Interior Angles in Convex Polygons

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Chapter 1. Interior Angles in Convex Polygons

Here you’ll learn how to find the measure of an interior angle of a convex polygon based on the number of sides the polygon has.

**Interior Angles in Convex Polygons**

The interior angle of a polygon is one of the angles on the inside, as shown in the picture below. A polygon has the same number of interior angles as it does sides.

The sum of the interior angles in a polygon depends on the number of sides it has. The **Polygon Sum Formula** states that for any \( n \)-gon, the interior angles add up to \((n - 2) \times 180\)°.

Once you know the sum of the interior angles in a polygon it is easy to find the measure of ONE interior angle if the polygon is regular: all sides are congruent and all angles are congruent. Just divide the sum of the angles by the number of sides.

**Regular Polygon Interior Angle Formula:** For any equiangular \( n \)-gon, the measure of each angle is \( \frac{(n-2) \times 180\,\text{°}}{n} \).
In the picture below, if all eight angles are congruent then each angle is \( \frac{(8-2) \times 180^\circ}{8} = \frac{6 \times 180^\circ}{8} = \frac{1080^\circ}{8} = 135^\circ \).

What if you were given an equiangular seven-sided convex polygon? How could you determine the measure of its interior angles?

**Examples**

**Example 1**

The interior angles of a pentagon are \( x^\circ, x^\circ, 2x^\circ, 2x^\circ, \) and \( 2x^\circ \). What is \( x \)?

From the Polygon Sum Formula we know that a pentagon has interior angles that sum to \((5 - 2) \times 180^\circ = 540^\circ\). Write an equation and solve for \( x \).

\[
x^\circ + x^\circ + 2x^\circ + 2x^\circ + 2x^\circ = 540^\circ \\
8x = 540 \\
x = 67.5
\]

**Example 2**

What is the sum of the interior angles in a 100-gon?

Use the Polygon Sum Formula. \((100 - 2) \times 180^\circ = 17,640^\circ\).

**Example 3**

The interior angles of a polygon add up to 1980°. How many sides does it have?

Use the Polygon Sum Formula and solve for \( n \).
\[(n - 2) \times 180^\circ = 1980^\circ\]
\[180^\circ n - 360^\circ = 1980^\circ\]
\[180^\circ n = 2340^\circ\]
\[n = 13\]

The polygon has 13 sides.

**Example 4**

How many degrees does each angle in an equiangular nonagon have?

First we need to find the sum of the interior angles; set \(n = 9\).

\[(9 - 2) \times 180^\circ = 7 \times 180^\circ = 1260^\circ\]

"Equiangular" tells us every angle is equal. So, each angle is \(\frac{1260^\circ}{9} = 140^\circ\).

**Example 5**

An interior angle in a regular polygon is 135°. How many sides does this polygon have?

Here, we will set the Regular Polygon Interior Angle Formula equal to 135° and solve for \(n\).

\[\frac{(n - 2) \times 180^\circ}{n} = 135^\circ\]
\[180^\circ n - 360^\circ = 135^\circ n\]
\[-360^\circ = -45^\circ n\]
\[n = 8\]

The polygon is an octagon.

**Review**

1. Fill in the table.

**Table 1.1:**

<table>
<thead>
<tr>
<th># of sides</th>
<th>Sum of the Interior Angles</th>
<th>Measure of Each Interior Angle in a Regular n–gon</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>60°</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>360°</td>
<td>108°</td>
</tr>
<tr>
<td>5</td>
<td>540°</td>
<td>120°</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2. What is the sum of the angles in a 15-gon?
3. What is the sum of the angles in a 23-gon?
4. The sum of the interior angles of a polygon is 4320°. How many sides does the polygon have?
5. The sum of the interior angles of a polygon is 3240°. How many sides does the polygon have?
6. What is the measure of each angle in a regular 16-gon?
7. What is the measure of each angle in an equiangular 24-gon?
8. Each interior angle in a regular polygon is 156°. How many sides does it have?
9. Each interior angle in an equiangular polygon is 90°. How many sides does it have?

For questions 10-18, find the value of the missing variable(s).
19. The interior angles of a hexagon are $x^\circ, (x + 1)^\circ, (x + 2)^\circ, (x + 3)^\circ, (x + 4)^\circ$, and $(x + 5)^\circ$. What is $x$?

**Review (Answers)**

To see the Review answers, open this [PDF file](https://www.ck12.org/flx/render/embeddedobject/1267) and look for section 6.1.

**Resources**

![Image of a hexagon with labeled angles.](https://www.ck12.org/flx/render/embeddedobject/1267)