Basic Algebra Flexbook Solution Key
Chapter10
Quadratic Equations and Functions

Lesson 10.1
Graphs of Quadratic Functions

1.

(1) The *vertex* is the point at which a parabola reaches it maximum or minimum (top or bottom).
(2) The *line of symmetry* is the imaginary line which runs through the vertex of a parabola dividing into equals halves.
(3) A *parabola* is the shape created by the graph of a quadratic function. It is U-shaped figure that can open either up or down.
(4) The minimum is the vertex of a parabola that opens upward (the lowest point).
(5) The maximum is the vertex of a parabola that opens downward (the highest point).

2. You can tell if $y=ax^2 + bx + c$ opens up or down by whether a is negative or positive.

3.

(1) vertex is (25, 610)
(2) y-intercept is (0,0)
(3) x-intercepts are (0,0) and (50,0)
(4) domain is all real numbers
(5) range is $y \le 610$
(6) line of symmetry is x=25
(7) *a* is negative
(8) $-1 < a < 1$

4.

(1)

$d(s) = \frac{1}{20}s^2 + s$

$d(45) = \frac{1}{20}(45)^2 + 45$

$d(45) = \frac{1}{20}(2025) + 45$

$d(45) = \frac{405}{4} + 45$

$d(45) = 101.25 + 45$

$d(45) = 146.25$

(2)

$d(s) = \frac{1}{20}s^2 + s$

$96 = \frac{1}{20}s^2 + s$

s	$\frac{1}{20}s^2 + s$	d
~~10~~	~~1/20(10)^2+10=1/20(100)+10=5+10=15~~	~~15~~
~~20~~	~~1/20(20)^2+20=1/20(400)+20=20+20=40~~	~~40~~
30	1/20(30)^2+30=1/20(900)+30=45+30=75	75
40	1/20(40)^2+40=1/20(1600)+40=80+40=120	120

s	$\frac{1}{20}s^2 + s$	d
~~32~~	~~1/20(32)^2+32=1/20(1024)+32=51.2+32=83.2~~	~~83.2~~
34	1/20(34)^2+34=1/20(1156)+34=57.8+34=91.8	91.8
36	1/20(36)^2+36=1/20(1296)+36=64.8+36=100.8	100.8

1/20(35)^2+35=1/20(1225)+35=61.25+35=96.25

d ≈ 35

5.

$D = 16t^2$

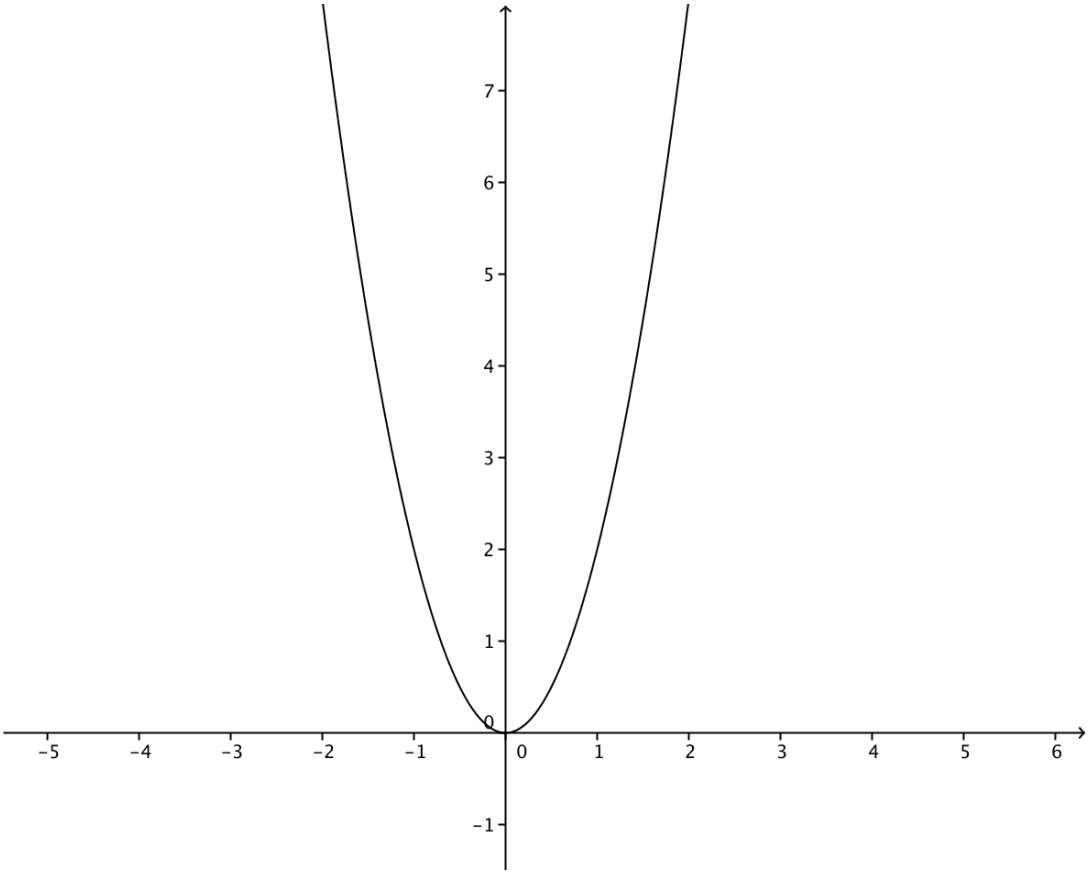
$D = 16(3.5)^2$

$D = 16(12.25)$

$D = 196$

6. $y=2x^2$

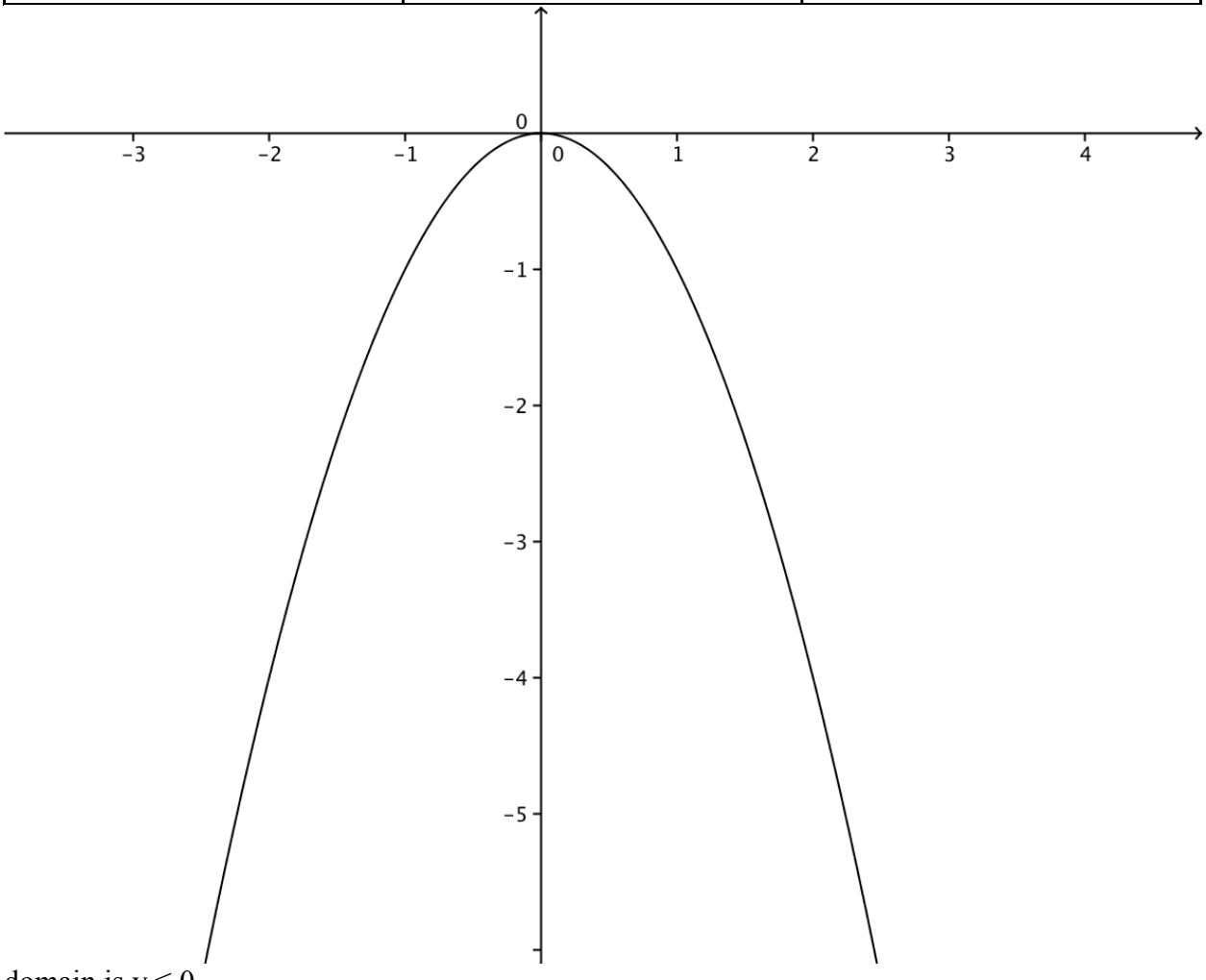
x	$2x^2$	y
-3	$2(-3)^2 = 2(9)=18$	18
-2	$2(-2)^2 = 2(4)=8$	8
-1	$2(-1)^2 = 2(1)=2$	2
0	$2(0)^2 = 2(0)=0$	0
1	$2(1)^2 = 2(1)=2$	2
2	$2(2)^2 = 2(4)=8$	8
3	$2(3)^2 = 2(9)=18$	18



domain is $y \geq 0$

7. $y= -x^2$

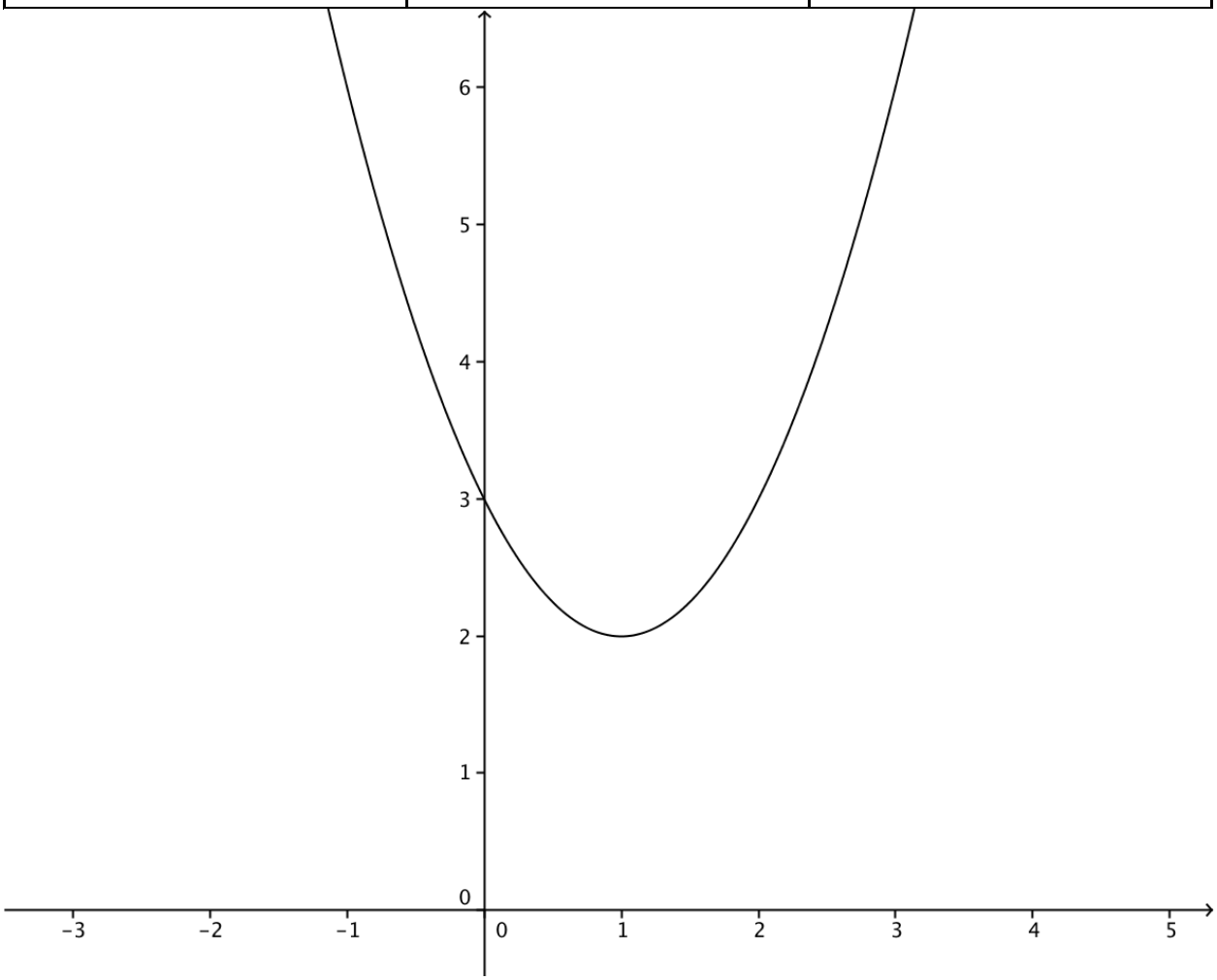
x	$-x^2$	y
-3	$-(-3)^2 = -9$	-9
-2	$-(-2)^2 = -4$	-4
-1	$-(-1)^2 = -1$	-1
0	$-(0)^2 = 0$	0
1	$-(1)^2 = -1$	-1
2	$-(2)^2 = -4$	-4
3	$-(3)^2 = -9$	-9



domain is $y \leq 0$

8. $y = x^2 - 2x + 3$

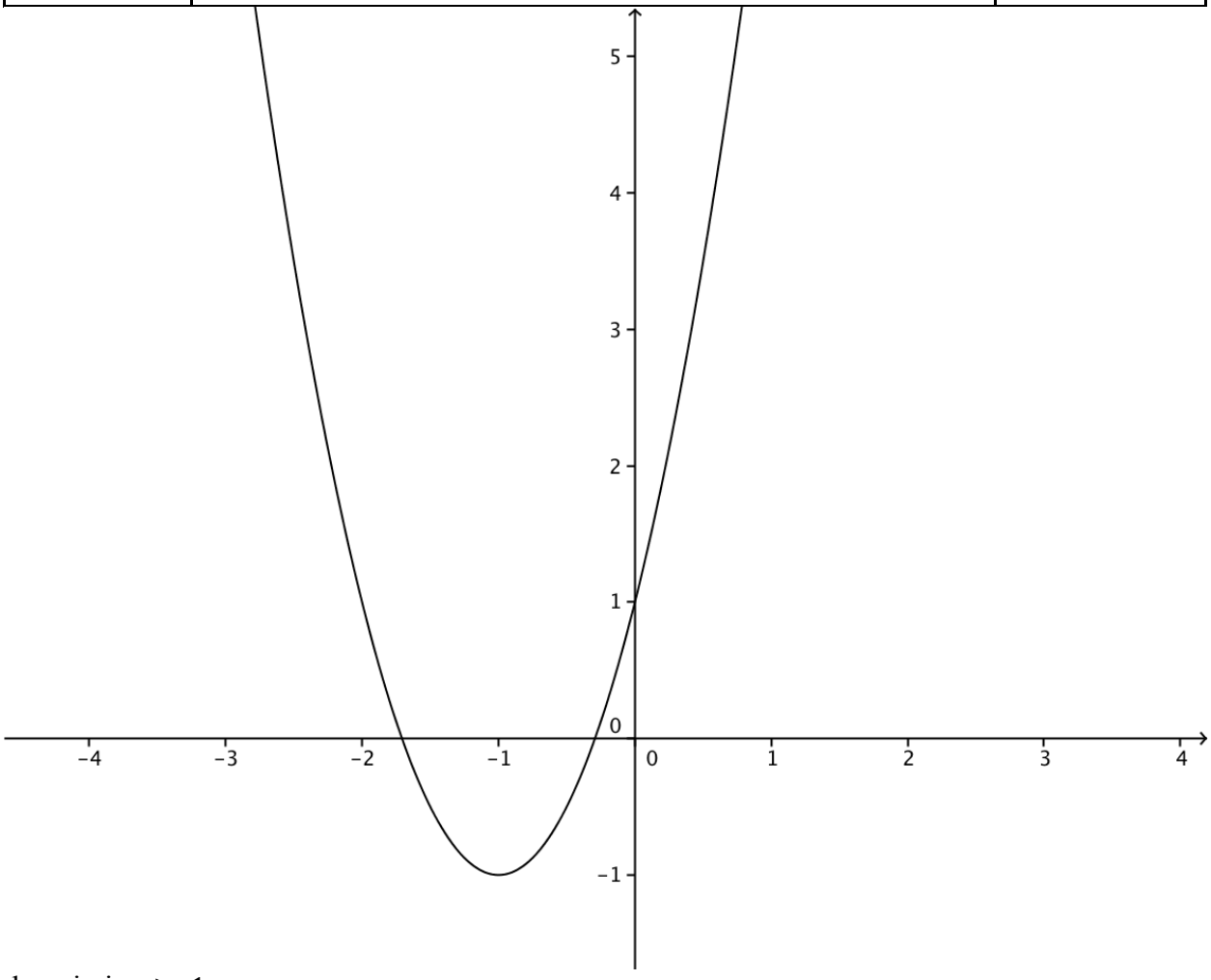
x	x^2-2x+3	y
-3	$(-3)^2 - 2(-3) + 3 = 9+6+3=18$	18
-2	$(-2)^2 - 2(-2) + 3 = 4+4+3=11$	11
-1	$(-1)^2 - 2(-1) + 3 = 1+2+3=6$	6
0	$(0)^2 - 2(0) + 3 = 0+0+3=3$	3
1	$(1)^2 - 2(1) + 3 = 1-2+3=2$	2
2	$(2)^2 - 2(2) + 3 = 4-4+3=3$	3
3	$(3)^2 - 2(3) + 3 = 9-6+3=6$	6



domain is $y \geq 2$

9. $y = 2x^2 + 4x + 1$

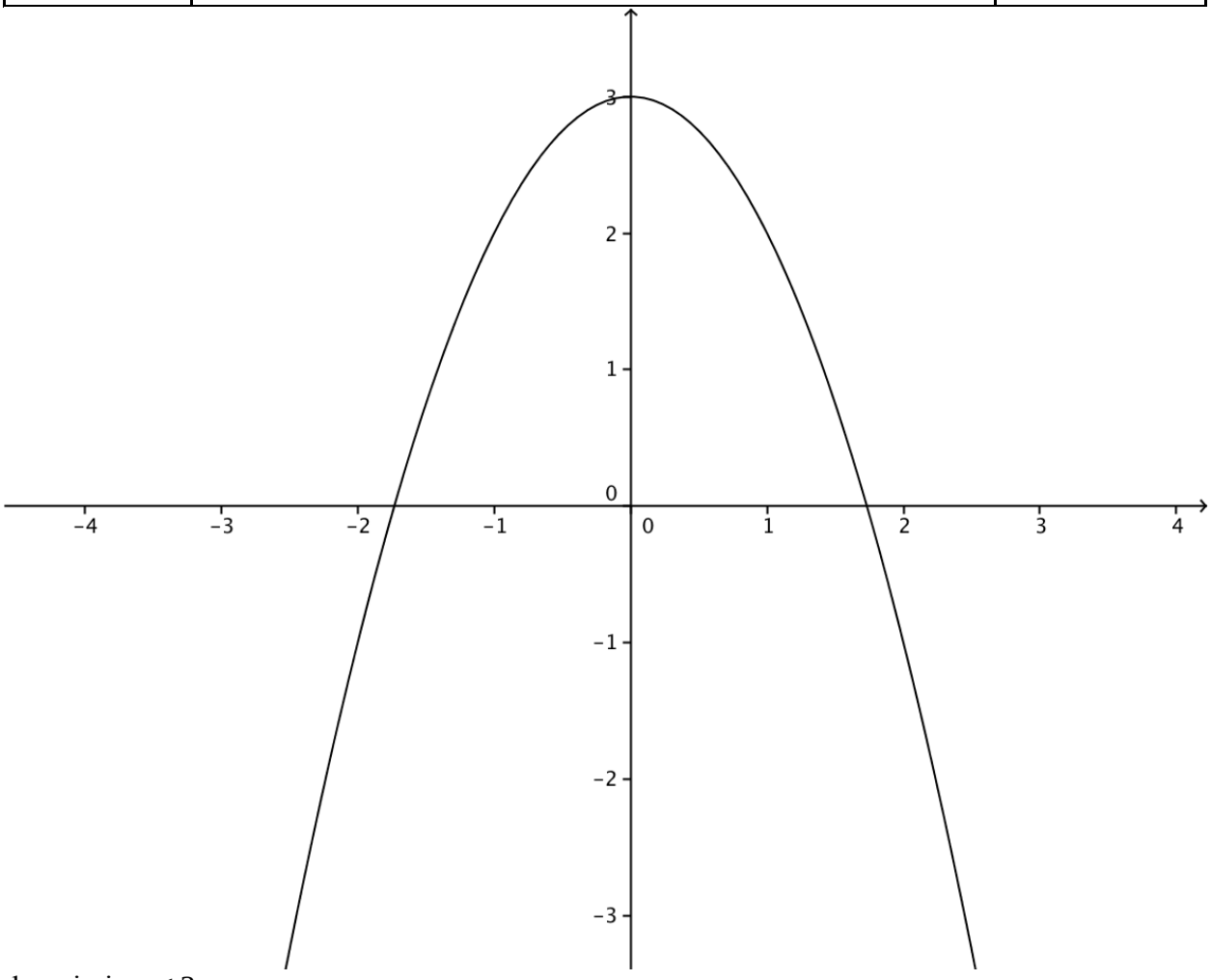
x	$2x^2+4x+1$	y
-3	$2(-3)^2 +4(-3) +1= 2(9)-12+1 =18-12+1= 7$	7
-2	$2(-2)^2 +4(-2) +1= 2(4)-8+1 =8-8+1= 1$	1
-1	$2(-1)^2 +4(-1) +1= 2(1)-4+1 =2-4+1= -1$	-1
0	$2(0)^2 +4(0) +1= 2(0)+0+1 =0+0+1= 1$	1
1	$2(1)^2 +4(1) +1= 2(1)+4+1 =2+4+1= 7$	7
2	$2(2)^2 +4(2) +1= 2(4)+8+1 =8+8+1= 17$	17
3	$2(3)^2 +4(3) +1= 2(9)+12+1 =18+12+1= 21$	21



domain is $y \geq -1$

10. $y = -x^2 + 3$

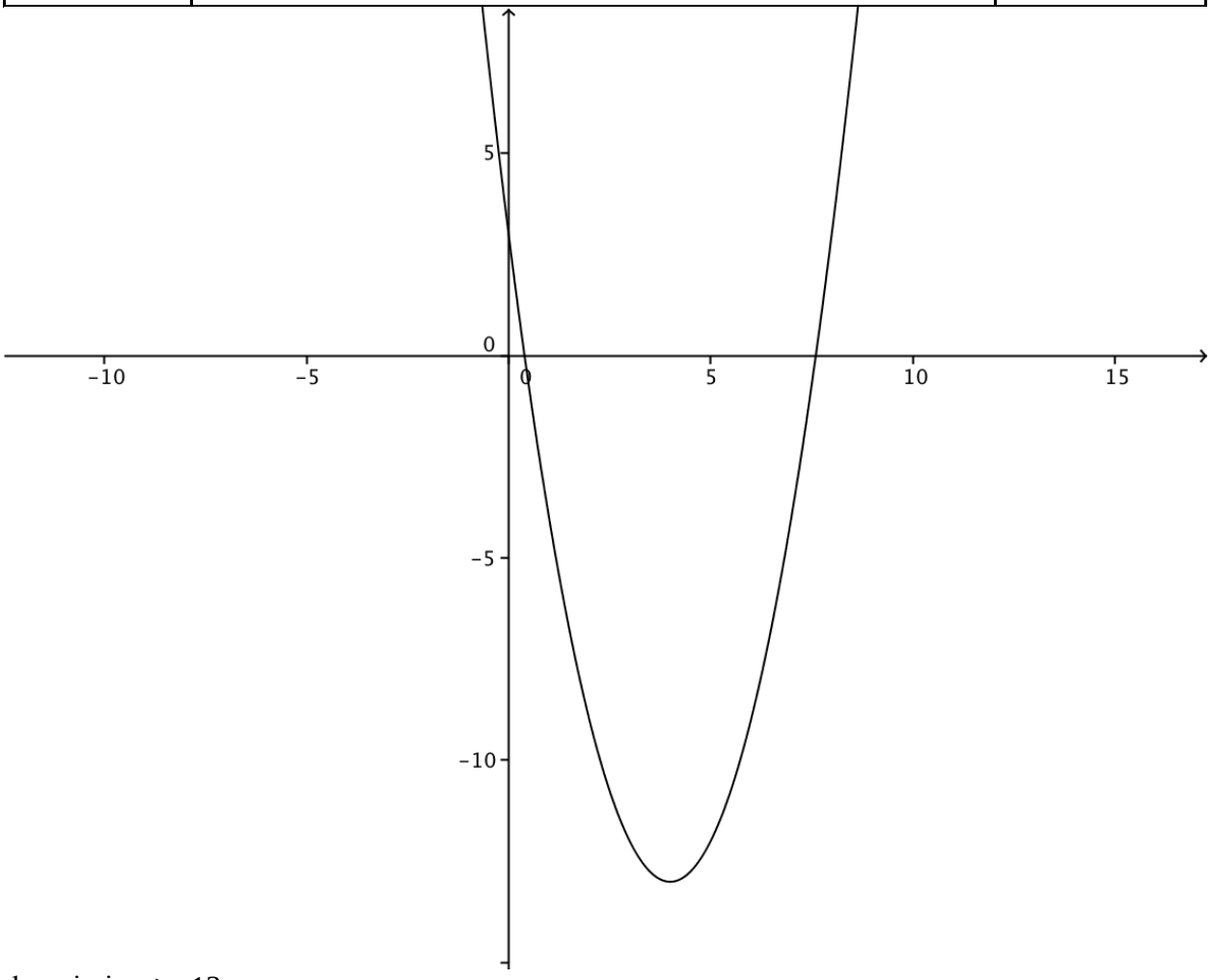
x	$-x^2+3$	y
-3	$-(-3)^2+3= -9+3= -6$	-6
-2	$-(-2)^2+3= -4+3= -1$	-1
-1	$-(-1)^2+3= -1+3= 2$	2
0	$-(0)^2+3= 0+3= 3$	3
1	$-(1)^2+3= -1+3= 2$	2
2	$-(2)^2+3= -4+3= -1$	-1
3	$-(3)^2+3= -9+3= -6$	-6



domain is $y \leq 3$

11. $y= x^2 -8x +3$

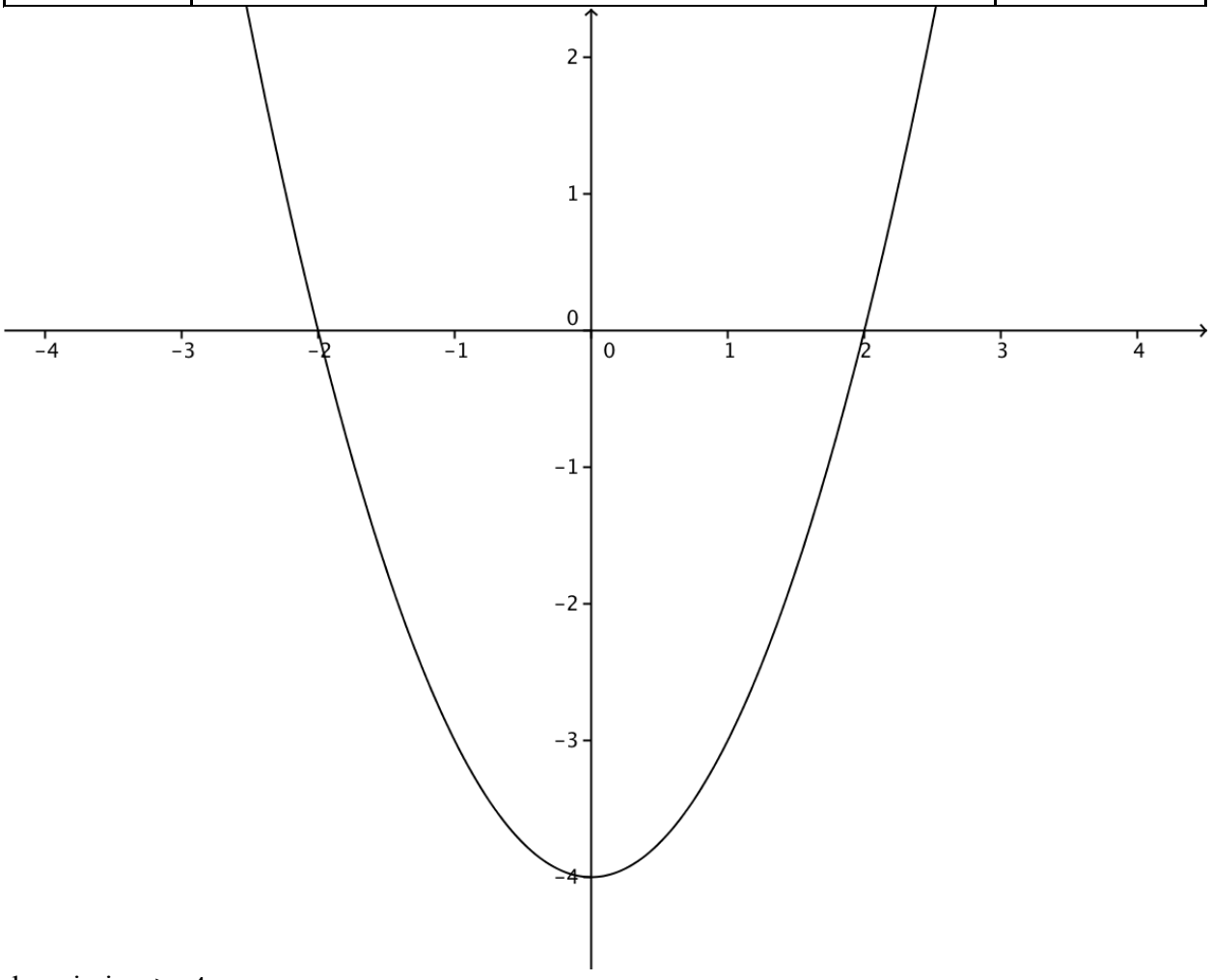
x	$x^2 -8x +3$	y
-3	$(-3)^2 -8(-3) +3= 9+24+3= 36$	36
-2	$(-2)^2 -8(-2) +3= 4+16+3=23$	23
-1	$(-1)^2 -8(-1) +3= 1+8+3= 12$	12
0	$(0)^2 -8(0) +3= 0-0+3= 3$	3
1	$(1)^2 -8(1) +3= 1-8+3= -4$	-4
2	$(2)^2 -8(2) +3= 4-16+3= -9$	-9
3	$(3)^2 -8(3) +3= 9-24+3= -12$	-12



domain is $y \geq -13$

12. $y = x^2-4$

x	$x^2 -4$	y
-3	$(-3)^2 -4= 9-4 = 5$	5
-2	$(-2)^2 -4= 4-4 = 0$	0
-1	$(-1)^2 -4= 1-4 = -3$	-3
0	$(0)^2 -4= 0-4 = -4$	-4
1	$(1)^2 -4= 1-4 = -3$	-3
2	$(2)^2 -4= 4-4 = 0$	0
3	$(3)^2 -4= 9-4 = 5$	5



domain is $y \geq -4$

13. the y-intercepts are the same (0,0)

14. the y-intercepts are the same (4,0)

15. the y-intercepts are the same (-2,0)

16. a is positive so the parabola opens upward
$-1 < a < 1$ so the parabola is wide about its line of symmetry
y-intercept = -8

17. a is negative so the parabola opens downward
$a < -1$ so the parabola is narrow about its line of symmetry
y-intercept = -21

18. a is positive so the parabola opens upward
$a > 1$ so the parabola is narrow about its line of symmetry
y-intercept = 4

19. the parabola opens down

20. the parabola opens up

21. the parabola opens down

22. $y=4x^2$ is more narrow around its line of symmetry because $a > 1$

23. both equations are equally narrow around their line of symmetry because a= -2 in both equations

24. both equations are equally narrow around their line of symmetry because a= 3 in both equations

25.

$y = 4x^2 - 4$

$y = (2x - 2)(2x + 2)$

$2x - 2 = 0$

$2x = 2$

$x = 1$

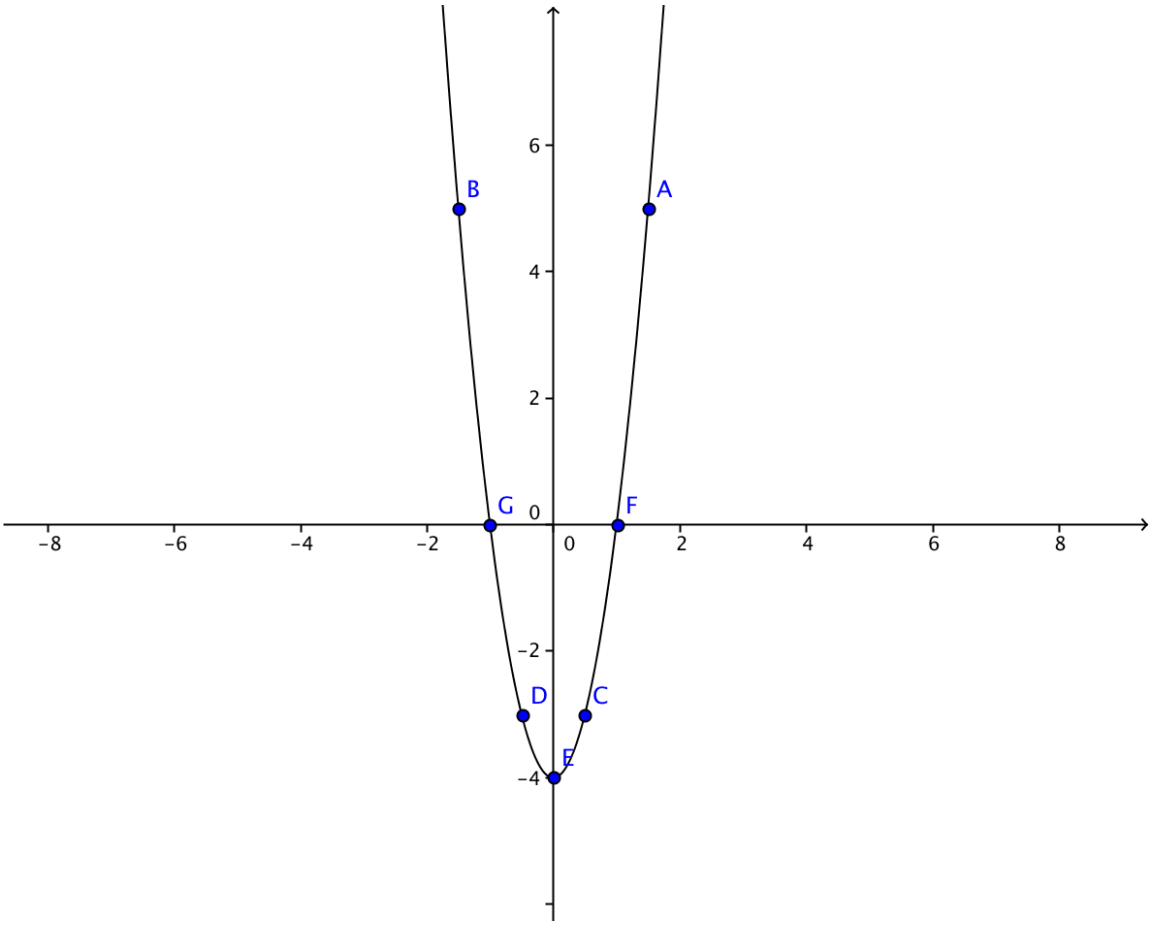
$2x + 2 = 0$

$2x = -2$

$x = -1$

x	$4x^2-4$	y
3/2	$4(3/2)^2-4=4(9/4)-4=9-4=5$	5
1		0
1/2	$4(1/2)^2-4=4(1/4)-4=1-4=-3$	-3
0		-4
-1/2	$4(-1/2)^2-4=4(1/4)-4=1-4=-3$	-3
-1		0
-3/2	$4(-3/2)^2-4=4(9/4)-4=9-4=5$	5

The x intercepts are 1 and -1, the y-intercept is -4.



$y = -x^2 + x + 12$

$y = -(x^2 - x - 12)$

$y = -(x - 4)(x + 3)$

$-(x - 4) = 0$

$-x + 4 = 0$

$-x = -4$

$x = 4$

$-(x + 3) = 0$

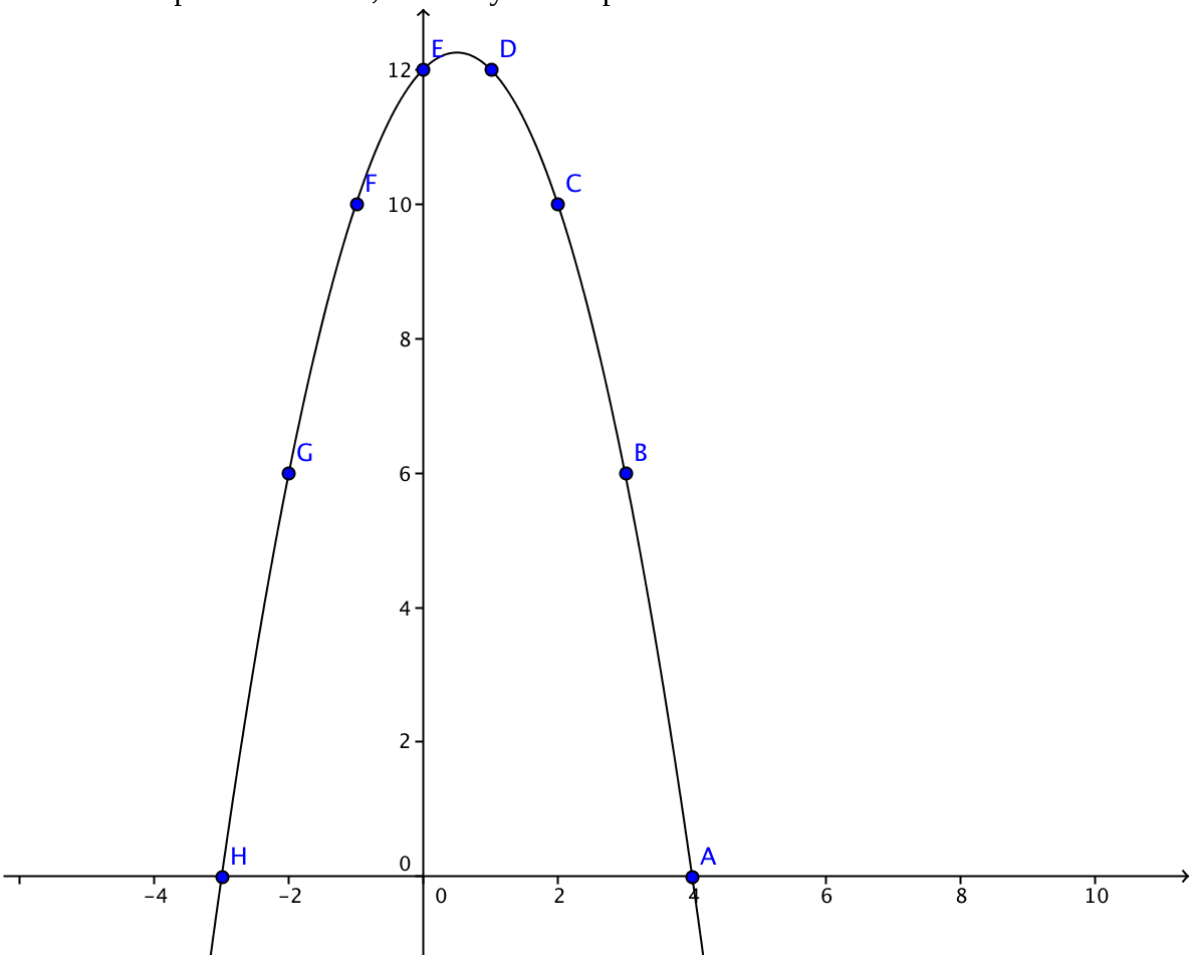
$-x - 3 = 0$

$-x = 3$

$x = -3$

x	$-x^2+x+12$	y
4		0
3	$-(3)^2+3+12=-9+3+12=6$	6
2	$-(2)^2+2+12=-4+2+12=10$	10
1	$-(1)^2+1+12=-1+1+12=12$	12
0		12
-1	$-(-1)^2-1+12=-1-1+12=10$	10
-2	$-(-2)^2-2+12=-4-2+12=6$	6
-3		0
-4	$-(-4)^2-4+12=-16-4+12=-8$	-8

The x-intercepts are -3 and 4, and the y-intercept is 12.



$y = 2x^2 + 10x + 8$

$y = 2(x^2 + 5x + 4)$

$y = 2(x + 4)(x + 1)$

$2(x + 4) = 0$

$2x + 8 = 0$

$2x = -8$

$x = -4$

$2(x + 1) = 0$

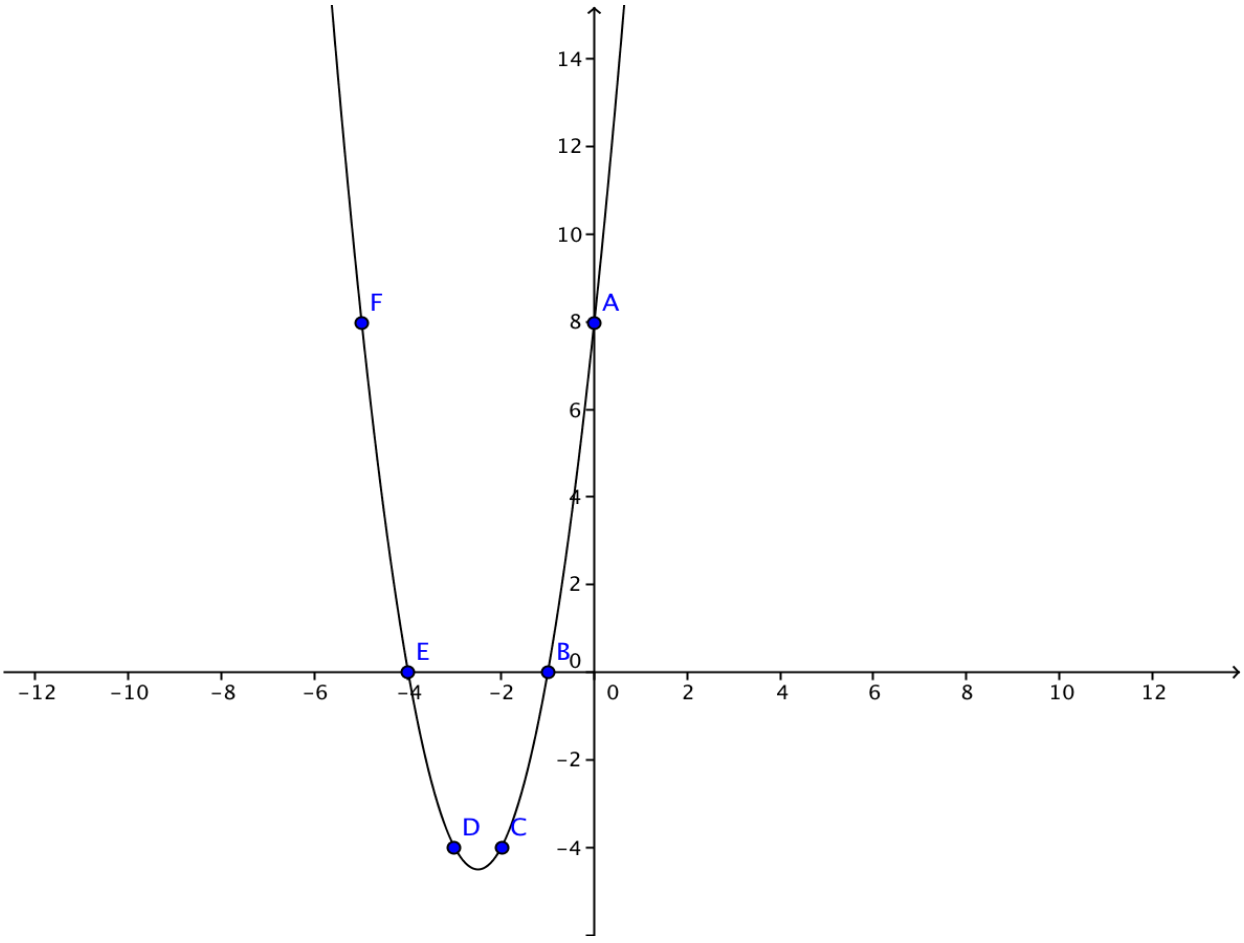
$2x + 2 = 0$

$2x = -2$

$x = -1$

x	$2x^2+10x+8$	y
0		8
-1		0
-2	$2(-2)^2+10(-2)+8=8-20+8=-4$	-4
-3	$2(-3)^2+10(-3)+8=18-30+8=-4$	-4
-4		0
-5	$2(-5)^2+10(-5)+8=50-50+8=8$	8

The x-intercepts are -4 and -1, and the y-intercept is 8.



.

$y = \frac{1}{2}x^2 - 2x$

$y = x(\frac{1}{2}x - 2)$

$x = 0$

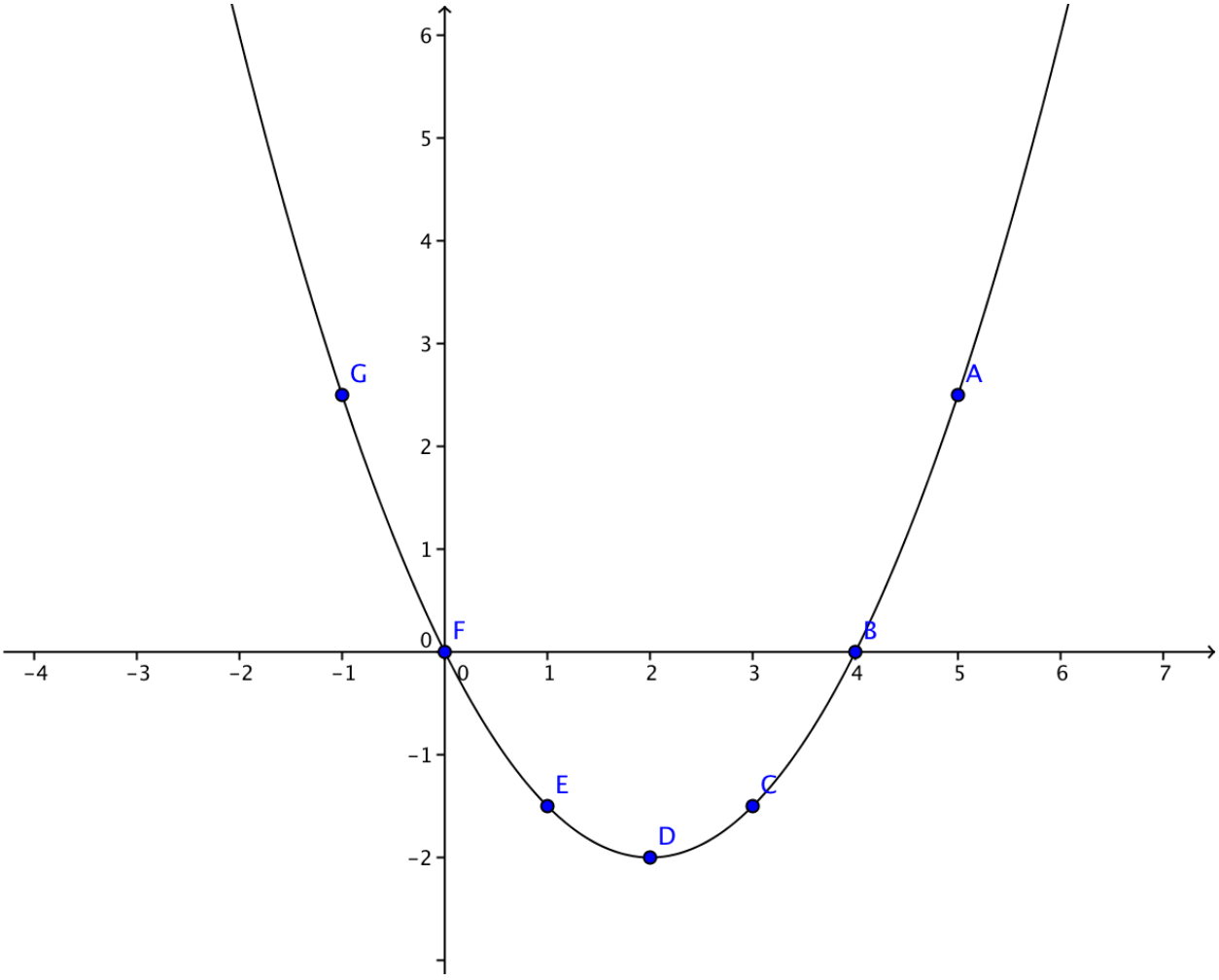
$\frac{1}{2}x - 2 = 0$

$\frac{1}{2}x = 2$

$x = 4$

The x-intercepts are 0 and 4, and the y-intercept is 0.

x	$1/2x^2$-2x	y
5	$1/2(5)^2$-2(5)=12.5-10=2.5	5/2
4		0
3	$1/2(3)^2$-2(3)=4.5-6=-1.5	-3/2
2	$1/2(2)^2$-2(2)=2-4=-2	-2
1	$1/2(1)^2$-2(1)=0.5-2=-1.5	-3/2
0		0
-1	$1/2(-1)^2$-2(-1)=0.5+2=2.5	5/2



29.

$y = x - 2x^2$

$y = -2x^2 + x$

$y = x(-2x + 1)$

$x = 0$

$-2x + 1 = 0$

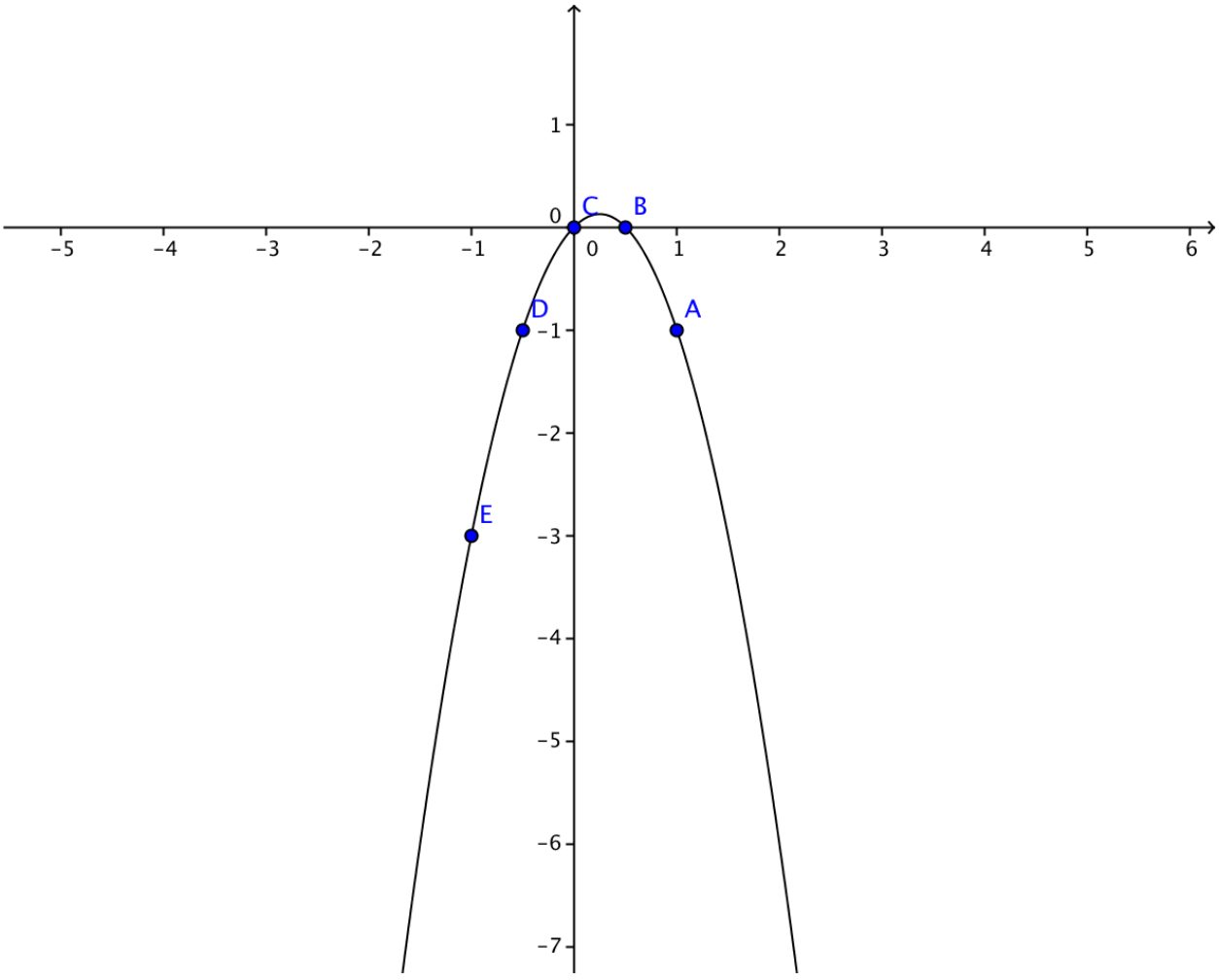
$-2x = -1$

$-x = -\frac{1}{2}$

$x = \frac{1}{2}$

x	$x-2x^2$	y
1	$(1)-2(1)^2=1-2=-1$	-1
1/2		0
0		0
-1/2	$(-1/2)-2(-1/2)^2=-1/2-1/2=-1$	-1
-1	$(-1)-2(-1)^2=-1-2=-3$	-3

The x-intercepts are 0 and 1/2, and the y-intercept is 0.



30.

$y = 4x^2 - 8x + 4$

$y = 4(x^2 - 2x + 1)$

$y = 4(x-1)^2$

$4(x-1) = 0$

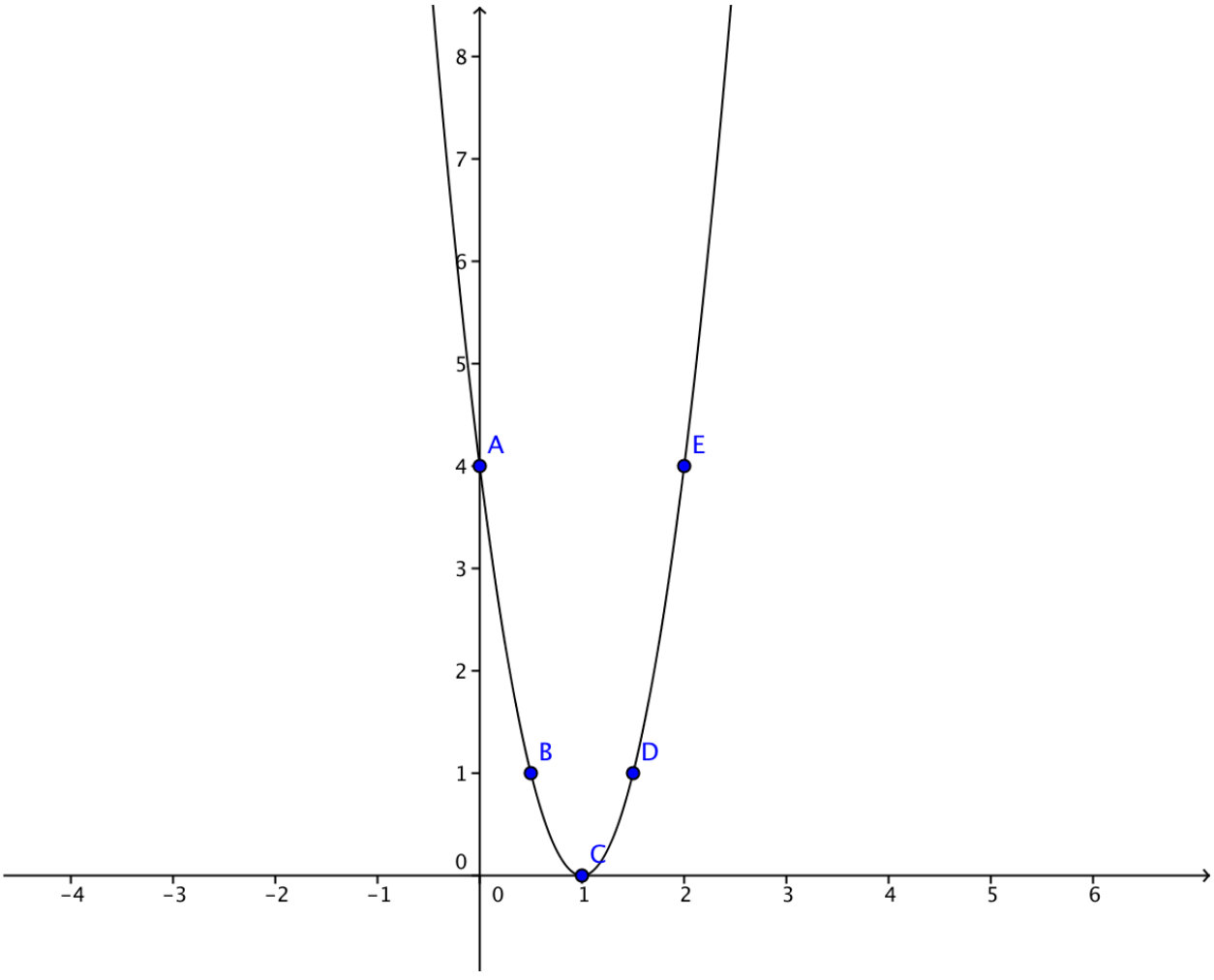
$4x - 4 = 0$

$4x = 4$

$x = 1$

x	$4x^2$-8x+4	y
0		4
1/2	$4(1/2)^2$-8(1/2)+4=1-4+4=1	1
1		0
3/2	$4(3/2)^2$-8(3/2)+4=9-12+4=1	1
2	$4(2)^2$-8(2)+4=16-16+4=4	4

The x-intercept is 1 and the y-intercept is 4.



31. The ball hits the ground 14 feet away from Nadia. The distance from Nadia at which the ball reaches its maximum height is just over 6 feet away (x ≈ 6). The maximum height is just over 10 feet (y ≈ 10).

32. The parabola reaches it maximum height when x=30.
If the width equals 30 then the length is 60 (*l*=120-2x=120-2(30)=60).
The dimensions would be 30 feet by 60 feet giving a total area of 1800 square feet.

33.

$$6u^2v - 11u^2v^2 - 10u^2v^3$$

$$u^2v\left(6 - 11v - 10v^2\right)$$

34.

$$3x^2 + 11x + 10$$

$$(3 \cdot x \cdot x) + (11 \cdot x) + (5 \cdot 2)$$

35.

$$-\frac{1}{9}(63)\left(-\frac{3}{7}\right) = -7\left(-\frac{3}{7}\right) = 3$$

36.

$$|b + 2| = 9$$

$$(b + 2) = 9$$

$$b = 7$$

$$-(b + 2) = 9$$

$$-b - 2 = 9$$

$$-b = 11$$

$$b = -11$$

$$b = 7, -11$$

37.

$$\left(4x^3y^2z\right)^3 = 4^3 \cdot x^{3\cdot3} \cdot y^{2\cdot3} \cdot z^{1\cdot3} = 64x^9y^6z^3$$

38.

$$7x + 4y = 9$$

$$4y = -7x + 9$$

$$y = -\tfrac{7}{4}x + \tfrac{9}{4}$$

$$y = -1.75x + 2.25$$

The slope is -1.75 and the y-intercept is 2.25.

Lesson 10.2
Solving Quadratic Equations by Graphing

1. Alternate names for the solution to a parabola are roots, zeros, or x-intercepts.

2.
1. h_0 is the initial height of the object.
2. t is the time (usually in seconds)
3. v_0 is the initial velocity
4. g is gravity due to acceleration (9.8 m/s or 32 ft/s)
5. h(t) is equation that explains the height as a function of time of a projectile

3.
$$h(t) = -\tfrac{1}{2}(g)t^2 + v_0 t + h_0$$
$$h(t) = -\tfrac{1}{2}(9.8)t^2 + 15t + 3$$
1. $h(t) = -4.9t^2 + 15t + 3$

Graph this polynomial to answer the following.
2. The maximum height is 14.48 meters. This height occurs at 1.53 seconds.
3. At four seconds the height is -16. This means that the rocket has already hit the ground and has stopped moving.
4. The rocket will hit the ground at 3.25 seconds.
5. The rocket will be 13 meters from the ground at 0.98 seconds and 2 seconds.

4. Two solutions

5. One solution

All solutions below are found by graphing the polynomial provided.

6. Two solutions

7. Two solutions

8. No solutions

9. The zero is 2.

10. The zeros are 1.67 and 0.

11. No solutions

12. The zeros are -1.19 and 1.69.

13. The zeros are 3 and -3.

14. The zero is -3.

15. The zero is 0.

16. No solutions

17. The zeros are 0.33 and 1.

18. The zeros are -1.5 and 1.5

19. The zeros are -0.27 and 7.27

20. The zero is -5.

21. The zeros are 0 and 1.5

22. No solutions

23. It takes 9.8 seconds to reach the ground.

24.
(a) The roots are -0.43 and -11.57
(b) The vertex is (-6,-31)

25.
(a) There are no roots to this equation.
(b) The vertex is (-1.5, 3.75)

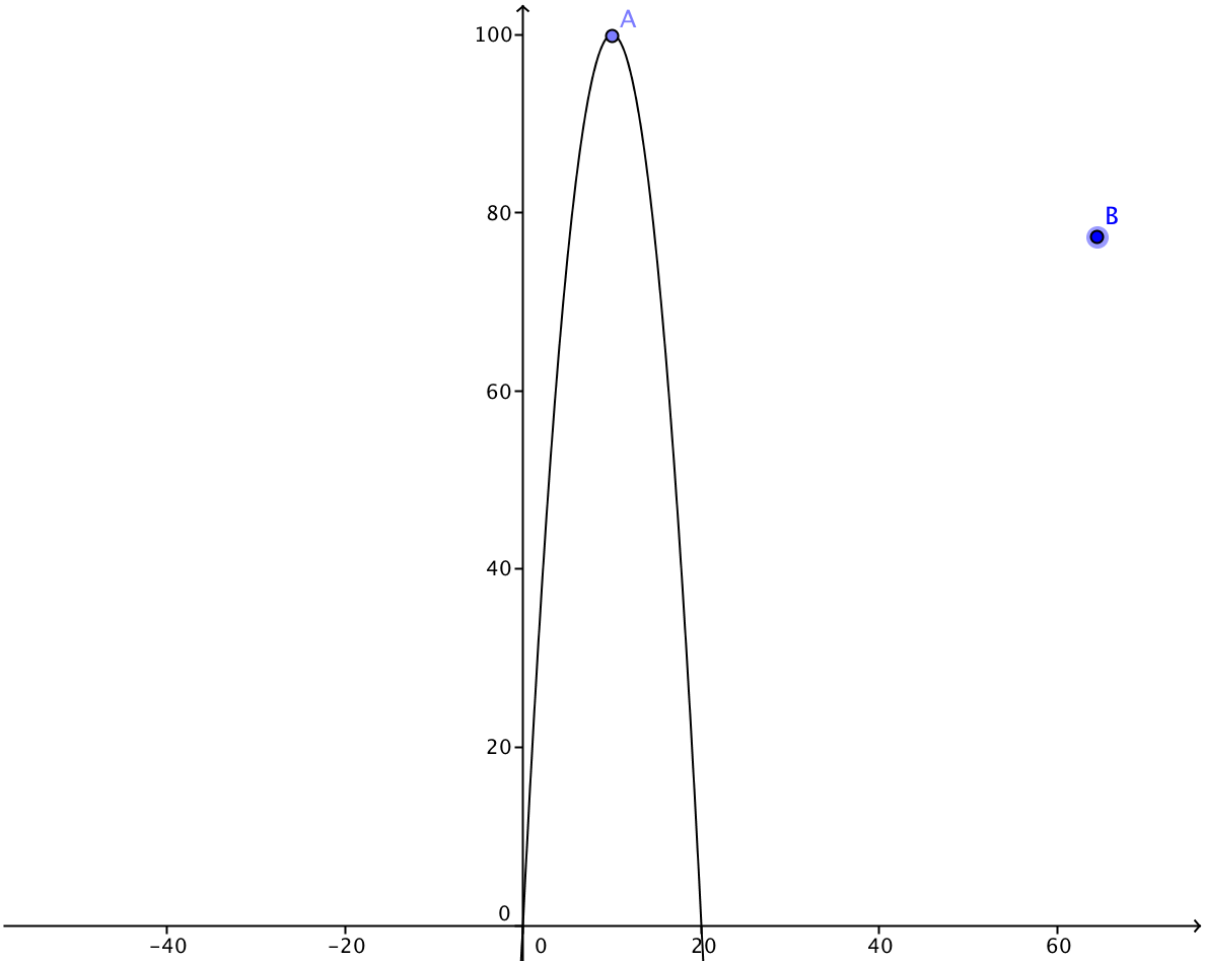
26.
(a) The roots are -4.85 and 1.85
(b) The vertex is (-1.5, 11.25)

27.

$$\begin{cases} A = l(w) \\ 40 = 2l + 2w \end{cases}$$

$$40 = 2l + 2w$$
$$40 - 21 = 2w$$
$$\frac{40 - 2l}{2} = w$$
$$20 - l = w$$

$$A = l(w)$$
$$A = l(20 - l)$$
$$A = 20l - l^2$$



Based on the graph the largest area is 100 ft, which is created by making a 10ft by 10 ft square.

28.

$h(t) = -\frac{1}{2}(g)t^2 + v_0 t + h_0$

1. $h(t) = -\frac{1}{2}(32)t^2 + (45)t + 5.2$

$h(t) = -\frac{1}{2}(g)t^2 + v_0 t + h_0$

$h(1) = -\frac{1}{2}(32)1^2 + (45)1 + 5.2$

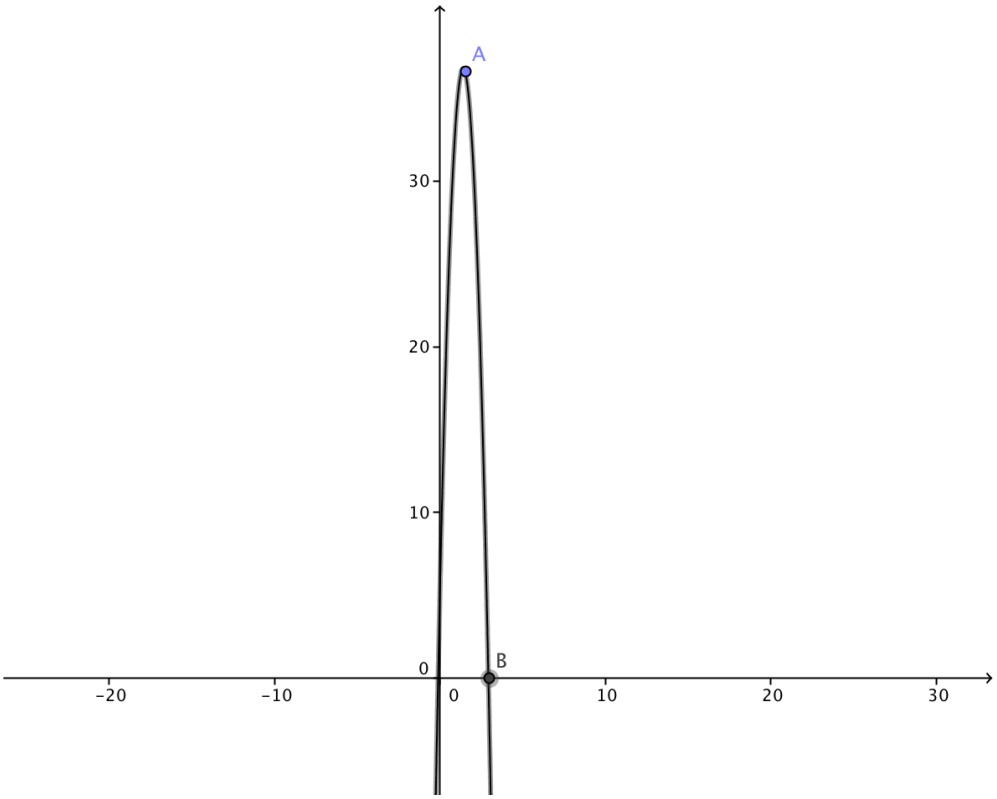
$h(1) = -(16)1 + 45 + 5.2$

$h(1) = -16 + 50.2$

2. $h(1) = 34.2$

The height of the ball after 1 second will be 34.2 ft.

3.



The maximum height of the ball is 36.69 feet.

4. The ball will hit the ground at 2.92 seconds

Mixed Review

$3r^2 - 4r + 1$

29. $(3r-1)(r-1)$

30.

$(2+\sqrt{3})(4+\sqrt{3})$

$8 + 2\sqrt{3} + 4\sqrt{3} + 3$

$11 + 6\sqrt{3}$

31.

$9 - 3x + 18y = 0$

$-3x + 18y = -9$

$18y = 3x - 9$

$y = \frac{3}{18}x - \frac{9}{18}$

$y = \frac{1}{6}x - \frac{1}{2}$

The slope is 1/6 and the y-intercept is -1/2.

32.

$y = a(1-r)^x$

$a = 100\% = 1$

$r = \frac{1}{2} = 0.5$

$x = 44 \div 16 = 2.75$

$y = 1(1-0.5)^{2.75}$

$y = (0.5)^{2.75}$

$y = 0.14865$

The percentage remaining is 14.865%.

33.

$0.00000009865 \times 123564.21$

0.012189609317

$1.2189609317 \times 10^{-2}$

34.
A= 12% chlorine mixture
B= 30% chlorine mixture

15% of 150mL = 22.5 mL

$$\begin{cases} A + B = 150 \\ .12A + .30B = 22.5 \end{cases}$$

$$A + B = 150$$
$$B = 150 - A$$

$$.12A + .30B = 22.5$$
$$.12A + .30(150 - A) = 22.5$$
$$.12A + 45 - .30A = 22.5$$
$$-0.18A + 45 = 22.5$$
$$-0.18A = -22.5$$
$$A = 125$$

$$150 - 125 = 25$$

There should be 125mL of the 12% solution and 25mL of the 30% solution in order to create 150mL with 15% chlorine.

Lesson 10.3
Solving Quadratic Equations Using Square Roots

1.
$x^2 = 196$
$\sqrt{x^2} = \sqrt{196}$
$x = \pm 14$

2.
$x^2 - 1 = 0$
$x^2 = 1$
$\sqrt{x^2} = \sqrt{1}$
$x = \pm 1$

3.
$x^2 - 100 = 0$
$x^2 = 100$
$\sqrt{x^2} = \sqrt{100}$
$x = \pm 10$

4.
$x^2 - 16 = 0$
$x^2 = 16$
$\sqrt{x^2} = \sqrt{16}$
$x = \pm 4$

5.
$9x^2 - 1 = 0$
$9x^2 = 1$
$x^2 = \frac{1}{9}$
$\sqrt{x^2} = \sqrt{\frac{1}{9}}$
$x = \pm \frac{1}{3}$

6.

$4x^2 - 49 = 0$

$4x^2 = 49$

$x^2 = 12.25$

$\sqrt{x^2} = \sqrt{12.25}$

$x = \pm 3.5$

7.

$64x^2 - 9 = 0$

$64x^2 = 9$

$x^2 = 0.140625$

$\sqrt{x^2} = \sqrt{0.140625}$

$x = \pm 0.375$

8.

$x^2 - 81 = 0$

$x^2 = 81$

$\sqrt{x^2} = \sqrt{81}$

$x = \pm 9$

9.

$25x^2 - 36 = 0$

$25x^2 = 36$

$x^2 = 1.44$

$\sqrt{x^2} = \sqrt{1.44}$

$x = \pm 1.2$

10.

$x^2 - 9 = 0$

$x^2 = 9$

$\sqrt{x^2} = \sqrt{9}$

$x = \pm 3$

11.

$x^2 - 16 = 0$

$x^2 = 16$

$\sqrt{x^2} = \sqrt{16}$

$x = \pm 4$

12.

$x^2 - 36 = 0$

$x^2 = 36$

$\sqrt{x^2} = \sqrt{36}$

$x = \pm 6$

13.

$16x^2 - 49 = 0$

$16x^2 = 49$

$x^2 = 3.0625$

$\sqrt{x^2} = \sqrt{3.0625}$

$x = \pm 1.75$

14.

$(x-2)^2 = 1$

$x - 2 = \sqrt{1}$

$x - 2 = \pm 1$

$x - 2 = 1$

$x = 3$

$x - 2 = -1$

$x = 1$

$x = 1, x = 3$

15.

$(x+5)^2 = 16$

$x+5 = \sqrt{16}$

$x+5 = \pm 4$

$x+5 = 4$

$x = -1$

$x+5 = -4$

$x = -9$

$x = -1, x = -9$

16.

$(2x-1)^2 - 4 = 0$

$(2x-1)^2 = 4$

$2x-1 = \sqrt{4}$

$2x-1 = \pm 2$

$2x-1 = 2$

$2x = 3$

$x = \frac{3}{2}$

$2x-1 = -2$

$2x = -1$

$x = -\frac{1}{2}$

$x = \frac{3}{2}, x = -\frac{1}{2}$

17.

$(3x+4)^2 = 9$

$3x+4 = \sqrt{9}$

$3x+4 = \pm 3$

$3x+4 = 3$

$3x = -1$

$3x = -\frac{1}{3}$

$3x+4 = -3$

$3x = -7$

$x = -\frac{7}{3}$

$x = -\frac{1}{3}, x = -\frac{7}{3}$

18.

$(x-3)^2 - 25 = 0$

$(x-3)^2 = 25$

$x-3 = \sqrt{25}$

$x-3 = \pm 5$

$x-3 = 5$

$x = 8$

$x-3 = -5$

$x = -2$

$x = 8, x = -2$

19.

$x^2 - 6 = 0$

$x^2 = 6$

$\sqrt{x^2} = \sqrt{6}$

$x = \sqrt{6} \approx 2.4495$

20.

$x^2 - 20 = 0$

$x^2 = 20$

$\sqrt{x^2} = \sqrt{20}$

$x = \sqrt{4 \cdot 5}$

$x = 2\sqrt{5} \approx 4.4721$

21.

$3x^2 - 14 = 0$

$3x^2 = 14$

$x^2 = \frac{14}{3}$

$\sqrt{x^2} = \sqrt{\frac{14}{3}}$

$x = \sqrt{\frac{14}{3}} \approx 2.1602$

22.

$(x - 6)^2 = 5$

$x - 6 = \sqrt{5}$

$x = 6 + \sqrt{5} \approx 8.2361$

23.

$$(4x+1)^2 - 8 = 0$$
$$(4x+1)^2 = 8$$
$$4x+1 = \sqrt{8}$$
$$4x = \sqrt{8} - 1$$
$$x = \frac{\sqrt{8}-1}{4} = \frac{\sqrt{8}}{4} - \frac{1}{4} \approx 0.4571$$

24.

$$(x+10)^2 = 2$$
$$x+10 = \sqrt{2}$$
$$x = \sqrt{2} - 10 \approx -8.5858$$

25.

$$2(x+3)^2 = 8$$
$$(x+3)^2 = 4$$
$$x+3 = \sqrt{4}$$
$$x+3 = \pm 2$$

$$x+3 = 2$$
$$x = -1$$

$$x+3 = -2$$
$$x = -5$$

$$x = -1, x = -5$$

26.

$h(t) = -\frac{1}{2}(g)t^2 + v_0 t + h_0$

$0 = -\frac{1}{2}(32)t^2 + (0)t + 25$

$0 = -16t^2 + 25$

$-25 = -16t^2$

$25 = 16t^2$

$\sqrt{25} = \sqrt{16t^2}$

$\pm 5 = \pm 4t$

$5 = 4t$

$t = \frac{5}{4}$

It takes 1.25 seconds for the ball to fall to the ground 25 feet below.

27.

$h(t) = -\frac{1}{2}(g)t^2 + v_0 t + h_0$

$h(t) = -\frac{1}{2}(32)t^2 + (0)t + 400$

$0 = -16t^2 + 400$

$16t^2 = 400$

$\sqrt{16t^2} = \sqrt{400}$

$\pm 4t = \pm 20$

$4t = 20$

$t = 5$

Susan will hear the splash 5 seconds later.

28.

$h(t) = -\frac{1}{2}(g)t^2 + v_0 t + h_0$

$h(t) = -\frac{1}{2}(9.8)(5.3)^2 + (0)(5.3) + h_0$

$0 = -4.9(28.09) + h_0$

$0 = -137.641 + h_0$

$h_0 = 137.641$

The cliff is 137.641 meters high.

29.

$h(t) = -\frac{1}{2}(g)t^2 + v_0 t + h_0$

$h(t) = -\frac{1}{2}(g)t^2 + v_0 t + h_0$

$h(t) = -\frac{1}{2}(32)t^2 + (0)t + 50$

$0 = -16t^2 + 50$

$16t^2 = 50$

$\sqrt{16t^2} = \sqrt{50}$

$4t = \sqrt{25 \cdot 2}$

$4t = 5\sqrt{2}$

$t = \frac{5\sqrt{2}}{4}$

$t \approx 1.7677$

$h(t) = -\frac{1}{2}(32)t^2 + (0)t + 40$

$0 = -16t^2 + 40$

$16t^2 = 40$

$\sqrt{16t^2} = \sqrt{40}$

$4t = \sqrt{4 \cdot 10}$

$4t = 2\sqrt{10}$

$t = \frac{2\sqrt{10}}{4}$

$t \approx 1.5811$

$1.5811 + 0.5 = 2.0811$

The rock Nisha dropped from the roof will hit the ground before the quarter Ashaan dropped from the top story window.

30.

$h(t) = -\frac{1}{2}(g)t^2 + v_0 t + h_0$

$h(t) = -\frac{1}{2}(g)t^2 + v_0 t + h_0$

$h(t) = -\frac{1}{2}(32)t^2 + (0)t + 120$

$0 = -16t^2 + 120$

$16t^2 = 120$

$\sqrt{16t^2} = \sqrt{120}$

$4t = \sqrt{4 \cdot 30}$

$4t = 2\sqrt{30}$

$t = \frac{2\sqrt{30}}{4}$

$t \approx 2.7386$

$h(t) = -\frac{1}{2}(32)t^2 + (0)t + 72$

$0 = -16t^2 + 72$

$16t^2 = 72$

$\sqrt{16t^2} = \sqrt{72}$

$4t = \sqrt{9 \cdot 8}$

$4t = 3\sqrt{8}$

$t = \frac{3\sqrt{8}}{4}$

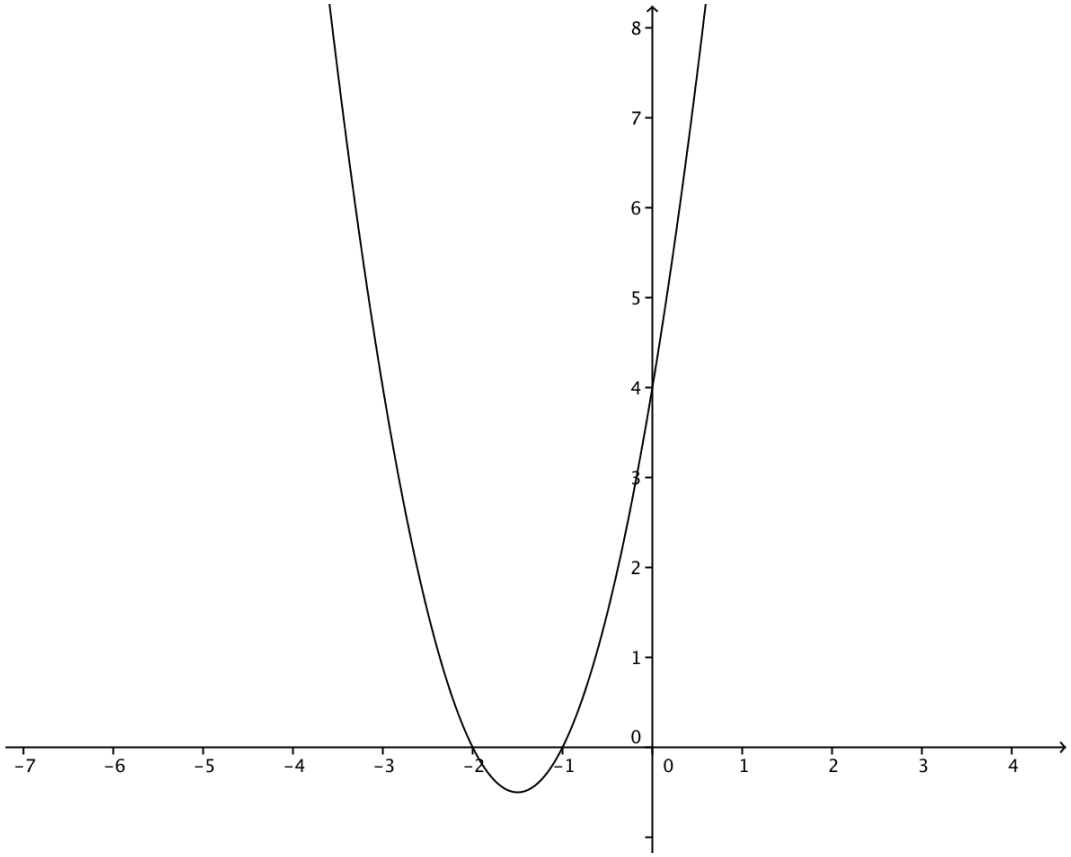
$t \approx 2.1213$

$2.1213 + 1 = 3.1213$

The apple dropped by Victor off the tenth floor will hit the ground before the orange dropped by Juan off the sixth floor. The time difference is 3.1213-2.7386=0.3827 of a second.

Mixed Review

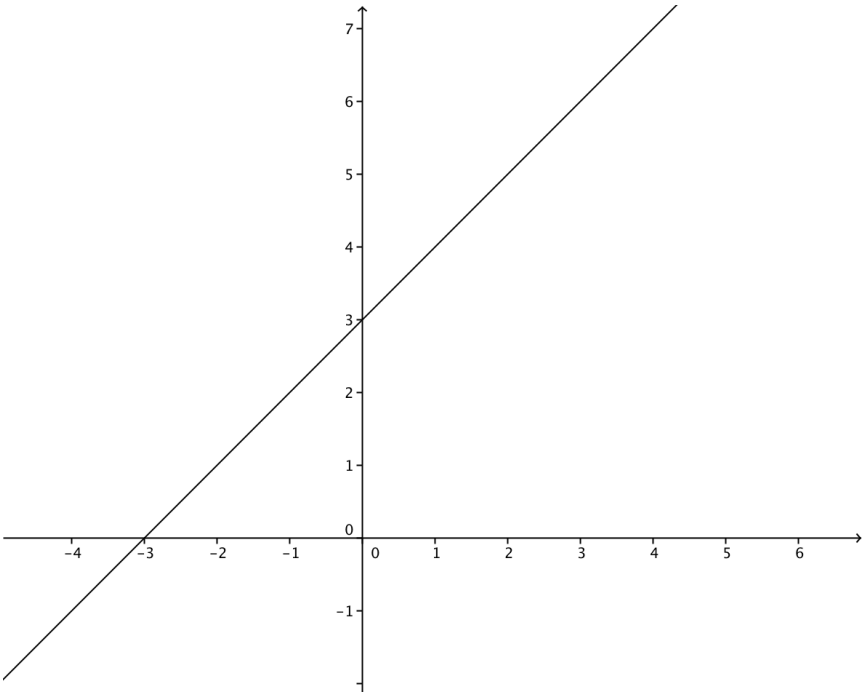
31.



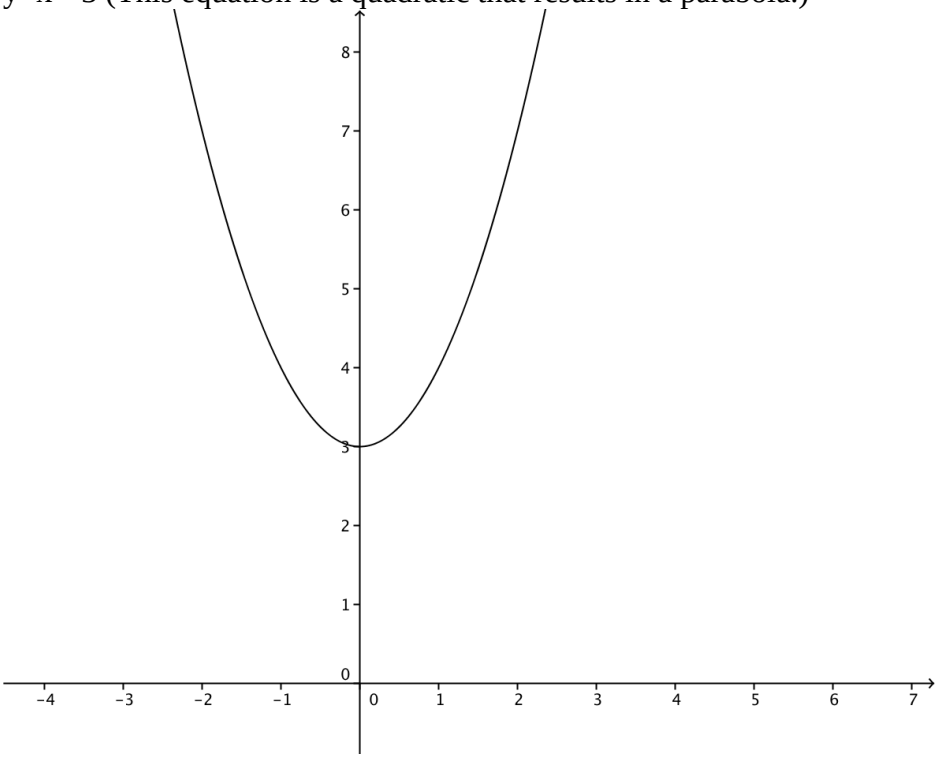
1. vertex (-1.5,-0.5)
2. x-intercepts = -1, -2
3. y-intercept= 4
4. axis of symmetry : x=-1.5

32.

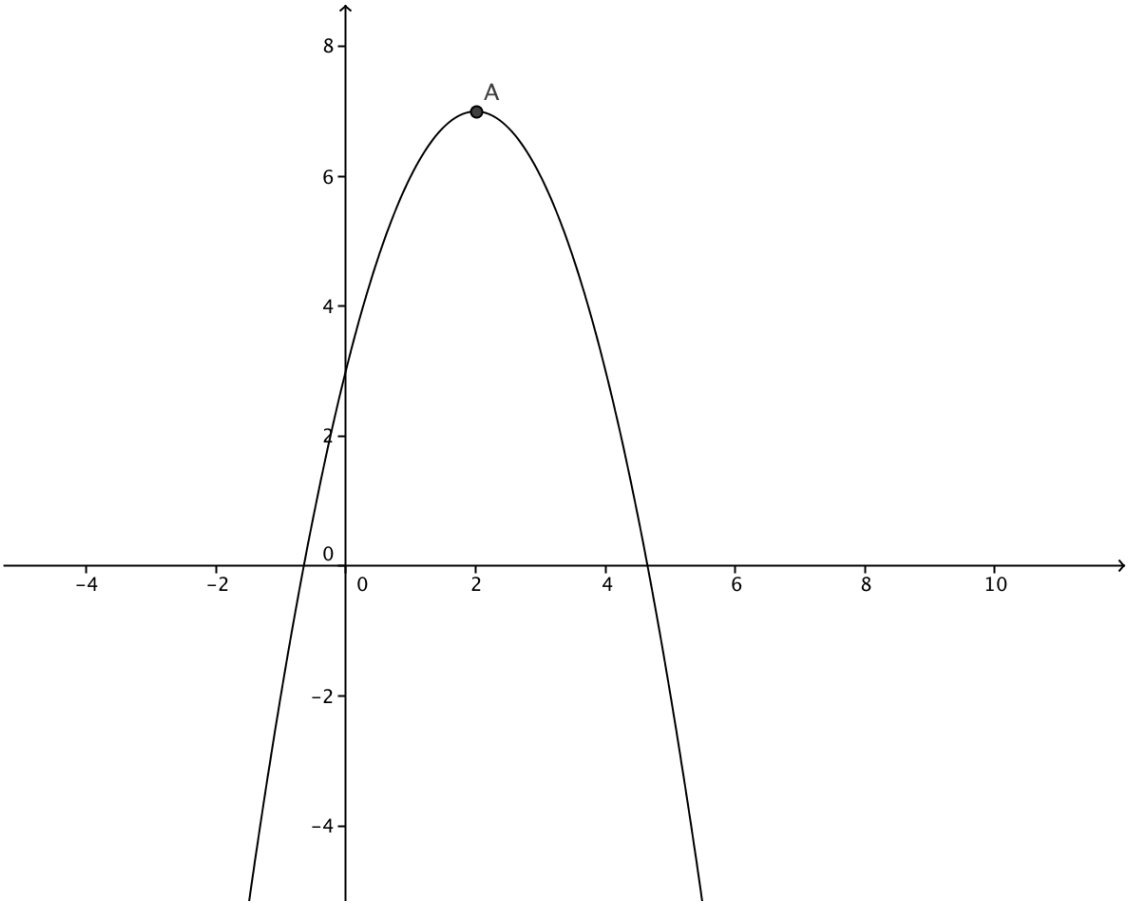
y=x+3 (This is a linear equation that results in a line.)



$y=x^2+3$ (This equation is a quadratic that results in a parabola.)



33.



domain = all real numbers
range= all real numbers less than or equal to 7

34.
h= the price of a hot dog
s= the price of a soda
Friday the club sells 112 times h and 70 times s to equal 154 (112h+70s=154)
Saturday the club sells 240 times h and 120 times s to equal 300 (240h +120s=300)
Now let's solve the system.

$$\begin{cases} 112h + 70s = 154 \\ 240h + 120s = 300 \end{cases}$$

$$240h + 120s = 300$$
$$120s = 300 - 240h$$
$$s = 2.5 - 2h$$

$$112h + 70s = 154$$
$$112h + 70(2.5 - 2h) = 154$$
$$112h + 175 - 140h = 154$$
$$175 - 28h = 154$$
$$-28h = -21$$
$$h = 0.75$$

$$240h + 120s = 300$$
$$240(0.75) + 120s = 300$$
$$180 + 120s = 300$$
$$120s = 120$$
$$s = 1$$

Hot dog cost $0.75 each and sodas cost $1.00 each.

Lesson 10.4
Solving Quadratic Equations by Completing the Square

1. "Completing the square" is the term used for turning a quadratic equation into a perfect square trinomial by adding or subtracting terms on both sides.

2. In order to change a quadratic equation into a perfect square trinomial (complete the square) one must do the following.
- recognize that the third term is missing
- utilize the formula for a perfect square $\left(a^2+2(ab)+b^2\right)$ to determine that the leading coefficient (a) will be x (if there is a coefficient in front of x, divide all terms on both sides by that number to get x alone)
- Now use the same formula to find 2(ab) or 2xb by setting the middle term equal to 2xb and solving the equation for b.
-We can also find b by dividing the coefficient of x in the middle term by 2, this also gives us b
- find the square of b (b^2) and place it as the third term in the equation
- check by ensuring that the equation factors to $(a+b)^2$

3.

$$h(t)=-\tfrac{1}{2}(g)t^2+v_0t+h_0$$
$$h(t)=-\tfrac{1}{2}(9.8)t^2+(50)t+2$$
$$h(t)=-4.9t^2+50t+2$$

a)

$$h(t)=-4.9t^2+50t+2$$
$$h(4)=-4.9\left(4^2\right)+50(4)+2$$
$$h(4)=-4.9(16)+200+2$$
$$h(4)=-78.4+202$$
$$h(4)=123.6$$

$$h(t)=-4.9t^2+50t+2$$
$$h(8)=-4.9\left(8^2\right)+50(8)+2$$
$$h(8)=-4.9(64)+400+2$$
$$h(8)=-313.6+402$$
$$h(8)=88.4$$

The height of the arrow after 4 seconds will be 123.6 meters. After 8 seconds the height of the arrow will be 88.4 meters.

b)

$h(t) = -4.9t^2 + 50t + 2$

$0 = -4.9t^2 + 50t + 2$

$0 = t^2 - \frac{500}{49}t - \frac{20}{49}$

$\frac{20}{49} = t^2 - \frac{500}{49}t$

$\frac{20}{49} + \frac{62500}{2401} = t^2 - \frac{500}{49}t + \frac{62500}{2401}$

$\frac{63480}{2401} = \left(t - \frac{250}{49}\right)^2$

$\sqrt{\frac{63480}{2401}} = \left(t - \frac{250}{49}\right)$

$5.1419 \approx t - 5.1020$

$t \approx 10.2439$

The arrow will hit the ground after approximately 10.2439 seconds.

4.

$y = a(x-h)^2 + k$

5.

$y = a(x-h)^2 + k$

$y = \frac{1}{3}(x-1)^2 + 1$

6.

$y = a(x-h)^2 + k$

$y = -2(x-(-5))^2 + 0$

$y = -2(x+5)^2$

7.

$y = a(x-h)^2 + k$

$y = 1(x-1)^2 + -2$

$y = (x-1)^2 + -2$

8.

$y = a(x-h)^2 + k$

$y = 1(x-(-3))^2 + 6$

$y = (x+3)^2 + 6$

9.

$x^2 + 5x$

$b = \frac{5}{2} = 2.5$

$b^2 = 6.25$

$x^2 + 5x + 6.25$

$(x + 2.5)^2$

10.

$x^2 - 2x$

$b = -\frac{2}{2} = -1$

$b^2 = 1$

$x^2 - 2x + 1$

$(x - 1)^2$

11.

$x^2 + 3x$

$b = \frac{3}{2} = 1.5$

$b^2 = 2.25$

$x^2 + 3x + 2.25$

$(x + 1.5)^2$

12.

$x^2 - 4x$

$b = -\frac{4}{2} = -2$

$b^2 = 4$

$x^2 - 4x + 4$

$(x-2)^2$

13.

$3x^2 + 18x$

$\frac{3}{3}x^2 + \frac{18}{3}x = \frac{0}{3}$

$x^2 + 6x = 0$

$b = \frac{6}{2} = 3$

$b^2 = 9$

$x^2 + 6x + 9$

$(x+3)^2$

$3x^2 + 18x + 27$

$3(x+3)^2$

14.

$2x^2 - 22x$

$\frac{2}{2}x^2 - \frac{22}{2}x = \frac{0}{2}$

$x^2 - 11x = 0$

$b = -\frac{11}{2} = -5.5$

$b^2 = 30.25$

$x^2 - 11x + 30.25$

$(x - 5.5)^2$

$2x^2 - 22x + 60.5$

$2(x - 5.5)^2$

15.

$8x^2 - 10x$

$\frac{8}{8}x^2 - \frac{10}{8}x = \frac{0}{8}$

$x^2 - 1.25x = 0$

$b = -1.25 / 2 = -0.625$

$b^2 = 0.390625$

$x^2 - 1.25x + 0.390625$

$(x - 0.625)^2$

$8x^2 - 10x + 3.125$

$8(x - 0.625)^2$

16.

$5x^2 + 12x$

$\frac{5}{5}x^2 + \frac{12}{5}x = \frac{0}{5}$

$x^2 + 2.4x = 0$

$b = 2.4 / 2 = 1.2$

$b^2 = 1.44$

$x^2 + 2.4x + 1.44$

$(x + 1.2)^2$

$5x^2 + 12x + 7.2$

$5(x + 1.2)^2$

17.

$x^2 - 4x = 5$

$x^2 - 4x + 4 = 5 + 4$

$x^2 - 4x + 4 = 9$

$(x - 2)^2 = 9$

$x - 2 = \sqrt{9}$

$x - 2 = \pm 3$

$x - 2 = 3$

$x = 5$

$x - 2 = -3$

$x = -1$

$x = -1, 5$

18.

$x^2 - 5x = 10$

$x^2 - 5x + 6.25 = 10 + 6.25$

$(x - 2.5)^2 = 16.25$

$x - 2.5 = \sqrt{16.25}$

$x = 2.5 + \sqrt{16.25}$

$x \approx 2.5 \pm 4.0311$

$x \approx 2.5 + 4.0311$

$x \approx 6.5311$

$x \approx 2.5 - 4.0311$

$x \approx -1.5311$

19.

$x^2 + 10x + 15 = 0$

$x^2 + 10x = -15$

$x^2 + 10x + 25 = -15 + 25$

$x^2 + 10x + 25 = 10$

$(x - 5)^2 = 10$

$x - 5 = \sqrt{10}$

$x = 5 + \sqrt{10}$

$x \approx 5 \pm 3.1623$

$x \approx 5 + 3.1623$

$x \approx 8.1623$

$x \approx 5 - 3.1623$

$x \approx 1.8377$

20.

$x^2 + 15x + 20 = 0x$

$x^2 + 15x = -20$

$x^2 + 15x + 56.25 = -20 + 56.25$

$x^2 + 15x + 56.25 = 36.25$

$(x - 7.5)^2 = 36.25$

$x - 7.5 = \sqrt{36.25}$

$x = \sqrt{36.25} + 7.5$

$x \approx \pm 6.0208 + 7.5$

$x \approx 6.0208 + 7.5$

$x \approx 13.5208$

$x \approx -6.0208 + 7.5$

$x \approx 1.4792$

21.

$2x^2 - 18x = 0$

$x^2 - 9x = 0$

$x^2 - 9x + 20.25 = 20.25$

$(x - 4.5)^2 = 20.25$

$x - 4.5 = \sqrt{20.25}$

$x = \sqrt{20.25} + 4.5$

$x = \pm 4.5 + 4.5$

$x = 4.5 + 4.5$

$x = 9$

$x = -4.5 + 4.5$

$x = 0$

22.

$4x^2 + 5x = -1$

$x^2 + \frac{5}{4}x = -\frac{1}{4}$

$x^2 + \frac{5}{4}x + \frac{25}{64} = -\frac{1}{4} + \frac{25}{64}$

$x^2 + \frac{5}{4}x + \frac{25}{64} = \frac{9}{64}$

$\left(x - \frac{5}{8}\right)^2 = \frac{9}{64}$

$x - \frac{5}{8} = \sqrt{\frac{9}{64}}$

$x = \frac{5}{8} + \sqrt{\frac{9}{64}}$

$x = .625 \pm .375$

$x = .625 + .375$

$x = 1$

$x = .625 - .375$

$x = .25$

23.

$10x^2 - 30x - 8 = 0$

$x^2 - 3x - .8 = 0$

$x^2 - 3x = .8$

$x^2 - 3x + 2.25 = .8 + 2.25$

$x^2 - 3x + 2.25 = 3.05$

$(x - 1.5)^2 = 3.05$

$x - 1.5 = \sqrt{3.05}$

$x = 1.5 + \sqrt{3.05}$

$x \approx 1.5 \pm 1.7464$

$x \approx 1.5 + 1.7464$

$x \approx 3.2464$

$x \approx 1.5 - 1.7464$

$x \approx -0.2464$

24.

$5x^2 + 15x - 40 = 0$

$x^2 + 3x - 8 = 0$

$x^2 + 3x = 8$

$x^2 + 3x + 2.25 = 8 + 2.25$

$(x - 1.5)^2 = 10.25$

$x - 1.5 = \sqrt{10.25}$

$x = 1.5 + \sqrt{10.25}$

$x \approx 1.5 \pm 3.2016$

$x \approx 1.5 + 3.2016$

$x \approx 4.7016$

$x \approx 1.5 - 3.2016$

$x \approx -1.7016$

25.

$y = x^2 - 6x$

$y + 9 = x^2 - 6x + 9$

$y + 9 = (x - 3)^2$

$y = (x - 3)^2 - 9$

26.

$y + 1 = -2x^2 - x$

$y = -2x^2 - x - 1$

$y = -2\left(x^2 + \frac{1}{2}x + \frac{1}{2}\right)$

$y = -2\left(x^2 + \frac{1}{2}x + \frac{1}{16}\right) + \frac{1}{2}(-2) - \frac{1}{16}(-2)$

$y = -2\left(x^2 + \frac{1}{4}\right)^2 - 1 + \frac{2}{16}$

$y = -2\left(x^2 + \frac{1}{4}\right)^2 - 1 + \frac{1}{8}$

$y = -2\left(x^2 + \frac{1}{4}\right)^2 - \frac{8}{8} + \frac{1}{8}$

$y = -2\left(x^2 + \frac{1}{4}\right)^2 - \frac{7}{8}$

27.

$y = 9x^2 + 3x - 10$

$y = 9\left(x^2 + \frac{1}{6}x - \frac{10}{9}\right)$

$y = 9\left(x^2 + \frac{1}{6}x + \frac{1}{36}\right) - \frac{10}{9}(9) - \frac{1}{36}(9)$

$y = 9\left(x + \frac{1}{6}\right)^2 - 10 - \frac{9}{36}$

$y = 9\left(x + \frac{1}{6}\right)^2 - 10 - \frac{3}{12}$

$y = 9\left(x + \frac{1}{6}\right)^2 - \frac{120}{12} - \frac{3}{12}$

$y = 9\left(x + \frac{1}{6}\right)^2 - \frac{123}{12}$

$y = 9\left(x + \frac{1}{6}\right)^2 - \frac{41}{4}$

28.

$y = 32x^2 + 60x + 10$

$y = 32(x^2 + \frac{60}{32}x + \frac{10}{32})$

$y = 32\left(x^2 + \frac{60}{32}x + \frac{900}{1024}\right) + \frac{900}{1024}(32) + \frac{10}{32}(32)$

$y = 32\left(x + \frac{30}{32}\right)^2 - \frac{900}{1024}(32) + 10$

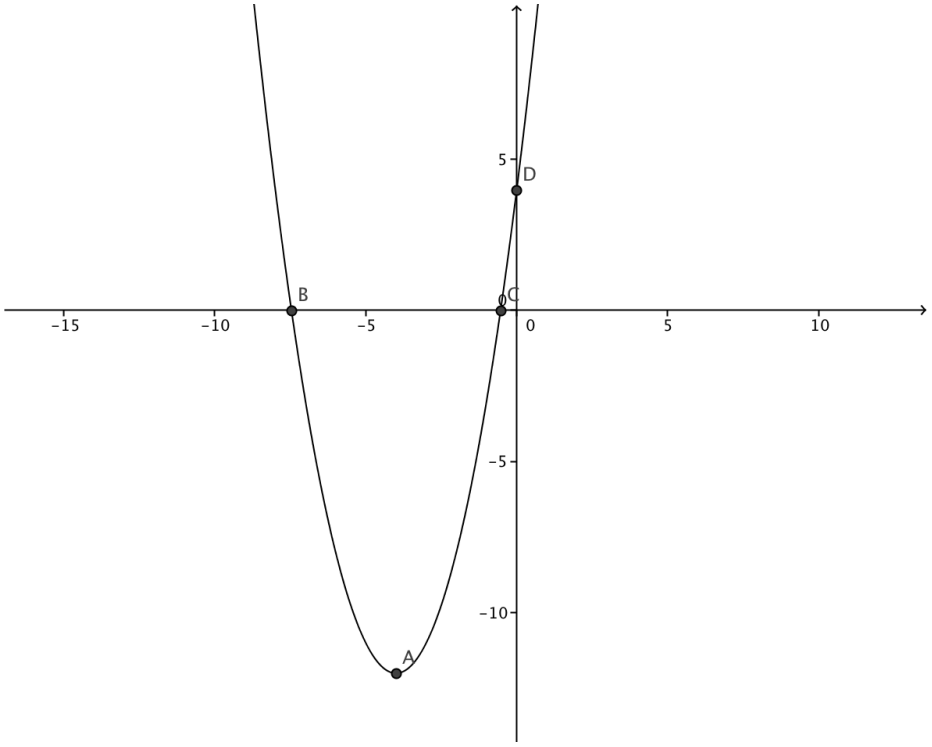
$y = 32\left(x + \frac{30}{32}\right)^2 - \frac{28800}{1024} + 10$

$y = 32\left(x + \frac{15}{16}\right)^2 - 28.125 + 10$

$y = 32\left(x + \frac{15}{16}\right)^2 - 18.125$

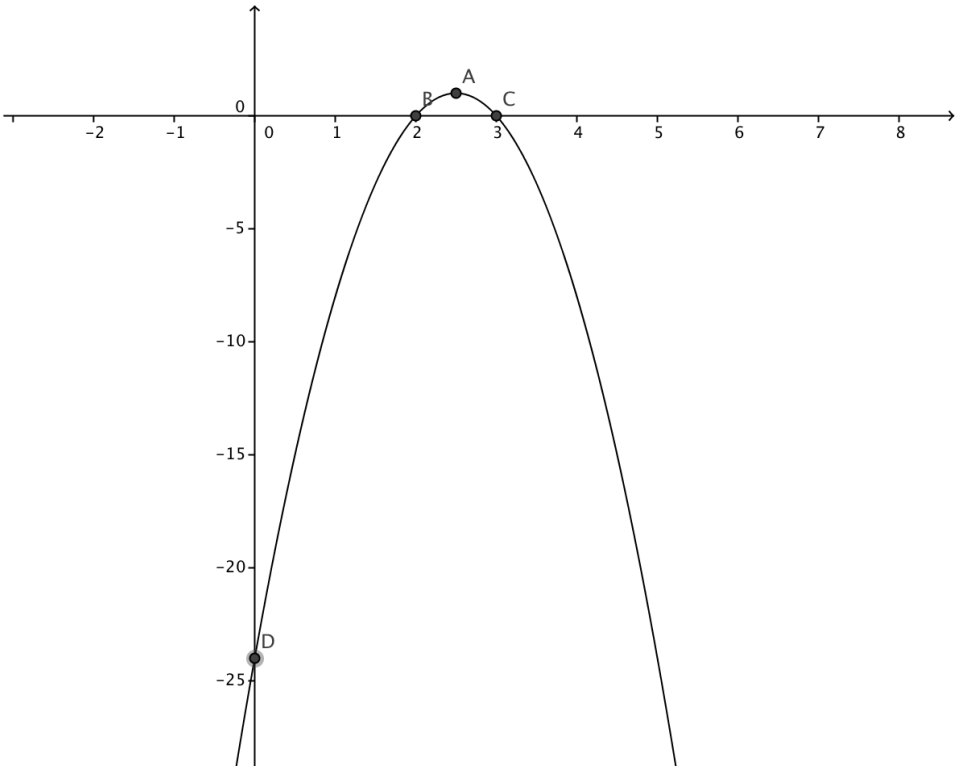
29. $y-4=x^2+8x$

1. vertex (-4,-12)
2. x-intercepts (-7.46,0) (-0.54,0)
3. y-intercept (0,4)
4. The parabola opens up.
5.



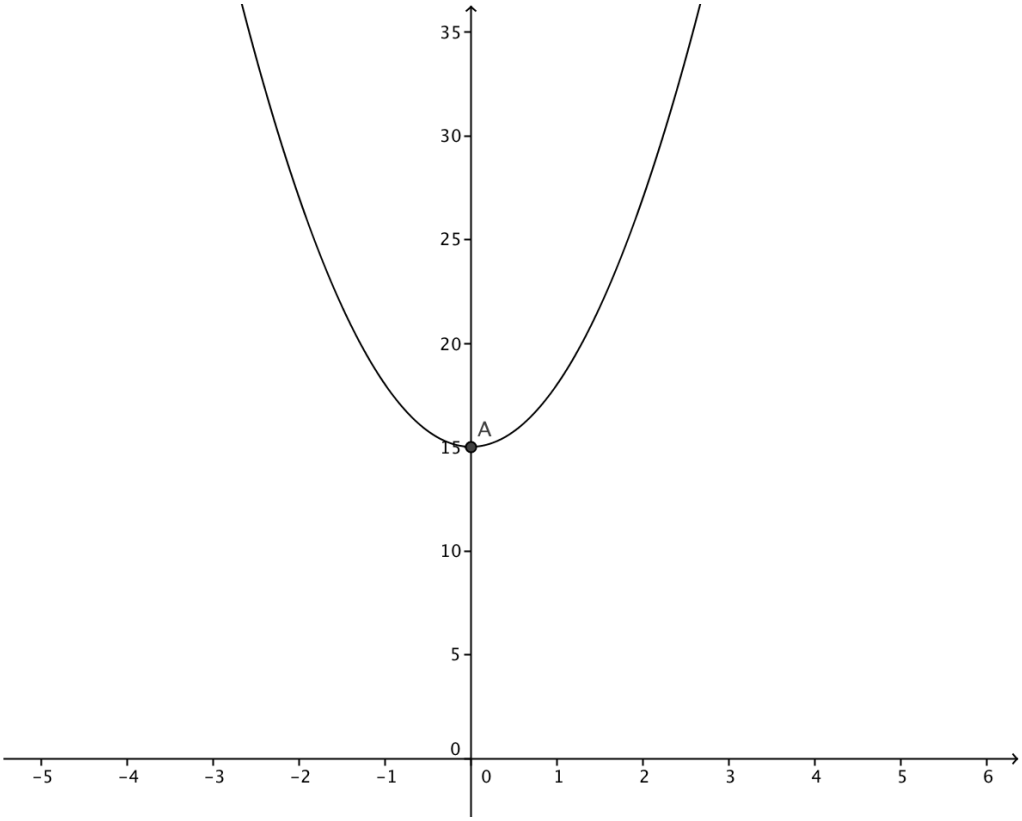
30. $y = -4x^2 + 20x - 24$

1. vertex (2.5,1)
2. x-intercepts (2,0) (3,0)
3. y-intercept (0,-24)
4. The parabola opens down.
5.



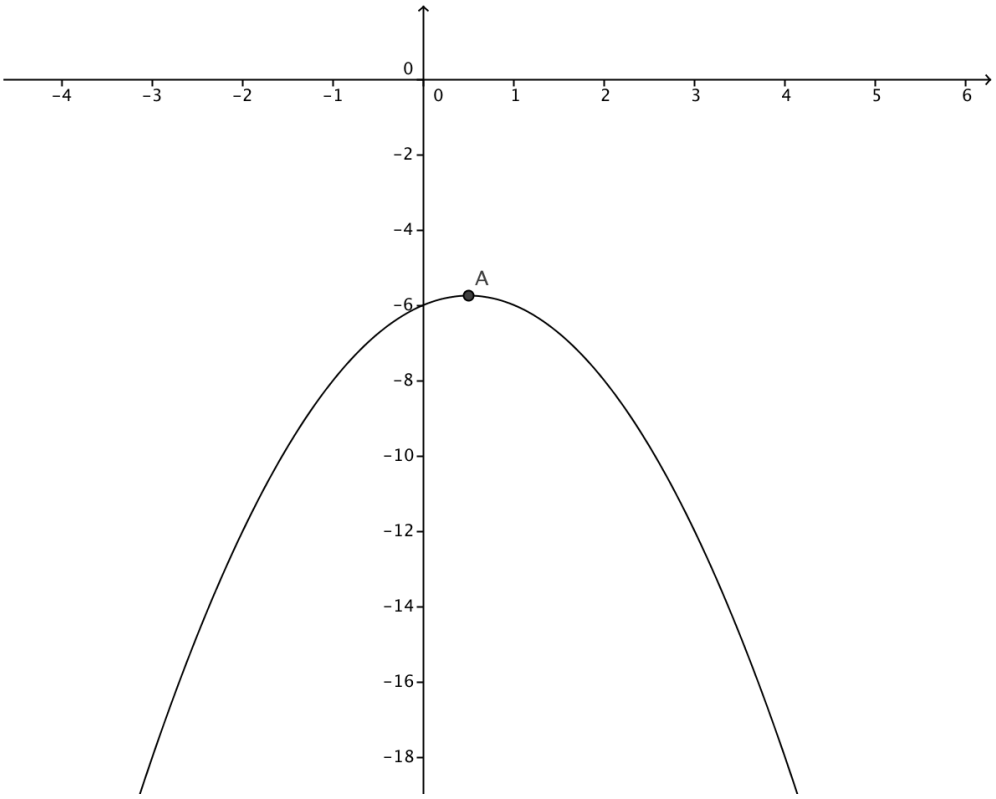
31. $y = 3x^2 + 15x$

1. vertex (0,15)
2. x-intercepts NONE
3. y-intercept (0,15)
4. The parabola opens up.
5.

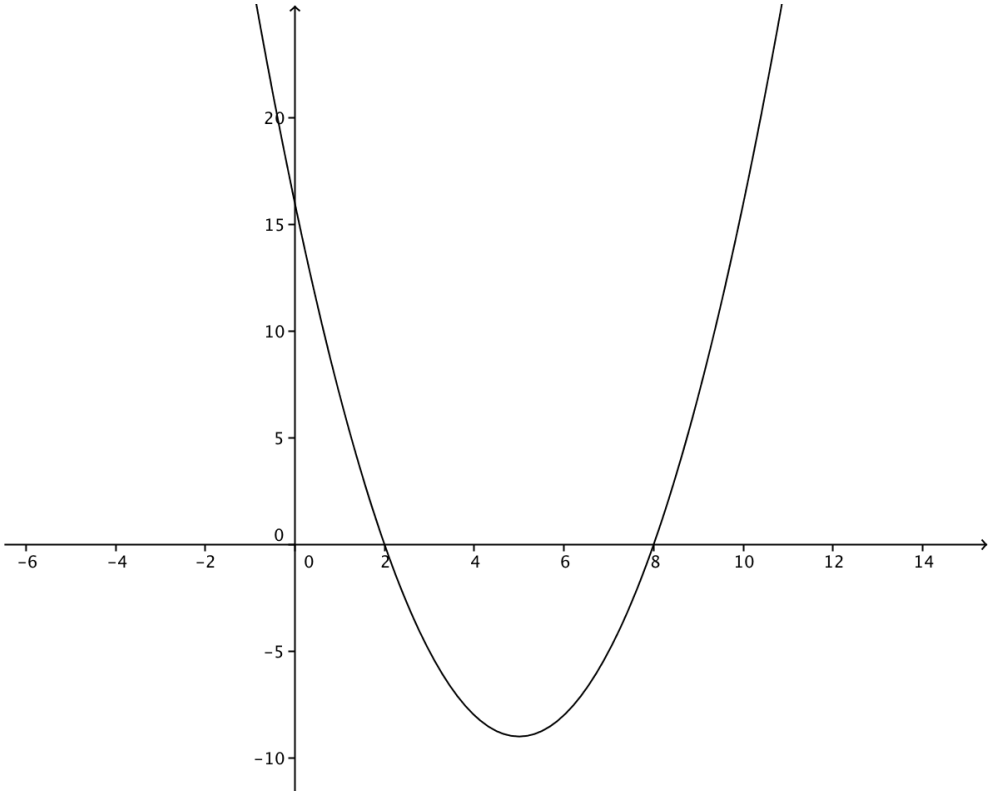


32. $y+6=-x^2+x$

1. vertex (0.5,-5.75)
2. x-intercepts NONE
3. y-intercept (0,-6)
4. The parabola opens down.
5.

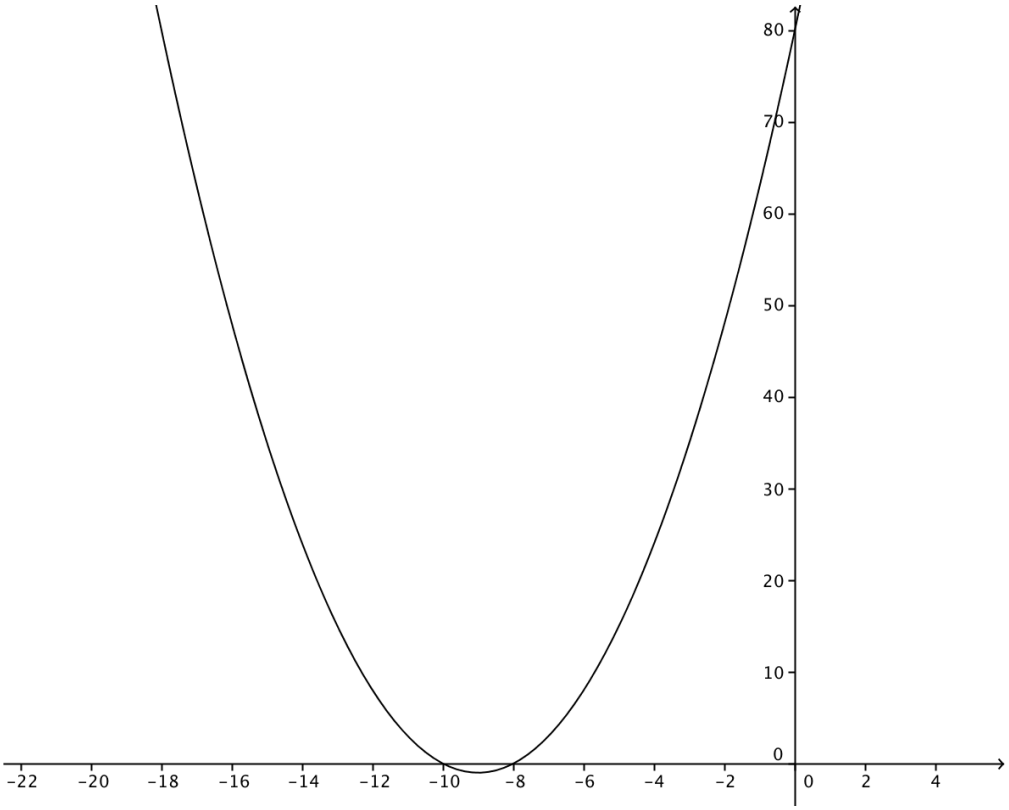


33. $x^2 - 10x + 25 = 9$

1. vertex (5,-9)
2. x-intercepts (2,0) (8,0)
3. y-intercept (0,16)
4. The parabola opens up .
5. 

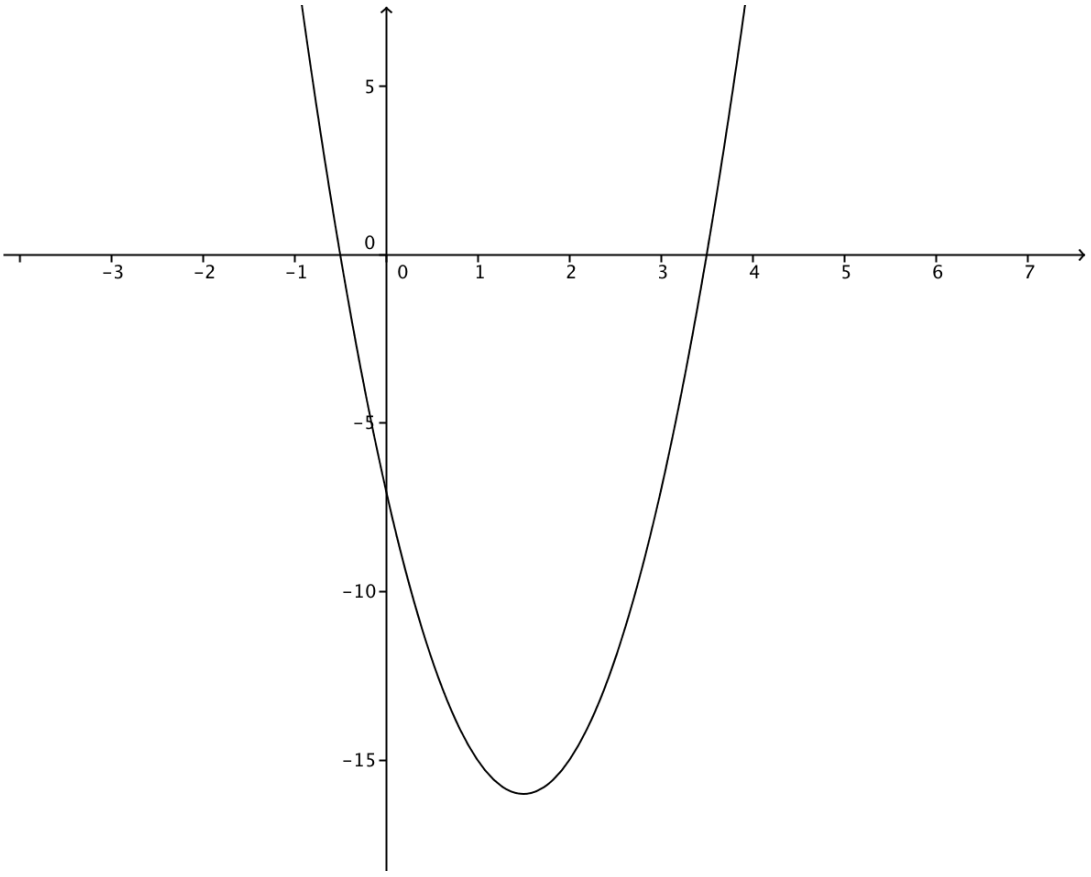
34. $x^2 + 18x + 81 = 1$

1. vertex (-9,-1)
2. x-intercepts (-10,0) (-8,0)
3. y-intercept (0,80)
4. The parabola opens up.
5.



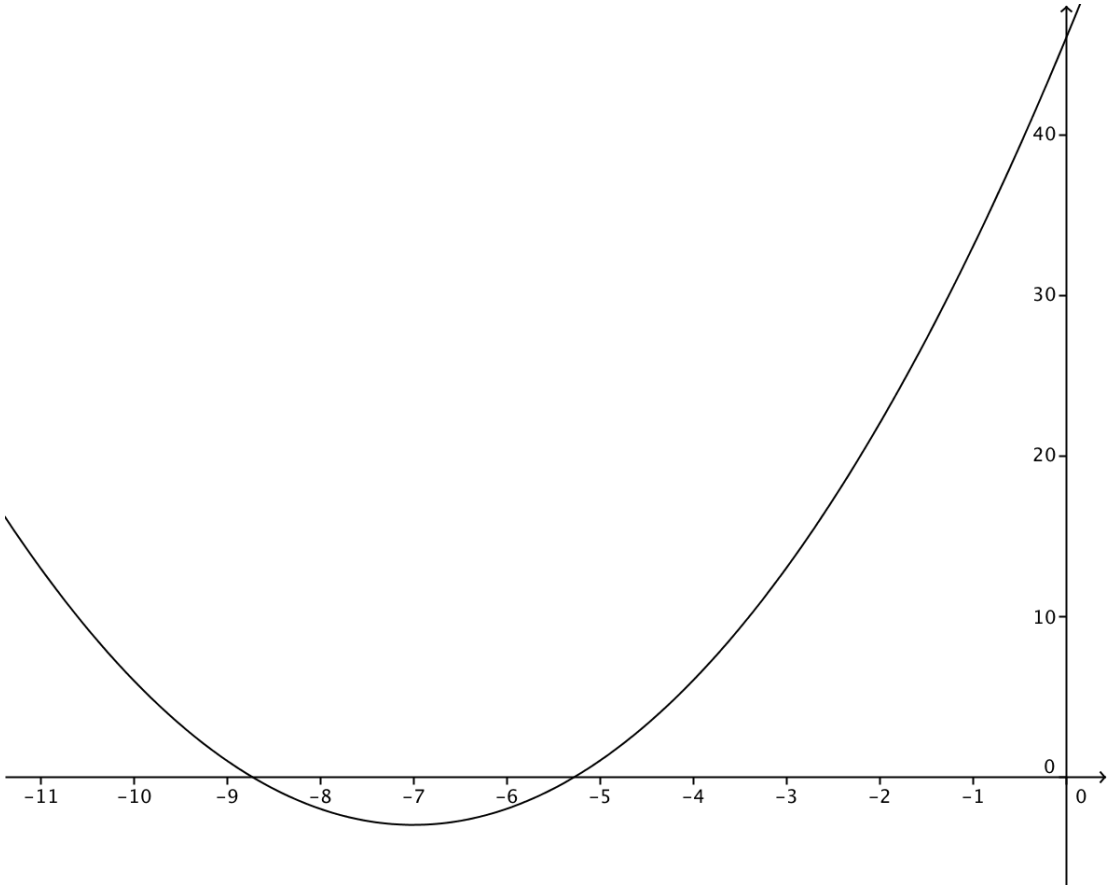
35. $4x^2 - 12x + 9 = 16$

1. vertex (1.5,-16)
2. x-intercepts (-0.5,0) (3.5,0)
3. y-intercept (0,-7)
4. The parabola opens up.
5.



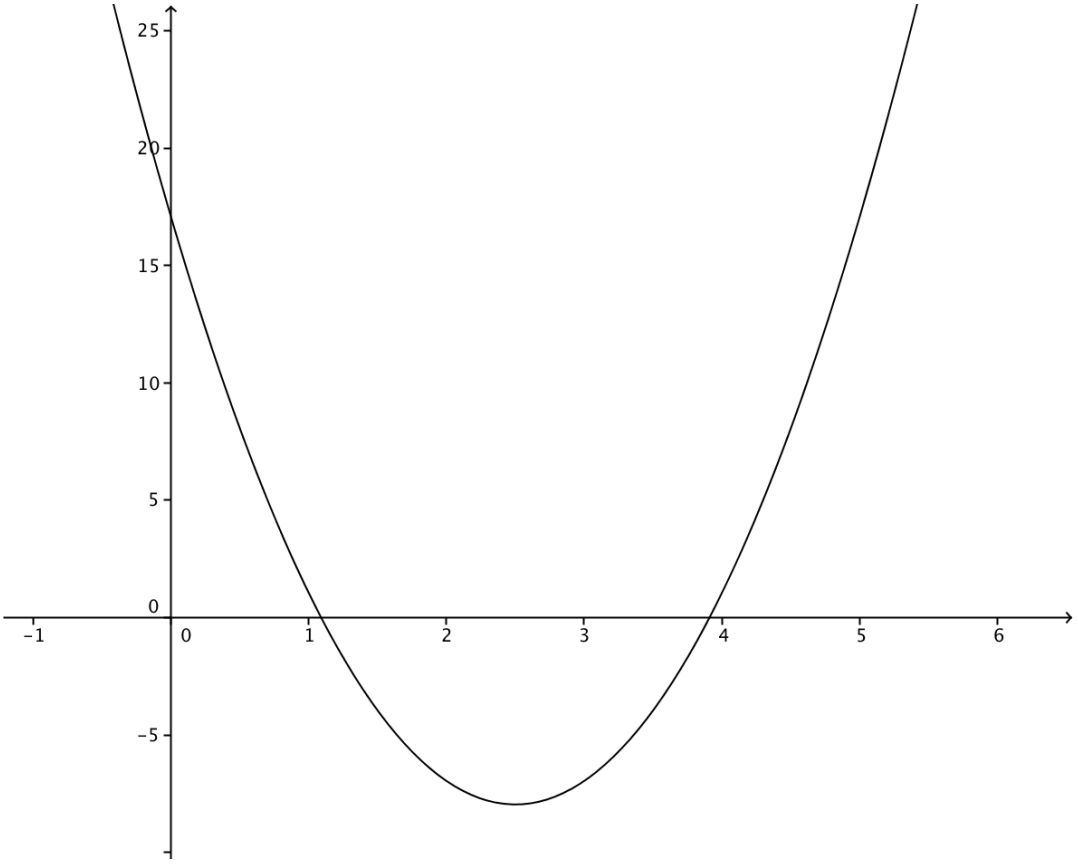
36. $x^2 + 14x + 49 = 3$

1. vertex (-7,-3)
2. x-intercepts (-8.732,0) (-5.268,0)
3. y-intercept (0,46)
4. The parabola opens up.
5.



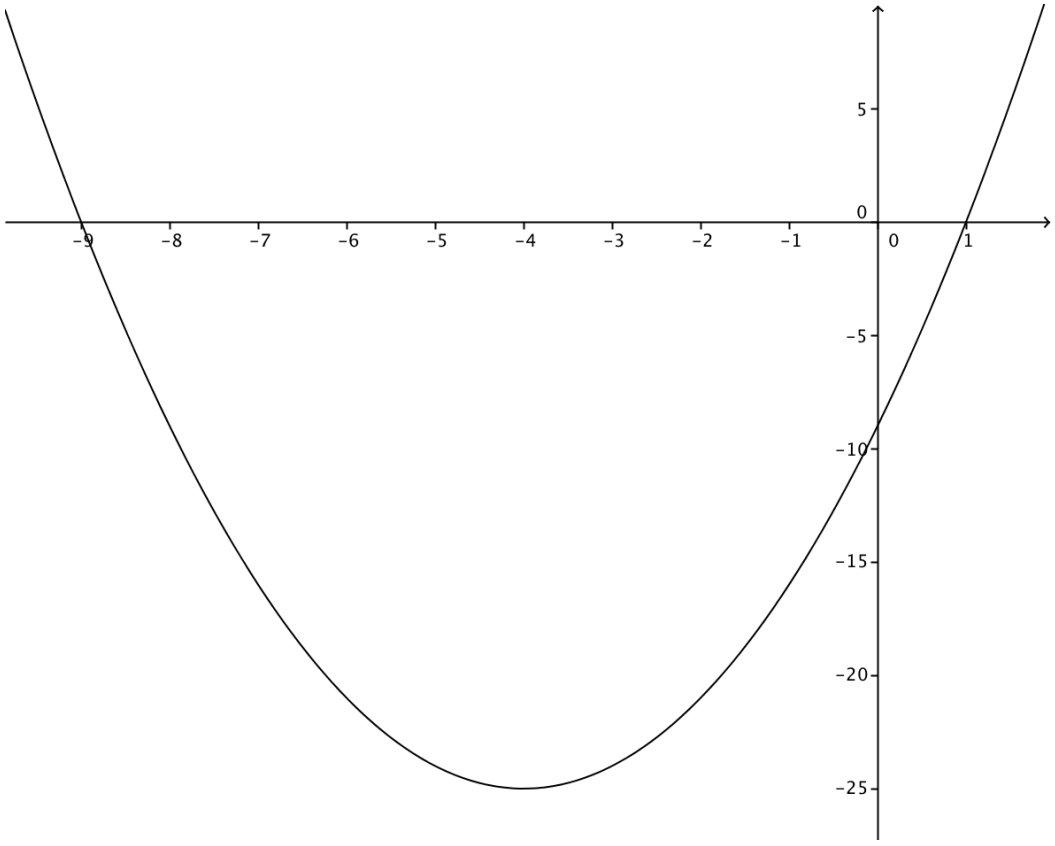
37. $4x^2 - 20x + 25 = 9$

1. vertex (2.5,-9)
2. x-intercepts (1,0) (4,0)
3. y-intercept (0,16)
4. The parabola opens up.
5.



38. $x^2 + 8x + 16 = 25$

1. vertex (-4,-25)
2. x-intercepts (-9,0) (1,0)
3. y-intercept (0,-9)
4. The parabola opens up.
5.



39.

$h(t) = -\frac{1}{2}(g)t^2 + v_0 t + h_0$

$h(t) = -\frac{1}{2}(32)t^2 + 16t + 25$

$h(t) = -16t^2 + 16t + 25$

$0 = -16t^2 + 16t + 25$

$0 - 25 = -16t^2 + 16t$

$-25 = -16t^2 + 16t$

$-25 = -16\left(t^2 - t\right)$

$-25 + \frac{1}{4}(-16) = -16\left(t^2 - t + \frac{1}{4}\right)$

$-25 - 4 = -16\left(t - \frac{1}{2}\right)^2$

$-29 = -16\left(t - \frac{1}{2}\right)^2$

$-29 \div -16 = \left(t - \frac{1}{2}\right)^2$

$\frac{29}{16} = \left(t - \frac{1}{2}\right)^2$

$\sqrt{\frac{29}{16}} = \sqrt{\left(t - \frac{1}{2}\right)^2}$

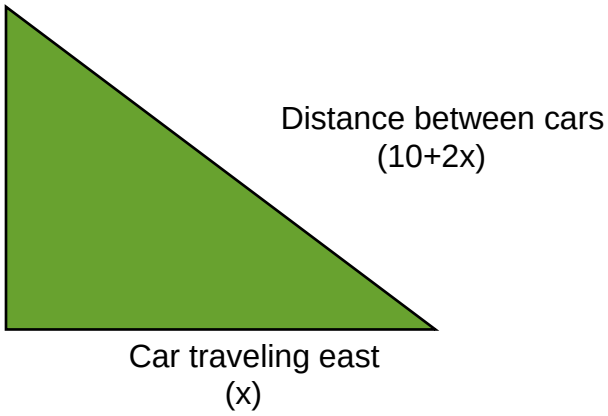
$\frac{\sqrt{29}}{4} = t - \frac{1}{2}$

$t = \frac{\sqrt{29}}{4} + \frac{1}{2}$

$t = \frac{\sqrt{29}}{4} + \frac{2}{4}$

$t \approx 1.8463$

40.



If the car traveling north goes 30 miles and the car traveling east goes x and the distance between the two cars is 10+2x, then it creates the right triangle above. We can find the distance between the two cars (the hypotenuse) by using the pythagorean theorem.

$a^2 + b^2 = c^2$

$x^2 + 30^2 = (10 + 2x)^2$

$x^2 + 900 = 4x^2 + 40x + 100$

$900 = 3x^2 + 40x + 100$

$0 = 3x^2 + 40x - 800$

$0 = 3(x^2 + \frac{40}{3}x - \frac{800}{3})$

$0 = 3(x^2 + \frac{40}{3}x + \frac{400}{9}) - \frac{400}{9}(3) - \frac{800}{3}(3)$

$0 = 3\left(x + \frac{20}{3}\right)^2 - \frac{400}{3} - 800$

$0 = 3\left(x + \frac{20}{3}\right)^2 - \frac{2800}{3}$

$\frac{2800}{3} = 3\left(x + \frac{20}{3}\right)^2$

$\frac{2800}{3} \div 3 = \left(x + \frac{20}{3}\right)^2$

$\frac{2800}{9} = \left(x + \frac{20}{3}\right)^2$

$\sqrt{\frac{2800}{9}} = \sqrt{\left(x + \frac{20}{3}\right)^2}$

$\frac{\sqrt{2800}}{3} = x + \frac{20}{3}$

$x = \frac{\sqrt{2800}}{3} - \frac{20}{3}$

$x \approx 10.9717$

$c = 10 + 2x$

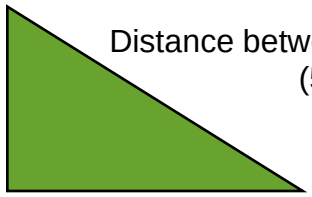
$c = 10 + 2(10.9717)$

$c = 10 + 21.9434$

$c = 31.9434$

The distance between the two cars is approximately 31.9434 miles.

41.



Distance walked by Amanda in 1/2 an hour
(x)

Distance biked by Dolvin in 1/2 an hour
(x+3)

If the distance walked by Amanda is x and the distance biked by Dolvin is x+3, with the distance between them being 5.5 miles, we have the right triangle seen above. We can use the pythagorean theorem to find x and determine the distances traveled by Amanda and Dolvin.

$a^2 + b^2 = c^2$

$x^2 + (x+3)^2 = (5.5)^2$

$x^2 + (x^2 + 6x + 9) = 30.25$

$2x^2 + 6x + 9 - 30.25 = 0$

$2x^2 + 6x - 21.25 = 0$

$2(x^2 + 3x - 10.625) = 0$

$2(x^2 + 3x + 2.25) + 2.25(2) - 10.625(2)$

$2(x+1.5)^2 + 4.5 - 21.25 = 0$

$2(x+1.5)^2 - 16.75 = 0$

$2(x+1.5)^2 = 16.75$

$(x+1.5)^2 = 8.375$

$\sqrt{(x+1.5)^2} = \sqrt{8.375}$

$x + 1.5 = \sqrt{8.375}$

$x = \sqrt{8.375} - 1.5$

$x \approx 1.3940$

$b = x + 3$

$b = 1.394 + 3$

$b = 4.394$

Amanda walked approximately 1.394 miles and Dolvin biked approximately 4.394 miles.

Mixed Review

42.

Original Height	1st Bounce	2nd Bounce	3rd Bounce	4th Bounce	5th Bounce
4ft	2.8ft	1.96 ft	1.372 ft	0.9604 ft	0.6723 ft

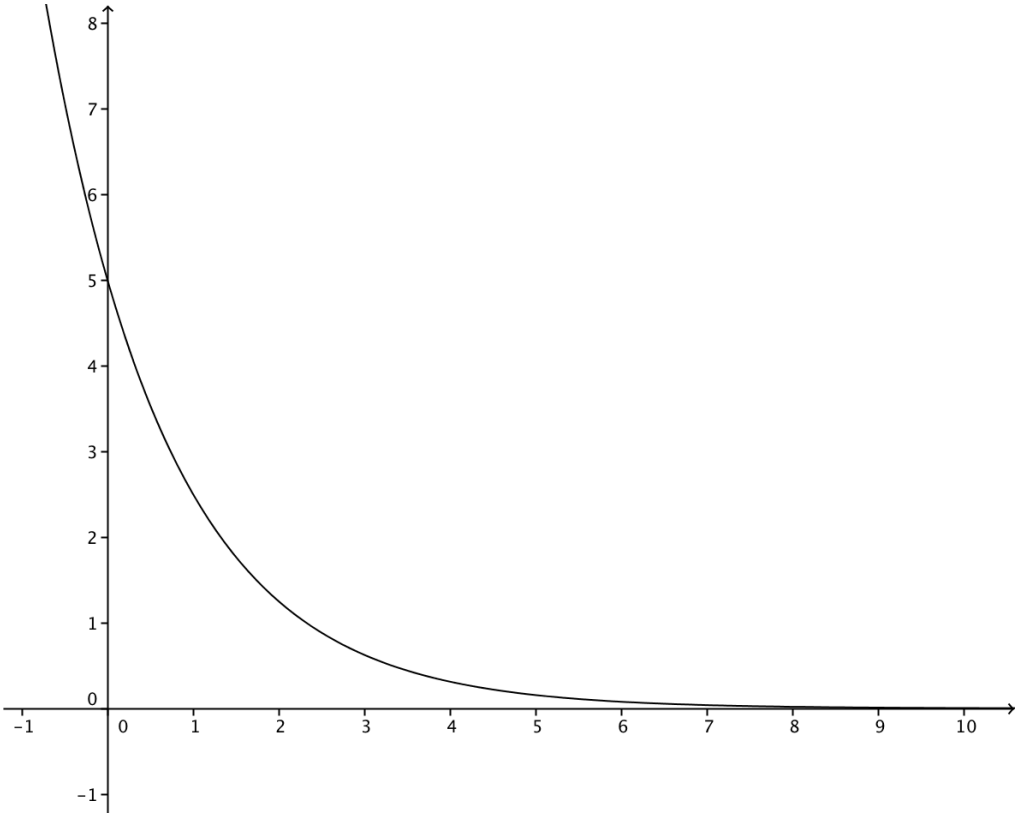
The ball will reach a height of 0.2306 ft on the 8th bounce.

43.

$y = \frac{2}{7}x - 11$

$-\frac{2}{7}x + y = 11$

44. $y = 5\left(\frac{1}{2}\right)^x$



This is exponential decay. The growth(decay) factor is 1/2.

45.

$|3r-4| \le 2$

$3r-4 \le 2$

$3r \le 6$

$r \le 6$

$3r-4 \ge -2$

$3r \ge 2$

$r \ge \frac{2}{3}$

$\frac{2}{3} \le r \le 6$

46.

$-2m+6=-8(5m+4)$

$-2m+6=-40m-32$

$-2m=-40m-32-6$

$-2m+40m=-38$

$38m=-38$

$m=-1$

47.

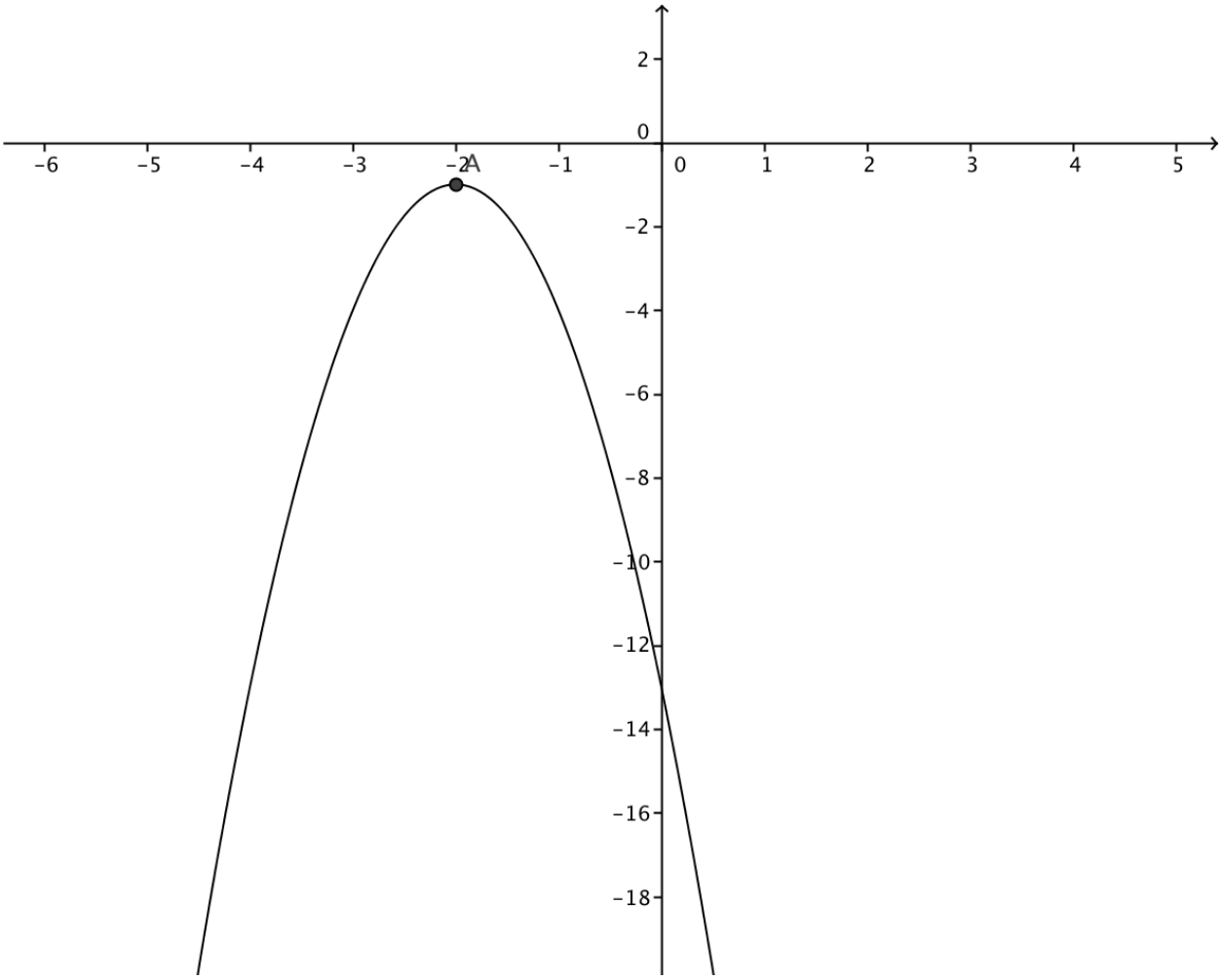
$4a^2+36a-40$

$4\left(a^2+9a-10\right)$

$4(a+10)(a-1)$

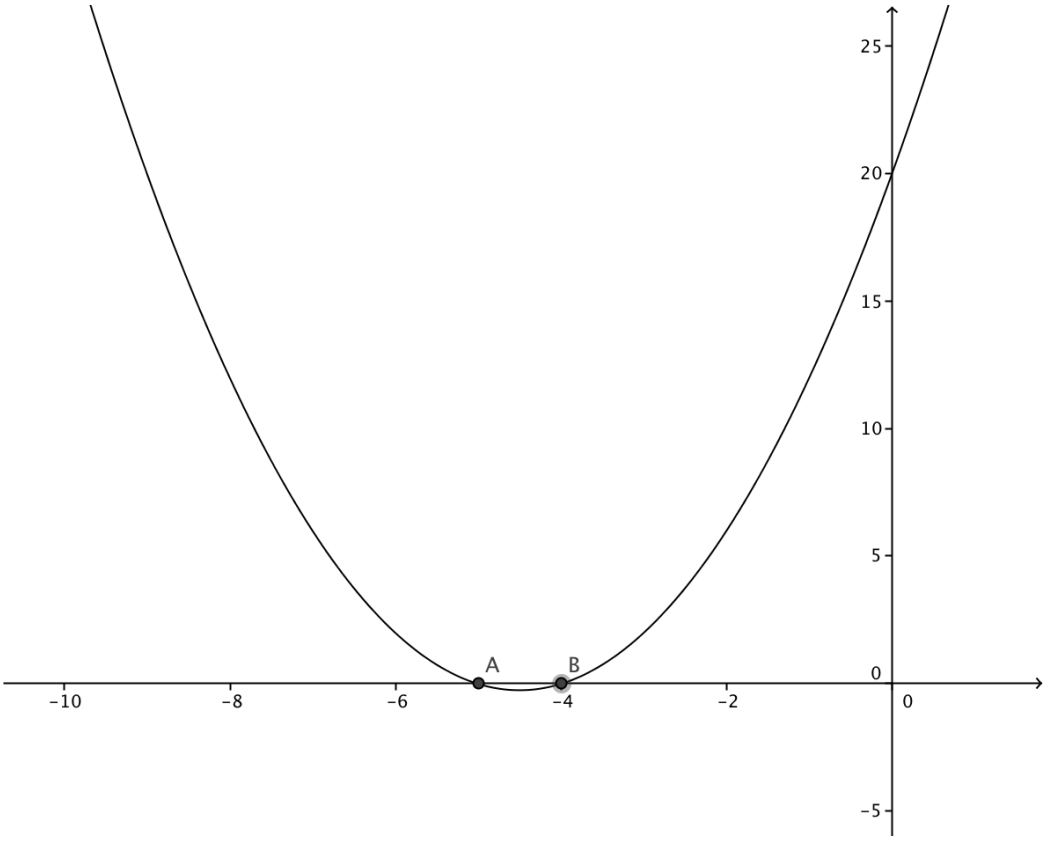
Quick Quiz

1. $y = -3x^2 - 12x - 13$



1. The vertex is (-2,-1).
2. The axis of symmetry is x=-2.
3. The domain is all real numbers. The range is y is less than or equal to -1.
4. The y-intercept is (0,-13).
5. There are no x-intercepts.

2. $y = x^2 + 9x + 20$



x= -4, -5

3.

$74 = x^2 - 7$

$x^2 - 7 = 74$

$x^2 = 74 + 7$

$x^2 = 81$

$\sqrt{x^2} = \sqrt{81}$

$x = \pm 9$

4.

$h(t) = -\frac{1}{2}(g)t^2 + v_0 t + h_0$

$h(t) = -\frac{1}{2}(32)t^2 + 100t + 5$

$h(t) = -16t^2 + 100t + 5$

$vertex = (h,k)$

$h = -\dfrac{b}{2a}$

$h(t) = -16t^2 + 100t + 5$

$b = 100$

$a = -16$

$h = -\dfrac{100}{2(-16)}$

$h = -\dfrac{100}{-32}$

$h = \dfrac{100}{32}$

$h = 3.125$

$h(t) = -16t^2 + 100t + 5$

$k = -16(3.125)^2 + 100(3.125) + 5$

$k = 161.25$

1. $vertex = (3.125, 161.25)$

$h(t) = -16t^2 + 100t + 5$

$0 = -16t^2 + 100t + 5$

$-5 = -16t^2 + 100t$

$-5 = -16\left(t^2 - \frac{100}{16}t\right)$

$-5 = -16\left(t^2 - \frac{25}{4}t\right)$

$-5 + \frac{625}{64}(-16) = -16\left(t^2 - \frac{25}{4}t + \frac{625}{64}\right)$

$-5 - \frac{10,000}{64} = -16\left(t - \frac{25}{8}\right)^2$

$-5 - \frac{625}{4} = -16\left(t - \frac{25}{8}\right)^2$

$-\frac{20}{4} - \frac{625}{4} = -16\left(t - \frac{25}{8}\right)^2$

$-\frac{645}{4} = -16\left(t - \frac{25}{8}\right)^2$

$-\frac{645}{4} = -16\left(t - \frac{25}{8}\right)^2$

$-\frac{645}{4} \div (-16) = \left(t - \frac{25}{8}\right)^2$

$\frac{645}{64} = \left(t - \frac{25}{8}\right)^2$

$\sqrt{\frac{645}{64}} = \sqrt{\left(t - \frac{25}{8}\right)^2}$

$\sqrt{\frac{645}{64}} = t - \frac{25}{8}$

$t = \frac{\sqrt{645}}{8} + \frac{25}{8}$

2. $t \approx 6.2996$

$h(t) = -16t^2 + 100t + 5$

$90 = -16t^2 + 100t + 5$

$85 = -16t^2 + 100t$

$85 = -16\left(t^2 - \frac{100}{16}t\right)$

$85 = -16\left(t^2 - \frac{25}{4}t\right)$

$85 + \frac{625}{64}(-16) = -16\left(t^2 - \frac{25}{4}t + \frac{625}{64}\right)$

$85 - \frac{10,000}{64} = -16\left(t - \frac{25}{8}\right)^2$

$85 - \frac{625}{4} = -16\left(t - \frac{25}{8}\right)^2$

$\frac{340}{4} - \frac{625}{4} = -16\left(t - \frac{25}{8}\right)^2$

$-\frac{285}{4} = -16\left(t - \frac{25}{8}\right)^2$

$-\frac{285}{4} \div (-16) = \left(t - \frac{25}{8}\right)^2$

$\frac{285}{64} = \left(t - \frac{25}{8}\right)^2$

$\sqrt{\frac{285}{64}} = \sqrt{\left(t - \frac{25}{8}\right)^2}$

$\sqrt{\frac{285}{64}} = t - \frac{25}{8}$

$t = \frac{\sqrt{285}}{8} + \frac{25}{8}$

3. $t \approx 5.2352$

5.

$v^2 - 20v + 25 = 6$

$v^2 - 20v = 6 - 25$

$v^2 - 20v = -19$

$v^2 - 20v + 100 = -19 + 100$

$(v - 10)^2 = 81$

$\sqrt{(v - 10)^2} = \sqrt{81}$

$v - 10 = \pm 9$

$v - 10 = 9$

$v = 19$

$v - 10 = -9$

$v = 1$

$v = 19,1$

Lesson 10.5
Solving Quadratic Equations Using the Quadratic Formula

1. The Quadratic Formula is a mathematical formula that can be used to solve an quadratic equation in standard form. Any quadratic equation can be solved using the quadratic formula,and it is the most common way to solve a quadratic equation. It is most useful when the equation is already in standard form. It is a good choice when graphing is too imprecise, when factoring is too difficult or complicated, or when square roots are not easily found.

2. The first known solution of a quadratic equation was recorded in 628 AD by an Indian mathematician named Brahmagupta.

3.

$x^2 - 14x + 45 = 0$

$a = 1$

$b = -14$

$c = 45$

$x = -\frac{b}{a}$

$x = -\frac{(-14)}{1}$

$x = 14$

4.

$8x^2 - 16x - 42 = 0$

$a = 8$

$b = -16$

$c = -42$

$x = -\frac{b}{a}$

$x = -\frac{(-16)}{8}$

$x = 2$

5.

$4x^2 + 16x + 12 = 0$

$a = 4$

$b = 16$

$c = 12$

$x = -\frac{b}{a}$

$x = -\frac{(16)}{4}$

$x = -4$

6.

$x^2 + 2x - 15 = 0$

$a = 1$

$b = 2$

$c = -15$

$x = -\frac{b}{a}$

$x = -\frac{(2)}{1}$

$x = -2$

7.

$$ax^2+bx+c=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x^2+4x-21=0$$

$$x=\frac{-4\pm\sqrt{\left(4^2\right)-4(1)(-21)}}{2(1)}$$

$$x=\frac{-4\pm\sqrt{16+84}}{2}$$

$$x=\frac{-4\pm\sqrt{100}}{2}$$

$$x=\frac{-4+\sqrt{100}}{2}=\frac{-4+10}{2}=\frac{6}{2}=3$$

$$x=\frac{-4-\sqrt{100}}{2}=\frac{-4-10}{2}=\frac{-14}{2}=-7$$

$$x=3,-7$$

8.

$$ax^2+bx+c=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x^2-6x=12$$

$$x^2-6x-12=0$$

$$x=\frac{-(-6)\pm\sqrt{\left(-6^2\right)-4(1)(-12)}}{2(1)}$$

$$x=\frac{6\pm\sqrt{36+48}}{2}$$

$$x=\frac{6\pm\sqrt{84}}{2}$$

$$x=\frac{6+\sqrt{84}}{2}\approx\frac{6+9.1652}{2}\approx\frac{15.1652}{2}\approx5.0551$$

$$x=\frac{6-\sqrt{84}}{2}\approx\frac{6-9.1652}{2}\approx\frac{-3.1652}{2}\approx-1.5826$$

$$x\approx5.0551,-1.5826$$

9.

$ax^2 + bx + c = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$3x^2 - \frac{1}{2}x = \frac{3}{8}$

$3x^2 - \frac{1}{2}x - \frac{3}{8} = 0$

$x = \dfrac{-\left(-\frac{1}{2}\right) \pm \sqrt{\left(-\frac{1}{2}^2\right) - 4(3)\left(-\frac{3}{8}\right)}}{2(3)}$

$x = \dfrac{\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{36}{8}}}{6}$

$x = \dfrac{0.5 \pm \sqrt{4.75}}{6}$

$x = \dfrac{0.5 + \sqrt{4.75}}{6} \approx \dfrac{0.5 + 2.1794}{6} = \dfrac{2.6794}{6} = 0.4466$

$x = \dfrac{0.5 - \sqrt{4.75}}{6} \approx \dfrac{0.5 - 2.1794}{6} = \dfrac{-1.6794}{6} = -0.2799$

$x = 0.4466, -0.2799$

10.

$ax^2 + bx + c = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$2x^2 + x - 3 = 0$

$x = \dfrac{-1 \pm \sqrt{\left(1^2\right) - 4(2)(-3)}}{2(2)}$

$x = \dfrac{-1 \pm \sqrt{1 + 24}}{4}$

$x = \dfrac{-1 \pm \sqrt{25}}{4}$

$x = \dfrac{-1 + \sqrt{25}}{4} = \dfrac{-1 + 5}{4} = \dfrac{4}{4} = 1$

$x = \dfrac{-1 - \sqrt{25}}{4} = \dfrac{-1 - 5}{4} = \dfrac{-6}{4} = -1.5$

$x = 1, -1.5$

11.

$$ax^2+bx+c=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$-x^2-7x+12=0$$

$$x=\frac{-(-7)\pm\sqrt{(-7^2)-4(-1)(12)}}{2(-1)}$$

$$x=\frac{7\pm\sqrt{49+48}}{-2}$$

$$x=\frac{7\pm\sqrt{97}}{-2}$$

$$x=\frac{7+\sqrt{97}}{-2}\approx\frac{7+9.8489}{-2}\approx\frac{16.8489}{-2}\approx-8.4245$$

$$x=\frac{7-\sqrt{97}}{-2}\approx\frac{7-9.8489}{-2}\approx\frac{-2.8489}{-2}\approx1.4245$$

$$x=-8.4245,1.4245$$

12.

$$ax^2+bx+c=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$-3x^2+5x=0$$

$$x=\frac{-(5)\pm\sqrt{(5^2)-4(-3)(0)}}{2(-3)}$$

$$x=\frac{-5\pm\sqrt{25+0}}{-6}$$

$$x=\frac{-5\pm\sqrt{25}}{-6}$$

$$x=\frac{-5+\sqrt{25}}{-6}=\frac{-5+5}{-6}=\frac{0}{-6}=0$$

$$x=\frac{-5-\sqrt{25}}{-6}=\frac{-5-5}{-6}=\frac{-10}{-6}\approx1.6667$$

$$x=0,1.6667$$

13.

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$4x^2 = 0$$

$$x = \frac{-(0) \pm \sqrt{(0^2) - 4(4)(0)}}{2(4)}$$

$$x = \frac{0 \pm \sqrt{0 + 0}}{8}$$

$$x = \frac{0 \pm \sqrt{0}}{8}$$

$$x = \frac{0}{8} = 0$$

$$x = 0$$

14.

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x^2 + 6x + 2 = 0$$

$$x = \frac{-(6) \pm \sqrt{(6^2) - 4(1)(2)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{36 - 8}}{2}$$

$$x = \frac{-6 \pm \sqrt{28}}{2}$$

$$x = \frac{-6 + \sqrt{28}}{2} \approx \frac{-6 + 5.2915}{2} \approx \frac{-0.7085}{2} \approx -0.3543$$

$$x = \frac{-6 - \sqrt{28}}{2} \approx \frac{-6 - 5.2915}{2} \approx \frac{-11.2915}{2} \approx -5.6458$$

$$x = -0.3543, -5.6458$$

For questions 15 through 26 the steps and process may differ according to the method chosen to solve the equation. The answers however, should always be the same. Listed are possible choices for the easiest method of solution.

15.

$x^2 - x = 6$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x - 3 = 0$

$x = 3$

$x + 2 = 0$

$x = -2$

$x = 3, -2$

16.

$x^2 - 12 = 0$

$x^2 = 12$

$\sqrt{x^2} = \sqrt{12}$

$x = \sqrt{12}$

$x \approx \pm 3.4641$

17.

$-2x^2+5x-3=0$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{-5\pm\sqrt{5^2-4(-2)(-3)}}{2(-2)}$

$x=\dfrac{-5\pm\sqrt{25-24}}{-4}$

$x=\dfrac{-5\pm\sqrt{1}}{-4}$

$x=\dfrac{-5+\sqrt{1}}{-4}=\dfrac{-5+1}{-4}=\dfrac{-4}{-4}=1$

$x=\dfrac{-5-\sqrt{1}}{-4}=\dfrac{-5-1}{-4}=\dfrac{-6}{-4}=1.5$

$x=1,1.5$

18.

$x^2+7x-18=0$

$x^2+7x=18$

$x^2+7x+12.25=18+12.25$

$(x+3.5)^2=30.25$

$\sqrt{(x+3.5)^2}=\sqrt{30.25}$

$x+3.5=\pm5.5$

$x+3.5=5.5$

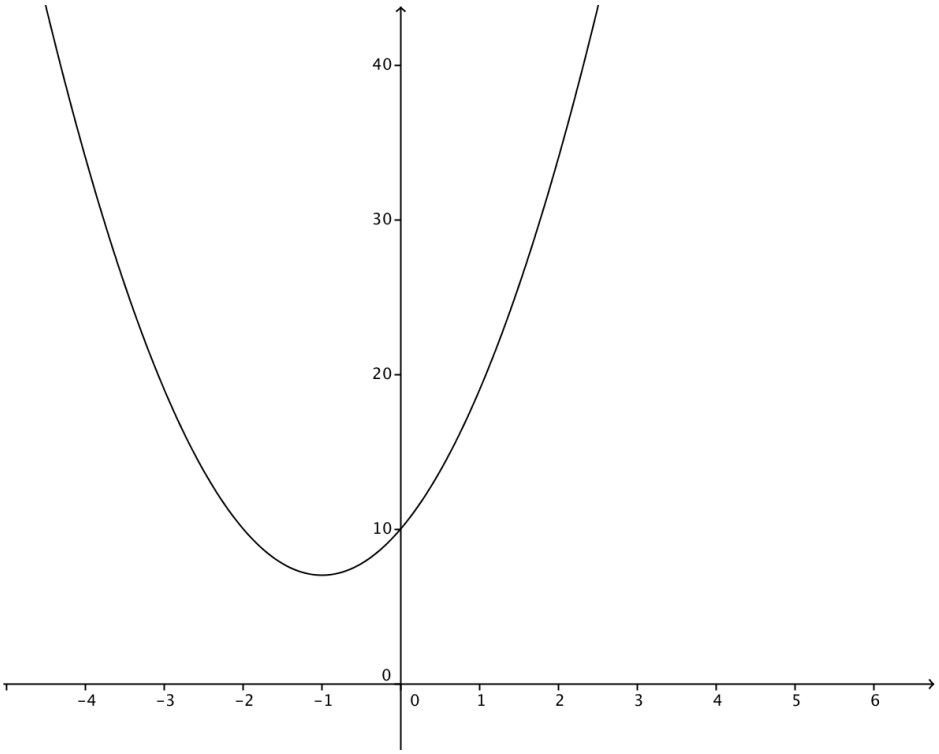
$x=2$

$x+3.5=-5.5$

$x=-9$

$x=2,-9$

19. $3x^2 + 6x = -10$



The graphs shows that the parabola does not intersect the x-axis, therefore there are no roots.

20.

$-4x^2 + 400x = 0$

$-4\left(x^2 - 100x\right) = 0$

$-4\left(x^2 - 100x + 2500\right) = 0 + 2500(-4)$

$-4(x - 50)^2 = -10000$

$(x - 50)^2 = 2500$

$\sqrt{(x - 50)^2} = \sqrt{2500}$

$x - 50 = \pm 50$

$x - 50 = 50$

$x = 100$

$x - 50 = -50$

$x = 0$

$x = 100, 0$

21.

$-3x^2+12x+1=0$

$-3x^2+12x=-1$

$-3(x^2-4x)=-1$

$-3(x^2-4x+4)=-1+4(3)$

$-3(x-2)^2=-12$

$(x-2)^2=4$

$\sqrt{(x-2)^2}=\sqrt{4}$

$x-2=\pm 2$

$x-2=2$

$x=4$

$x-2=-2$

$x=0$

$x=4,0$

22.

$x^2+6x+9=0$

$(x+3)(x+3)=0$

$(x+3)^2=0$

$x+3=0$

$x=-3$

23.

$81x^2+1=0$

$\sqrt{81x^2+1}=\sqrt{0}$

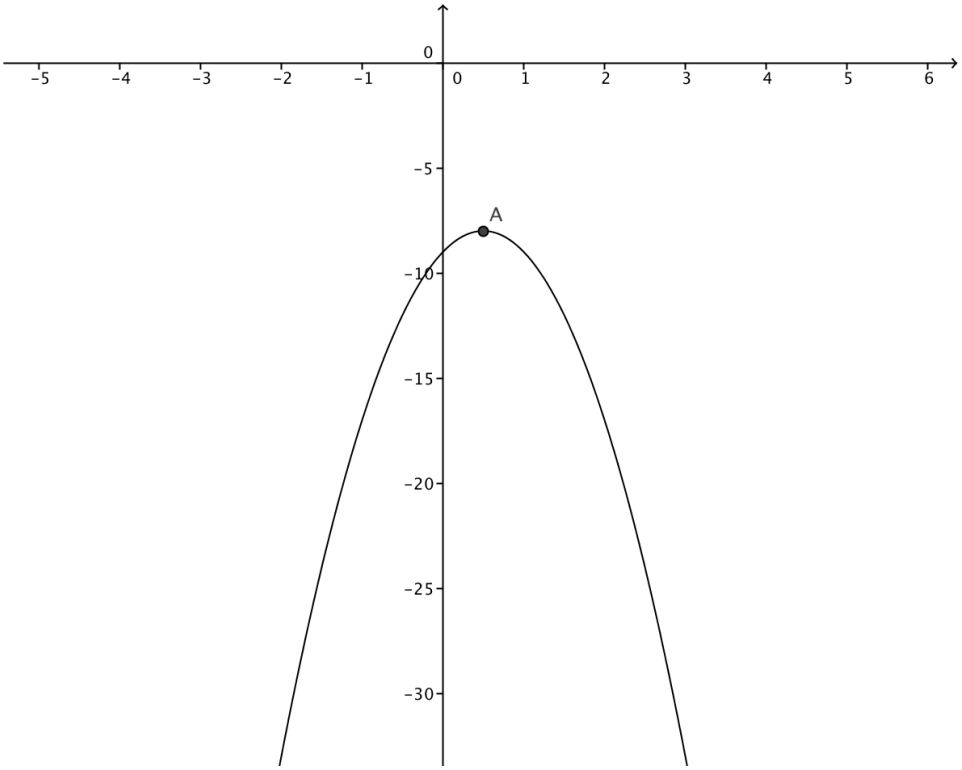
$(9x+1)=0$

$9x+1=0$

$9x=-1$

$x=-\frac{1}{9}$

24. $-4x^2 + 4x = 9$



The graphs shows that the parabola does not intersect the x-axis, therefore there are no roots.

25.

$36x^2 - 21 = 0$

$\sqrt{36x^2 - 21} = \sqrt{0}$

$(6x - 4.5826) = 0$

$6x - 4.5826 = 0$

$6x = 4.5826$

$x \approx 0.7638$

26.

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x + 3 = 0$

$x = -3$

$x - 1 = 0$

$x = 1$

$x = -3, 1$

27.

If we call one number x, then the second number is x+1. We know their product is 72, therefore:

$x(x+1) = 72$

$x^2 + x = 72$

$\left(x^2 + x + \frac{1}{4}\right) = 72 + \frac{1}{4}$

$\left(x + \frac{1}{2}\right)^2 = \frac{289}{4}$

$\sqrt{(x+0.5)^2} = \sqrt{72.25}$

$x + 0.5 = \pm 8.5$

$x + 0.5 = 8.5$

$x = 8$

$x + 0.5 = -8.5$

$x = -9$

(There are two possibilities for x: 8 and -9. However -9 does not meet the requirements, because the next number consecutively is -10 and their product is not 72.)

The two consecutive integers are 8 and 9.

28.
If we call one number x, then the second number is x+2. Their product is 3(x+(x+2))-1.
Therefore:

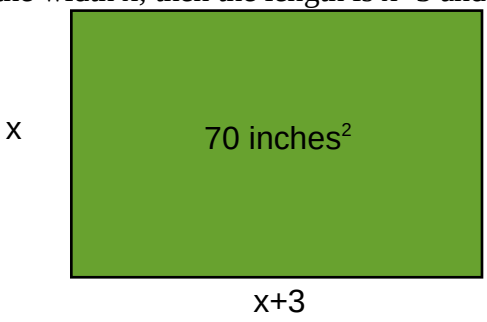
$$x(x+2)=3(x+(x+2))-1$$
$$x^2+2x=3(2x+2)-1$$
$$x^2+2x=6x+6-1$$
$$x^2+2x=6x+5$$
$$x^2+2x-6x-5=0$$
$$x^2-4x-5=0$$

$$x^2-4x-5=0$$
$$x^2-4x=5$$
$$x^2-4x+4=5+4$$
$$x^2-4x+4=9$$
$$(x-2)^2=9$$
$$\sqrt{(x-2)^2}=\sqrt{9}$$
$$x-2=3$$
$$x=3+2$$
$$x=5$$

The two consecutive odd integers are 5 and 7.

29.

If we call the width x, then the length is x+3 and the rectangle looks like this:



The Area of a rectangle is equal to its length times its width, therefore:

$$x(x+3)=70$$

$$x^2+3x=70$$

$$x^2+3x-70=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-3\pm\sqrt{3^2-4(1)(-70)}}{2(1)}$$

$$x=\frac{-3\pm\sqrt{9+280}}{2}$$

$$x=\frac{-3\pm\sqrt{289}}{2}$$

$$x=\frac{-3+\sqrt{289}}{2}=\frac{-3+17}{2}=\frac{14}{2}=7$$

$$x=\frac{-3-\sqrt{289}}{2}=\frac{-3-17}{2}=\frac{-20}{2}=-10$$

$$x=7,-10$$

$$x+3=l$$

$$7+3=l$$

$$l=10$$

(There are two possible solutions for x, but since we are talking about distance the answer cannot be negative.)

The dimensions of the rectangle are 7inches by 10inches.

30.
The problem tells us that the width of the entire garden is x and the length of the entire garden is 2x. It also tells us that the area of the entire garden is 200 sq ft. By using these pieces of information we can solve for x and then compute the amount of fence needed.

$$x(2x) = 200$$

$$2x^2 = 200$$

$$x^2 = 100$$

$$\sqrt{x^2} = \sqrt{100}$$

$$x = 10$$

Now we know that the width equals 10 ft and therefore the length equals 20 ft. We also know that the additional fencing needed to close off the three sections are also each equal to the width. Now we add together all the pieces of fence.

$$x + x + x + x + 2x + 2x = y$$

$$4x + 4x = y$$

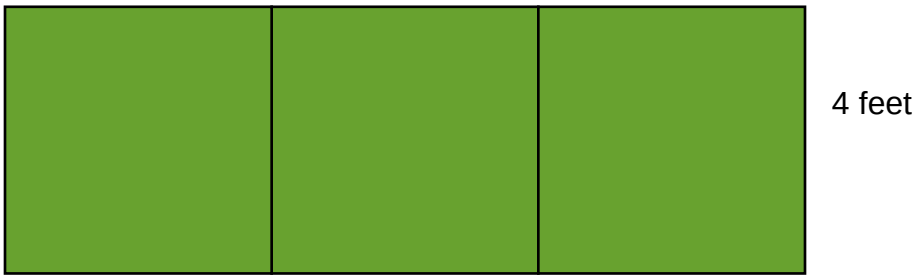
$$8x = y$$

$$8(10) = y$$

$$y = 80$$

Suzie will need 80 feet of fencing to enclose her entire garden and separate the sections.

31.



The rectangle above represents the larger board that Angel is cutting. It is divided into 3 equal squares. The area of the overall rectangle is 8 * 4 = 32 sq feet. The cut off square is one third of this. The area of a square is side squared. Therefore:

$$\tfrac{1}{3} * 32 \approx 10.6667$$

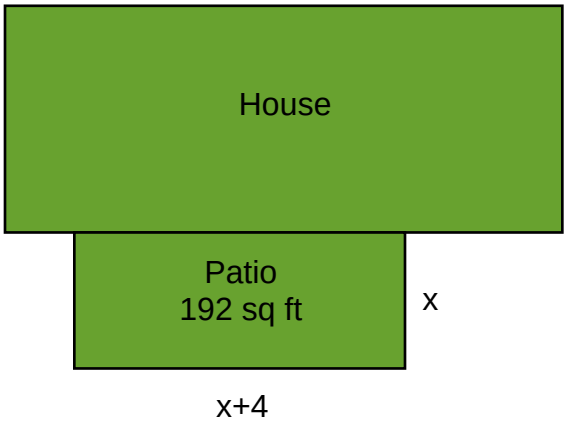
$$x^2 \approx 10.6667$$

$$\sqrt{x^2} \approx \sqrt{10.6667}$$

$$x \approx 3.266$$

The length of the side of the square is approximately 3.266 feet.

32.



The diagram above shows what the house and patio would look like. We can call the width of the patio x and the length x+4. We know the total area of the patio is 192 sq ft, therefore:

$$x(x+4)=192$$

$$x^2+4x=192$$

$$x^2+4x-192=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-4\pm\sqrt{4^2-4(1)(-192)}}{2(1)}$$

$$x=\frac{-4\pm\sqrt{16+768}}{2}$$

$$x=\frac{-4\pm\sqrt{784}}{2}$$

$$x=\frac{-4+\sqrt{784}}{2}=\frac{-4+28}{2}=\frac{24}{2}=12$$

$$x=\frac{-4-\sqrt{784}}{2}=\frac{-4-28}{2}=\frac{-32}{2}=-16$$

Because this is a distance problem, we only use the positive value of x, which is 12. If Mike is fencing in three sides then he needs:

$$x+x+x+4=y$$

$$3x+4=y$$

$$3(12)+4=y$$

$$36+4=y$$

$$y=40$$

Mike needs 40 feet of fencing to enclose the three sides of the patio.

Mixed Review

33.
Let's call the # of balcony seats x. This means the # of floor seats is 4x. The number of box seats is x+200. We also know that the total number of seats is 1100, therefore:

$$x + 4x + (x + 200) = 1100$$
$$6x + 200 = 1100$$
$$6x = 900$$
$$x = 150$$

$$4x = 4(150) = 600$$
$$x + 200 = 150 + 200 = 350$$

The theatre has 150 balcony seats, 600 floor seats, and 350 box seats.

34.
$$(10, 65) \,\&\, (5, 30)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{30 - 65}{5 - 10} = \frac{-35}{-5} = 7$$
$$y - 65 = 7(x - 10)$$
$$y - 65 = 7x - 70$$
$$y = 7x - 5$$

35.
$$120\%(x) = 60$$
$$1.2x = 60$$
$$x = 50$$

36.
$$\sqrt{16} = 4$$
4 is a Positive Real Number, an Integer, a Rational Number and a Natural or Whole Number.

37.
$$6\tfrac{1}{7} \div -2\tfrac{3}{4} = \frac{43}{7} \div -\frac{11}{4} = \frac{43}{7} \times -\frac{4}{11} = -\frac{172}{77} = -2\tfrac{18}{77}$$

38.
The most appropriate domain for a set of books in a library is whole numbers. It is not possible to have negative numbers of books, nor is it possible to have a fraction of a book, therefore the answer must be whole numbers.

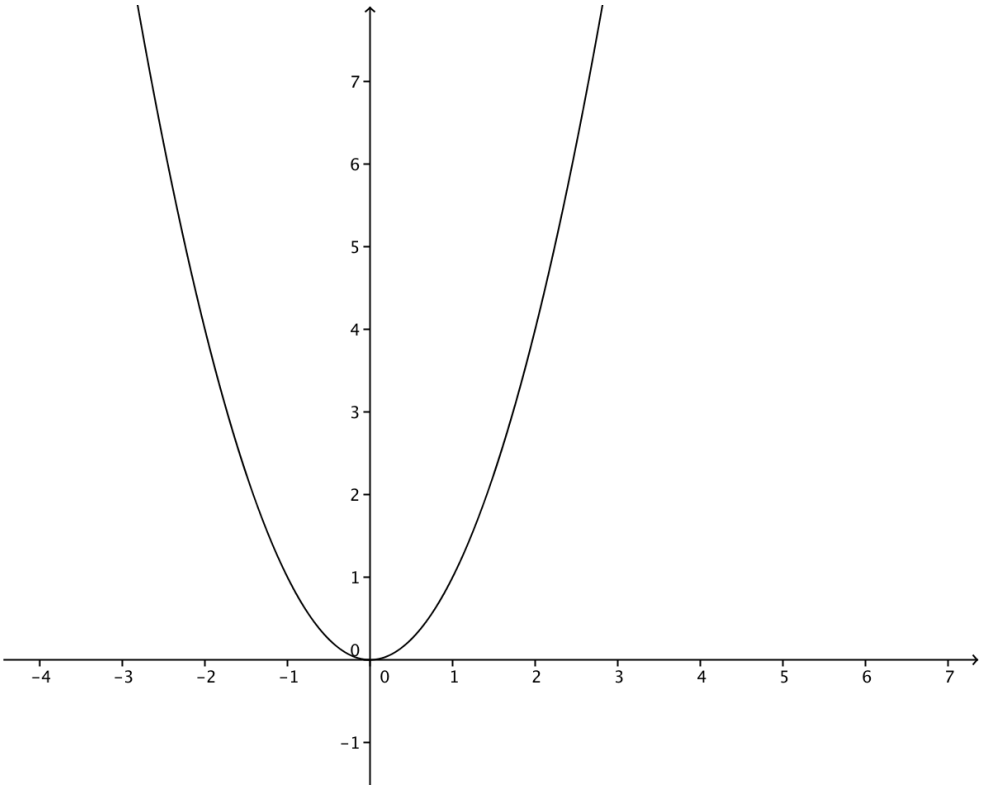
Lesson 10.6
The Discriminant

1. The Discriminant is the value under the radical in the Quadratic Formula. It tells us how many real roots the quadratic equation has.

2. $D = b^2 - 4ac$

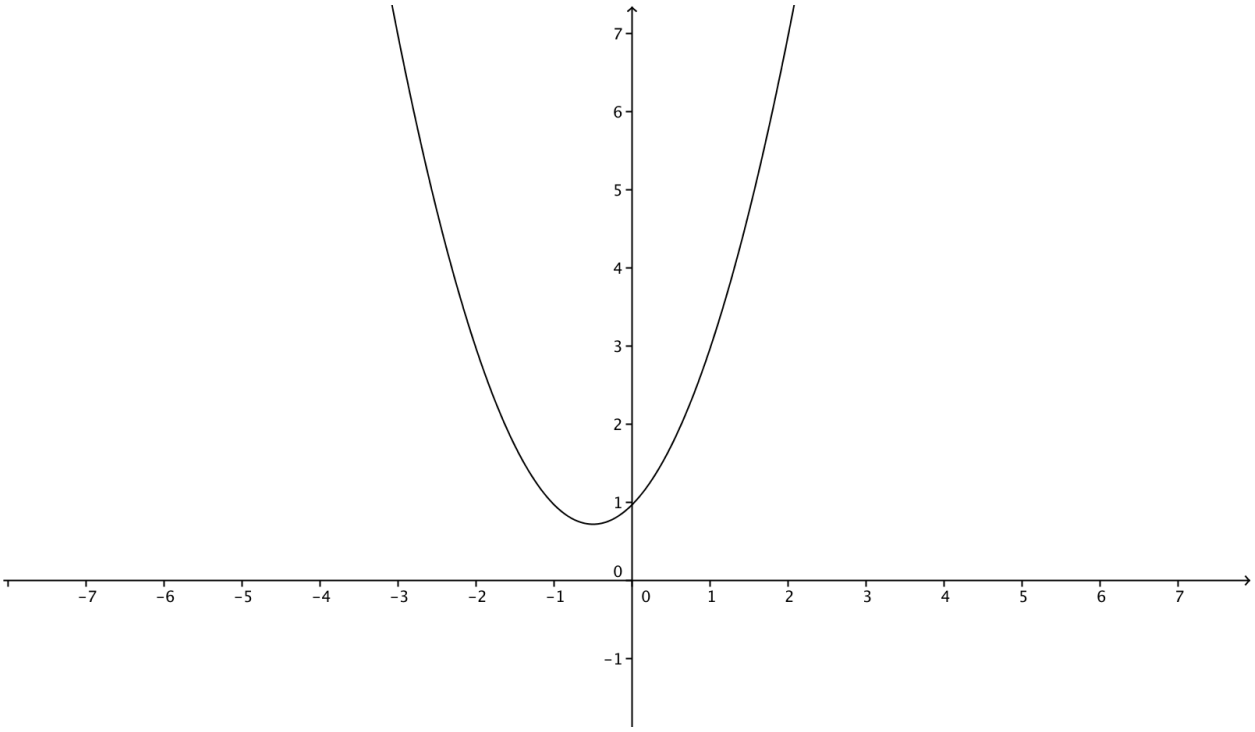
3. No you cannot find the discriminant of a linear equation. You need a quadratic equation in standard form to determine what the values of a, b, and c are. In a linear equation these values do not exist.

4.



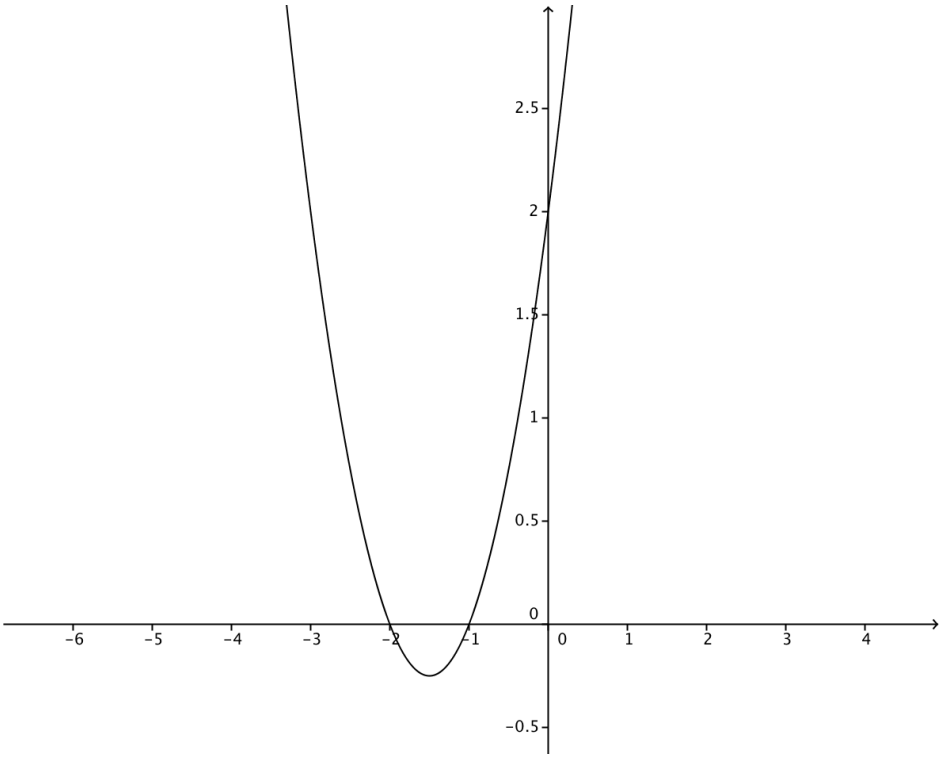
When D=0 there is one real solution.

5.



When D=-2.85 then the equation has no real solutions.

6.



When D>0 then there will be two real solutions.

7.

$$2x^2 - 4x + 5 = 0$$
$$D = b^2 - 4ac$$
$$D = (-4)^2 - 4(2)(5)$$
$$D = 16 - 40$$
$$D = -24$$

8.

$$x^2 - 5x = 8$$
$$x^2 - 5x - 8 = 0$$
$$D = b^2 - 4ac$$
$$D = (-5)^2 - 4(1)(-8)$$
$$D = 25 + 32$$
$$D = 57$$

9.

$$4x^2 - 12x + 9 = 0$$
$$D = b^2 - 4ac$$
$$D = (-12)^2 - 4(4)(9)$$
$$D = 144 - 144$$
$$D = 0$$

10.

$$x^2 + 3x + 2 = 0$$
$$D = b^2 - 4ac$$
$$D = (3)^2 - 4(1)(2)$$
$$D = 9 - 8$$
$$D = 1$$

11.

$x^2 - 16x = 32$

$x^2 - 16x - 32 = 0$

$D = b^2 - 4ac$

$D = (-16)^2 - 4(1)(32)$

$D = 256 - 128$

$D = 128$

12.

$-5x^2 + 5x - 6 = 0$

$D = b^2 - 4ac$

$D = (5)^2 - 4(-5)(-6)$

$D = 25 - 120$

$D = -95$

13.

$-x^2 + 3x - 6 = 0$

$D = b^2 - 4ac$

$D = (3)^2 - 4(-1)(-6)$

$D = 9 - 24$

$D = -15$

This quadratic equation will have no real solutions.

14.

$5x^2 = 6x$

$5x - 6x = 0$

$D = b^2 - 4ac$

$D = (-6)^2 - 4(5)(0)$

$D = 36 - 0$

$D = 36$

This quadratic equation will have two real solutions.

15.

$41x^2 - 31x - 52 = 0$

$D = b^2 - 4ac$

$D = (-31)^2 - 4(41)(-52)$

$D = 961 + 8528$

$D = 9489$

This quadratic equation will have two real solutions.

16.

$x^2 - 8x + 16 = 0$

$D = b^2 - 4ac$

$D = (-8)^2 - 4(1)(16)$

$D = 64 - 64$

$D = 0$

This quadratic equation will have one real solution.

17.

$-x^2 + 3x - 10 = 0$

$D = b^2 - 4ac$

$D = (3)^2 - 4(-1)(10)$

$D = 9 - 40$

$D = -31$

This quadratic equation will have no real solutions.

18.

$x^2 - 64 = 0$

$D = b^2 - 4ac$

$D = (0)^2 - 4(1)(-64)$

$D = 0 + 256$

$D = 256$

This quadratic equation will have two real solutions.

19.

$x^2 = -4x + 20$

$x^2 + 4x - 20 = 0$

$D = b^2 - 4ac$

$D = (4)^2 - 4(1)(-20)$

$D = 16 + 80$

$D = 96$

The Discriminant is NOT a perfect square, so the solutions will be irrational numbers.

20.

$x^2 + 2x - 3 = 0$

$D = b^2 - 4ac$

$D = (2)^2 - 4(1)(-3)$

$D = 4 + 12$

$D = 16$

The Discriminant IS a perfect square, so the solutions will be rational numbers.

21.

$3x^2 - 11x = 10$

$3x^2 - 11x - 10 = 0$

$D = b^2 - 4ac$

$D = (-11)^2 - 4(3)(-10)$

$D = 121 + 120$

$D = 241$

The Discriminant is NOT a perfect square, so the solutions will be irrational numbers.

22.

$\frac{1}{2}x^2 + 2x + \frac{2}{3} = 0$

$D = b^2 - 4ac$

$D = (2)^2 - 4\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)$

$D = 4 - \frac{4}{3}$

$D = \frac{8}{3}$

The Discriminant is NOT a perfect square, so the solutions will be irrational numbers.

23.

$x^2 - 10x + 25 = 0$

$D = b^2 - 4ac$

$D = (-10)^2 - 4(1)(25)$

$D = 100 - 100$

$D = 0$

The Discriminant equals 0, so the solution will be a rational number.

24.

$x^2 = 5x$

$x^2 - 5x = 0$

$D = b^2 - 4ac$

$D = (-5)^2 - 4(1)(0)$

$D = 25 - 0$

$D = 25$

The Discriminant IS a perfect square, so the solutions will be rational.

25.

$h(t) = -32t^2 + 55t + 4$

$36 = -32t^2 + 55t + 4$

$0 = -32t^2 + 55t + 4 - 36$

$0 = -32t^2 + 55t - 32$

$D = b^2 - 4ac$

$D = (55)^2 - 4(-32)(-32)$

$D = 3025 - 4096$

$D = -1071$

The Discriminant is a negative number so the equation has no real solutions. Therefore: when Marty throws the phone it will not reach Yolanda.

26.

$R = x(200 - 0.4x)$

$R = 200x - 0.4x^2$

$-0.4x^2 - 200x = R$

$-0.4x^2 - 200x = 20,000$

$-0.4x^2 - 200x - 20,000 = 0$

$D = b^2 - 4ac$

$D = (-200)^2 - 4(-0.4)(-20,000)$

$D = 40,000 - 32,000$

$D = 8,000$

The Discriminant of the equation is a positive number so there are two real solutions.
Yes, there are two possible ways in which Bryson's business can generate a revenue of $20,000 in the month of July.

27.

$y = -\frac{32}{6400}x^2 + x$

$10 = -\frac{32}{6400}x^2 + x$

$0 = -\frac{32}{6400}x^2 + x - 10$

$D = b^2 - 4ac$

$D = (1)^2 - 4\left(-\frac{32}{6400}\right)(-10)$

$D = 1 - \frac{1}{5}$

$D = \frac{4}{5}$

The Discriminant of the equation is a positive number so there are two real solutions.
Yes, Marcus kicked the ball hard enough to go over the goal post.

Mixed Review

28.

$6x^2 - x - 12$

$(3x + 4)(2x - 3)$

29.

$y = a(x-h)^2 + k$

$y = -\frac{1}{4}x^2 - 3x - 12$

$12 = -\frac{1}{4}x^2 - 3x$

$12 = -\frac{1}{4}(x^2 + 12x)$

$12 + 36\left(-\frac{1}{4}\right) = -\frac{1}{4}(x^2 + 12x + 36)$

$12 - 9 = -\frac{1}{4}(x+6)^2$

$3 = -\frac{1}{4}(x+6)^2$

$0 = -\frac{1}{4}(x+6)^2 - 3$

$y = -\frac{1}{4}(x+6)^2 - 3$

vertex= (-6,-3)

30.

$-4x^2 - 15 = -4x$

$-4x^2 - 15 + 4x = 0$

$-4x^2 + 4x - 15 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-4 \pm \sqrt{4^2 - 4(-4)(-15)}}{2(-4)}$

$x = \frac{-4 \pm \sqrt{16 - 240}}{-8}$

$x = \frac{-4 \pm \sqrt{-224}}{-8}$

The quadratic equation has no real solutions.

31.

1 fathom = 6 feet

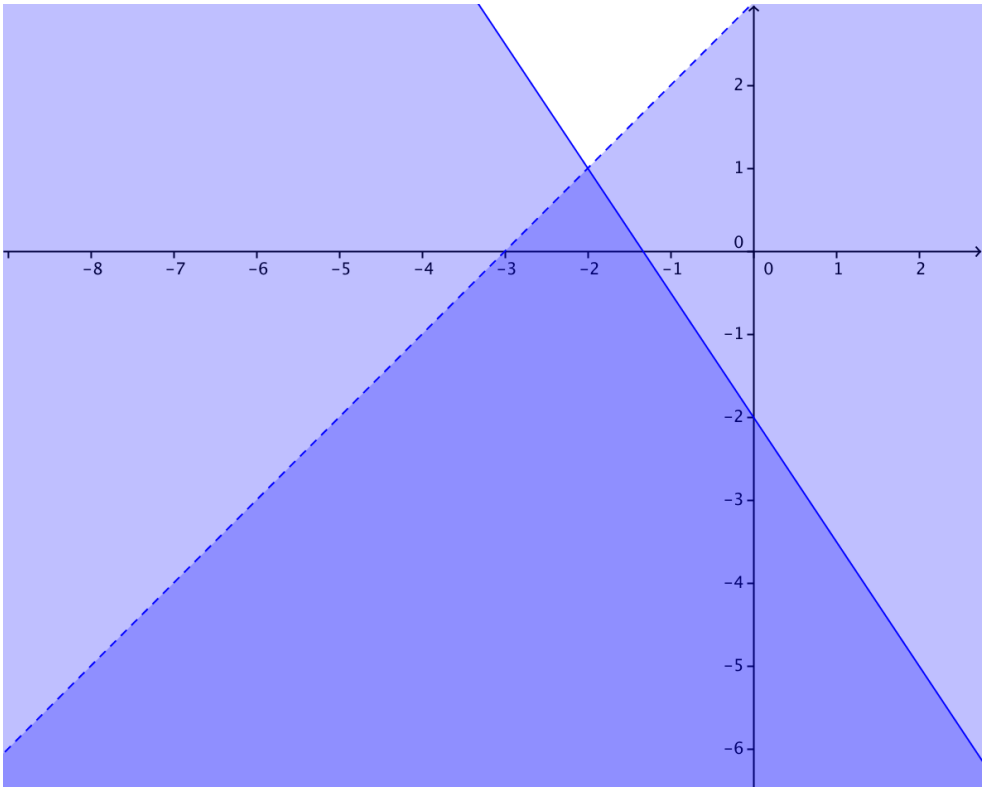
4 fathoms= 6*4= 24 feet

1 foot=30.48 cm

24 ft=30.48 * 24 = 731.52 cm

4 fathoms = 731.52 cm

32. $\begin{cases} 3x + 2y \leq -4 \\ x - y > -3 \end{cases}$



33.
Assuming that the order does not matter, and that you CAN repeat toppings:
7 * 7 * 7 = 343 ways.

Assuming that the order does not matter and that you CANNOT repeat toppings:
${}_7C_3 = 35$

Lesson 10.7
Linear, Exponential, and Quadratic Models

1. If the second set of differences have the same value it can be concluded that the equation is a quadratic.

2. If you have to find the difference 5 times you can safely assume that the degree of the polynomial is 5. It is an equation with a degree of 5.

3. You would test the ratio of differences to determine if the equation is exponential.

4. If you have a cubic function you can conclude that you would have to take the differences 3 times.

5. The first difference is always 3, therefore the data can be modeled by a linear function.

6. Neither the first or second differences are equal so the equation is neither linear nor quadratic.

7. The first difference is always 25, therefore the data can be modeled by a linear function.

8. The second difference is always 5, therefore the data can be modeled by a quadratic function.

9. The second difference is always 2, therefore the data can be modeled by a quadratic function.

10. Neither the first or second differences are equal so the equation is neither linear nor quadratic.

11. This data CANNOT be modeled by an exponential function because the ratios are not equal.

12. This data CAN be modeled by an exponential function because the ratios are equal.

13. This data CAN be modeled by an exponential function because the ratios are equal.

14.
500/400=1.25
625/500=1.25
781.25/625=1.25
876.5625/781.25=1.25
Because the ratios are equal, this is an exponential function.

$y = a(b)^x$

$a = 400$

$b = 1.25$

$y = 400(1.25)^x$

15.
(-3)-(-2)=-1
(-2)-(-1)=-1
(-1)-0=-1
0-1=-1
1-2=-1
Because the first set of differences are equal, this is a linear function.

$y = mx + b$

$m = 0.5$

$b = 1.5$

$y = 0.5x + 1.5$

16.
14-4=10
4-(-2)=6
(-2)-(-4)=2
(-4)-(-2)=-2
(-2)-4=-6
4-14=-10

10-6=4
6-2=4
2-(-2)=4
(-2)-(-6)=4
(-6)-(-10)=4
Because the second set of differences are equal, this is a quadratic function.

$y = ax^2 + bx + c$

$a = 2$

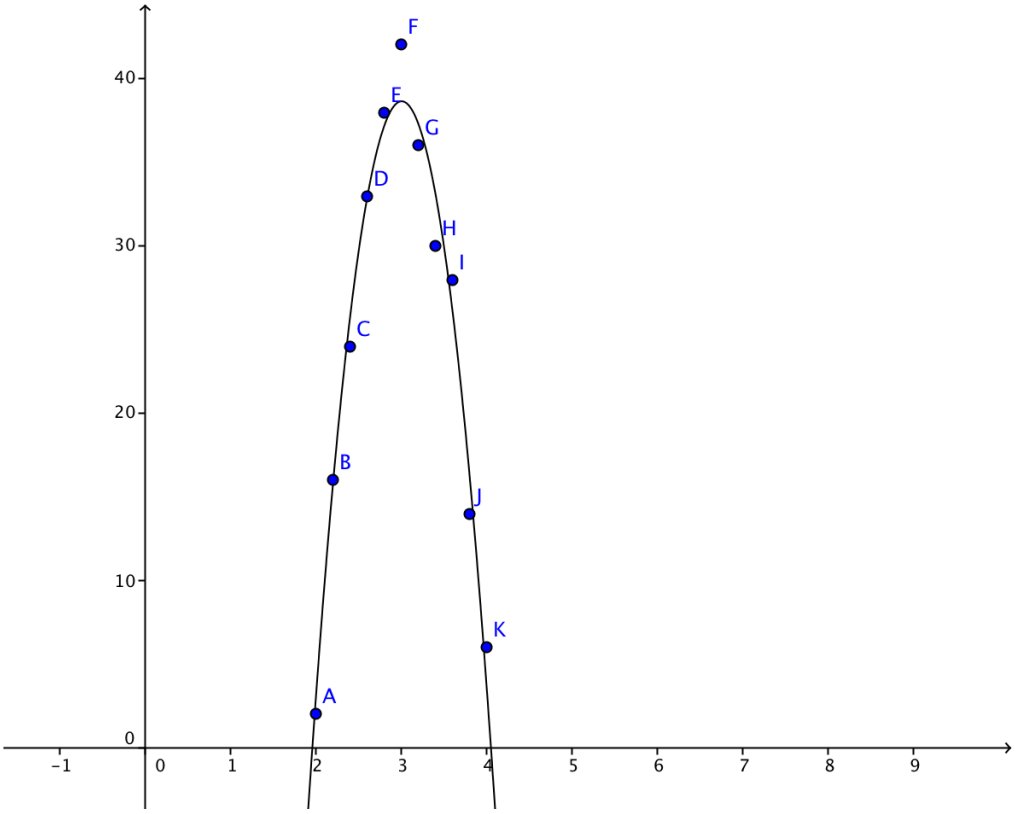
$b = 0$

$c = -4$

$y = 2x^2 - 4$

17.

1. & 3.



2. $y = -35.43x^2 + 213.31x - 282.44$

4. The maximum height is 42.

$y = -35.43x^2 + 213.31x - 282.44$

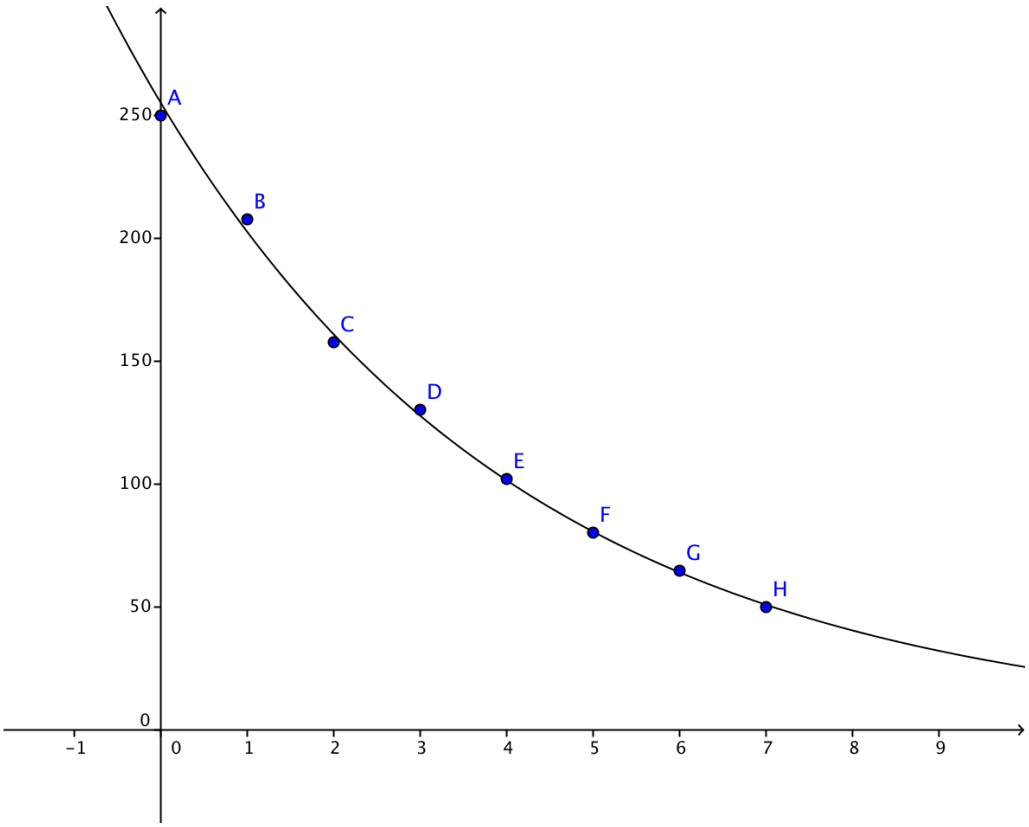
$y = -35.43(2.5)^2 + 213.31(2.5) - 282.44$

$y = -221.4375 + 533.275 - 282.44$

5. $y = 29.3975$

At 2.5 seconds the ball will be at approximately 29.3975.

18.

1. 

2. An exponential function best suits the data.

$y = a(b)^x$

$a = 255.25$

$b = 0.79$

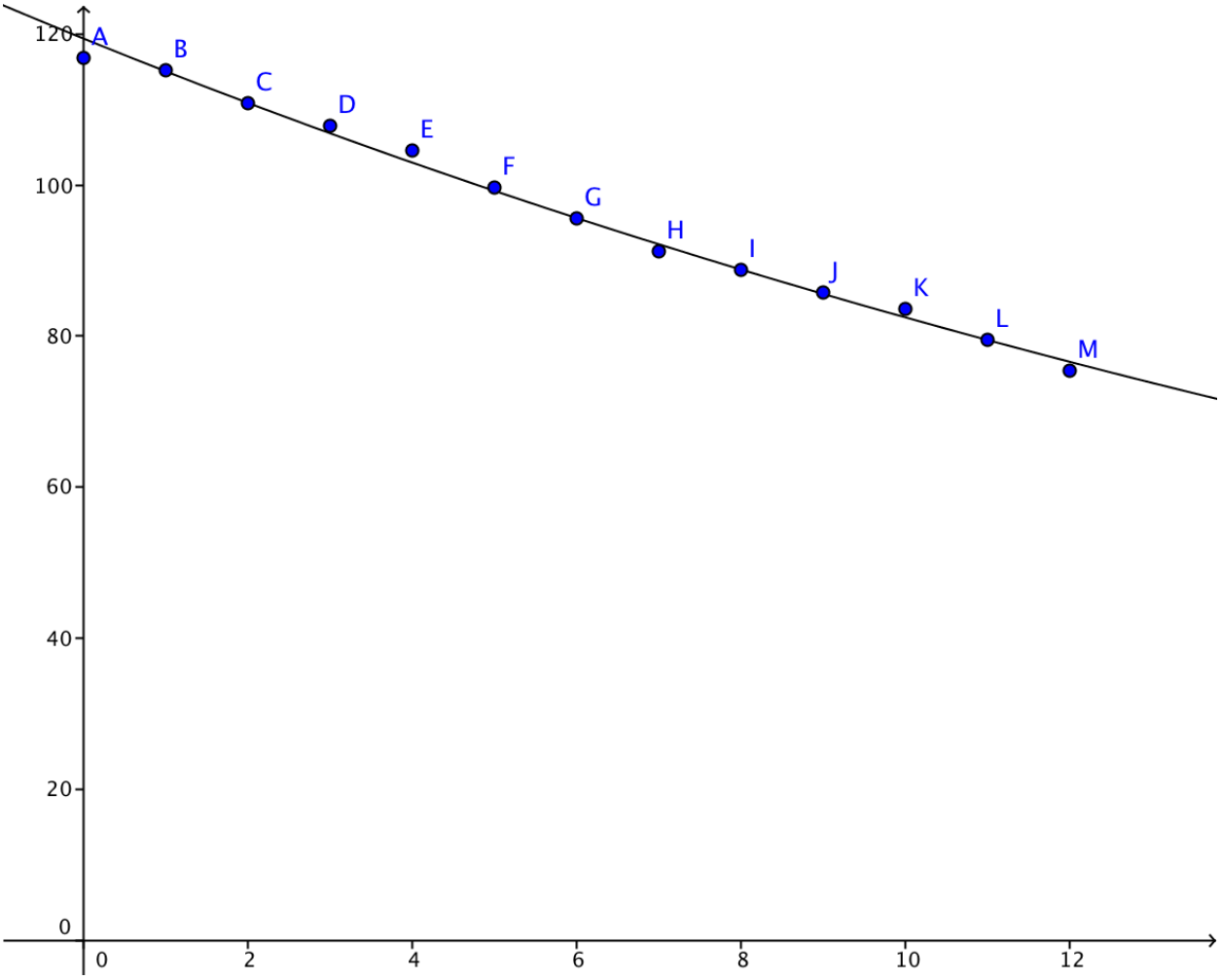
3. $y = 255.25(0.79)^x$

$y = 255.25(0.79)^x$

$y = 255.25(0.79)^{10}$

4. $y = 24.1678$

19.



$y = a(b)^x$

$a = 119.41$

$b = 0.96$

$y = 119.41(0.96)^x$

The model that best fits the data is the exponential function $y = 119.41(0.96)^x$

$y = 119.41(0.96)^x$

$y = 119.41(0.96)^{20}$

$y = 52.7795$

Based on this model the rate of pregnancy in 2010 will be 52.7795

Mixed Review

20.

$y = mx + b$

$y = 2.5x$

$16 = 2.5x$

$x = 6.4$

The equation for this scenario is y=2.5x.
Cam will run out of flour after 6.4 loaves of bread.

21.

$h(t) = -\frac{1}{2}(g)t^2 + v_0 t + h_0$

$h(t) = -\frac{1}{2}(32)t^2 + 10t + 7$

1. $h(t) = -16t^2 + 10t + 7$

$h(t) = -16t^2 + 10t + 7$

$0 = -16t^2 + 10t + 7$

$-7 = -16t^2 + 10t$

$-7 = -16(t^2 - \frac{10}{16}t)$

$-7 + \frac{100}{1024}(-16) = -16(t^2 - \frac{10}{16}t + \frac{100}{1024})$

$-7 - \frac{100}{64} = -16\left(t - \frac{10}{32}\right)^2$

$-8.5625 = -16(t - 0.3125)^2$

$y = -16(t - 0.3125)^2 + 8.5625$

2. $(h,k) = (0.3125, 8.5625)$

The maximum height the ball reaches is 8.5625 ft.

3. The y-intercept is (0,7). It represents the initial height the basketball is being shot from.

$h(t) = -16t^2 + 10t + 7$

$0 = -16t^2 + 10t + 7$

$-7 = -16t^2 + 10t$

$-7 = -16(t^2 - \frac{10}{16}t)$

$-7 + \frac{100}{1024}(-16) = -16(t^2 - \frac{10}{16}t + \frac{100}{1024})$

$-7 - \frac{100}{64} = -16\left(t - \frac{10}{32}\right)^2$

$-8.5625 = -16(t - 0.3125)^2$

$0.5352 = (t - 0.3125)^2$

$\sqrt{0.5352} = \sqrt{(t - 0.3125)^2}$

$0.7316 = t - 0.3125$

4. $t \approx 1.0441$

The ball will hit the ground at approximately 1.044 seconds.

$h(t) = -16t^2 + 10t + 7$

$11 = -16t^2 + 10t + 7$

$0 = -16t^2 + 10t + 7 - 11$

$0 = -16t^2 + 10t - 4$

$a = -16$

$b = 10$

$c = -4$

$D = b^2 - 4ac$

$D = 10^2 - 4(-16)(-4)$

$D = 100 - 256$

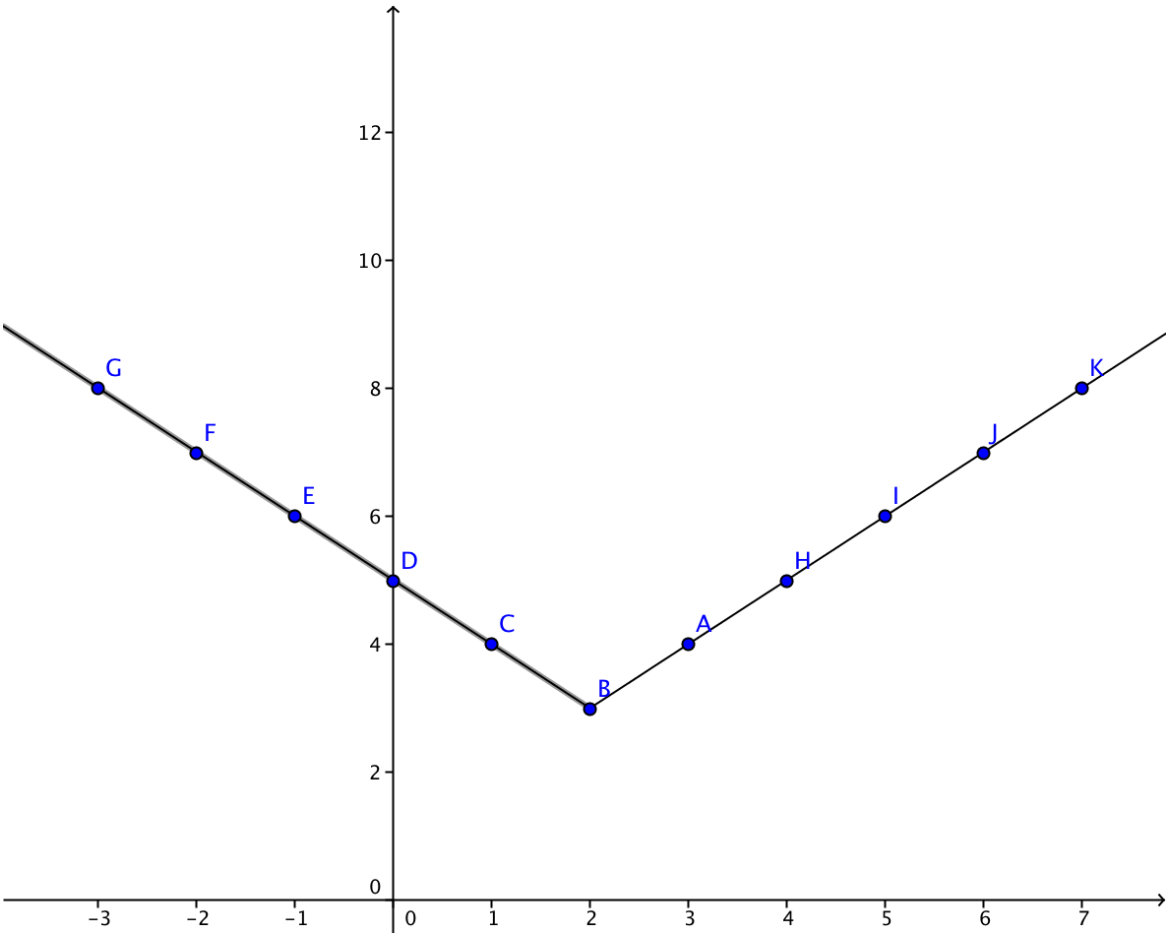
5. $D = -156$

The ball will NOT reach 11 feet because the discriminant is a negative number, meaning that this equation has no solutions.

22.

$y = |x - 2| + 3$

x	|x-2|+3	y
3	|3-2|+3=1+3=4	4
2	|2-2|+3=0+3=3	3
1	|1-2|+3=1+3=4	4
0	|0-2|+3=2+3=5	5
-1	|-1-2|+3=3+3=6	6
-2	|-2-2|+3=4+3=7	7
-3	|-3-2|+3=5+3=8	8



Domain= all Real numbers

Range= all Real numbers >3

23.

$6 \geq -5(c+4)+10$

$6 \geq -5c-20+10$

$6 \geq -5c-10$

$6+10 \geq -5c$

$16 \geq -5c$

$-3.2 \geq c$

$c \leq -3.2$

24.

Yes this relation is a function. Its domain is $2 \geq x \geq -6$. Its range is $5 \geq y \geq -3$.

25.

Graphing Quadratic Equations: by graphing a quadratic function we can determine the vertex, x-intercepts (solutions, roots) and y-intercept

Solving Quadratic Equations Using Square Roots: when a function is in the form $0=ax^2-c$ then you can solve by taking moving the c to the other side of the equation and taking the square roots of both sides

Completing the Square: any quadratic equation can be solved by completing the square. You must create a perfect square trinomial which looks like $a^2+2ab+b^2=(a+b)^2$. You do this by eliminating any leading coefficient through division so that a = x and then dividing b by 1/2 and adding the square of b to the entire equation. (Remember whatever you do to one side of the equation you have to do to the other side.)

Solving Using the Quadratic Formula: for any quadratic equation in standard form, $y=ax^2+bx+c$, you solve by plugging a, b, and c into the quadratic formula.

$$y=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Using the Discriminant: the Discriminant (D) is the value under the radical sign in the quadratic formula ($D=b^2-4ac$) If D is negative, then your function has no solutions. If D is 0 then your function has only one solution. If D is positive, then your function has 2 solutions. If D is a perfect square, then your solutions will be rational. If D is not a perfect square, then your solutions will be irrational.

Lesson 10.8
Problem Solving Strategies: Choose a Function Model

1.

1. An exponential function seems to be the best model for this situation.

2.

$y = mx + b$

$m \approx -2.736$

$b \approx 43.3989$

$y = -2.736x + 43.3989$

$y = -2.736x + 43.3989$

$y = -2.736(4.2) + 43.3989$

$y = 31.9077$

3.

$y = ax^2 + bx + c$

$a \approx 0.0748$

$b \approx -3.8682$

$c \approx 49.9791$

$y = 0.0748x^2 - 3.8682x + 49.9791$

$y = 0.0748x^2 - 3.8682x + 49.9791$

$y = 0.0748(4.2)^2 - 3.8682(4.2) + 49.9791$

$y = 35.0522$

4.

$y = ax^3 + bx^2 + cx + d$

$a \approx -4.6 \times 10^{-14}$

$b \approx 0.0748$

$c \approx -3.8682$

$d \approx 49.9791$

$y = (-4.6 \times 10^{-14})x^3 + 0.0748x^2 - 3.8682x + 49.9791$

$y = (-4.6 \times 10^{-14})x^3 + 0.0748x^2 - 3.8682x + 49.9791$

$y = (-4.6 \times 10^{-14})(4.2)^3 + 0.0748(4.2)^2 - 3.8682(4.2) + 49.9791$

$y = 35.0522$

5. The quadratic or cubic regressions seem to be the models that best fit.

6.

$y = 0.0748x^2 - 3.8682x + 49.9791$

$y = 0.0748(5)^2 - 3.8682(5) + 49.9791$

$y = 32.5081$

The height of the water at 5 seconds is about 32.5081 cm.

7.

$y = 0.0748x^2 - 3.8682x + 49.9791$

$y = 0.0748(13)^2 - 3.8682(13) + 49.9791$

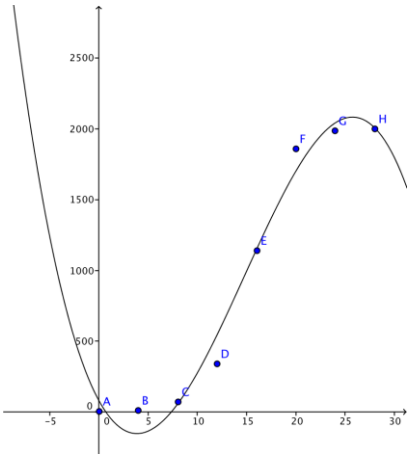
$y = 12.3337$

The height of the water at 13 seconds is about 12.3337 cm.

2.

1. Of the three options, quadratic seems the best fit.

2. $y = -0.4264x^3 + 18.9454x^2 - 127.2956x + 80.2576$



3. $n(g) = -0.4264g^3 + 18.9454g^2 - 127.2956g + 80.2576$

4.

$n(g) = -0.4264g^3 + 18.9454g^2 - 127.2956g + 80.2576$

$n(10) = -0.4264\left(10^3\right) + 18.9454\left(10^2\right) - 127.2956(10) + 80.2576$

$n(10) = -0.4264(1000) + 18.9454(100) - 1272.956 + 80.2576$

$n(10) = -426.4 + 1894.54 - 1272.956 + 80.2576$

$n(10) = 275.4416$

5.

$n(g) = -0.4264g^3 + 18.9454g^2 - 127.2956g + 80.2576$

$n(25) = -0.4264\left(25^3\right) + 18.9454\left(25^2\right) - 127.2956(25) + 80.2576$

$n(25) = -0.4264(15625) + 18.9454(625) - 3182.39 + 80.2576$

$n(25) = -6662.5 + 11840.875 - 3182.39 + 80.2576$

$n(25) = 2076.2426$

3.

$y = -4.92x^2 + 34.7x + 1.2$

$0 = -4.92x^2 + 34.7x + 1.2$

$-1.2 = -4.92x^2 + 34.7x$

$-1.2 = -4.92(x^2 - 7.0528x)$

$-1.2 + 12.4355(-4.92) = -4.92(x^2 - 7.0528x + 12.4355)$

$-1.2 - 61.1827 = -4.92(x - 3.5264)^2$

$-62.3827 = -4.92(x - 3.5264)^2$

$0 = -4.92(x - 3.5264)^2 + 62.3827$

$y = a(x - h)^2 + k$

$y = -4.92(x - 3.5264)^2 + 62.3827$

$(h, k) = (3.5264, 62.3827)$

The maximum height of the golf ball is approximately 62.3827 meters.

4.

$y = -4.92x^2 + 34.7x + 1.2$

$x = 5.2$

$y = -4.92(5.2)^2 + 34.7(5.2) + 1.2$

$y = -4.92(27.04) + 180.44 + 1.2$

$y = -133.0368 + 180.44 + 1.2$

$y = 48.6032$

At 5.2 seconds the height of the golf ball will be approximately 48.6032 meters.

Mixed Review

5.

$2 \div 6 * 5 + 3^2 - 11 * 9\frac{1}{2}$

$2 \div 6 * 5 + 9 - 11 * 9\frac{1}{2}$

$(2 \div 6) * 5 + -11 * 9\frac{1}{2}$

$(\frac{2}{6} * 5) + -11 * 9\frac{1}{2}$

$\frac{10}{6} + (-11 * 9\frac{1}{2})$

$\frac{10}{6} + \frac{209}{2} = \frac{10}{6} + \frac{627}{6} = \frac{637}{6} = 106\frac{1}{6}$

6.

$(60,812)(115,1126)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{1126 - 812}{115 - 60} = \frac{314}{55} \approx 5.7091$$

$$y - y_1 = m(x - x_1)$$

$$y - 812 = 5.7091(x - 60)$$

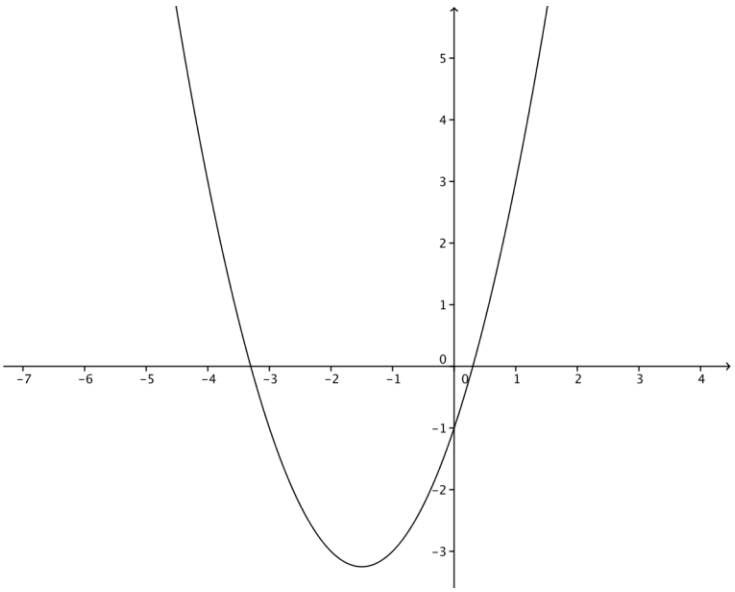
$$y - 812 = 5.7091x - 342.546$$

$$y = 5.7091x - 342.546 + 812$$

$$y = 5.7091x + 469.454$$

1. start up cost = $469.45
2. slope = 5.0791 It represents the cost per shirt ($5.08)

7.



$$y = x^2 + 3x - 1$$

$$x = 0.3, -3.3$$

8.

$$\frac{\frac{6}{7}}{\frac{1}{2}} = \frac{6}{7} \div \frac{1}{2} = \frac{6}{7} \times \frac{2}{1} = \frac{12}{7}$$

9.

$$F = m \cdot a$$

$$\frac{F}{a} = m$$

$$m = \frac{F}{a}$$

$$m = \frac{300}{70} = \frac{30}{7} \approx 4.2857$$

$$m \approx 4.2857$$

10.

$$A = s^2$$

$$256 = s^2$$

$$\sqrt{256} = \sqrt{s^2}$$

$$16 = s$$

The length of one side equals 16 inches,

11. $\frac{3}{1000} = 0.003$

Lesson 10.9
Chapter 10 Review

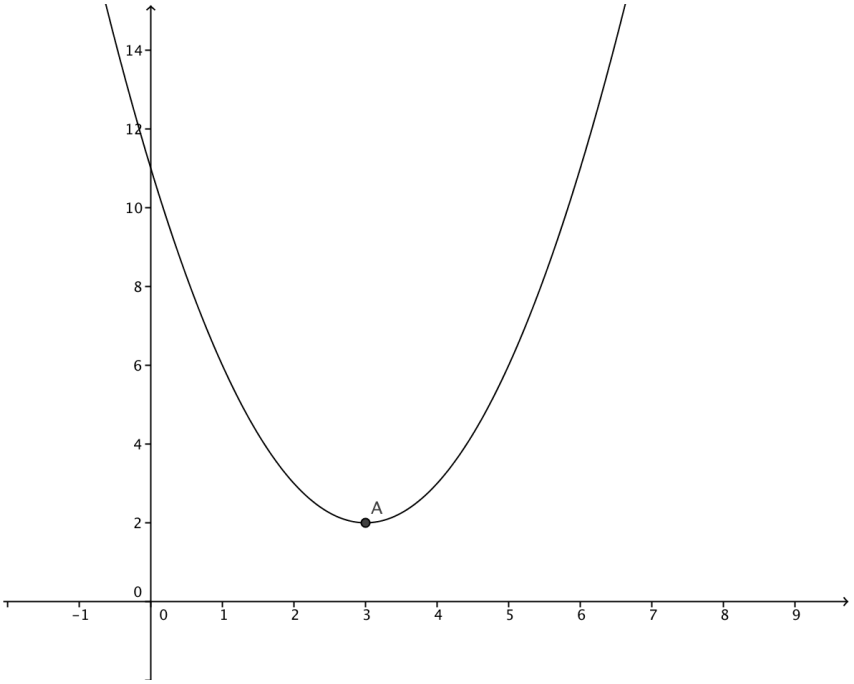
1. Vertex: The vertex is the point which represents the maximum or minimum value of a parabola. The line of symmetry goes through the vertex and the parabola rises or falls symmetrically on both sides of the vertex.

2. Standard form for a quadratic equation: $y = ax^2 + bx + c$

3. Model: the function that best fits a set of data

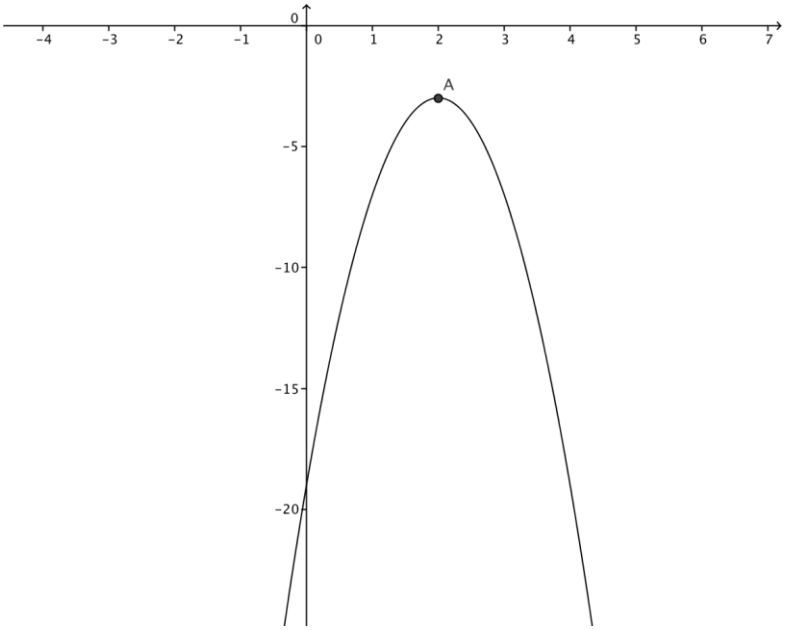
4. Discriminant: the value under the radical sign in the quadratic formula, $D = b^2 - 4ac$. The discriminant tells us how many solutions the function will have and whether the solutions will be rational or irrational.

5. $y = x^2 - 6x + 11$



Vertex= (3,2)
Range: $y \geq 2$

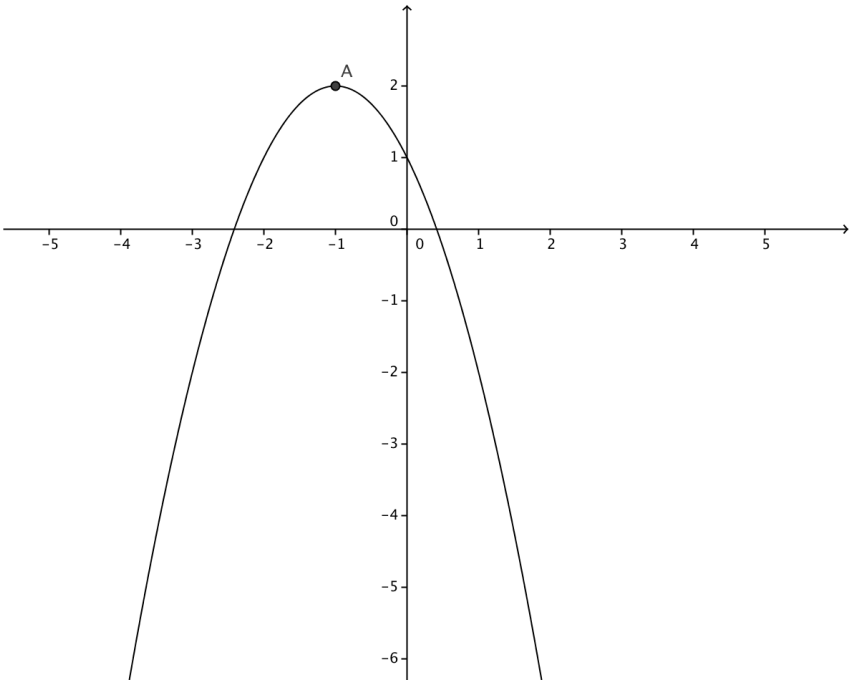
6. $y = -4x^2 + 16x - 19$



Vertex = (2, -3)

Range: $y \leq -3$

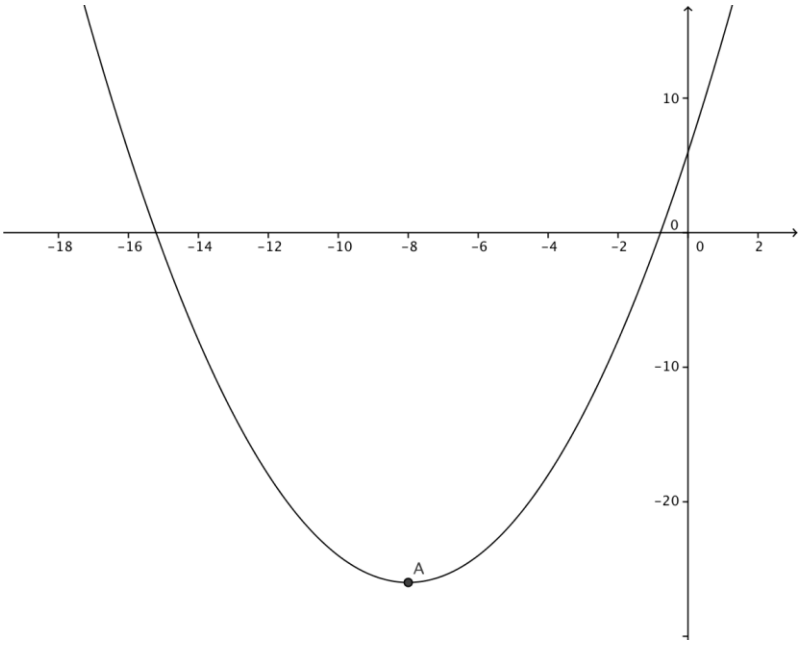
7. $y = -x^2 - 2x + 1$



Vertex= (-1,2)

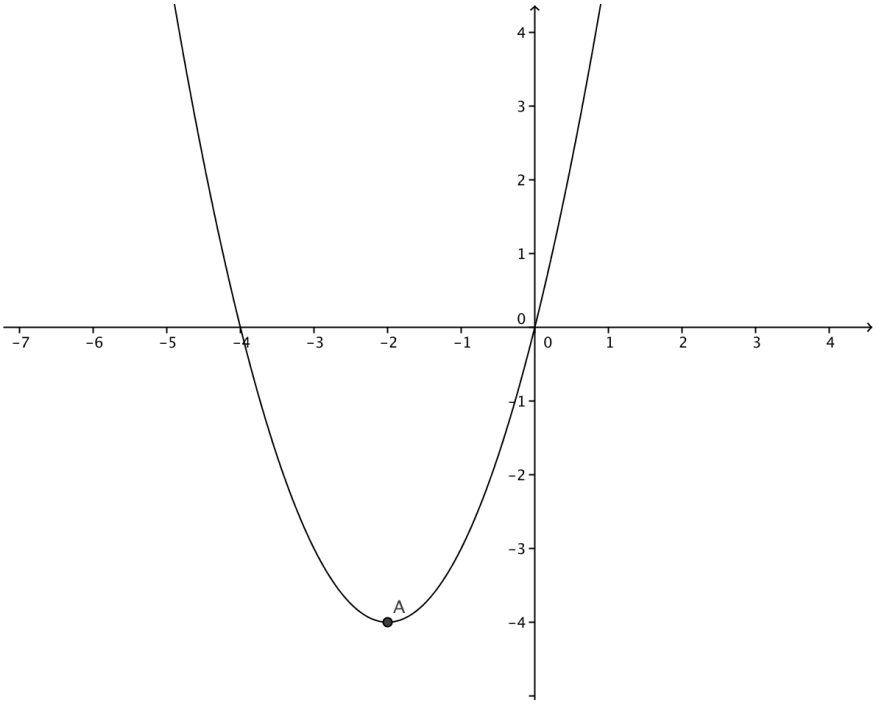
Range: $y \leq 2$

8. $y = \frac{1}{2}x^2 + 8x + 6$



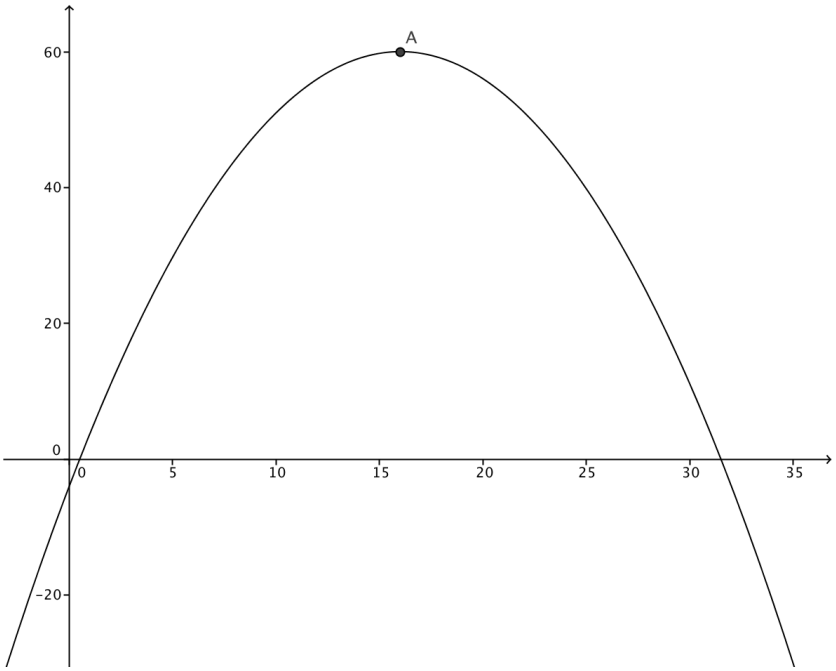
Vertex= (-8,-26)
Range: $y \geq -26$

9. $y = x^2 + 4x$



Vertex= (-2,-4)
Range: $y \geq -4$

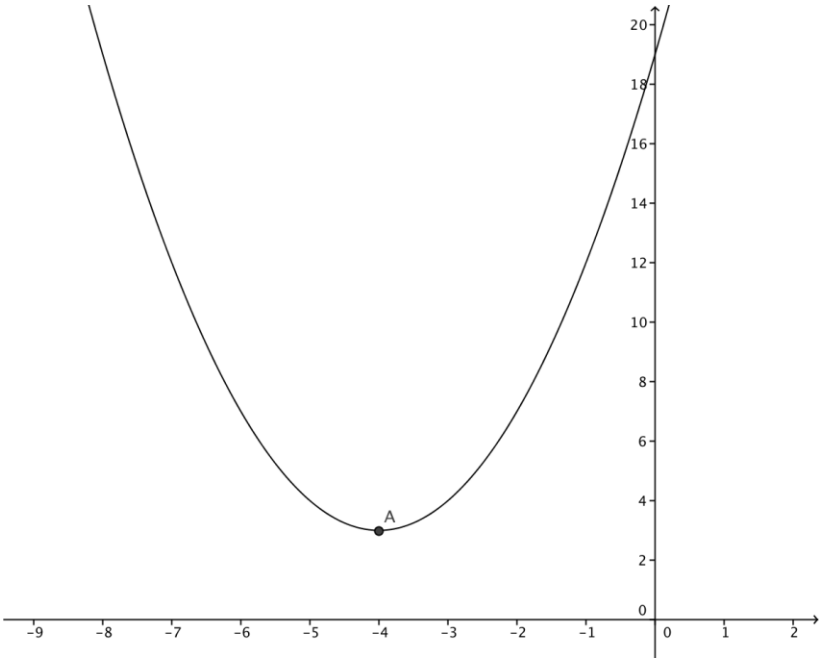
10. $y = -\frac{1}{4}x^2 + 8x - 4$



Vertex= (16,60)

Range: $y \leq 60$

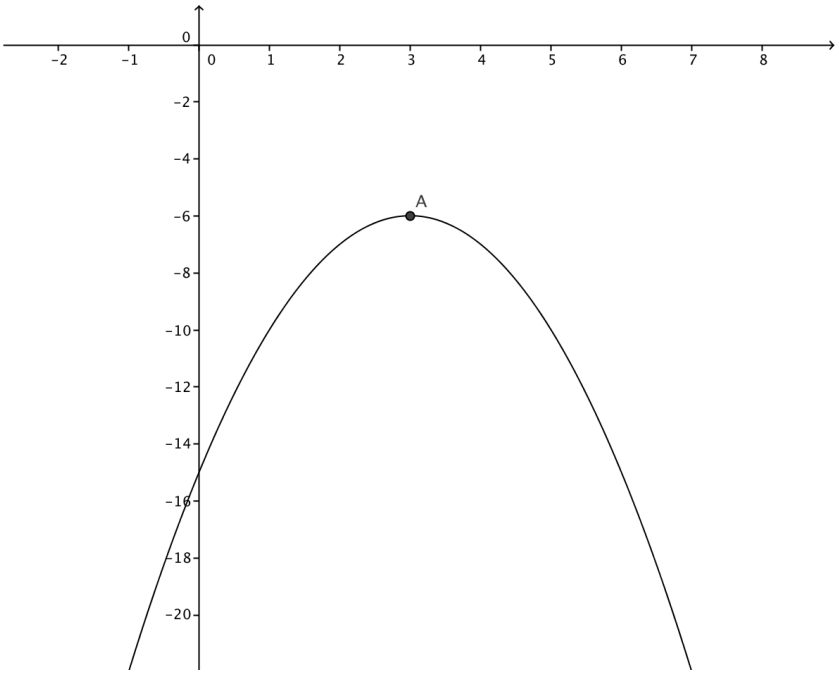
11. $y = (x+4)^2 + 3$



Vertex= (-4,3)

Range: $y \geq 3$

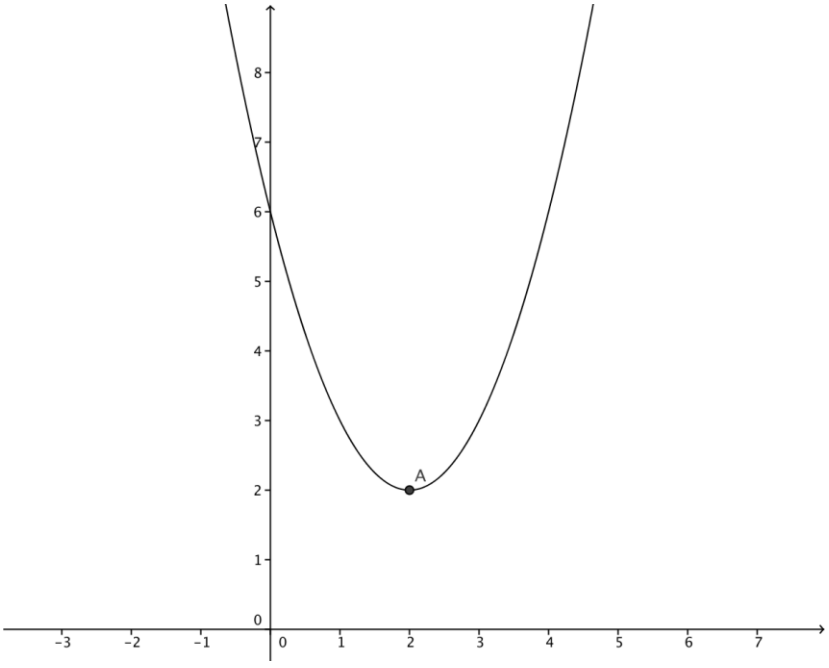
12. $y = -(x-3)^2 - 6$



Vertex= (3,-6)

Range: $y \leq -6$

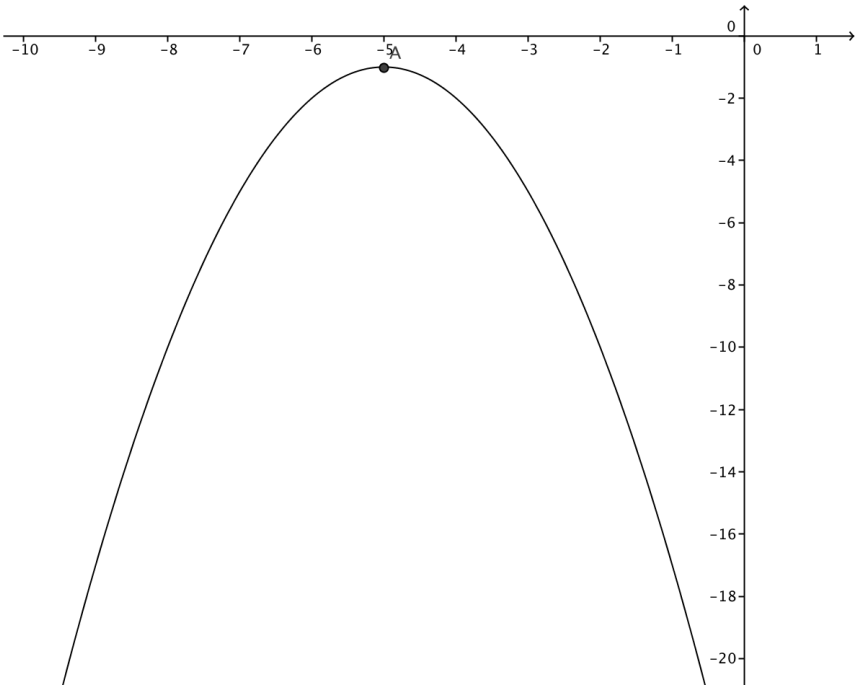
13. $y = (x-2)^2 + 2$



Vertex= (2,2)

Range: $y \geq 2$

14. $y=-(x+5)^2-1$



Vertex= (-5,-1)

Range: $y \le -1$

15.

$x-24=-5x^2$

$x-24+5x^2=0$

$5x^2+x-24=0$

$y=5x^2+x-24$

16.

$5+4a=a^2$

$0=a^2-5-4a$

$y=a^2-4a-5$

17.

$-6-18a^2=-528$

$-18a^2=-528+6$

$-18a^2=-522$

$0=18a^2-522$

$y=18a^2-522$

18.

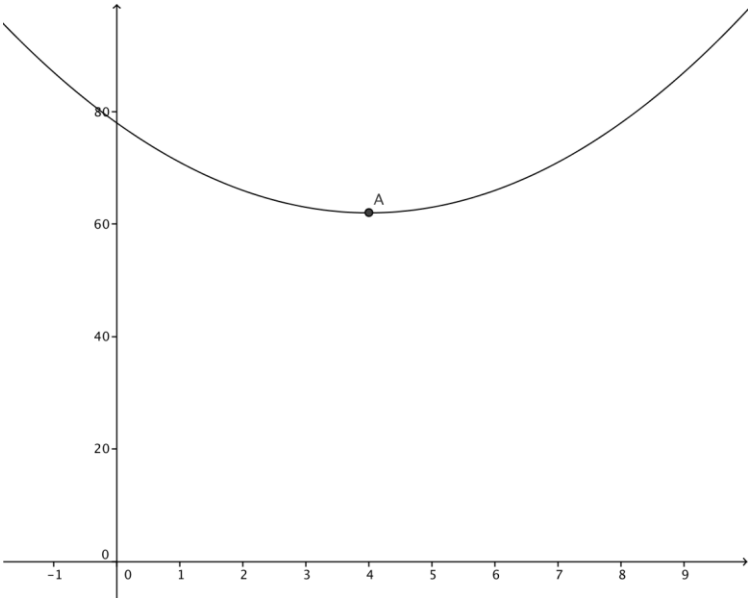
$y=-(x+4)^2+2$

$y=-(x^2+4x+4x+16)+2$

$y=-x^2-8x-16+2$

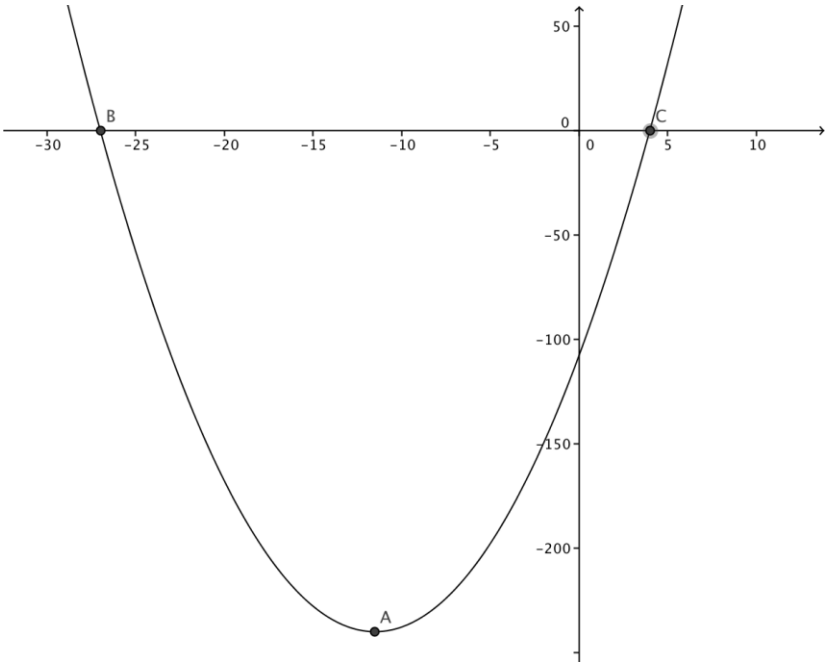
$y=-x^2-8x-14$

19. $x^2-8x+87=9$



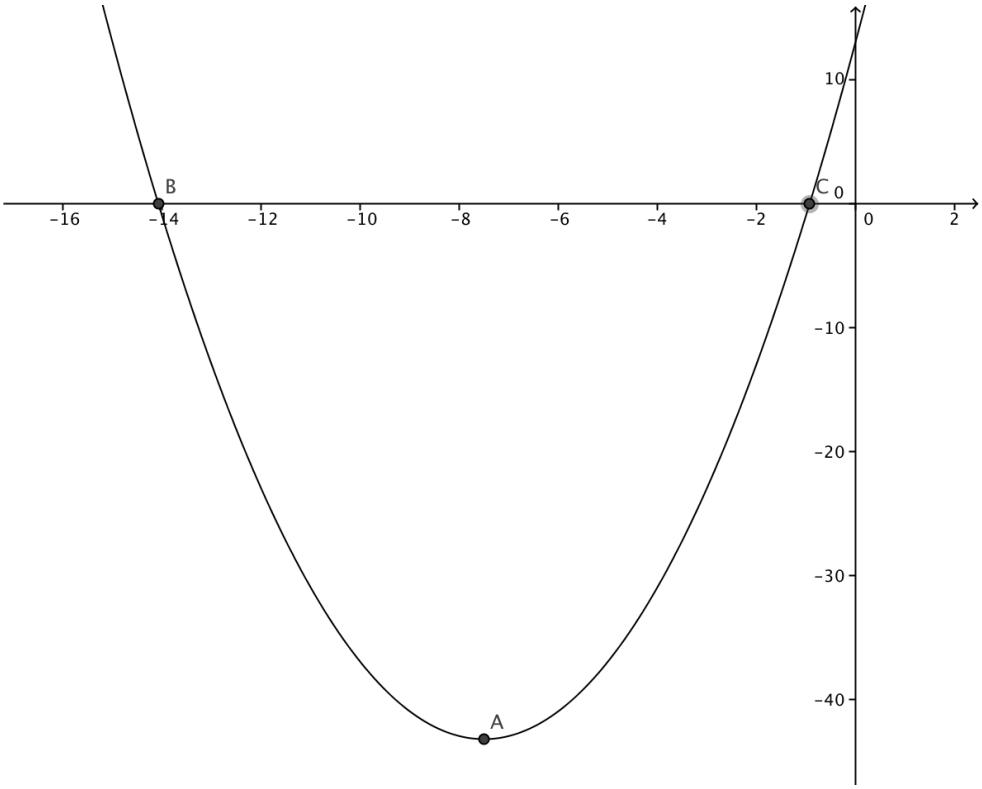
There are no solutions.

20. $23x + x^2 - 104 = 4$



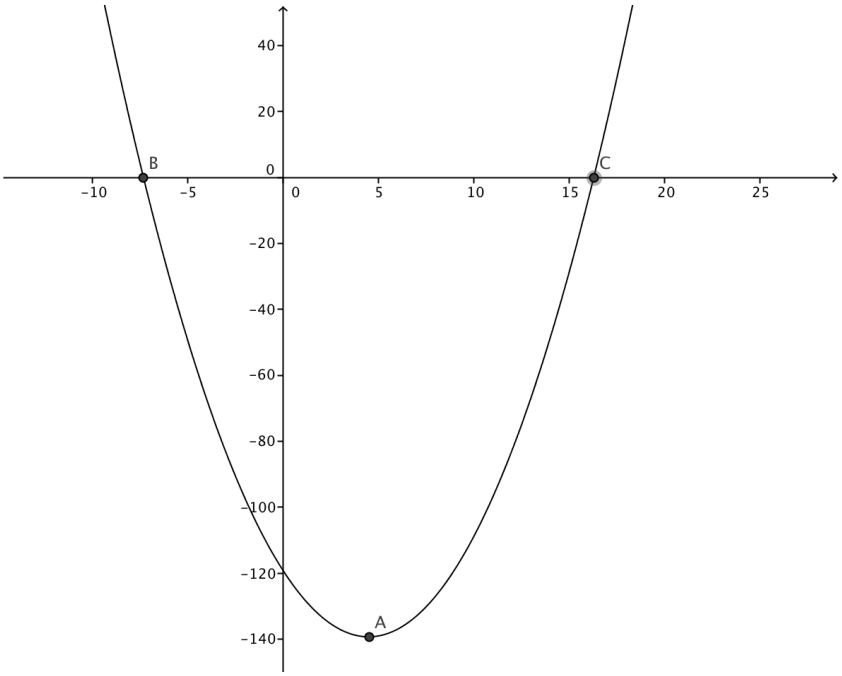
The solutions are x=4 and x=-27

21. $13 + 26x = -x^2 + 11x$



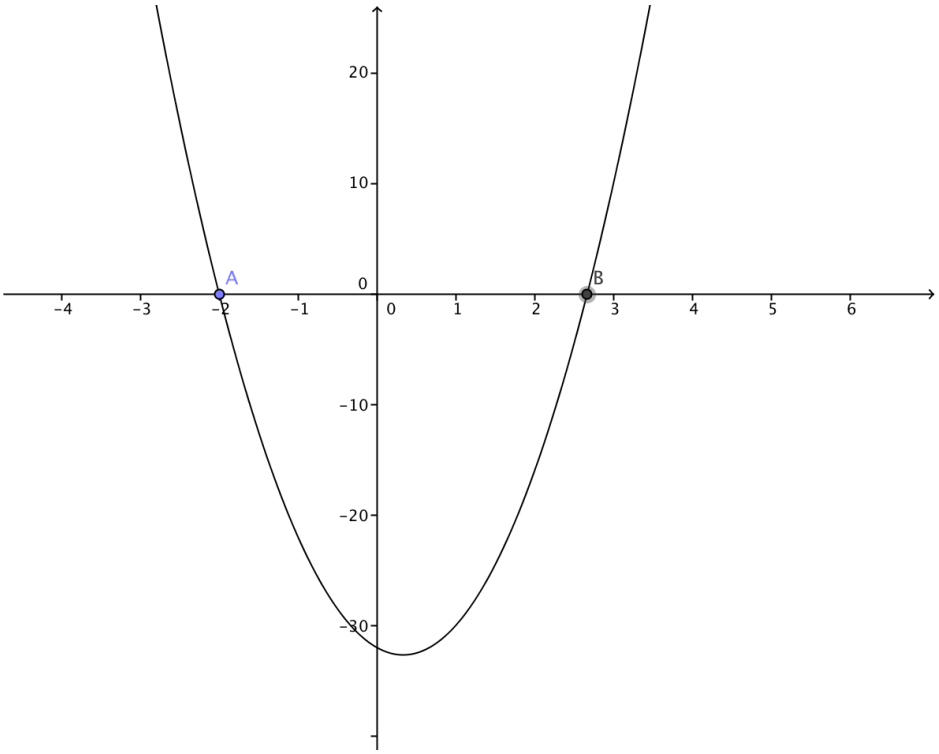
The solutions are x=-14.08 and x=-0.92

22. $x^2 - 9x = 119$



The solutions are x=-7.3 and x=16.3

23. $-32 + 6x^2 - 4x = 0$



The solutions are x=-2 and x=2.67

24.

$x^2 = 225$

$\sqrt{x^2} = \sqrt{225}$

$x = \pm 15$

25.

$x^2 - 2 = 79$

$x^2 = 81$

$\sqrt{x^2} = \sqrt{81}$

$x = \pm 9$

26.

$x^2 + 100 = 200$

$x^2 = 100$

$\sqrt{x^2} = \sqrt{100}$

$x = \pm 10$

27.

$8x^2 - 2 = 262$

$8x^2 = 264$

$x^2 = 33$

$\sqrt{x^2} = \sqrt{33}$

$x = \sqrt{33}$

$x \approx \pm 5.7446$

28.

$-6 - 4x^2 = -65$

$-4x^2 = -59$

$x^2 = 14.75$

$\sqrt{x^2} = \sqrt{14.75}$

$x = \sqrt{14.75}$

$x \approx \pm 3.8406$

29.

$703 = 7x^2 + 3$

$700 = 7x^2$

$100 = x^2$

$\sqrt{100} = \sqrt{x^2}$

$x = \pm 10$

30.

$10 + 6x^2 = 184$

$6x^2 = 174$

$x^2 = 29$

$\sqrt{x^2} = \sqrt{29}$

$x = \sqrt{29}$

$x \approx \pm 5.3852$

31.

$2 + 6x^2 = 152$

$6x^2 = 150$

$x^2 = 25$

$\sqrt{x^2} = \sqrt{25}$

$x = \pm 5$

32.

$n^2 - 4n - 3 = 9$

$n^2 - 4n = 12$

$n^2 - 4n + 4 = 12 + 4$

$(n-2)^2 = 16$

$\sqrt{(n-2)^2} = \sqrt{16}$

$n - 2 = \pm 4$

$n - 2 = 4$

$n = 6$

$n - 2 = -4$

$n = -2$

$n = -2, 6$

33.

$h^2+10h+1=3$
$h^2+10h=2$
$h^2+10h+25=2+25$
$(h+5)^2=27$
$\sqrt{(h+5)^2}=\sqrt{27}$
$h+5=\sqrt{27}$
$h=\sqrt{27}-5$
$h=\sqrt{27}-5$
$h\approx\pm5.1962-5$
$h\approx5.1962-5$
$h\approx0.1962$
$h\approx-5.1962-5$
$h\approx-10.1962$
$h\approx0.1962,-10.1962$

34.

$x^2+14x-22=10$
$x^2+14x=32$
$x^2+14x+49=32+49$
$(x+7)^2=81$
$\sqrt{(x+7)^2}=\sqrt{81}$
$x+7=\pm9$
$x+7=\pm9$
$x+7=9$
$x=2$
$x+7=-9$
$x=-16$
$x=2,-16$

35.

$t^2-10t=-9$
$t^2-10t+25=-9+25$
$(t-5)^2=16$
$\sqrt{(t-5)^2}=\sqrt{16}$
$t-5=\pm4$
$t-5=\pm4$
$t-5=4$
$t=9$
$t-5=-4$
$t=1$
$t=1,9$

36.

$x^2 - 20x + 28 = -8$

$x^2 - 20x = -36$

$x^2 - 20x + 100 = -36 + 100$

$(x-10)^2 = 64$

$(x-10)^2 - 64 = 0$

$y = (x-10)^2 - 64$

$y = a(x-h)^2 + k$

$(h,k) = (10,-64)$

The minimum point of the function is (10,-64)

37.

$a^2 + 2a - 63 = -5$

$a^2 + 2a = 58$

$a^2 + 2a + 1 = 58 + 1$

$(a+1)^2 = 59$

$(a+1)^2 - 59 = 0$

$y = (a+1)^2 - 59$

$y = a(x-h)^2 + k$

$(h,k) = (-1,-59)$

The minimum point of the function is (-1,-59)

38.

$x^2 + 6x - 33 = 4$

$x^2 + 6x = 37$

$x^2 + 6x + 9 = 37 + 9$

$(x+3)^2 = 46$

$(x+3)^2 - 46 = 0$

$y = (x+3)^2 - 46$

$y = a(x-h)^2 + k$

$(h,k) = (-3,-46)$

The minimum point of the function is (-3,-46)

39.

$4x^2 - 3x = 45$

$4x^2 - 3x - 45 = 0$

$y = 4x^2 - 3x - 45$

$y = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(-45)}}{2(4)}$

$y = \dfrac{3 \pm \sqrt{9 + 720}}{8}$

$y = \dfrac{3 \pm \sqrt{729}}{8}$

$y = \dfrac{3 + \sqrt{729}}{8} = \dfrac{3 + 27}{8} = \dfrac{30}{8} = 3.75$

$y = \dfrac{3 - \sqrt{729}}{8} = \dfrac{3 - 27}{8} = \dfrac{-24}{8} = -3$

$y = 3.75, -3$

40.

$-5x + 11x^2 = 15$

$11x^2 - 5x - 15 = 0$

$y = 11x^2 - 5x - 15$

$y = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(11)(-15)}}{2(11)}$

$y = \dfrac{5 \pm \sqrt{25 + 660}}{22}$

$y = \dfrac{5 \pm \sqrt{685}}{22}$

$y = \dfrac{5 + \sqrt{685}}{22} \approx \dfrac{5 + 26.1725}{22} \approx \dfrac{31.1725}{22} \approx 1.4169$

$y = \dfrac{5 - \sqrt{685}}{22} \approx \dfrac{5 - 26.1725}{22} \approx \dfrac{-21.1725}{22} \approx -0.9624$

$y \approx 1.4169, -0.9624$

41.

$$-3r = 12r^2 - 3$$

$$0 = 12r^2 - 3 + 3r$$

$$y = 12r^2 + 3r - 3$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-3 \pm \sqrt{(3)^2 - 4(12)(-3)}}{2(12)}$$

$$y = \frac{-3 \pm \sqrt{9 + 144}}{24}$$

$$y = \frac{-3 \pm \sqrt{153}}{24}$$

$$y = \frac{-3 + \sqrt{153}}{24} \approx \frac{-3 + 12.3693}{24} \approx \frac{9.3693}{24} \approx 0.3904$$

$$y = \frac{-3 - \sqrt{153}}{24} \approx \frac{-3 - 12.3693}{24} \approx \frac{-15.3693}{24} \approx -0.6404$$

$$y \approx 0.3904, -0.6404$$

42.

$$2m^2 + 10m = 8$$

$$2m^2 + 10m - 8 = 0$$

$$y = 2m^2 + 10m - 8$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-10 \pm \sqrt{10^2 - 4(2)(-8)}}{2(2)}$$

$$y = \frac{-10 \pm \sqrt{100 + 64}}{4}$$

$$y = \frac{-10 \pm \sqrt{164}}{4}$$

$$y = \frac{-10 + \sqrt{164}}{4} \approx \frac{-10 + 12.8062}{4} \approx \frac{2.8062}{4} \approx 0.7016$$

$$y = \frac{-10 - \sqrt{164}}{4} \approx \frac{-10 - 12.8062}{4} \approx \frac{-22.8062}{4} \approx -5.7016$$

$$y \approx 0.7016, -5.7016$$

43.

$7c^2 + 14c - 28 = -7$

$7c^2 + 14c - 21 = 0$

$y = 7c^2 + 14c - 21$

$y = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = \dfrac{-14 \pm \sqrt{14^2 - 4(7)(-21)}}{2(7)}$

$y = \dfrac{-14 \pm \sqrt{196 + 588}}{14}$

$y = \dfrac{-14 \pm \sqrt{784}}{14}$

$y = \dfrac{-14 + \sqrt{784}}{14} = \dfrac{-14 + 28}{14} = \dfrac{14}{14} = 1$

$y = \dfrac{-14 - \sqrt{784}}{14} = \dfrac{-14 - 28}{14} = \dfrac{-42}{14} = -3$

$y = 1, -3$

44.

$3w^2 - 15 = -3w$

$3w^2 - 15 + 3w = 0$

$y = 3w^2 + 3w - 15$

$y = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = \dfrac{-3 \pm \sqrt{3^2 - 4(3)(-15)}}{2(3)}$

$y = \dfrac{-3 \pm \sqrt{9 + 180}}{6}$

$y = \dfrac{-3 \pm \sqrt{189}}{6}$

$y = \dfrac{-3 + \sqrt{189}}{6} \approx \dfrac{-3 + 13.7477}{6} \approx \dfrac{10.7477}{6} \approx 1.7913$

$y = \dfrac{-3 - \sqrt{189}}{6} \approx \dfrac{-3 - 13.7477}{6} \approx \dfrac{-16.7477}{6} \approx -2.7913$

$y \approx 1.7913, -2.7913$

45.

$4x^2 - 4x + 1 = 0$

$D = b^2 - 4ac$

$D = (-4)^2 - 4(4)(1)$

$D = 16 - 16$

$D = 0$

The equation will have only 1 solution and it will be rational.

46.

$2x^2 - x - 3 = 0$

$D = b^2 - 4ac$

$D = (-1)^2 - 4(2)(-3)$

$D = 1 + 24$

$D = 25$

The equation will have 2 solutions and they will be rational.

47.

$-2x^2 - x - 1 = -2$

$-2x^2 - x + 1 = 0$

$D = b^2 - 4ac$

$D = (-1)^2 - 4(-2)(1)$

$D = 1 + 8$

$D = 9$

The equation will have 2 solutions and they will be rational.

48.

$4x^2 - 8x + 4 = 0$

$D = b^2 - 4ac$

$D = (-8)^2 - 4(4)(4)$

$D = 64 - 64$

$D = 0$

The equation will have only 1 solution and it will be rational.

49.

$-5x^2+10x-5=0$

$D=b^2-4ac$

$D=(10)^2-4(-5)(-5)$

$D=100-100$

$D=0$

The equation will have only 1 solution and it will be rational.

50.

$4x^2+3x+6=0$

$D=b^2-4ac$

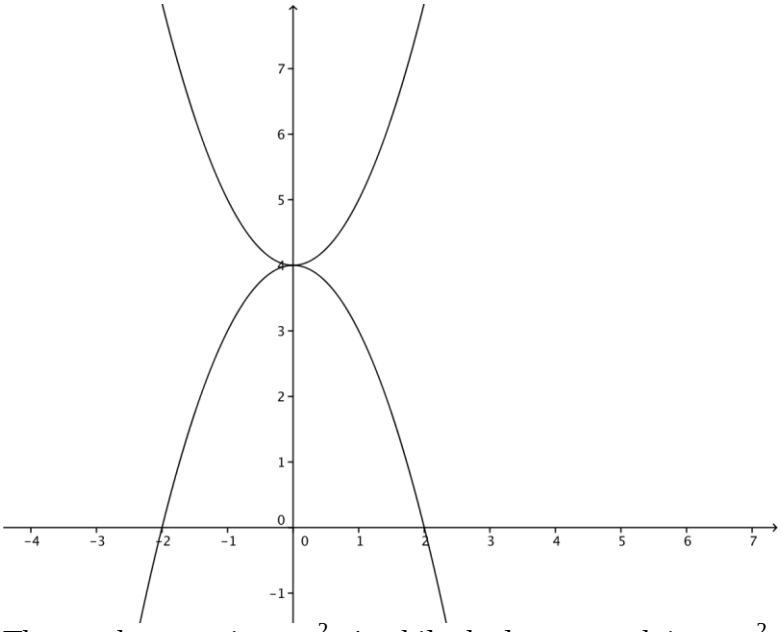
$D=(3)^2-4(4)(6)$

$D=9-96$

$D=-87$

The equation will have no real solutions.

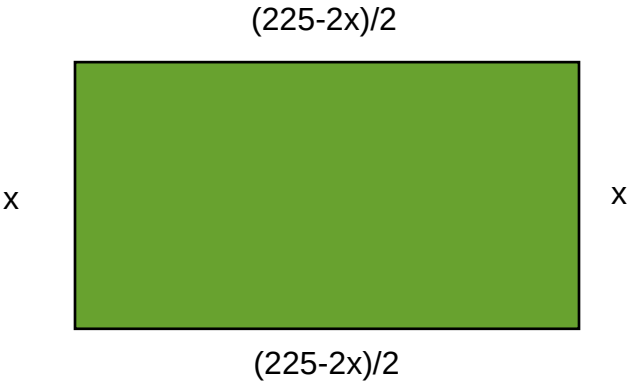
51.



The graph on top is $y= x^2+4$, while the lower graph is $y=-x^2+4$

While they have the same vertex and y-intercept, the first parabola opens upward with its vertex as a minimum and the second parabola opens downward with its vertex as a maximum. As a result of their opposite orientations the first equation will have no solutions but the second equation will have two.

52.



$$A = x * \frac{(225 - 2x)}{2}$$
$$A = \tfrac{1}{2} * x(225 - 2x)$$
$$A = \tfrac{1}{2} * 225x - 2x^2$$
$$A = 112.5x - x^2$$
$$0 = -x^2 + 112.5x$$
$$0 = -1(x^2 - 112.5x)$$
$$0 + 3164.0625(-1) = -1(x^2 - 112.5x + 3164.0625)$$
$$-3164.0625 = -1(x - 56.25)^2$$
$$3164.0625 = (x - 56.25)^2$$
$$\sqrt{3164.0625} = \sqrt{(x - 56.25)^2}$$
$$56.25 = x - 56.25$$
$$112.5 = x$$

The dimensions of the garden are 112.5 ft by 112.5 ft

53.

$h(t) = -\frac{1}{2}(g)t^2 + v_0 t + h_0$

$h(t) = -\frac{1}{2}(9.8)t^2 + 0t + 70$

1. $h(t) = -4.9t^2 + 70$

2. The leading coefficient is -4.9 It tells us that the parabola will open downward (because it is negative) and will be narrow about its line of symmetry (because it is less than -1).

$h(t) = -4.9t^2 + 70$

$0 = -4.9t^2 + 0t + 70$

$-70 = -4.9t^2 + 0t$

$-70 = -4.9(t^2 - 0t)$

$-70 + 0 = -4.9(t^2 - 0t + 0)$

$-70 = -4.9(t - 0)^2$

$0 = -4.9(t - 0)^2 + 70$

$y = a(x - h) + k$

3. $(h, k) = (0, 70)$

The maximum height of the ball is 70 meters.

$h(t) = -4.9t^2 + 70$

$h(t) = -4.9(0.65)^2 + 70$

$h(t) = -4.9(.4225) + 70$

$h(t) = -2.07025 + 70$

4. $h(t) = 67.92975$

After 0.65 seconds the ball will be 67.92975 meters from the ground.

$h(t) = -4.9t^2 + 70$

$0 = -4.9t^2 + 70$

$-70 = -4.9t^2$

$4.9t^2 = 70$

$t^2 = 14.2857$

$\sqrt{t^2} = \sqrt{14.2857}$

5. $t = 3.7796$

The ball will reach the ground at approximately 3.7796 seconds.

54.

1. The best function for this data seems to be a linear regression.

2.

$y = mx + b$

$m = 9.8286$

$b = -914.9524$

$y = 9.8286x - 914.9524$

The function of best fit is $y = 9.8286x - 914.9524$

3.

$y = 9.8286x - 914.9524$

$x = 2012 = 112$

$y = 9.8286(112) - 914.9524$

$y = 1100.8032 - 914.9524$

$y = 185.8508$

Based on the equation, in 2012 a person would be playing approximately 186 hours of video games.

4. Yes this seems like a viable answer, both because the math fits with the data we already have, (a linear increasing trend) and because of the reality of the greater availability and popularity of video games in recent years. One would expect an substantial increase.

55.

1.

$y = mx + b$

$m = 1.5617$

$b = 15.22$

$y = 1.5617x + 15.22$

$x = 2008 = 18$

$y = 1.5617(18) + 15.22$

$y = 28.1106 + 15.22$

$y = 43.3306$

Based on this model the amount spent in 2008 would be approximately 43.3 billion.

2.

$y = ax^2 + bx + c$

$a = 0.0919$

$b = 0.8266$

$c = 16.0776$

$y = 0.0919x^2 + 0.8266x + 16.0776$

$x = 2008 = 18$

$y = 0.0919(18)^2 + 0.8266(18) + 16.0776$

$y = 0.0919(324) + 14.8788 + 16.0776$

$y = 29.7756 + 14.8788 + 16.0776$

$y = 60.732$

Based on this model the amount spent in 2008 would be approximately 60.7 billion.

3. The linear model seems more accurate.

$y = 1.5617x + 15.22$

$x = 2012 = 22$

$y = 1.5617(22) + 15.22$

$y = 34.3574 + 15.22$

$y = 49.5774$

Based on this model the amount spent in 2012 would be approximately 49.6 billion.

4. The amount of money spent in 2012 could be changed by any number of factors. More or less books printed. The price of books going up or down. The rate of books being bought changes dramatically making the equation an inaccurate predictor.

56.

1.

$y = ax^2 + bx + c$

$a = -1.6607$

$b = 6549.6902$

$c = -6450876.9468$

$y = -1.6607x^2 + 6549.6902x - 6,450,876.9468$

2.

$y = -1.6607x^2 + 6549.6902x - 6,450,876.9468$

$0 = -1.6607x^2 + 6549.6902x - 6,450,876.9468$

$6,450,876.9468 = -1.6607x^2 + 6549.6902x$

$6,450,876.9468 = -1.6607(x^2 - 3943.9334x)$

$6,450,876.9468 + 3888652.6659(-1.6607) = -1.6607(x^2 - 3943.9334x + 3888652.6659)$

$-7008.5355 = -1.6607(x - 1971.9667)$

$0 == -1.6607(x - 1971.9667) + 7008.5355$

$y = a(x-h)^2 + k$

$(h,k) = (1972, 7009)$

BASED ON THE MODEL the maximum number of hospitals is 7009.

3. BASED ON THE MODEL the maximum occurs in 1972.

4.

$y = -1.6607x^2 + 6549.6902x - 6,450,876.9468$

$7000 = -1.6607x^2 + 6549.6902x - 6,450,876.9468$

$0 = -1.6607x^2 + 6549.6902x - 6,450,876.9468 - 7000$

$0 = -1.6607x^2 + 6549.6902x - 6457876.9468$

$y = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = \dfrac{-6549.6902 \pm \sqrt{6549.6902^2 - 4(-1.6607)(-6457876.9468)}}{2(-1.6607)}$

$y = \dfrac{-6549.6902 \pm \sqrt{42898441.716 - 42898384.9822}}{-3.3214}$

$y = \dfrac{-6549.6902 \pm \sqrt{56.7338}}{-3.3214}$

$y = \dfrac{-6549.6902 + \sqrt{56.7338}}{-3.3214} \approx \dfrac{-6549.6902 + 7.5322}{-3.3214} \approx \dfrac{-6542.158}{-3.3214} \approx 1969.6989$

$y = \dfrac{-6549.6902 - \sqrt{56.7338}}{-3.3214} \approx \dfrac{-6549.6902 - 7.5322}{-3.3214} \approx \dfrac{-6557.2224}{-3.3214} \approx 1974.2344$

$y \approx 1970, 1974$

5. The trend appears to be that the # of hospitals was increasing before the 1980's and has been decreasing since.

57.

1. The best model for the data seems to be an exponential function.

2.

$y = ax^2 + bx + c$

$a = 0.8764$

$b = -10.4074$

$c = 34.1418$

$y = 0.8764x^2 - 10.4074x + 34.1418$

$x = 7$

$y = 0.8764(7^2) - 10.4074(7) + 34.1418$

$y = 42.9436 - 72.8518 + 34.1418$

$y = 4.2336$

Based on this model, the length of the seventh swing is approximately 4.2336

3.

$y = a(b)^x$

$a = 38.4606$

$b = 0.65$

$y = 38.4606(0.65)^x$

$x = 7$

$y = 38.4606(0.65)^7$

$y = 38.4606(0.049)$

$y = 1.8846$

Based on this model, the length of the seventh swing is approximately 1.8846

Lesson 10.10
Chapter 10 Test

1. False, the vertex determines the range of the quadratic function.

2. When $a = -\frac{1}{3}$ the parabola opens downward and the vertex is a maximum (because a is negative). In addition we know that the parabola will be wide about its line of symmetry (because $-1 < a < 1$).

3.

$0 = -2x^2 + 3x - 2$

$a = -2$

$b = 3$

$c = -2$

$D = b^2 - 4ac$

$D = 3^2 - 4(-2)(-2)$

$D = 9 - 16$

$D = -7$

This equation has NO real solutions.

4.

1.

$h(t) = -\frac{1}{2}(g)t^2 + v_0 t + h_o$

$h(t) = -\frac{1}{2}(32)t^2 + 45t + 4$

$h(t) = -16t^2 + 45t + 4$

2.

$h(t) = -16t^2 + 45t + 4$

$0 = -16t^2 + 45t + 4$

$-4 = -16t^2 + 45t$

$-4 = -16(t^2 - 2.8125t)$

$-4 + 1.9775(-16) = -16(t^2 - 2.8125t + 1.9775)$

$-33.6625 = -16(t - 1.4063)^2$

$0 = -16(t - 1.4063)^2 + 33.6625$

$y = -16(t - 1.4063)^2 + 33.6625$

$y = a(x - h)^2 + k$

$(h, k) = (1.4063, 33.6625)$

The maximum height of the ball is 33.6525 feet.

3.

$h(t) = -16t^2 + 45t + 4$

$10 = -16t^2 + 45t + 4$

$0 = -16t^2 + 45t - 6$

$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = \frac{-45 \pm \sqrt{45^2 - 4(-16)(-6)}}{2(-16)}$

$y = \frac{-45 \pm \sqrt{2025 - 384}}{-32}$

$y = \frac{-45 \pm \sqrt{1641}}{-32}$

$y = \frac{-45 + \sqrt{1641}}{-32} \approx \frac{-45 + 40.5093}{-32} \approx \frac{-4.4907}{-32} \approx 0.1403$

$y = \frac{-45 - \sqrt{1641}}{-32} \approx \frac{-45 - 40.5093}{-32} \approx \frac{-85.5093}{-32} \approx 2.6722$

$y \approx 0.1403, 2.6722$

The ball will reach 10 ft at 0.1403 and 2.6722 seconds.

4. No the ball will not reach 36.7 ft because the maximum height of the ball is 33.6525 ft.

$$h(t) = -16t^2 + 45t + 4$$
$$0 = -16t^2 + 45t + 4$$
$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$y = \frac{-45 \pm \sqrt{45^2 - 4(-16)(4)}}{2(-16)}$$
$$y = \frac{-45 \pm \sqrt{2025 + 256}}{-32}$$
$$y = \frac{-45 \pm \sqrt{2281}}{-32}$$
$$y = \frac{-45 + \sqrt{2281}}{-32} \approx \frac{-45 + 47.7598}{-32} \approx \frac{50.5196}{-32} \approx -1.5787$$
$$y = \frac{-45 - \sqrt{2281}}{-32} \approx \frac{-45 - 47.7598}{-32} \approx \frac{-92.7598}{-32} \approx 2.8987$$

5. $y \approx -1.5787, 2.8987$

The ball will hit the ground at 2.8987

5.

$$2x^2 = 2x + 40$$
$$2x^2 - 2x - 40 = 0$$
$$2(x^2 - x - 20) = 0$$
$$2(x - 5)(x + 4) = 0$$

$$x - 5 = 0$$
$$x = 5$$
$$x + 4 = 0$$
$$x = -4$$
$$x = 5, -4$$

6.

$$11j^2 = j + 24$$

$$11j^2 - j - 24 = 0$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(11)(-24)}}{2(11)}$$

$$y = \frac{1 \pm \sqrt{1 + 1056}}{22}$$

$$y = \frac{1 \pm \sqrt{1057}}{22}$$

$$y = \frac{1 + \sqrt{1057}}{22} \approx \frac{1 + 32.5115}{22} \approx \frac{33.5115}{22} \approx 1.5233$$

$$y = \frac{1 - \sqrt{1057}}{22} \approx \frac{1 - 32.5115}{22} \approx \frac{-31.5115}{22} \approx -1.4323$$

$$y \approx 1.5233, -1.4323$$

7.

$$g^2 = 1$$

$$\sqrt{g^2} = \sqrt{1}$$

$$g = \pm 1$$

8.

$$-11r^2 - 5 = -178$$

$$-11r^2 = -173$$

$$r^2 = \frac{173}{11}$$

$$\sqrt{r^2} = \sqrt{\frac{173}{11}}$$

$$r = \sqrt{\frac{173}{11}}$$

$$r \approx \pm 3.9658$$

9.

$x^2 + 8x - 65 = -8$

$x^2 + 8x - 57 = 0$

$x^2 + 8x = 57$

$x^2 + 8x + 16 = 57 + 16$

$(x+4)^2 = 73$

$\sqrt{(x+4)^2} = \sqrt{73}$

$x + 4 = \sqrt{73}$

$x + 4 \approx \pm 8.544$

$x + 4 \approx 8.544$

$x \approx 4.544$

$x + 4 \approx -8.544$

$x \approx -12.544$

$x \approx 4.544, -12.544$

10.

$y = -(x-6)^2 + 5$

$y = a(x-h) + k$

$a = -1$

$h = 6$

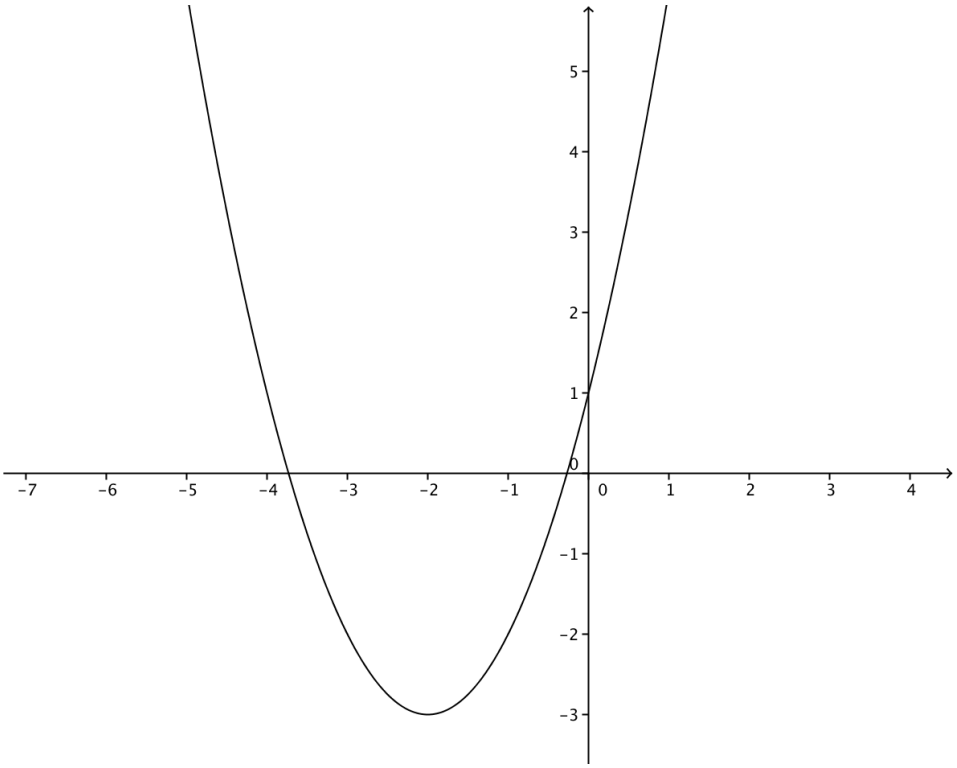
$k = 5$

Vertex= (6,5)

The parabola opens downward.

The vertex is a maximum.

11. $y=(x+2)^2-3$



12.

$-5x^2-6x=1$

$-5x^2-6x-1=0$

$D=b^2-4ac$

$D=(-6)^2-4(-5)(-1)$

$D=36-20$

$D=16$

The equation has two real solutions.

13. If D=-14 then the quadratic equations has no real solutions.

14.

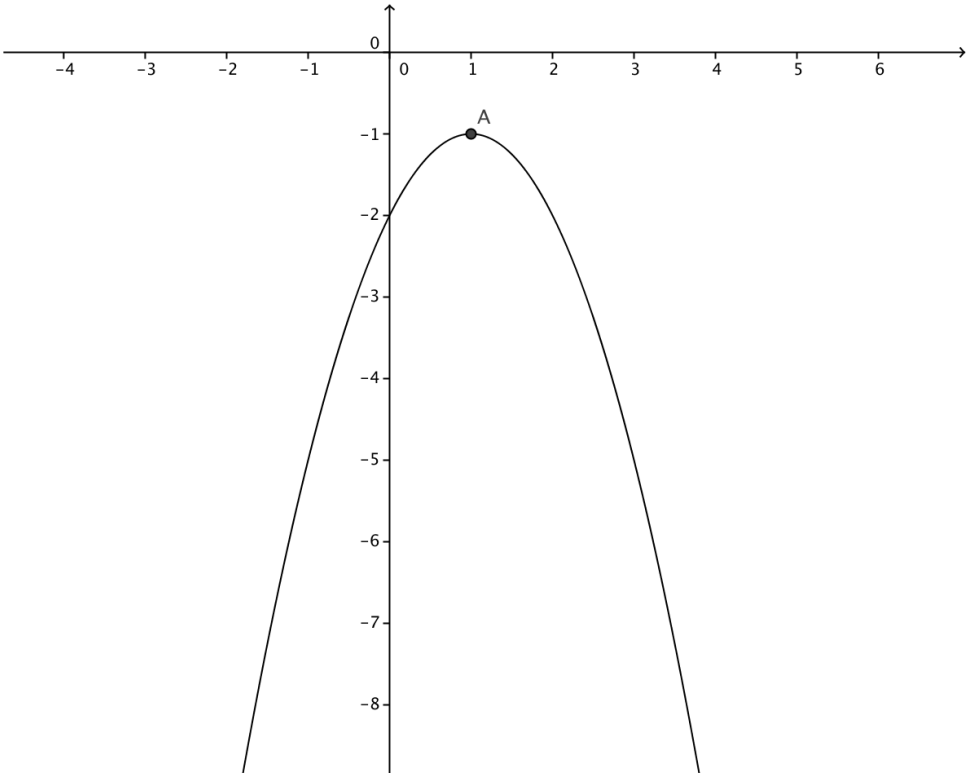
$y-7=-2(x+1)^2$

$y-7=-2(x^2+2x+1)$

$y-7=-2x^2-4x-2$

$y=-2x^2-4x+5$

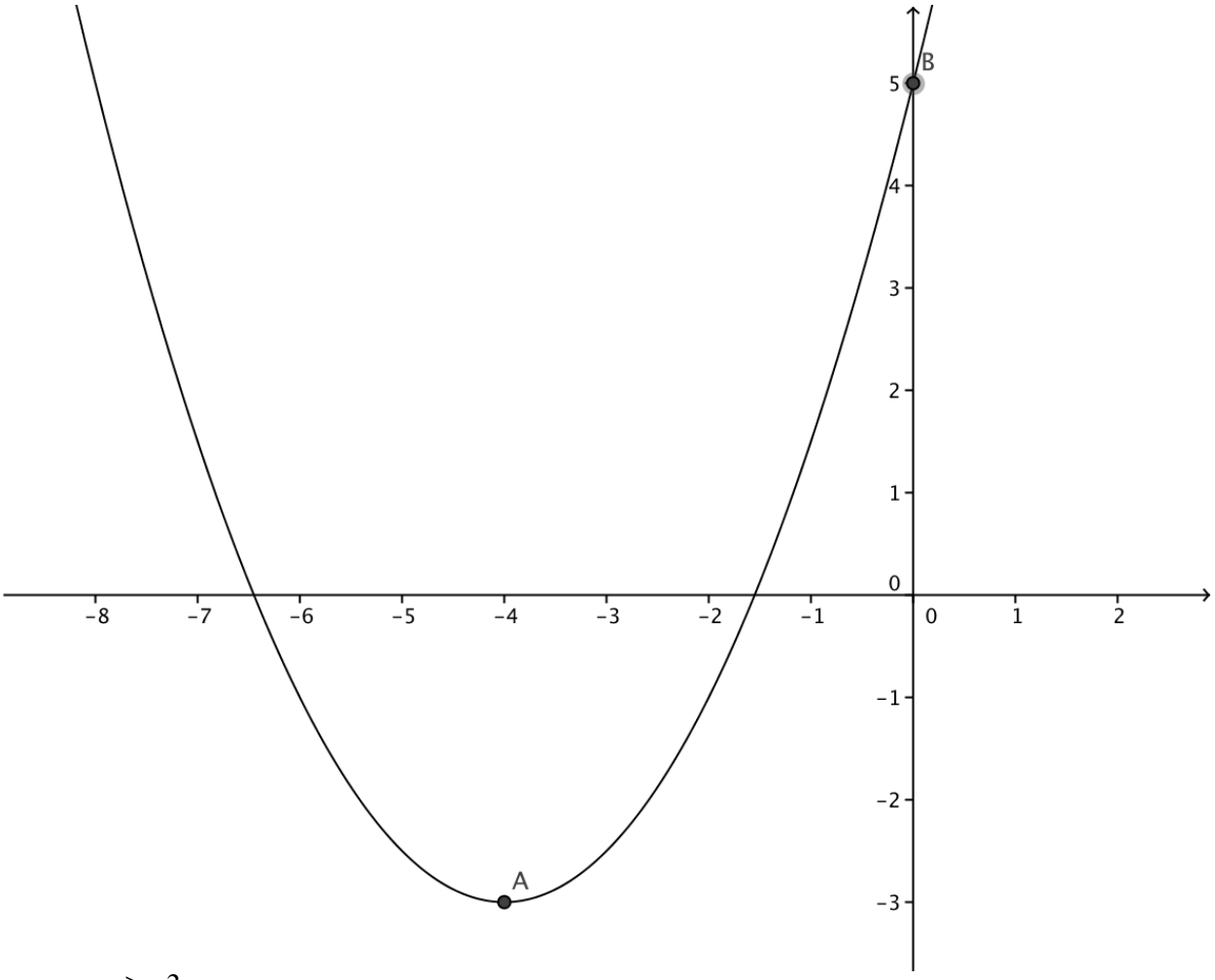
15. $y = -x^2 + 2x - 2$



Vertex = (1,-1)

Range: $y \leq -1$

16. $y = \frac{1}{2}x^2 + 4x + 5$



Range: $y \geq -3$

y-intercept: (0,5)

17.

1.

$y = mx + b$

$m = -0.1391$

$b = 222.2557$

$y = -0.1391x + 222.2557$

$x = 1999 = 99$

$y = -0.1391(99) + 222.2557$

$y = -13.7709 + 222.2557$

$y = 208.4848$

2.

$y = ax^2 + bx + c$

$a = -0.1763$

$b = 31.0581$

$c = -1151.7613$

$y = -0.1763x^2 + 31.0581x - 1151.7613$

$x = 1999 = 99$

$y = -0.1763\left(99^2\right) + 31.0581(99) - 1151.7613$

$y = -0.1763(9801) + 3074.7519 - 1151.7613$

$y = -1727.9163 + 3074.7519 - 1151.7613$

$y = 195.0743$

3.

$y = a(b)^x$

$a = 222.9965$

$b = 0.9993$

$y = 222.9965(0.9993)^x$

$x = 1999 = 99$

$y = 222.9965(0.9993)^{99}$

$y = (222.9965)(0.933)$

$y = 208.0557$

4. The quadratic function seems to be the best fit for this model.