Trigonometric Functions and Identities

Graphing Trigonometric Functions

Review Queue Answers

1. $\frac{\sqrt{2}}{2}$ 2. $-\frac{\sqrt{3}}{2}$ 3. -1 4. $\frac{\sqrt{3}}{2}$

Graphing Sine and Cosine

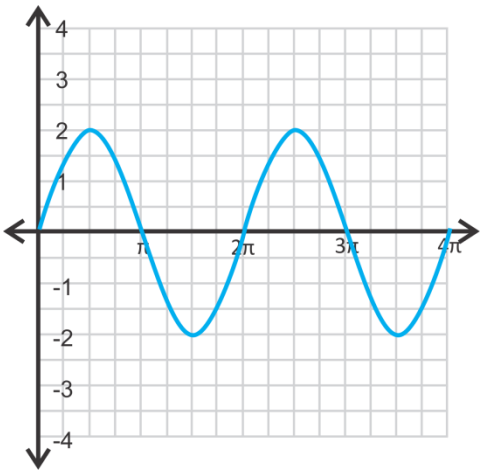
1. A. $\left(\frac{\pi}{2},1\right)$ B. $(\pi,-1)$ C. $\left(\frac{3\pi}{2},0\right)$ D. $\left(\frac{11\pi}{6},-\frac{1}{2}\right)$

   E. $(2\pi,\ 1)$ F. $\left(\frac{11\pi}{4},\frac{\sqrt{2}}{2}\right)$ G. $\left(\frac{7\pi}{2},-1\right)$ H. $\left(\frac{11\pi}{3},-\frac{\sqrt{2}}{2}\right)$

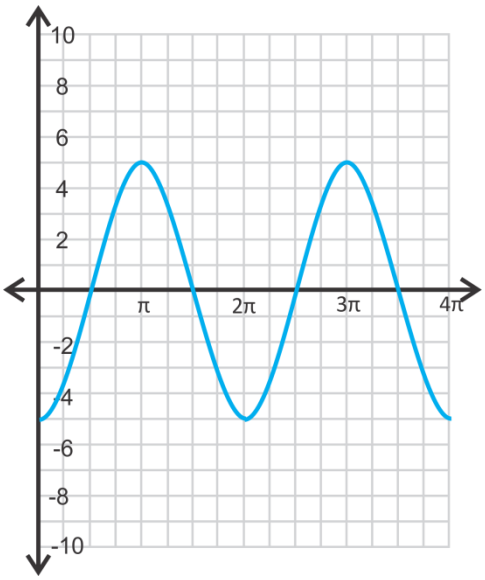
2. $\left(\frac{\pi}{4},\ \frac{\sqrt{2}}{2}\right), \left(\frac{5\pi}{4},\ -\frac{\sqrt{2}}{2}\right), \left(\frac{9\pi}{4},\ \frac{\sqrt{2}}{2}\right), \left(\frac{\pi}{4},\ -\frac{\sqrt{2}}{2}\right)$

3. $\left(\frac{\pi}{3},\frac{\sqrt{3}}{2}\right)$ and $\left(\frac{5\pi}{3},-\frac{\sqrt{3}}{2}\right)$

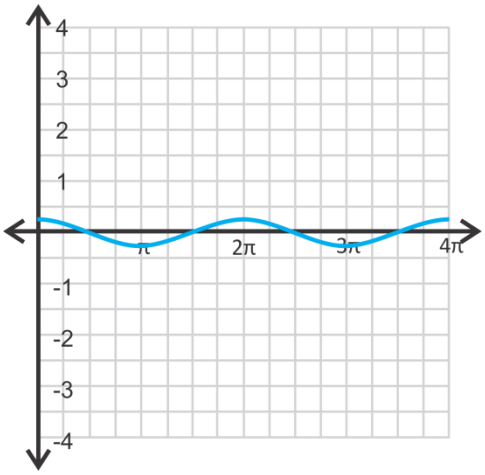
4. ans-1401-01



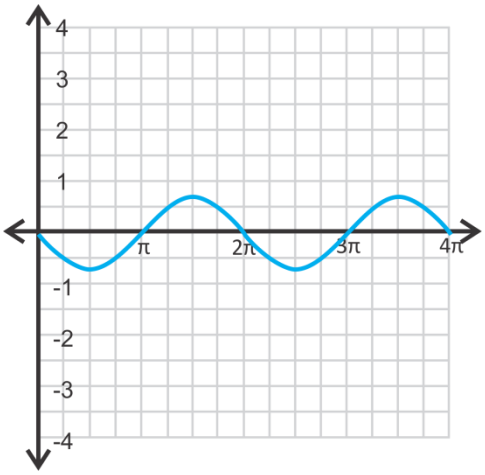
5. ans-1401-02



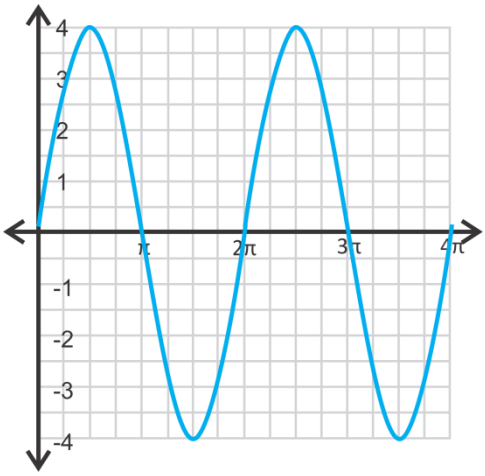
6. ans-1401-03



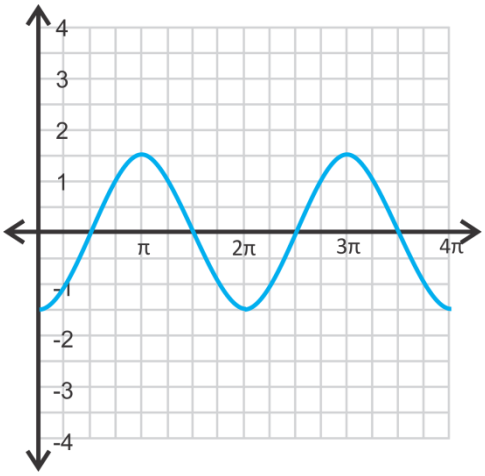
7. ans-1401-04



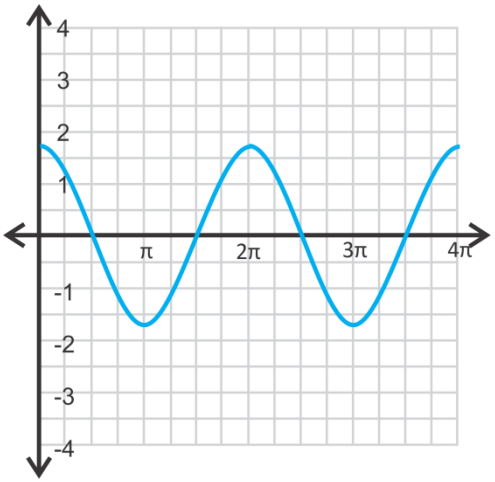
8. ans-1401-05



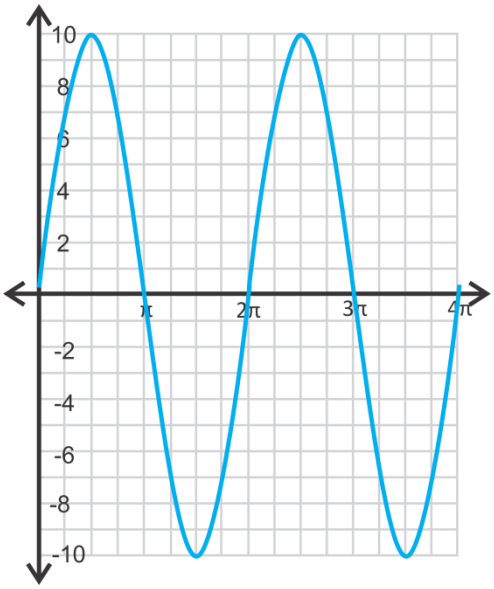
9. ans-1401-06



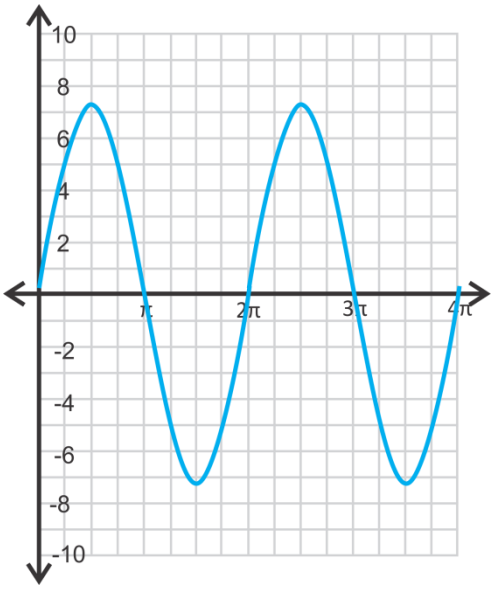
10. ans-1401-07



11. ans-1401-08



12. ans-1401-09



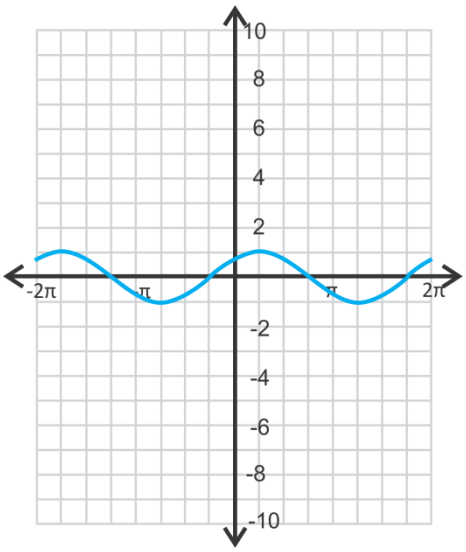
13. $\frac{\pi}{2}$ units 14. $\frac{\pi}{2}$ units 15. $y = 2.5\sin x$ 16. $y = 1.75\cos x$

Translating Sine and Cosine Functions

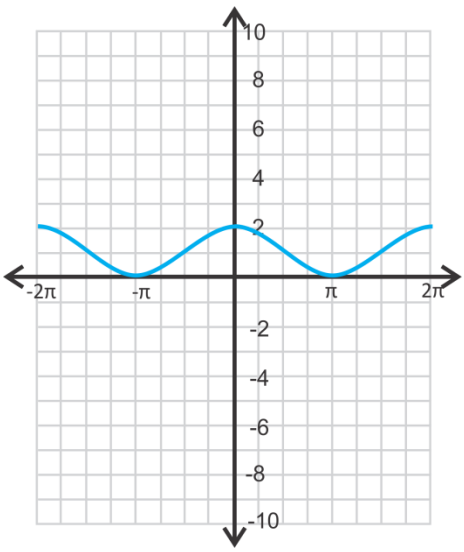
1. C 2. A 3. D 4. B

5. a) C b) D 6. a) $\frac{\pi}{2}$ b) $\frac{\pi}{2}$

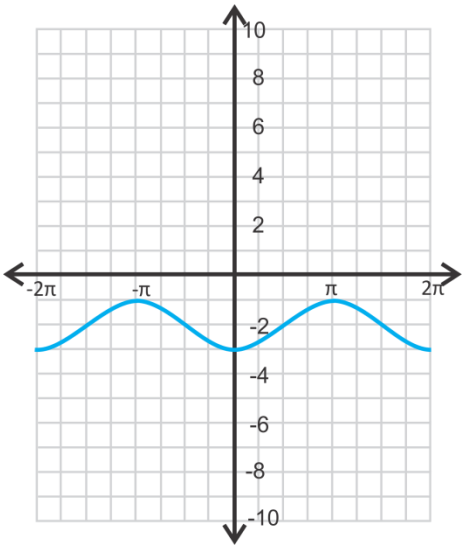
7. ans-1401-10



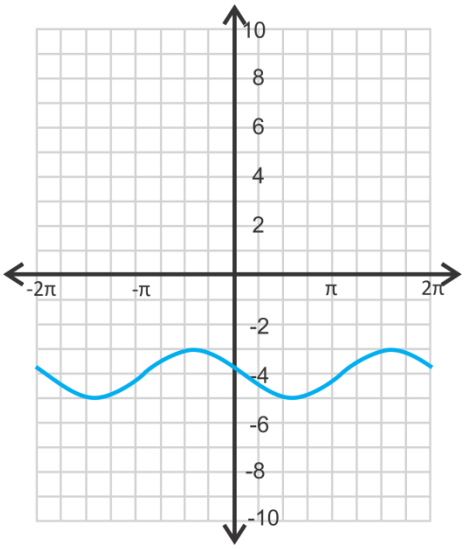
8. ans-1401-11



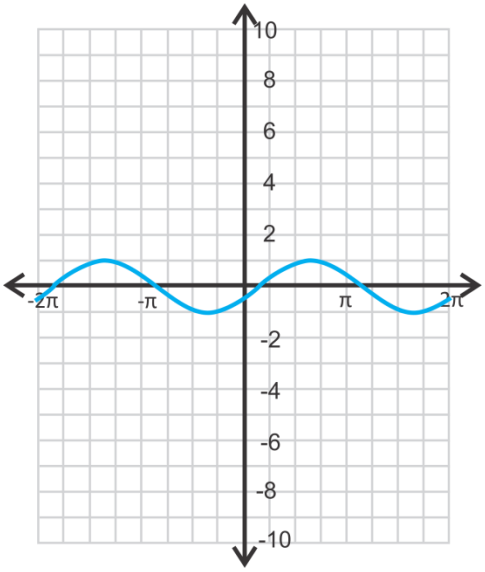
9. ans-1401-12



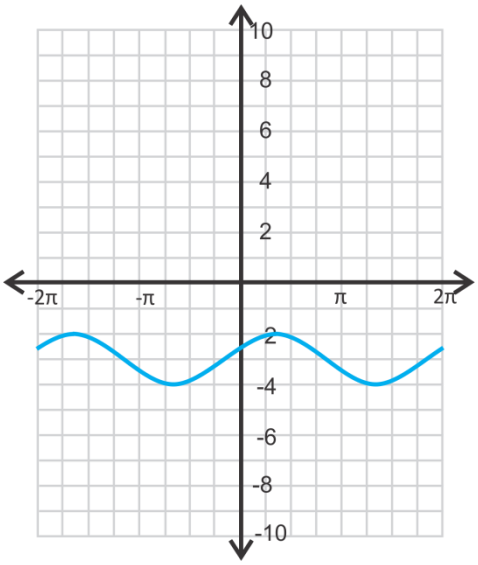
10. ans-1401-13



11. ans-1401-14



12. ans-1401-15

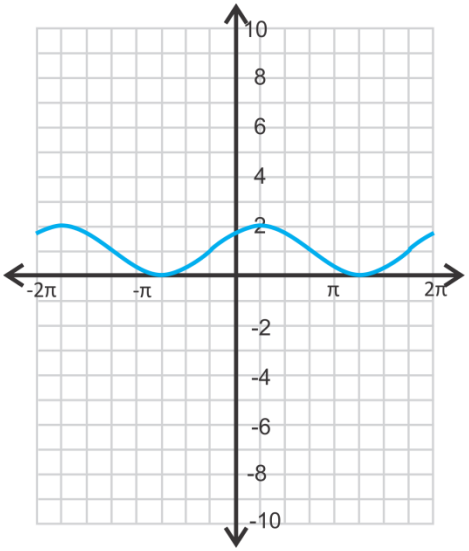


13. Yes, explanations will vary.

Putting it all Together

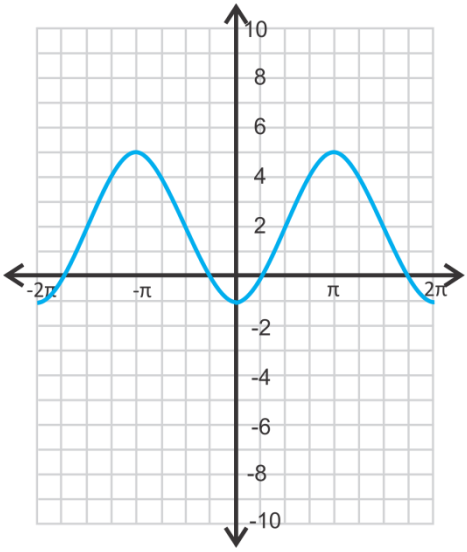
1. T 2. T 3. F 4. F 5. T

6. ans-1401-16



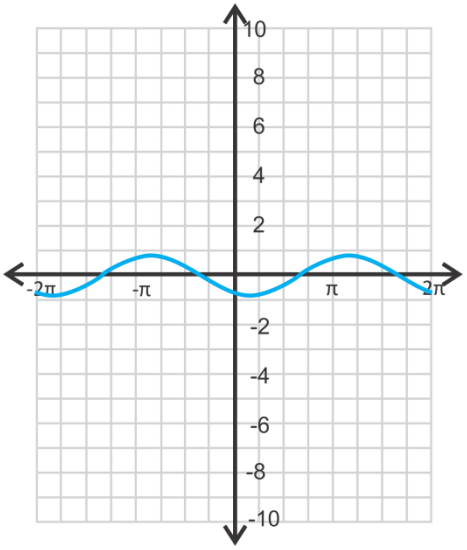
D: □ , R: $y \in [2,\ 0]$

7. ans-1401-17



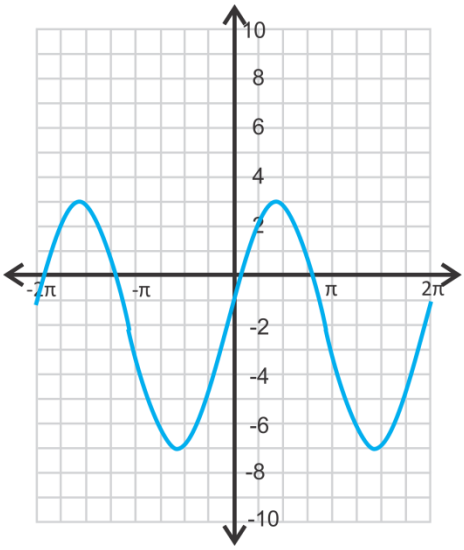
D: □ , R: $y \in [5,\ -1]$

8. ans-1401-18



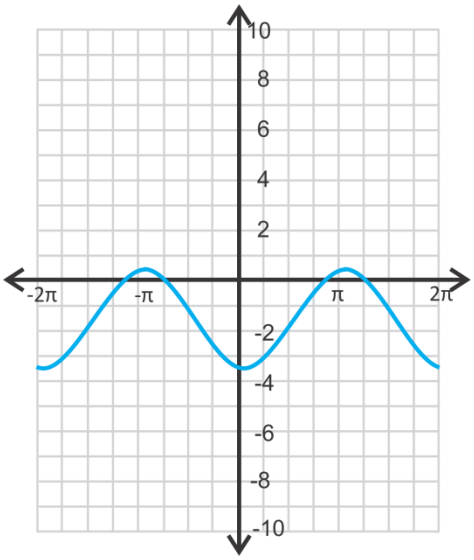
D: □ , R: $y \in \left[\frac{3}{4},\ -\frac{3}{4}\right]$

9. ans-1401-19



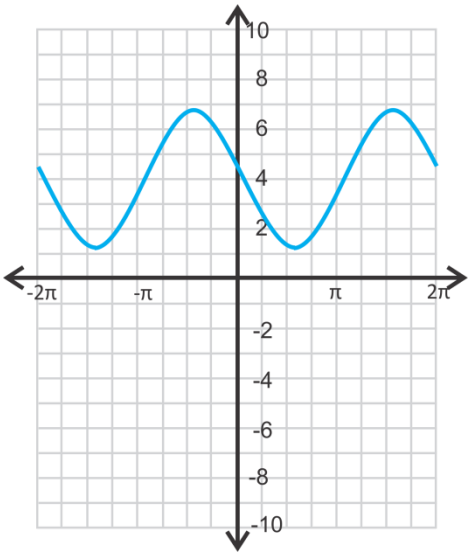
D: □ , R: $y \in [3,\ -7]$

10. ans-1401-20



D: □ , R: $y \in [0.5,\ -3.5]$

11. ans-1401-21



D: □ , R: $y \in [6.8,\ 1.2]$

12. $y = 2.5\sin x + 1.5$

13. $y = 2.5\cos\left(x - \frac{\pi}{2}\right) + 1.5$

$y = 2.5\sin\left(x - 2\pi\right) + 1.5$

14. every 2π

15. $y = 2.5\cos\left(x - \frac{5\pi}{2}\right) + 1.5$

16. $y = \sin\left(x - \frac{\pi}{2}\right) - 1$

17. $y = \cos\left(x - \pi\right) - 1$

18. $y = -\sin\left(x + \frac{\pi}{2}\right) - 1$

19. $y = -\cos x - 1$

20. Answers will vary.

Changes in the Period of a Sine and Cosine Function

1. $\frac{2\pi}{3}$

2. $\frac{\pi}{2}$

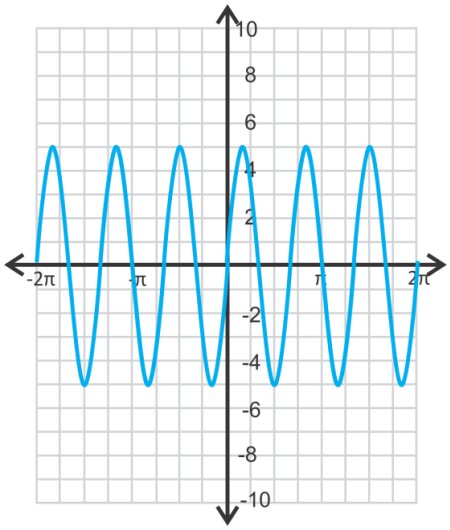
3. π

4. 2π

5. $\frac{4\pi}{5}$

6. $\frac{2\pi}{3}$

7. D: $\mathbb{R}$, R: $y \in [5, -5]$ ans-1401-22

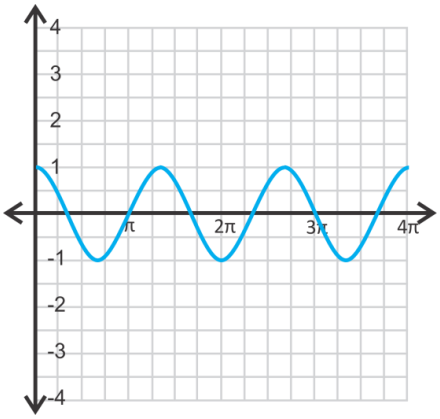


8. minimums: $\left(\frac{\pi}{6} \pm \frac{2\pi}{3}n,\ 5\right)$

maximums: $\left(\frac{\pi}{2} \pm \frac{2\pi}{3}n,\ -5\right)$

9. $x = 0,\ \frac{\pi}{3},\ \frac{2\pi}{3},\ \pi,\ \frac{4\pi}{3},\ \frac{5\pi}{3},\ 2\pi$

10. D: $\mathbb{R}$, R: $y \in [1, -1]$ ans-1401-23

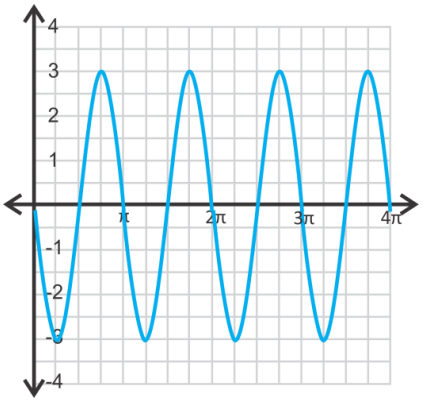


11. minimums: $\left(\frac{8\pi}{3} \pm \frac{8\pi}{3}n,\ 1\right)$ and $(0, 1)$

maximums: $\left(\frac{4\pi}{3} \pm \frac{8\pi}{3}n,\ -1\right)$

12. $x = \frac{2\pi}{3},\ 2\pi$

13. D: $\mathbb{R}$, R: $y \in [3, -3]$ ans-1401-24



14. minimums: $\left(\frac{3\pi}{4} \pm \pi n,\ 3\right)$

maximums: $\left(\frac{\pi}{4} \pm \pi n,\ -3\right)$

15. $x = 0,\ \frac{\pi}{2},\ \pi,\ \frac{3\pi}{2},\ 2\pi$

16. D: $\mathbb{R}$, R: $y \in \left[\,|a|+k,\ -|a|+k\,\right]$

17. $y = -2\sin\frac{8}{3}x$

18. $y = \frac{3}{5}\sin\frac{2}{5}x$

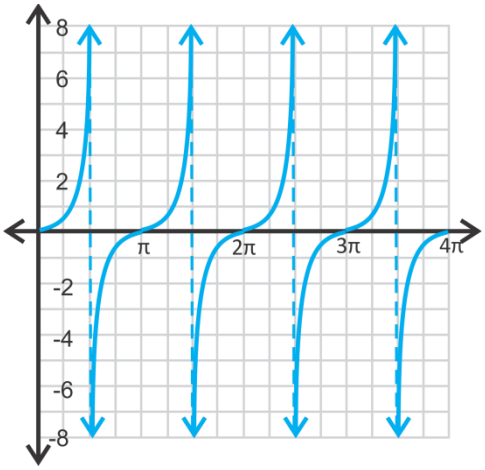
19. $y = 9\sin\frac{\pi}{3}x$

20. $x = \frac{\pi}{3},\ \frac{2\pi}{3},\ \pi,\ \frac{4\pi}{3},\ \frac{5\pi}{3}$

Graphing Tangent

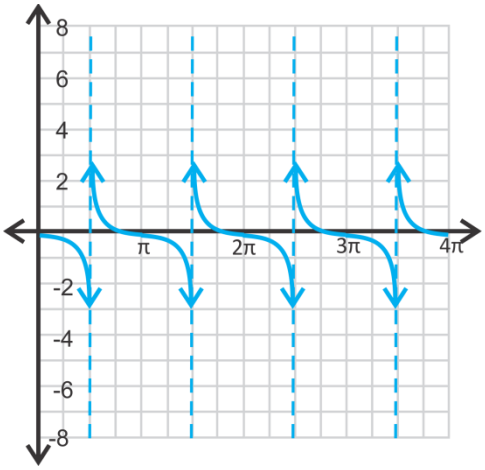
*n is any integer.

1. $p = \pi$, D: $\mathbb{R}$; $x \neq \frac{\pi}{2} \pm \pi n$, R: $\mathbb{R}$



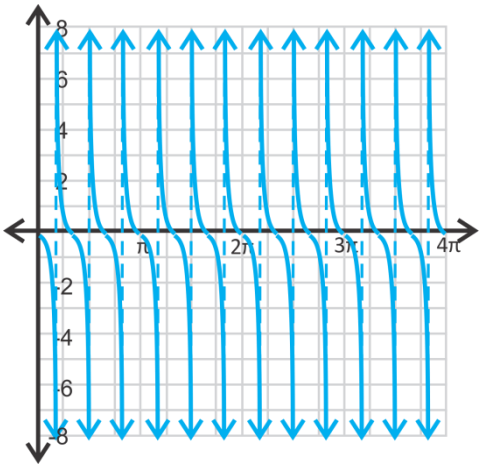
ans-1402-01

2. $p = \pi$, D: $\mathbb{R}$; $x \neq \frac{\pi}{2} \pm \pi n$, R: $\mathbb{R}$



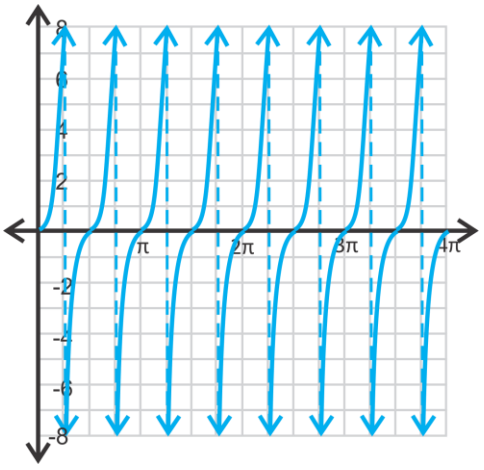
ans-1402-02

3. $p = \frac{\pi}{3}$, D: $\mathbb{R}$; $x \neq \frac{\pi}{6} \pm \frac{\pi}{3} n$, R: $\mathbb{R}$



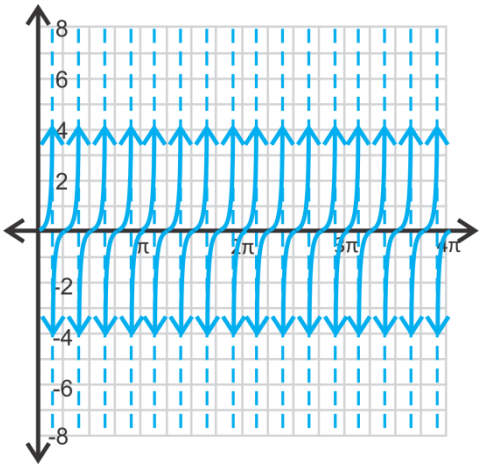
ans-1402-03

4. $p = \frac{\pi}{2}$, D: $\mathbb{R}$; $x \neq \frac{\pi}{4} \pm \frac{\pi}{2} n$, R: $\mathbb{R}$



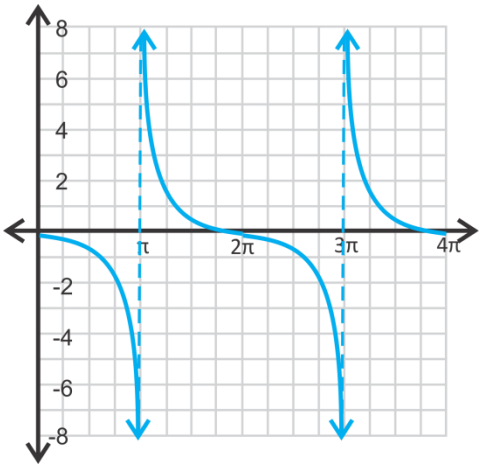
ans-1402-04

5. $p = \frac{\pi}{4}$, D: ; $x \notin \frac{\pi}{8} \pm \frac{\pi}{4} n$, R:



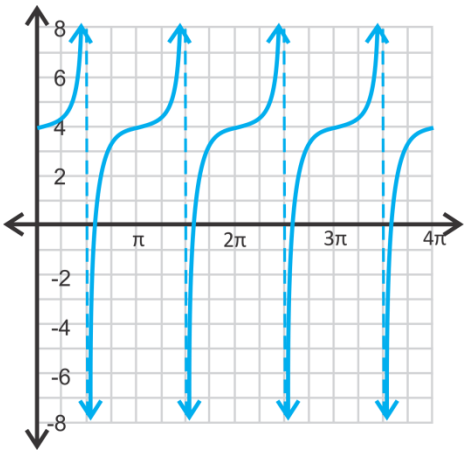
ans-1402-05

6. $p = 2\pi$, D: ; $x \notin \pi \pm 2\pi n$, R:



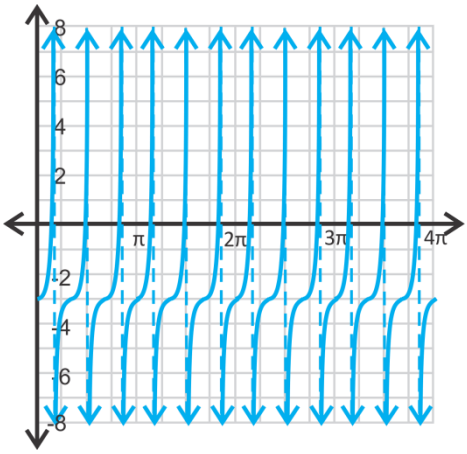
ans-1402-06

7. $p = \pi$, D: ; $x \notin \frac{\pi}{2} \pm \pi n$, R:



ans-1402-07

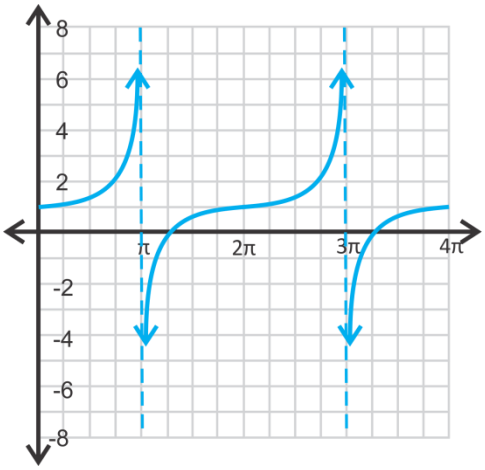
8. $p = \frac{\pi}{3}$, D: ; $x \notin \frac{\pi}{6} \pm \frac{\pi}{3} n$, R:



ans-1402-08

9. $p = \pi$, D: ; $x \notin \frac{\pi}{2} \pm \pi n$, R:

ans-1402-09



10. $x = \pm \pi n$

11. $x = \pm \frac{\pi}{3} n$

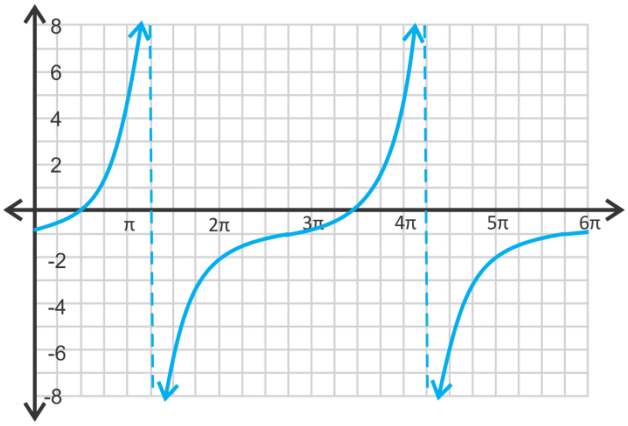
12. $x = \frac{\pi}{4} n$

13. $y = 3\tan\frac{2}{3}x$

14. $y = \frac{1}{4}\tan\frac{1}{2}x$

15. $y = -2.5\tan\frac{\pi}{8}x$

16. $p = 3\pi$, D: □ ; $x \notin \frac{5\pi}{4} \pm 3\pi n$, ans-1402-10



Using Trigonometric Identities

Review Queue Answers

1. $\frac{\sqrt{3}}{2}$

2. $-\frac{\sqrt{3}}{2}$

3. 0

4. 1

5. $\frac{1}{2}$

6. $\sqrt{3}$

Introduction to Trig Identities

1. Student needs to show proof.

2. Student needs to show proof.

3. Student needs to show proof.

4. The graphs overlap.

5. Student needs to show proof.

6. Hint: Sine is odd and cosine is even.

7. Hint: Change everything to sine and cosine.

8. Hint: Change everything to sine and cosine.

9. Hint: Change cosecant using the Reciprocal Identity.

10. Hint: Change cotangent to tangent using the Reciprocal Identity.

11. Hint: Change everything to sine and cosine.

12. Hint: Use the Negative Angle Identity for sine.

13. Hint: Plug in value for θ into the Pythagorean Identity.

14. Hint: Plug in value for θ into the Pythagorean Identity.

15. Hint: Plug in value for θ into the Pythagorean Identity.

16. Odd: Sine, Tangent, Cosecant, Cotangent. Even: Cosine, Secant.

Using Identities to Find Exact Trigonometric Values

1. I and II. III and IV.
2. I and IV. II and III.
3. I and III. II and IV.

4. $\cos\theta=\frac{15}{17},\ \tan\theta=\frac{8}{15},\ \csc\theta=\frac{17}{8},\ \sec\theta=\frac{17}{15},\ \cot\theta=\frac{15}{8}$

5. $\sin\theta=\frac{\sqrt{11}}{6},\ \tan\theta=-\frac{\sqrt{11}}{5},\ \csc\theta=\frac{6\sqrt{11}}{11},\ \sec\theta=-\frac{6}{5},\ \cot\theta=-\frac{5\sqrt{11}}{11}$

6. $\cos\theta=\frac{4\sqrt{19}}{19},\ \sin\theta=\frac{\sqrt{57}}{19},\ \csc\theta=\frac{\sqrt{57}}{3},\ \sec\theta=\frac{\sqrt{19}}{4},\ \cot\theta=\frac{4\sqrt{3}}{3}$

7. $\sin\theta=-\frac{40}{41},\ \cos\theta=-\frac{9}{41},\ \tan\theta=\frac{40}{9},\ \csc\theta=-\frac{41}{40},\ \cot\theta=\frac{9}{40}$

8. $\cos\theta=\frac{5\sqrt{3}}{14},\ \tan\theta=-\frac{11\sqrt{3}}{15},\ \csc\theta=-\frac{14}{11},\ \sec\theta=\frac{14\sqrt{3}}{15},\ \cot\theta=-\frac{5\sqrt{3}}{11}$

9. $\sin\theta=\frac{\sqrt{2}}{2},\ \tan\theta=1,\ \csc\theta=\sqrt{2},\ \sec\theta=\sqrt{2},\ \cot\theta=1$

10. $\sin\theta=-\frac{\sqrt{6}}{6},\ \cos\theta=-\frac{\sqrt{30}}{6},\ \tan\theta=\frac{\sqrt{5}}{5},\ \sec\theta=-\frac{\sqrt{30}}{5},\ \csc\theta=-\sqrt{6}$

11. $\sin\theta=\frac{1}{4},\ \cos\theta=-\frac{\sqrt{15}}{4},\ \tan\theta=-\frac{\sqrt{15}}{15},\ \sec\theta=-\frac{4\sqrt{15}}{15},\ \cot\theta=-\sqrt{15}$

12. $\cos\theta=\frac{10\sqrt{149}}{149},\ \sin\theta=-\frac{7\sqrt{149}}{149},\ \csc\theta=-\frac{\sqrt{149}}{7},\ \sec\theta=\frac{\sqrt{149}}{10},\ \cot\theta=-\frac{10}{7}$

13. The Pythagorean Theorem.
14. $\frac{5\sqrt{3}}{11}$
15. $\frac{\sqrt{89}}{8}$

Simplifying Trigonometric Expressions

1. $\cos x$
2. $-\cos x\sin x$
3. $-\cot x$
4. $\cos^2 x$
5. $\csc x$
6. $\sin^2 x$

7. $\cos^2 x$ 8. $\sec^2 x$ 9. -1

10. $\csc^2 x$ 11. $\sin x$ 12. $\tan x$

13. $-\cos^2 x$ 14. -1 15. $\sec x$

Verifying a Trigonometric Identity

1. Hint: Use the Reciprocal Identities.
2. Hint: Use the Reciprocal Identities.
3. Hint: Change everything to sine and cosine.
4. Hint: Change everything to sine and cosine.
5. Hint: Use the Cofunction Identities.
6. Hint: Use the Cofunction Identities.
7. Hint: Change everything into sine and cosine.
8. Hint: Use the Pythagorean Identities.
9. Hint: FOIL.
10. Hint: Combine like terms.
11. Hint: Start with the Pythagorean Identities.
12. Hint: Change right hand side into terms of sine and cosine.
13. Hint: Find a common denominator for the left hand side.
14. Hint: Use the Pythagorean Identities.
15. Hint: Change left hand side into terms of sine and cosine. You may also need to find a common denominator and/or FOIL.

Using Trigonometric Equations

Review Queue Answers

1. x = 9, -2
2. $x = \pm\frac{2\sqrt{3}}{2}, \pm\frac{2i\sqrt{3}}{2}$
3. $x = \frac{\pi}{4}$

Solving Trigonometric Equations Using Algebra

*n is any integer.

1. yes
2. no
3. yes

4. $x = 2\pi n$
5. $x = \frac{\pi}{6} \pm \pi n$
6. $x = \frac{\pi}{3} \pm 2\pi n, \ \frac{5\pi}{3} \pm 2\pi n$

7. no solution

8. $x = \frac{\pi}{3} \pm 2\pi n,\ \frac{5\pi}{3} \pm 2\pi n$

9. $x = \frac{\pi}{3} \pm \frac{\pi}{3} n$, where n is not a multiple of 3.

10. $x = \frac{\pi}{4},\ \frac{5\pi}{4}$

11. $x = \frac{4\pi}{3},\ \frac{5\pi}{3}$

12. no solution

13. $x = 0.775, 5.508$

14. $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

15. $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Solving Trigonometric Equations Using Quadratic Techniques

1. $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

2. $x = \frac{3\pi}{2}, 3.3943, 6.0305$

3. $x = 0, \frac{\pi}{4}, \pi$

4. $x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

5. $x = \frac{\pi}{2}, 3.4814, 5.9433$

6. $x = 0.2527, \frac{\pi}{2}, 2.8889$

7. $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

8. $x = \frac{3\pi}{4}, \frac{7\pi}{4}$

9. $x = 1.1593, 1.9823, \pi$

10. $x = \frac{\pi}{2}$

11. $x = 0$

12. $x = \frac{\pi}{2}, \frac{3\pi}{2}$

13. $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

14. (0.3919, 0.1459), (2.7497, 0.1459)

15. (4.1461, -3.1416), (5.5234, 1.9)

Sum and Difference Formulas

Review Queue Answers

1. 0.9659

2. 0.9659

3. -3.7321

4. -0.2588

Finding Exact Trig Values using Sum and Difference Formulas

1. $\frac{\sqrt{6} - \sqrt{2}}{4}$

2. $\frac{\sqrt{6} - \sqrt{2}}{4}$

3. $-2 + \sqrt{3}$

4. $\frac{\sqrt{2} - \sqrt{6}}{4}$

5. $\frac{\sqrt{2} - \sqrt{6}}{4}$

6. $\frac{-\sqrt{6} - \sqrt{2}}{4}$

7. $\frac{\sqrt{6}+\sqrt{2}}{4}$ 8. $\frac{2-\sqrt{3}}{2}$ 9. $\frac{\sqrt{2}-\sqrt{6}}{4}$

10. Yes 11. $\frac{-\sqrt{6}-\sqrt{2}}{4}$ 12. Answers will vary.

13. 0.6157 14. Any combination that adds up to 142° will work.

15. Students must provide proof.

Simplifying Trig Expressions using Sum and Difference Formulas

1. $\frac{15-8\sqrt{3}}{34}$ 2. $-\frac{15\sqrt{3}+8}{34}$ 3. $-\frac{15+8\sqrt{3}}{34}$

4. $-\frac{480+289\sqrt{3}}{611}$ 5. $\frac{8-15\sqrt{3}}{34}$ 6. $-\frac{480-289\sqrt{3}}{611}$

7. $-\sin x$ 8. $\cos x$ 9. $-\cos x$

10. $-\sin x$ 11. $\tan x$ 12. $\tan x$

13. $\frac{1}{2}\left(\cos x-\sqrt{3}\sin x\right)$ 14. $\frac{1+\tan x}{1-\tan x}$ 15. $\frac{1}{2}\left(\cos x+\sqrt{3}\sin x\right)$

16. F 17. T 18. F

Solving Trig Equations using Sum and Difference Formulas

1. $x=\frac{\pi}{4},\frac{3\pi}{4}$ 2. $x=\pi$ 3. $x=0,\pi$

4. $x=\frac{\pi}{3},\frac{5\pi}{3}$ 5. $x=\frac{5\pi}{4},\frac{7\pi}{4}$ 6. $x=0,\pi$

7. $x=\frac{\pi}{2}$ 8. $x=0$ 9. $x=0,\frac{\pi}{3},\pi,\frac{5\pi}{3}$

10. $x=0,\pi$ 11. no solution 12. $x=\frac{3\pi}{4},\frac{7\pi}{4}$

13. $x=\frac{3\pi}{2}$ 14. $x=0$ 15. At 5.7 sec and 1.14 min.

Double and Half Angle Formulas

Review Queue Answers

1. 0.3827
2. -0.4142
3. 0
4. $\frac{\sqrt{6}-\sqrt{2}}{4}$

Finding Exact Trig Values using Double and Half Angle Formulas

1. $-\frac{\sqrt{2+\sqrt{3}}}{2}$
2. $\sqrt{2}-1$
3. $\frac{\sqrt{2-\sqrt{3}}}{2}$
4. $-\frac{\sqrt{2+\sqrt{3}}}{2}$
5. $\frac{\sqrt{1+\sqrt{2}}}{2}$
6. $-2-\sqrt{3}$
7. $-\frac{\sqrt{1+\sqrt{2}}}{2}$
8. $\frac{\sqrt{2-\sqrt{3}}}{2}$
9. $-\frac{120}{169}$
10. $-\sqrt{\frac{9}{13}}$
11. $-\frac{2}{3}$
12. $-\frac{119}{169}$
13. $\frac{16\sqrt{57}}{7}$
14. $\sqrt{\frac{11-\sqrt{57}}{22}}$
15. $\sqrt{\frac{11+\sqrt{57}}{22}}$
16. $-\frac{16\sqrt{57}}{121}$

Simplifying Trig Expressions using Double and Half Angle Formulas

1. $1+\cos x$
2. $\sec^2 x$
3. $\frac{2\sin x}{\cos x-\sin x}$
4. $1-5\sin^2 x$
5. $\sin 2x$
6. $\sin x(1+\cos x)$
7. Hint: Change $\cot\frac{x}{2}$ to $\frac{1}{\tan\frac{x}{2}}$ or $\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}$
8. Hint: Cross-multiply.
9. Hint: Expand $\sin 2x$ and $\cos 2x$
10. Hint: FOIL
11. Hint: Rewrite the half-angles
12. Hint: Rewrite $\csc 2x$ in terms of sine
13. Hint: $\cos 3x=\cos(x+2x)$
14. Hint: Use $\cos 2x=\cos^2 x-\sin^2 x$
15. Hint: Expand the double-angles
16. Hint: Factor

Solving Trig Equations using Double and Half Angle Formulas

1. $x = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$

2. $x = 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}$

3. $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

4. $x = 0, \pi$

5. x = 2.237, 5.379

6. $x = \frac{\pi}{2}, \frac{3\pi}{2}$

7. x = 2.6516

8. $x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}$

9. $x = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$

10. $x = 0, \pi$

11. no solution

12. $x = 0, \pi$

13. $x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$

14. $x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}, \frac{3\pi}{2}$

15. no solution

16. infinitely many solutions