$
$$(-2)^3=\left(\sqrt[3]{t-6}\right)^3$$
$$-8=t-6$$
$$t=-2$$

Check

$$-2=\sqrt[3]{t-6}$$
$$-2=\sqrt[3]{(-2)-6}$$
$$-2=\sqrt[3]{-8}$$
$$-2=-2$$

34.

$$5\sqrt{10} = 6\sqrt{w}$$

$$\left(5\sqrt{10}\right)^2 = \left(6\sqrt{w}\right)^2$$

$$25 * 10 = 36w$$

$$250 = 36w$$

$$w \approx 6.9444$$

Check

$$5\sqrt{10} = 6\sqrt{w}$$

$$5\sqrt{10} = 6\sqrt{6.9444}$$

$$5 * 3.1623 = 6 * 2.6352$$

$$15.8115 \approx 15.8112$$

35.

$$\sqrt{x^2+3x}=2$$
$$\left(\sqrt{x^2+3x}\right)^2=2^2$$
$$x^2+3x=4$$
$$x^2+3x+2.25=4+2.25$$
$$(x+1.5)^2=6.25$$
$$\sqrt{(x+1.5)^2}=\sqrt{6.25}$$
$$x+1.5=\pm 2.5$$
$$x+1.5=2.5$$
$$x=1$$
$$x+1.5=-2.5$$
$$x=-4$$

Check

$$\sqrt{x^2+3x}=2$$
$$\sqrt{1^2+3(1)}=2$$
$$\sqrt{1+3}=2$$
$$\sqrt{4}=2$$
$$2=2$$

$$\sqrt{x^2+3x}=2$$
$$\sqrt{(-4)^2+3(-4)}=2$$
$$\sqrt{16-12}=2$$
$$\sqrt{4}=2$$
$$2=2$$

36.

$\sqrt[4]{t} = 5$

$\left(\sqrt[4]{t}\right)^4 = 5^4$

$t = 625$

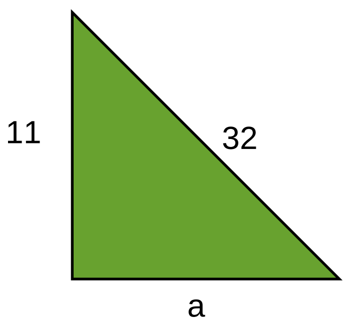
Check

$\sqrt[4]{t} = 5$

$\sqrt[4]{625} = 5$

$5 = 5$

37.



$a^2 + b^2 = c^2$

$a^2 + 11^2 = 32^2$

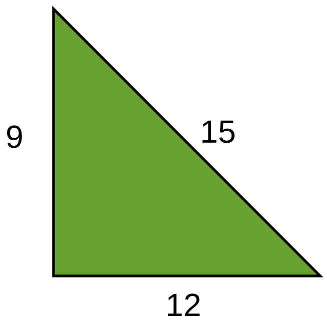
$a^2 + 121 = 1024$

$a^2 = 903$

$a = \sqrt{903}$

$a \approx 30.05$

38.



$a^2 + b^2 = c^2$

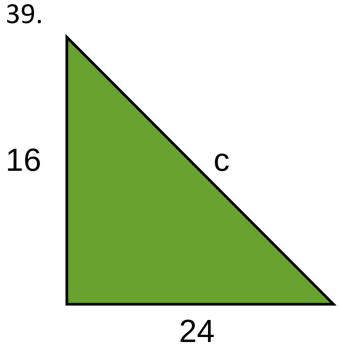
$9^2 + 12^2 = 15^2$

$81 + 144 = 225$

$225 = 225$

Yes 9,12, 15 is a right triangle.
(Also 9,12,15 is a version of a 3,4,5 right triangle.)

39.



$a^2 + b^2 = c^2$

$16^2 + 24^2 = c^2$

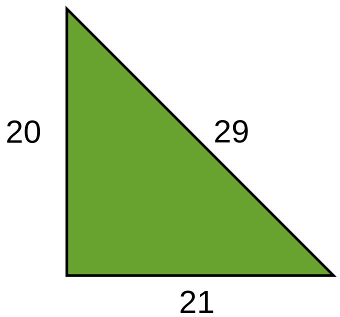
$256 + 576 = c^2$

$c^2 = 832$

$c = \sqrt{832}$

$c \approx 28.8444$

40.



$a^2 + b^2 = c^2$

$20^2 + 21^2 = 29^2$

$400 + 441 = 841$

$841 = 841$

Yes 20,21,29 is a right triangle.

41.

$(0,2)\,\&\,(-5,4)$

$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

$d = \sqrt{(4-2)^2 + (-5-0)^2}$

$d = \sqrt{2^2 + (-5)^2}$

$d = \sqrt{4 + 25}$

$d = \sqrt{29}$

$(x_m, y_m) = \frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}$

$(x_m, y_m) = \frac{(0 + (-5))}{2}, \frac{(2 + 4)}{2}$

$(x_m, y_m) = \frac{-5}{2}, \frac{6}{2}$

$(x_m, y_m) = (-2.5, 3)$

42.

$(7,-3)\,\&\,(4,-3)$

$d=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$

$d=\sqrt{(-3-(-3))^2+(4-7)^2}$

$d=\sqrt{0^2+(-3)^2}$

$d=\sqrt{0+9}$

$d=\sqrt{9}$

$d=3$

$(x_m,y_m)=\frac{(x_1+x_2)}{2},\frac{(y_1+y_2)}{2}$

$(x_m,y_m)=\frac{(7+4)}{2},\frac{(-3+(-3))}{2}$

$(x_m,y_m)=\frac{11}{2},\frac{-6}{2}$

$(x_m,y_m)=(5.5,-3)$

43.

$(4,6)\,\&\,(-3,0)$

$d=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$

$d=\sqrt{(0-6)^2+(-3-4)^2}$

$d=\sqrt{(-6)^2+(-7)^2}$

$d=\sqrt{36+49}$

$d=\sqrt{85}$

$(x_m,y_m)=\frac{(x_1+x_2)}{2},\frac{(y_1+y_2)}{2}$

$(x_m,y_m)=\frac{(4+(-3))}{2},\frac{(6+0)}{2}$

$(x_m,y_m)=\frac{1}{2},\frac{6}{2}$

$(x_m,y_m)=(0.5,3)$

44.

$$(8,-3) \,\&\, (-7,-6)$$

$$d=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$$

$$d=\sqrt{(-6-(-3))^2+(-7-8)^2}$$

$$d=\sqrt{(-3)^2+(-15)^2}$$

$$d=\sqrt{9+225}$$

$$d=\sqrt{234}$$

$$d=\sqrt{9}\cdot\sqrt{26}$$

$$d=3\sqrt{26}$$

$$(x_m,y_m)=\frac{(x_1+x_2)}{2},\frac{(y_1+y_2)}{2}$$

$$(x_m,y_m)=\frac{(8+(-7))}{2},\frac{(-3+(-6))}{2}$$

$$(x_m,y_m)=\frac{1}{2},\frac{-9}{2}$$

$$(x_m,y_m)=(0.5,-4.5)$$

45.

$(-8,-7)\,\&\,(6,5)$

$d=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$

$d=\sqrt{(5-(-7))^2+(6-(-8))^2}$

$d=\sqrt{(12)^2+(14)^2}$

$d=\sqrt{144+196}$

$d=\sqrt{340}$

$d=\sqrt{4}\cdot\sqrt{85}$

$d=2\sqrt{85}$

$(x_m,y_m)=\dfrac{(x_1+x_2)}{2},\dfrac{(y_1+y_2)}{2}$

$(x_m,y_m)=\dfrac{(-8+6)}{2},\dfrac{(-7+5)}{2}$

$(x_m,y_m)=\dfrac{-2}{2},\dfrac{-2}{2}$

$(x_m,y_m)=(-1,-1)$

46.

$$(-6,6)\ \&\ (0,8)$$

$$d=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$$

$$d=\sqrt{(8-6)^2+(0-(-6))^2}$$

$$d=\sqrt{(2)^2+(6)^2}$$

$$d=\sqrt{4+36}$$

$$d=\sqrt{40}$$

$$d=\sqrt{4}\cdot\sqrt{10}$$

$$d=2\sqrt{10}$$

$$(x_m,y_m)=\frac{(x_1+x_2)}{2},\frac{(y_1+y_2)}{2}$$

$$(x_m,y_m)=\frac{(-6+0)}{2},\frac{(6+8)}{2}$$

$$(x_m,y_m)=\frac{-6}{2},\frac{14}{2}$$

$$(x_m,y_m)=(-3,7)$$

47.

$(2,6)(x,4)$

$d=6$

$d=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$

$6=\sqrt{(4-6)^2+(x-2)^2}$

$6=\sqrt{-2^2+(x-2)^2}$

$6=\sqrt{4+x^2-4x+4}$

$6=\sqrt{x^2-4x+8}$

$6^2=\left(\sqrt{x^2-4x+8}\right)^2$

$36=x^2-4x+8$

$28=x^2-4x$

$28+4=x^2-4x+4$

$32=(x-2)^2$

$\sqrt{32}=\sqrt{(x-2)^2}$

$\sqrt{32}=x-2$

$x=2+\sqrt{32}$

$x\approx 2\pm 5.6569$

$x\approx 2+5.6569$

$x\approx 7.6569$

$x\approx 2-5.6569$

$x\approx -3.6569$

The two possibilities are approximately (7.6959,4) and (-3.6959,4)

48.

$$(9,0)(x,3)$$
$$d=5$$
$$d=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$$
$$5=\sqrt{(3-0)^2+(x-9)^2}$$
$$5=\sqrt{3^2+(x-9)^2}$$
$$5=\sqrt{9+x^2-18x+81}$$
$$5=\sqrt{x^2-18x+90}$$
$$5^2=\left(\sqrt{x^2-18x+90}\right)^2$$
$$25=x^2-18x+90$$
$$-65=x^2-18x$$
$$-65+81=x^2-18x+81$$
$$16=(x-9)^2$$
$$\sqrt{16}=\sqrt{(x-9)^2}$$
$$\pm4=x-9$$
$$x=9\pm4$$

$$x=9+4$$
$$x=13$$

$$x=9-4$$
$$x=5$$

The two possibilities are (13,3) and (5,3)

49.

$$\left(x_m, y_m\right) = \frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}$$

$$\left(7.5, 1.5\right) = \frac{(-5 + x)}{2}, \frac{(-6 + y)}{2}$$

$$7.5 = \frac{(-5 + x)}{2}$$

$$15 = -5 + x$$

$$20 = x$$

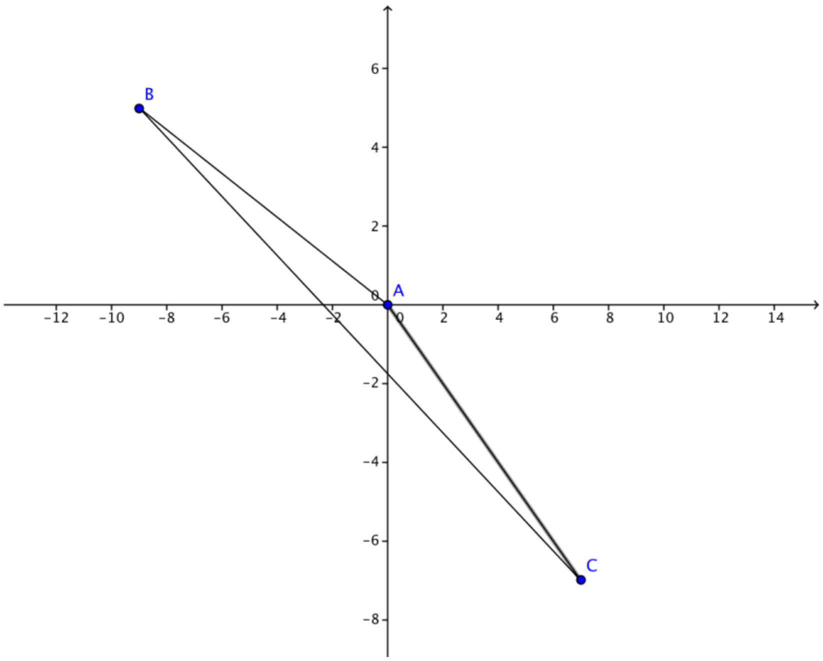
$$1.5 = \frac{(-6 + y)}{2}$$

$$3 = -6 + y$$

$$9 = y$$

The other end of the segment is (20,9)

50.



Point A (0,0) is the center of town.
Point B (-9,5) is Maggie's first stop.
Point C (7,-7) is Maggie's final stop.

$(7,-7)\&(0,0)$

$d=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$

$d=\sqrt{(0-(-7))^2+(0-7)^2}$

$d=\sqrt{7^2+(-7)^2}$

$d=\sqrt{49+49}$

$d=\sqrt{98}$

$d=\sqrt{49}\cdot\sqrt{2}=7\sqrt{2}$

$(x_m,y_m)=\frac{(x_1+x_2)}{2},\frac{(y_1+y_2)}{2}$

$(x_m,y_m)=\frac{(0+7)}{2},\frac{(0+(-7))}{2}$

$(x_m,y_m)=\frac{7}{2},\frac{-7}{2}$

$(x_m,y_m)=(3.5,-3.5)$

51.

$SA = 6s^2$

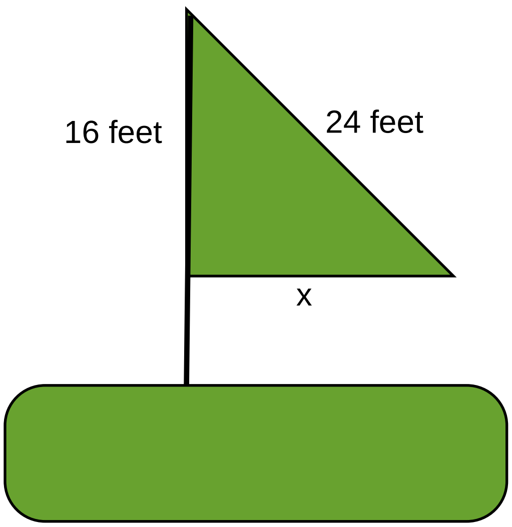
$337.50 = 6s^2$

$56.25 = s^2$

$\sqrt{56.25} = \sqrt{s^2}$

$s = 7.5$

52.



To find the area we need to multiply 1/2 times 16 feet times x. We will use pythagorean theorem to find x.

$a^2 + b^2 = c^2$

$16^2 + x^2 = 24^2$

$256 + x^2 = 576$

$x^2 = 320$

$x = \sqrt{320}$

$x \approx 17.8885$

$A = \frac{1}{2}lh$

$A = \frac{1}{2}(17.8885)(16)$

$A = 143.108$

The area of the sail is approximately 143.108 square feet.

53.

$$70 = \frac{63+65+80+84+73+x}{6}$$

$$70 = \frac{365+x}{6}$$

$$420 = 365 + x$$

$$x = 55$$

54.

$$Mean = \overline{x} = average$$

$$\overline{x} = \frac{\begin{array}{l}11+12+11+11+11+\\13+13+12+12+11+\\12+13+13+12+13+\\11+12+12+13\end{array}}{19}$$

$$\overline{x} = \frac{228}{19}$$

$$\overline{x} = 12$$

$$Median = middle\#$$

$$Med = 11,11,11,11,11,11,12,12,12,(12),12,12,12,13,13,13,13,13,13$$

$$Med = 12$$

$$Mode = \#\, appears > times$$

$$Mode = 11,11,11,11,11,11,12,12,12,12,12,12,12,13,13,13,13,13,13$$

$$Mode = 12$$

$$Range = highest - lowest$$

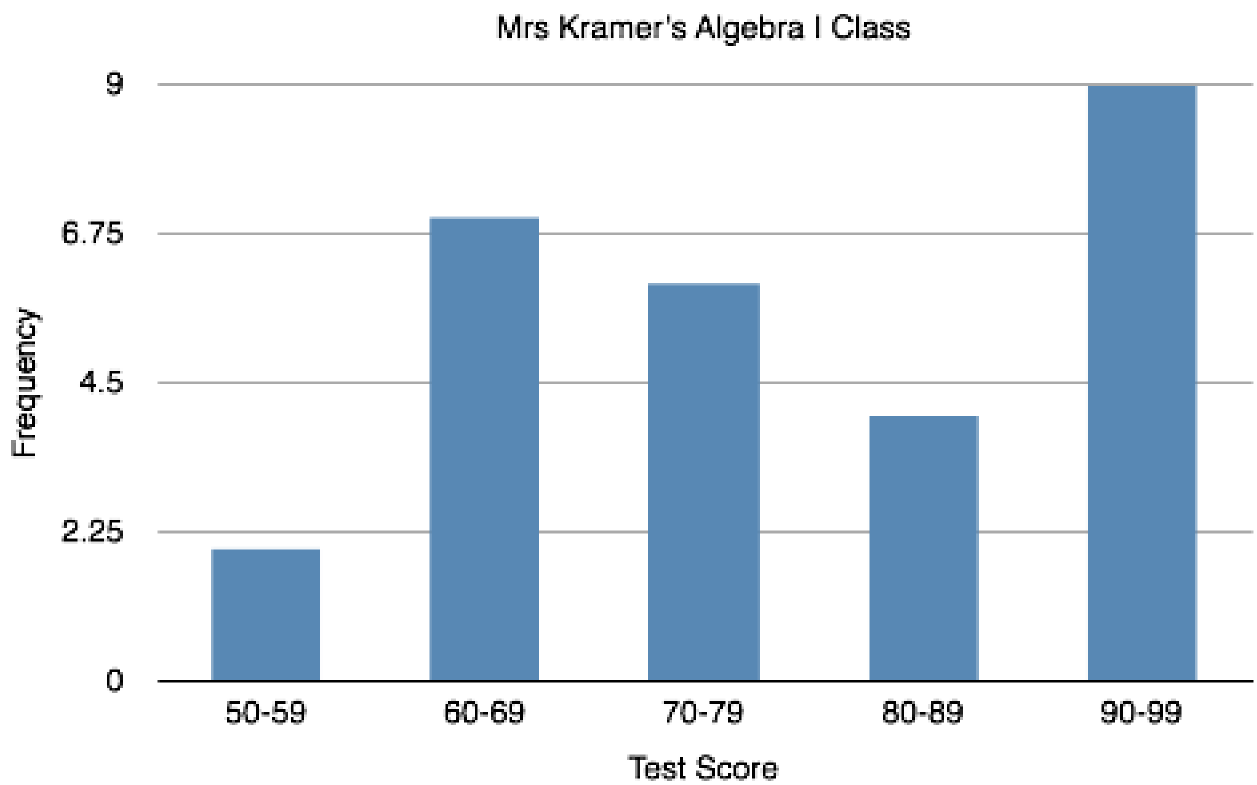
$$R = 13 - 11$$

$$R = 2$$

55.

1. Most likely the mean was used to describe this situation.
(Any large value data items raises the mean.)
2. The median should be used to describe this situation.
(The median is more accurate when there are very large or very small data items that don't fit with most of the data.)

56.

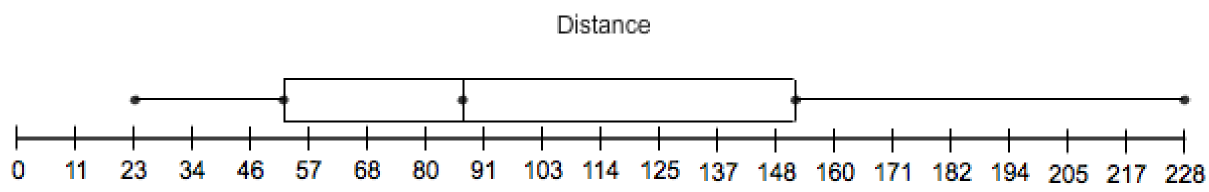


The mode based just on the data is 90. The mode based just on the histogram is 90-99.
From the data we can conclude that more students scored above average on the test than failed.

57.

Tips Earned	
Stem	**Leaf**
1	7
2	4,7
3	2,8
4	2
5	2,8
6	9
7	3

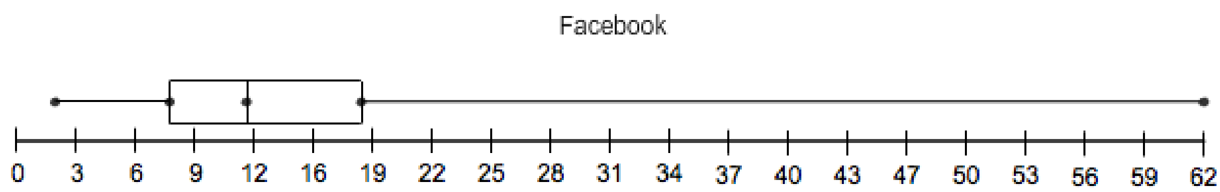
58.



(The number line on this plot has the values multiplied by 10.)

59. Disadvantages to a box-and-whisker plot are that it does not show any specific values for the data, nor does it tell how many values there are in the data.

60.



2 and 62 are outliers.
11 is the median value.
About half the students checked Facebook between 11 and 18 times that day.
Approximately 4 to 5 students checked Facebook between 2 and 7 times that day.
Approximately 4 to 5 students checked Facebook between 7 and 11 times that day.
Approximately 4 to 5 students checked Facebook between 11 and 18 times that day.
Approximately 4 to 5 students checked Facebook between 18 and 62 times that day.

61. An outlier causes a box-and-whisker plot to have a long whisker.

62. The median always represents: 4. The 50th percentile

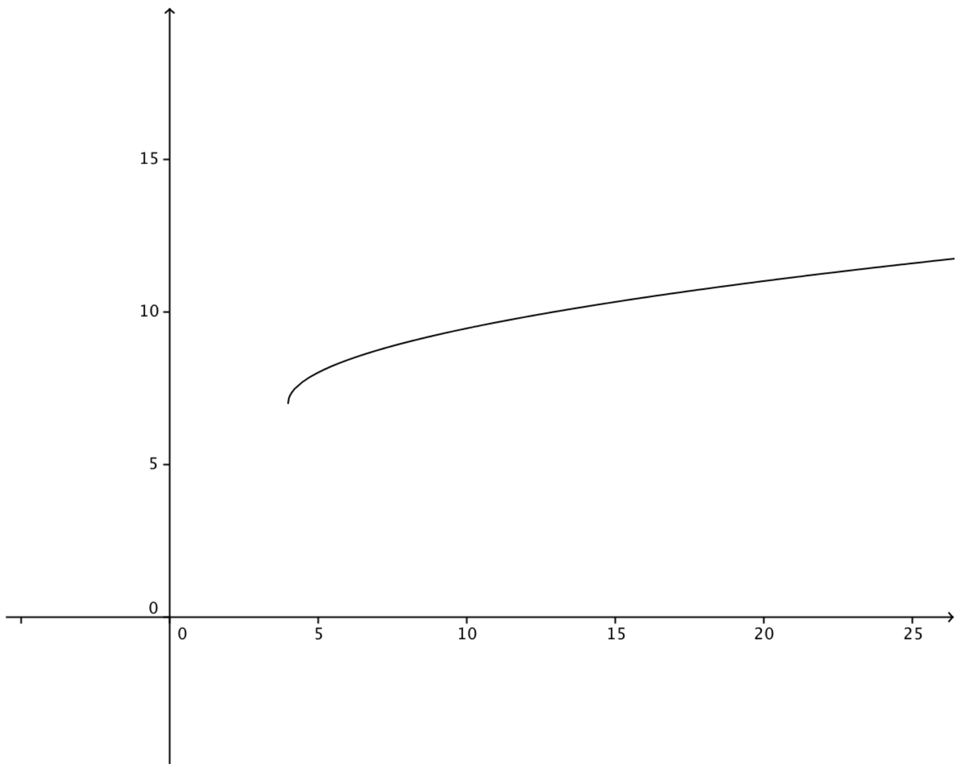
Lesson 11.10
Chapter 11 Test

1.
Stem-and-Leaf Plots: is an organization of numerical data into categories based on place value. For a stem-and-leaf plot, each number will be divided into two parts using place value. The stem is the left-hand column and will contain the digits in the largest place. The right-hand column will be the leaf and it will contain the digits in the smallest place.
One advantage is that they allow you to quickly view the median and mode of the data.
One disadvantage is that they require sorting the data manually.

Histograms: is a bar chart that describes a frequency distribution.
The horizontal axis of the histogram is separated into equal intervals. The vertical bars represent how many items are in each interval.
One advantage of histograms is that they are easy to read and allow use to make conclusions about the data quickly.
One disadvantage of histograms is that they only show frequency distribution and may not include individual data values.

Box-and-Whisker Plots: is another type of graph used to display data. It shows how the data are dispersed around a median. A box-and-whisker plot is a graph based upon medians. It shows the minimum value, the lower median, the median, the upper median and the maximum value of a data set. It is also known as a box plot.
One advantage of this type of graph is it often used when the number of data values is large or when two or more data sets are being compared.
One disadvantage is that it does not show specific values in the data. It does not show a distribution in as much detail as does a stem-and-leaf plot or a histogram.

2. $f(x) = 7 + \sqrt{x-4}$



Domain: $x \geq 4$

Range: $y \geq 7$

Origin = (4,7)

3. False: The upper quartile is the MEDIAN of the upper half of the data.

4. Domain: $x \geq 0$

5.

$$-6 = 2\sqrt[3]{c+5}$$

$$-3 = \sqrt[3]{c+5}$$

$$(-3)^3 = \left(\sqrt[3]{c+5}\right)^3$$

$$-27 = c+5$$

$$c = -32$$

Check

$$-6 = 2\sqrt[3]{c+5}$$

$$-6 = 2\sqrt[3]{-32+5}$$

$$-6 = 2\sqrt[3]{-27}$$

$$-6 = 2(-3)$$

$$-6 = -6$$

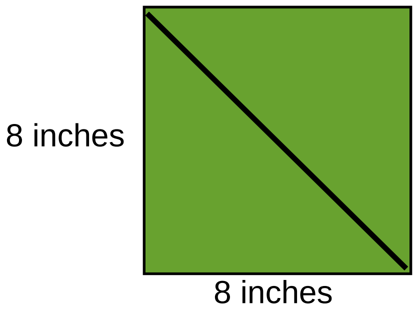
6.

$$\frac{4}{\sqrt{48}} = \frac{4}{\sqrt{48}} * \frac{4}{\sqrt{48}} = \frac{4\sqrt{48}}{48} = \frac{\sqrt{48}}{12} = \frac{\sqrt{16} * \sqrt{3}}{12} = \frac{\pm 4\sqrt{3}}{12} = \frac{\sqrt{3}}{3}$$

7.

$$\sqrt[3]{3} \times \sqrt[3]{81} = \sqrt[3]{243} = \sqrt[3]{27} * \sqrt[3]{9} = 3\sqrt[3]{9}$$

8.



The diagonal of a square baking dish creates the hypotenuse of a right triangle. Let's use Pythagorean Theorem.

$a^2 + b^2 = c^2$

$8^2 + 8^2 = c^2$

$64 + 64 = c^2$

$128 = c^2$

$c = \sqrt{128}$

$c = \sqrt{64} * \sqrt{2}$

$c = 8\sqrt{2}$

The length of the diagonal is $8\sqrt{2}$

Now let's find the area of the triangle created by this diagonal.

$A = \frac{1}{2}bh$

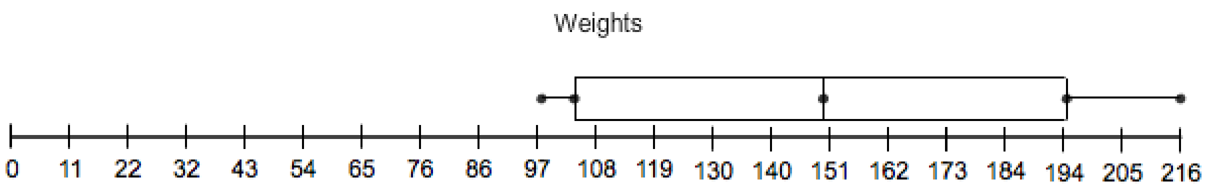
$A = \frac{1}{2}(8)(8)$

$A = 32$

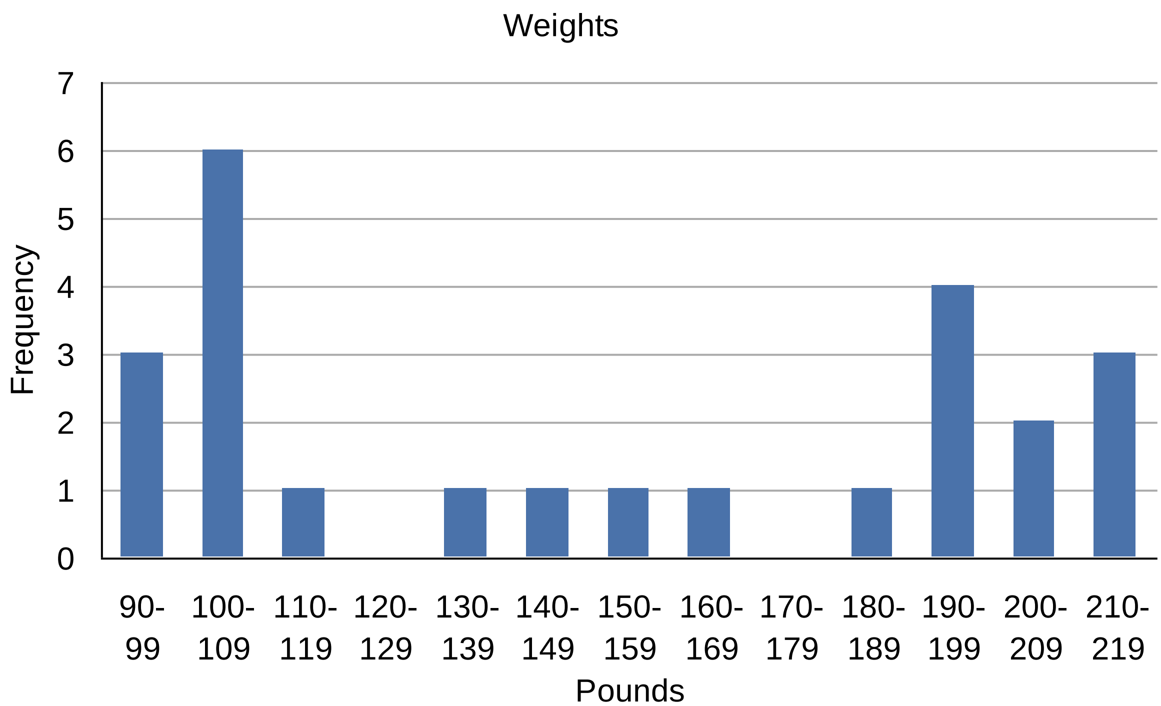
The area of half the pan is 32 square inches.

9.

1.



Weights	
Stem	**Leaf**
9	8, 8, 9
10	0, 0, 0, 4, 6, 8
11	6
12	-----
13	5
14	2
15	0
16	0
17	-----
18	0
19	5, 5, 5, 5
20	6, 6
21	0, 2, 6



2. Which method is better depends on what you are using the data for, but in general the stem and leaf plot gives the most detail without any difficulty reading it.

3. No: the lowest data is represented more than once and so is the highest data. The range is large, but accurate.

4. There is great variance in the weights of HS students with most of the students falling in the lower end or the higher end with less in the middle.

10.

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$d = \sqrt{(-2-(-9))^2 + (-6-5)^2}$$

$$d = \sqrt{7^2 + (-11)^2}$$

$$d = \sqrt{49 + 121}$$

$$d = \sqrt{170}$$

$$d \approx 13.0384$$

11.

1.

$$d=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$$
$$d=\sqrt{(80.224-70.269)^2+(25.79-43.665)^2}$$
$$d=\sqrt{9.955^2+(-17.875)^2}$$
$$d=\sqrt{99.102025+319.515625}$$
$$d=\sqrt{418.61765}$$
$$d\approx 20.4601$$

2.

$$(x_m,y_m)=\frac{(x_1+x_2)}{2},\frac{(y_1+y_2)}{2}$$
$$(x_m,y_m)=\frac{(43.665+25.79)}{2},\frac{(70.269+80.224)}{2}$$
$$(x_m,y_m)=\frac{69.455}{2},\frac{150.493}{2}$$
$$(x_m,y_m)=(34.7275,75.2465)$$

12.

$$s^2=3.5B^3$$
$$26^2=3.5B^3$$
$$676=3.5B^3$$
$$193.1429\approx B^3$$
$$B\approx\sqrt[3]{193.1429}$$
$$B\approx 5.7804$$

13.

$$(2,2)\ \&\ (5,y)$$
$$d=10$$
$$d=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$$
$$10=\sqrt{(2-y)^2+(2-5)^2}$$
$$10=\sqrt{(2-y)^2+(-3)^2}$$
$$10=\sqrt{y^2-4y+4+9}$$
$$10=\sqrt{y^2-4y+13}$$
$$10^2=\left(\sqrt{y^2-4y+13}\right)^2$$
$$100=y^2-4y+13$$
$$87=y^2-4y$$
$$87+4=y^2-4y+4$$
$$91=(y-2)^2$$
$$\sqrt{91}=\sqrt{(y-2)^2}$$
$$\sqrt{91}=y-2$$
$$y=2+\sqrt{91}$$
$$y\approx 2\pm 9.5394$$

$$y\approx 2+9.5394$$
$$y\approx 11.5394$$

$$y\approx 2-9.5394$$
$$y\approx -7.5394$$

The two possibilities are (5,11.5394) and (5,-7.5394)

14.

1.

$$Mean = \overline{x} = average$$

$$\overline{x} = \frac{9036 + 505 + 1870 + 826 + 628 + 187 + 452 + 1106 + 390 + 719}{10}$$

$$\overline{x} = \frac{15719}{10}$$

$$\overline{x} = 1571.9$$

$$Median = middle\#$$

$$Med = 187, 390, 452, 505, (628, 719), 826, 1106, 1870, 9036$$

$$Med = \frac{628 + 719}{2}$$

$$Med = \frac{1347}{2}$$

$$Med = 673.5$$

$$Mode = \#\, appears > times$$

$$Mode = 187, 390, 452, 505, 628, 719, 826, 1106, 1870, 9036$$

$$Mode = NONE$$

$$Range = highest - lowest$$

$$R = 9036 - 187$$

$$R = 8849$$

$$\sigma = deviation$$

$$\sigma = \sqrt{\frac{(x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + \ldots (x_n - \overline{x})^2}{n}}$$

$$\sigma = \sqrt{\frac{(187 - 1571.9)^2 + (390 - 1571.9)^2 + (452 - 1571.9)^2 + (505 - 1571.9)^2 + (628 - 1571.9)^2 + (719 - 1571.9)^2 + (826 - 1571.9)^2 + (1106 - 1571.9)^2 + (1870 - 1571.9)^2 + (9036 - 1571.9)^2}{10}}$$

$$\sigma = \sqrt{\frac{63900754.9}{10}}$$

$$\sigma = \sqrt{6390075.49}$$

$$\sigma \approx 2527.8599$$

2. Yes there are outliers. This raises the mean and the range.