

## 1.1 Relations and Functions

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### Answers

1. A function is a statement defining a single result for each question, or a single output of each input.
2. Yes
3. Yes
4. Yes
5. Answers vary, should mention how the function does not always have the same output for a given input.
6. Function
7. Not a function
8. Not a function
9. Function
10. Function
11. Not a Function
12. Not a function
13. Function
14. Function
15. Yes, this is a function. One boy, one corsage/date
16. It does change the answer, now it is not a function. Cory is one boy with two corsages/dates

## 1.2 Domain and Range

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### Answers

“Determine the Domain and the Range of the following relations.”

1. Domain : 0, 3, 90 : Range: 4, 20, 33
2. Domain: 3, 6, 10, -31 : Range: -4, 37, -10, 2
3. To calculate how far Tina can drive, the input value (the domain) would be the number of gallons: domain = {10, 11, 12} and the output (the range) would be the number of miles based on 30mpg: range = {300, 330, 360}.
4. To calculate the total cost of the outing, the input (domain) is the number of games played: domain = {4, 8}. The range is the possible cost,  $\$2.75x$ : range = { $\$11$ ,  $\$22$ }
5. The independent variable (the input) would be the number of hours he worked each week, “h”. The dependent variable (the output) would be the salary “S.” The domain would be between 20 and 25, and the range would be between  $\$200$  and  $\$250$ .
6. The ordered pair (20, 200) would represent a week when Bob worked 20 hours, and therefore earned  $\$200$
7. The ordered pairs are in the form (boy, girl), therefore the boys (in the “x” position) are the domain or input, and the girls (in the “y” position) are the range or output.
8. In the function:  $y = |x|$  the domain is any real number, while the range is only positive numbers. This is because any number may be input for “x”, but since the square root of a negative does not exist, any output will be positive.
9. In the function:  $|y| = x$  the domain is only positive numbers, while the range is any real number. This is because only positive numbers may be input for  $x$ , since the root of a negative does not exist.
10. In the function:  $y = x^2$  the domain is any real number, while the range is only positive numbers. This is because any number may be input for “x”, but since any number squared becomes positive, any output will be positive.

11.  $x = y^2$  the domain is only positive numbers, while the range can be any number. This is because only positive numbers may be input for “x”, since “x” must associate with a number that exists, and any number squared becomes positive.
12. The domain is: {A, B, C}, and the range {A, B, C, D}.
13. The domain is all of the “x” values: {1, 2.5, 3.75, 5, 6.5, 7.75}, and the range the “y” values {2, 5}.
14. The range is only positive numbers, while the domain is any number.
15. The domain is all of the red symbols, and the range is all of the green ones.

## 1.3 Intervals and Interval Notation

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### Answers

1.  $[-3, 1)$
2.  $(0, 2)$
3.  $(-3, +\infty)$
4.  $(-\infty, 2]$

"Solve and put your answer in interval notation."

5.  $x > 1 \therefore (1, +\infty)$
6.  $x \leq -2 \therefore (-\infty, 2]$

"For each number line, write the given set of numbers in interval notation."

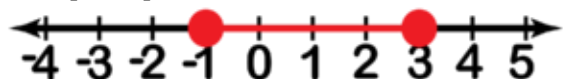
7.  $(-\infty, -3)(0, +\infty)$
8.  $(-\infty, -5)$
9.  $(-3, 5]$
10.  $(-\infty, 2)(2, +\infty)$

"Name the domain and range for each relation using interval notation"

11. Domain:  $(-4, 6)$  Range:  $(-2, 7)$
12. Domain:  $(-5, 7]$  Range:  $[-1.25, 8)$ ,  $[-1, 8)$  is also acceptable

"Express the following sets using interval notation, then sketch them on a number line"

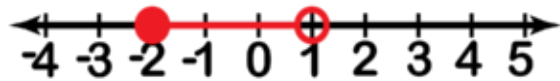
13.  $[-1, 3]$



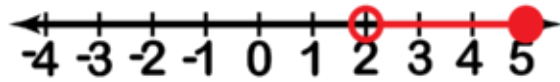
## Chapter 1 – Analyzing Functions

## Answer Key

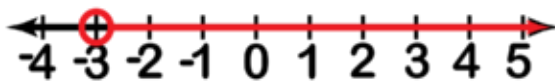
14.  $[-2, 1)$



15.  $(2, 5]$



16.  $(-3, +\infty)$



## 1.4 Average Rate of Change

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### Answers

For  $y = x^3$  find the average rate of change as:

- At  $x = 1, y = 1$ , and at  $x = 3, y = 27$ , the average rate of change is:  $\frac{26}{2} = 13$  or  $\frac{13}{1}$
- At  $x = -4, y = -64$ , and at  $x = -1, y = -1$ : the average rate of change is:  $\frac{63}{3} = 21$  or  $\frac{21}{1}$
- If  $f(1) = 2$  and the average rate of change of  $f$  between 1 and 5 is 3,  
Then:  $f(2) = 5 : f(3) = 8 : f(4) = 11$ , and  $f(5) = 14$ .
- Jamie's average speed during the first half of the trip was:  $\frac{21km}{1.5hr} = 14 km/hr$ . During the last half it was  $\frac{15km}{1.5hr} = 10 km/hr$ . His entire trip was  $\frac{36km}{3hr} = 12 km/hr$ . Jamie averaged more than 11.5 km/hr.
- The difference in price is \$0.35 total, the time change was 5 days, therefore the gas price averaged a change of \$0.07 per day.
- The population difference was 1,317 over a period of 21 years. The average rate of change was  $\frac{1,317}{21} \approx 62.7$  persons per year.
- The change in price is \$335 over an increase of 18  $yd^3$ . The average is  $\frac{335}{18} = \$18.6/yd^3$ .
- The increase in length is 2in, the increase in weight is 4lbs. The average change in length is  $\frac{2in}{4lbs} = \frac{1in}{2lbs} = \frac{.5in}{1lb}$ .
- The change in speed is 15mph, the increase in efficiency is 5mpg. The average increase in efficiency is  $\frac{15mph}{5mpg} = 3 \frac{mph}{mpg}$  or  $\frac{1mpg}{3mph}$ .
- If  $y = f(x) = x^2 + x + 2$ , then when  $x = -1, y = -2$ , and

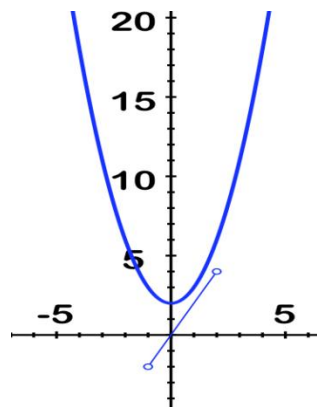
## Chapter 1 – Analyzing Functions

## Answer Key

when  $x = 2$ ,  $y = 4$ .

a) Average rate of change:  $\frac{\Delta y=6}{\Delta x=3} = 2$

b) Graph of  $f$  and the graph of the secant line through  $(-1, -2)$  and  $(2, 4)$ :



11. a) Amy's average *driving* speed for the trip was:

$$\frac{85mi}{115min-10min} = \frac{85mi}{105mins} = \frac{0.8mi}{min} = 48 \text{ mi/hr.}$$

- b) Her average speed for the *entire trip* (including the stop at McDonald's) was:

$$\frac{85mi}{115mins} = 44.5 \text{ mph}$$

12. If the weight  $w(t)$  (in grams) of a tumor,  $t$  weeks after it forms is given by  $w(t) = \frac{t^2}{15}$ , then the average rate at which the tumor is growing during the fifth week after it was formed is the increase in weight "during" week 5. Subtract the weight at week 6 from the weight at week 5:  $\frac{36}{15} - \frac{25}{15} \approx 0.75g$ .



## 1.5 Minimums and Maximums

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### Answers

1. The cost per chair should be minimized. The profit (a function of the selling price) should be maximized.
2.  $P = 2x + 40/x$
3. When  $x = 4.472$ , the perimeter is about 17.889 feet.
4. Answers will vary. Example: A maximum is the highest point on the graph, or the greatest "y" value of the function.
5. Answers will vary. Example: After graphing a function, you need to look for the highest or lowest point on the graph. It is important to explore a large subset of the domain, unless you are very familiar with a particular kind of function.
6. The rectangle with maximum area is a 6x6 square, with area 36 in<sup>2</sup>.
7.  $r \approx 1.68$  inches.
8. If  $b < 0$ , the function will have a maximum.
9. If you examine the graph for negative  $x$  values, you see that the graph has more than one "piece". The piece that we did not consider in the example in the text goes above and below the point we identified in the example in the text. Also, note that the coefficient of  $x^2$  is positive.
10.  $A(x) = x\left(\frac{P}{2} - x\right)$  If you choose values for  $P$  and examine the graphs, you will see that the rectangle with maximum area is always a square, like the one in problem.
11.  $A = x(80 - 2x)$   
 $A = -2(x^2 + 40x)$   
 $A = -2(x - 20)^2 + 800$   
Vertex: (20, 800)  
Maximum area = 800ft<sup>2</sup>
12.  $y = x^2$  is a parabola with vertex at the origin. The minimum is at (0, 0)
13.  $y = x^3$  is a cubic polynomial, it has no minimum or maximum.
14.  $y = |x|$  is a "V"-shape with vertex at the origin, the minimum is at (0, 0)
15.  $y = x + 3$  is a straight line, it has no minimum or maximum

## 1.6 Discrete and Continuous Functions

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### Answers

Identify each of the following variables as being either discrete or continuous.

1. Discrete: once it connects, it is a call.
2. Continuous: oranges can be added to produce any combination of weights.
3. Continuous: the rope can be cut to any length.
4. Continuous: the truck can speed up or slow down at any rate.
5. Discrete: each arrest is a whole unit, no fractions.
6. Discrete: if it's a flaw, it's a flaw.
7. Discrete: each Bald Eagle is a complete unit.
8. Continuous: time passes smoothly, not in discernable chunks.
9. The graph is continuous, no breaks or skips.
10. The equation  $f = 0.305m$  is continuous, as it can be used to convert any number or fraction of meters into feet.
11. The formula to calculate height in inches  $y$  would be:  $y = 72in + 1.25(x)in$   
The function is continuous, as the plant grows smoothly, not in jumps.
12. The total spent could be modeled by:  $y = \$12(t) + \$15(h)$ . The function is discrete, as each hat and t-shirt purchase is a complete unit with a specified price.
13. The table data is discrete, as each plastic container is counted in whole units.
14. Any equation written based on the data in the table would be suspect at best, as there is no correlation between the number of cans and the house number.
15. There is no way to predict the number of containers a 6th house would submit based on the data in the table, as the table does not suggest a correlation between increasing house number and number of submitted containers.

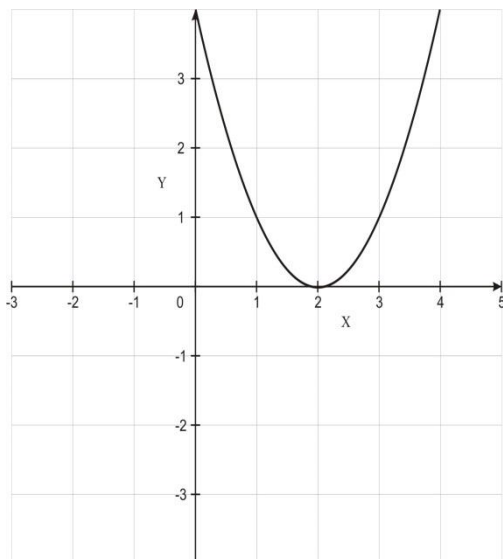
## 1.7 Increasing and Decreasing Functions

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### Answers

1.  $(-2.7, 2.75)$
2.  $(0, -2)$ .
3. If  $m > 0$ , the function will be increasing. If  $m < 0$ , the function will be decreasing
4. Increasing:  $(-\infty, -2.7)$  and  $(0, \infty)$ . Decreasing:  $(-2.7, 0)$ .
5. Answers will vary.

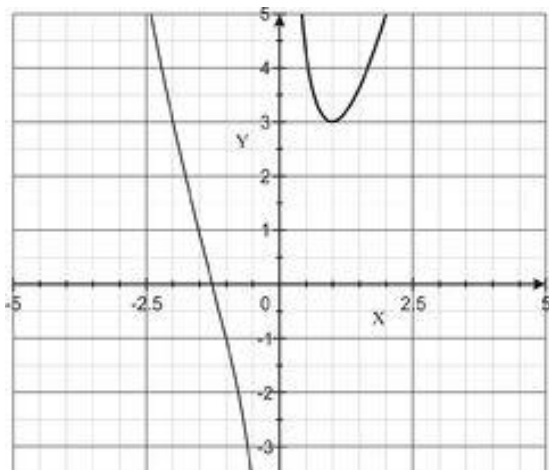
Example:



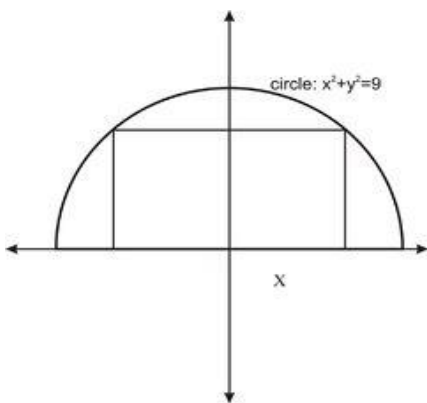
6. Answers will vary.

Example: a global maximum is the greatest function value, or the highest point on the graph of the function. A global minimum is the least function value, or the lowest point on the graph of a function. A relative max or min is the highest or lowest point within some interval in the domain of the function.

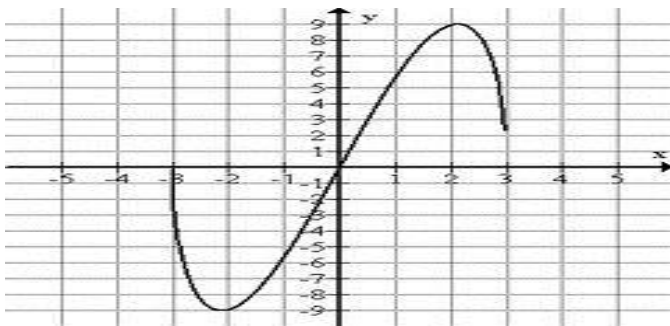
7. The function has a global minimum at approximately  $(-1.3, -1.5)$ , a relative maximum at approximately  $(0.17, 2.0)$ , and a relative minimum at  $(1.1, 1)$ .
8. The graph has a relative minimum at  $(1, 3)$ .



9. a)  $A = 2x\sqrt{9 - x^2}$



- b) The graph has a maximum at  $(2.12, 9)$ . So the maximum area of the rectangle is 9.



10. A graph of  $y = x^6$  should have 5 local extrema, since the number of extrema should be one less than the degree of the equation.
11. Answer will vary, but the sketched line should be straight and rising to the right on the interval  $[-5, 7]$ , and continue to infinity on the positive side.
12. Answer will vary, but the sketched line should have two "bumps" on the interval  $(-4, 9]$ , and no global extrema.
13. The graph should start at the origin, and rise to the right on a straight line to  $(40, 300)$ , ending there. The local extrema are the endpoints of the line, e.g.  $(0, 0)$  and  $(40, 300)$
14. If Sam is working a week with overtime, then the global and local minimum is  $(40\text{hrs}, \$300)$ , since he must have already worked 40 regular hours to qualify. The local and global maximum is now  $\$300 + \$168.75 = \$468.75$ .
15. Answers will vary, but the graph should show a straight line from the origin to the point  $(40, 300)$ , then continue at a steeper angle to  $(55, 468.75)$ . There should be marked extrema at  $(0, 0)$ ,  $(40, 300)$  and at  $(55, 468.75)$

## 1.8 Limits and Asymptotes

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### Answers

- Answers will vary, but should include a definition that describes a line that represents a value toward which the graph of a function will approach, but never quite touch.
- $\lim_{x \rightarrow -6} f(x) = \infty$  describes a graph which climbs upward from the left and becomes so steep by  $x = -6$  that the corresponding  $y$  value approaches infinity.  $\lim_{x \rightarrow \infty} f(x) = -6$  describes a graph which is limited in vertical travel, as the graph goes in the extreme right-hand direction, its vertical movement does not pass -6.
- $\lim_{x \rightarrow \infty} f(x) = 200$  describes a function with a horizontal asymptote at 200, meaning that the vertical travel of the function is limited by 200.
- $\lim_{x \rightarrow 175} f(x) = 175$  means that the function  $f(x)$  approaches the point (175, 175) as  $x$ -values increase.
- Since the expression  $\lim_{t \rightarrow \infty} \frac{3t^2 - 7t}{t - 8}$  has a greater power in the numerator, it will grow without bound, and since all the variables are positive, it will grow in the positive direction.
- The limit of  $\lim_{t \rightarrow \infty} 3$  is 3. The value of  $t$  does not affect  $f(t)$
- The function  $\lim_{t \rightarrow \infty} (t^2 - t^4)$  will grow without bound in the negative direction, as any significantly large positive or negative value for  $t$  will result in a negative number
- The limit of  $\lim_{x \rightarrow \infty} \sqrt{x^2 + 2x}$  is positive infinity
- The horizontal and vertical asymptotes of  $h(g) = \frac{5g^2 - 7g + 9}{g^2 - 2g - 3}$  are approximately 17.5 and 1.145 respectively. A graph of the function can be found at: Desmos Graphing Calculator <https://www.desmos.com/calculator/47hucnmrs7>
- The horizontal and vertical asymptotes are at  $y = 2$  and  $x = 4$ , respectively. Near the vertical asymptote,  $f$  grows without bound in the positive direction as approached from the right, and in the negative direction as approached from the left. A graph of the function can be found at: Desmos Graphing Calculator <https://www.desmos.com/calculator/csg0kfabsc>
- The only real root ( $x$ -intercept) is at  $f(x) = \frac{3}{4}$ . The  $y$ -intercept is at  $x = 3$ . There is a hole (an excluded point) in the graph at  $(-2, \frac{5}{6})$ .

12. If  $n > 0$ , the limit is 0, because the fraction  $\frac{1}{t^n}$  becomes infinitely small when  $t$  become infinitely huge.
13. If  $n < 0$  the limit is  $+\infty$
14. If  $n = 0$  the value of  $\frac{1}{t^n}$  remains 1, because  $t^0 = 1$
15. If the degree of G is less than the degree of H, the limit is 0, because the denominator will become bigger more quickly than the numerator, resulting in an ever-smaller fraction.
16. If the degree of G is greater than the degree of H, the function will grow without bound.
17. If the degree of G is the same as the degree of H, the limit is 1, because the coefficients of  $\frac{G(x)}{H(x)}$  will matter less and less as  $x$  becomes massively large.
18. A pool contains 8000 L of water. An additive that contains 30g of salt per liter of water is added to the pool at a rate of 25 L per minute.
- a) The concentration of salt after  $t$  minutes in grams per liter is:  $C(t) = \frac{(t)30g \times 25}{8000l + 25(t)l}$  because the portion of the additive that is salt increases the salt in the pool by 30 g/min, while the volume of the pool increases by 25 l/min, since this is the volume of water added each min.
- b) The concentration as time increases to  $\infty$  approaches 1 g/l. Physically, this makes sense because although there are initially more grams of salt being added than liters of water, as  $t$  becomes huge, the other numbers become less important, leaving  $\frac{t}{t}$ .

## 1.9 Infinite and Non-existent Limits

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### Answers

What are the three possible types of outcomes of  $\lim_{x \rightarrow \infty} f(x)$ ?

1. Output approaches a single value or number.
2. Output approaches zero.
3. Output grows positively or negatively larger without bound.
4. Examples vary, but should be a function which either grows without bound or does not approach a single number.

Assuming that  $f(x)$  is a rational function:

5. The  $\lim_{x \rightarrow \infty} f(x)$  when the degree of the numerator is less than the degree of the denominator is zero, because the fraction becomes infinitely small as the denominator outgrows the numerator.
6. The  $\lim_{x \rightarrow \infty} f(x)$  when the degrees of the numerator and the denominator are equal is 1, because any other terms or coefficients become progressively less important as  $x$  becomes huge.
7. The  $\lim_{x \rightarrow \infty} f(x)$  when the degree of the numerator is greater than the degree of the denominator does not exist, because the function grows without bound.
8. In general, if  $r$  is a positive real number, the  $\lim_{x \rightarrow \infty} \frac{1}{x^r}$  is "0".
9. In general, if  $r$  is a positive real number, the  $\lim_{x \rightarrow \infty} x^r$  is  $\infty$  because the function grows without bound.

Let  $a$  and  $b$  be real numbers and let  $t$  be a positive integer. Complete each of the following properties of limits.

10.  $\lim_{x \rightarrow a} x^t = a^t$ : The limit of a constant is the constant.
11. If  $f$  is a polynomial,  $\lim_{x \rightarrow a} f(x) = f(a)$
12.  $\lim_{x \rightarrow a} k \cdot f(x) = k \lim_{x \rightarrow a} f(x)$



13.  $\lim_{x \rightarrow \infty} t^x = \infty$

**Evaluate:**

14.  $\lim_{x \rightarrow -5} \frac{(5+x)^2 - 25}{x}$

15.  $\lim_{x \rightarrow -3} \frac{x^3 - 6x + 2}{x^2 + 2x - 3} = \pm\infty$

16.  $\lim_{x \rightarrow 0} \frac{(3+3y)^{-1}}{-3y^{-1}x} = \infty$

## 1.10 Linear and Absolute Value Function Families

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### Answers

1.  $y = |x - 7|$  Belongs to the absolute value function family
2.  $y = 3x - 4$  Belongs to the linear function family
3.  $f(x) = |x^2|$  Belongs to the absolute value function family
4.  $|x| - 2 = y$  Belongs to the absolute value function family
5.  $f(x) = x + \frac{3x}{2}$  Belongs to the linear function family
6. The graph should be in the shape of a large “V”, with the vertex at the origin, there should be a small O at the origin, since the value (0, 0) is undefined.
7.
  - a) What is the shape of the graph? The graph is in the shape of a wide “V”.
  - b) Compare the graph to the graph in the problem above. What is the difference between the two graphs? The two graphs are the same shape, but the second is two units lower than the first, and the first has a “hole” at (0, 0).
  - c) What is the slope of the two lines that create the graph? Both graphs have a slope of 1 on the right, and -1 on the left. A graph of the two equations is completed at:  
<https://www.desmos.com/calculator/onthtan9qm>
8.  $f(x) = |6x|$  The vertex occurs at (0, 0), because there is nothing "added or subtracted" to the "6x".
9.  $f(x) = |x - 6| + 8$  The vertex has been shifted 6 units to the right, and 8 units up from the origin.

10.  $f(x) = |x + 7| - 8$  The vertex has been shifted 7 units to the left and 8 units down from the origin.
11.  $f(x) = |x + 5|$  The vertex has been shifted 5 units to the left of the origin.
12. The graph of  $p(x) = |x|$  is a "V", with the vertex at the origin. If  $t(x) = -|x| - 3$ , the graph of  $t(x)$  is different from the graph of  $p(x)$  by being inverted and then shifted 3 units down.
13.  $f(x) = |x - 3|$ : The graph should be a "V" shape, shifted 3 units to the right, the table of values should reflect that all  $f(x)$  values are 3 less than the positive versions of the matching  $x$  values.
14. The graph should be a "V" shape, shifted 3 units to the left, the table of values should reflect that all  $f(x)$  values are 3 more than the positive versions of the matching  $x$  values.
15. a)  $f(x) = x - 7$
- b)  $f(x) = x - 2$
- c)  $f(x) = x + 1$
- d)  $f(x) = x + 5$
- e)  $f(x) = x + 10$

Parent Function:  $f(x) = ax + b$  Linear Function Family

Similarities: All graphs are straight lines with the same slope

Differences: Lines are shifted vertically

16. a)  $f(x) = \frac{2}{11}x$

b)  $f(x) = \frac{1}{2}x$

c)  $f(x) = \frac{2}{3}x$

Parent Function:  $f(x) = ax + b$  Linear Function Family

Similarities: All graphs are straight lines

Differences: The slopes are different

17. a)  $f(x) = x - 7$

b)  $f(x) = 2x$

c)  $f(x) = 4x$

d)  $f(x) = 2x + 5$

e)  $f(x) = 6x - 10$

Parent Function:  $f(x) = ax + b$  - Linear Function Family

Similarities: All graphs are straight lines

Differences: Some are shifted vertically, all but  $b$  and  $d$  have different slopes.

18. The  $a$  value changes the slope of the graph

19. The  $b$  value affects the vertical location of the graph

20. None of the functions have limited domains
  
21. None of the functions have limited ranges
  
22. Neither the  $a$  nor  $b$  values affect the domain
  
23. Neither the  $a$  nor  $b$  values affect the range

## 1.11 Square and Cube Function Families

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### Answers

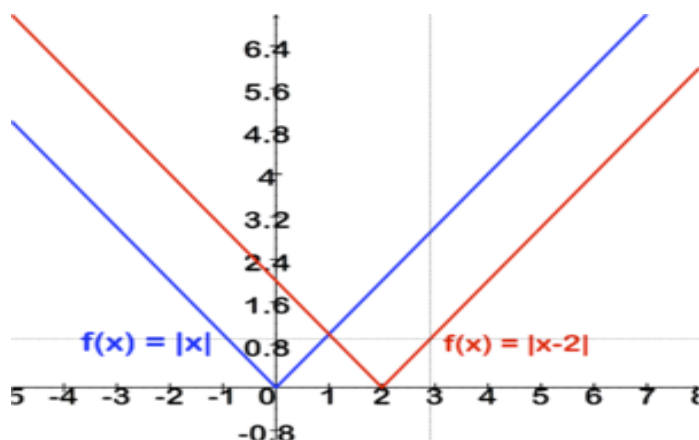
1. A Square Function is any function that has a second degree equation, meaning it has an  $x^2$  term and no greater powers of  $x$ .
2. A Cube function is a third degree equation, meaning it has an  $x^3$  term, and no greater powers of  $x$ .
3. A cubed function grows faster than a squared function by a multiple of  $x$
4. principal root
5. Because  $y = 0$  is excluded from the range.
6. The graph of these functions is here: <https://www.desmos.com/calculator/bckp9w1fmt>  
Similarities: width, direction, end behavior, domain, degree  
Differences: x-intercept, y-intercept, range
7. The graph of this set of functions is here: <https://www.desmos.com/calculator/1n9hrhtviq>  
Similarities: direction, end behavior, range, increasing/decreasing intervals, x-intercept, degree  
Differences: domain, width, y-intercept,
8. The graph of these functions is here: <https://www.desmos.com/calculator/ob9szkhnz3>  
Similarities: domain, x and y-intercept, end behavior, degree, width  
Differences: direction, range

9. The value of  $a$  determines the width (if  $0 < a < 1$  then the graph is wide, if  $a > 1$  the graph is narrow) and direction (if  $a < 0$  then the graph is reflected down)
10. The value of  $h$  determines the left/right location: if  $h > 0$  then the graph shifts left.
11. The value of  $k$  determines the up/down location: if  $k < 0$  then the graph shifts down.
12. All domains are  $(-\infty, \infty)$ ,  $(\infty, \infty)$  or All Real Numbers.
13. If  $a > 0$  then the range is  $[\#, +\infty)$  or  $y > \#$ ; if  $a < 0$  then the range is  $(-\infty, \#]$
14. None of the values affect the domain.
15.  $a$  and  $k$  affect the range.
16. The graphs are: <https://www.desmos.com/calculator/uxsbkzydiq>  
 Similarities: end behavior, increasing/ decreasing intervals, domain, range, degree, width, direction  
 Differences: y-intercept, x-intercepts
17. The graphs are: <https://www.desmos.com/calculator/jtagvq8ha1>  
 Similarities: y-intercept, x-intercept, domain, end behavior, range, degree, width  
 Differences: increasing/decreasing interval

## 1.12 Vertical and Horizontal Transformations

### Answers

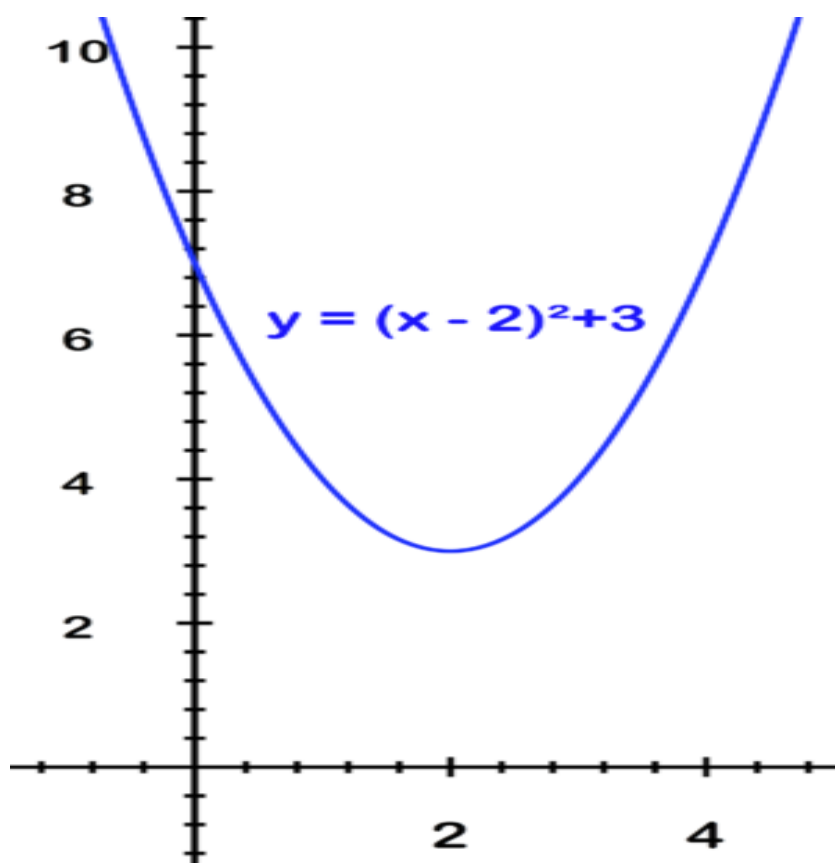
1. The graph of the function  $f(x) = 2|x - 1| - 3$  is:



2. The vertex is at (1, 3)
3. The graph opens upward because the  $a$  term is positive.
4. Up
5. Down
6. Right
7. Left
8. The slopes of the two rays are  $+a$ , so changing the  $a$  will make the graph wider or narrower.
9. This is a V-shaped graph shifted down 5 units
10. This is a steeper (slopes of  $+5$ ) V-shaped graph shifted left 7 units.
11.  $h(x) = \frac{1}{x} + 2$
12.  $h(x) = \frac{1}{x-4}$
13.  $h(x) = \frac{2}{x}$
14.  $f(x) = |x| - 4$
15.  $f(x) = |x + 6|$
16.  $f(x) = |x - 2| + 1$



17. The graph shifts up, and looks like a large "V" with the vertex at (0, 2).
18. The graph shifts down, and looks like a large "V" with the vertex at (0, -4).
19. The graph shifts right, and looks like a large "V" with the vertex at (4, 0)
20. The graph shifts left, and looks like a large "V" with the vertex at (-2, 0)
21. Graph looks like a large "V", significantly narrower than the ones above.
22. Graph looks like the one in Q #21, but slightly narrower yet.
23. Graph looks like a large "V", significantly wider than the reference  $y = |x|$
24. Graph looks like the one above, but slightly narrower.
25.  $g(x) = (x - 2)^2 + 3$  : Graph should look like this:



26.  $F(t) = H(t) + 1$
27.  $S(t) = H(t - 1)$

## 1.13 Stretching and Reflecting Transformations

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### Answers

1. It will either be stretched or compressed.
2. It creates a horizontal compression, which looks like a vertical stretch.
3. It creates a horizontal stretch, which looks like a vertical compression.
4. We have to negate (make negative)  $x$  to get a reflection over the  $y$  axis.
5. We have to negate the function (the output, or  $y$ ).
6.  $f(x) = 2x^2 + 3$
7.  $f(x) = \frac{1}{4}x^2 - 6$
8.  $f(x) = \sqrt{x}$
9.  $f(x) = \sqrt{-x}$
10. Shift down 2
11. Shift left 3
12. Shift left 1
13. Reflect over the  $y$  axis and stretch vertically
14. Shift left 3 and up 1
15. Vertically stretched ( $1/3$  as wide), shift right 3, up 1
16. Reflect over  $x$ , vertical stretch by factor of 4, shift right 1 (1st left by 1, then back right by 2)
17. Horizontal compress ( $2/3$  of standard), shift right 2, up  $1/4$
18. A line segment from  $(-1, 5)$  to  $(2, 6)$
19. A line segment from  $(-1, 6)$  to  $(2, 7)$
20. A line segment from  $(-1, -4)$  to  $(2, -5)$
21. A line segment from  $(-1, 5)$  to  $(2, 6)$  (same as Q# 18)
22. The reference line shifted up by 3
23. The reference line shifted right by 2
24. The reference line reflected over the  $x$  axis

## 1.14 Combining Transformations

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### Answers

1. The +1 shifts the graph vertically
2. The leading – symbol before the  $f(x)$  reflects the reference graph across the  $x$ -axis
3. The – symbol outside of the parenthesis in  $g(x)$  makes the graph shift downward.
4. Choice “c” is correct
5. Choice “a” is correct
6. The +1 shifts the graph horizontally
7. The leading – reflects the graph over the  $x$  axis
8. The "2.0" shifts the graph horizontally
9. The "3.0" shifts the graph vertically
10. The order of reflection does not matter
11.  $g(x) = -8x^3$
12. Choice "d" is correct
13. Choice "a" is correct
14. Choice "c" is correct
15. Choice "c" is correct
16.  $g(x) = -|x| + 1$
17. Choice "b" is correct

## 1.15 Operations on Functions

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### Answers

1.  $(x^5 + x^4)/(x + 1)$

2. -8

3.  $(x^2)/(x^2 + 2x + 1)$

4.  $(f + g)(x) = 5x - 3$

5.  $(g / h)(x) = (11/12x) + 19$

6.  $(f + g)(x) = x^2 - 2x - 13$

7.  $(g - f)(x) = -2x^2 + 8x - 13$

8.  $3/2 g(x) = -9x + 12$

9.  $(g * h)(x) = -6x^3 - 12x^2 - 9x - 18$

10.  $(f + g)(3) = -7$

11.  $(g + h)(12) = 24$

12.  $(g - h)(2) = 18$

13.  $5g(6) = 4230$

14.  $2h(-5) = 54$

15.  $(f * g)(1) = -15$

16.  $(h * g)(-2) = -224$

17.  $(f + g - h)(x) = 12x + 9$

18.  $(f * g * h)(x) = 3x^3 + 8x^2 - 16x$

19.  $(h)(x) = 5x - 4$

## 1.16 Composition of Functions

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### Answers

1. 17

2. 99

3. 145

4. 29

5.  $f(g(12)) = -25$

6.  $g(h(\frac{1}{3})) = -102$

7.  $g(f(10)) = 8$

8.  $h(g(4)) = -216$

9. no solution

10.  $f(g(h(1))) = -2698$

11.  $f(h(g(18)))$

12. No, compositions are not commutative.

13.  $f(h(x)) = -18x^2 - 39x - 18$

14.  $g(x) = x - 3$

15.  $r(n(p)) = -156.25p^2 - 375p + 775$

## 1.17 Linear and Quadratic Models

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### Answers

1.  $y = 900 + 100t$
2. Vertex at (1.5, 6.5) - Graph is a narrow parabola opening upward from the vertex
3. (-5, -2): Graph is a narrow parabola opening upward from the vertex
4.  $h = -16t^2 + 80(t)$
5. (2.5, 100)
6. 100ft
7. 2.5 secs
8. 4.5 sec.
9. After .5 s and after 5.5 s
10. 2165
11. -2

## 1.18 Cubic Models

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### Answers

- $v(x) = (m - 2x)(n - 2x)(x)$
  - The side length of the square must be less than  $\frac{m}{2}$
- The ideal dimension is apx 11.25in square - giving a maximum area of apx 126.5sq in
  - Graphing the function  $22.5w - w^2$  and finding the vertex solves the problem
- Yes
- No:  $x$  is not an exponent
- Yes
- No, the exponent is a constant (-5), not a variable
- 4 cm
- $f = \frac{1}{d^2}$
- $y \approx \frac{7.93}{x^2}$
- The intensity of the light is ultimately approximately inversely related to the square of the distance from it.
- $y \approx \frac{7.93}{(2.75)^2} \approx 1.05$
- $y = 3.6492 - 0.1007x$
- 2.1387
- We know that this model is no longer valid after the year 2005.