7.1 Sampling Distributions

Answers

1. The data would be skewed toward older pennies because there are more new pennies.
2. Histograms would vary but the distribution would be skewed to the ages of the older pennies.
3. Answers will vary.
4. If students had a group of 100 pennies and selecting samples 5 at a time, they would have 20 samples. Since the class of 30 is separated into groups of 5 and each group can have 20 samples, there would be a total of 180 samples.
5. The sample mean would be the same as the population mean.
6. If students had a group of 100 pennies and selecting samples 10 at a time, they would have 10 samples. Since the class of 30 is separated into groups of 5 and each group can have 10 samples, there would be a total of 60 samples.
7. As the sample size increases the distribution would tend to look more like a normal distribution.
8. Yes
9. Yes
10. An estimator is a statistical parameter that provides an estimation of a population parameter. A point estimator is a single numerical estimate of a population parameter. The sample mean is a point estimator for the population mean. The larger the sample size the more dilution of untypical scores there will be and the more the sample means would cluster around the population mean and decrease error.
11. Sampling error is a function of the sample size. As the sample size increases, the error will decrease. The degree of error is determined using the formula $s = \sqrt{\frac{P \times Q}{n}}$ where P and Q are population parameters and n is the sample size. As the sample size is increased by a factor of 4, for example, the standard error is decreased by ½.

7.2 Central Limit Theorem

Answers

1. a) 4038
   b) 5671
   c) 49
2. As sample size increases, the distribution of data approaches normal distribution.
3. a) Yes
   b) No
   c) Yes
4. Normal Distribution
5. The distribution should look like a bell curve be because *n* is sufficiently large (*n* > 30).
6. The distribution would be approximately normal with the mean near the population mean of 160 and a standard deviation of 10.20.
7. The distribution would be approximately normal with the mean near the population mean of $45,632 and a standard deviation of 618.12.
8. (a)
9. The sample standard deviation for Sample A would be greater than the sample standard deviation for Sample B.
10. (b)
11. a) The first investigator.
    b) The first investigator.
12. (b)
13. (d)
14. False. The Central Limit Theorem guarantees the sampling distribution of the sample means is approximately normal when the sample size is greater than 30.
15. a) $P(5.7 < \bar{x} < 8.1) = P(2.1 < z < 2.7) = 1.5\%$
    b) $\bar{x} < 5.35$

16. True. The sample size of 30 would have a higher standard deviation.

17. a) mean of the sample population should be the same as the population mean.

b) If the sample standard deviation were given for the population, the standard deviation would be $\sigma x = \frac{\sigma}{\sqrt{n}} = \frac{\sigma}{\sqrt{200}} = \frac{\sigma}{14.14}$

c) and d) There is information missing in this question.

18. The spread of the sampling distribution is smaller than the spread of the population values. If the population was skewed to the right, the sampling means will be skewed as well but not as much and without outliers.

19. Since the population mean and the sample mean are so close, the recommendation to the management of the plant is that the dispensing machine is working correctly.

7.3 Confidence Intervals

Answers

1. a) The interval is from 0.488 to 0.648 or from 48.8% to 64.8%

   b) The 99% confidence interval can be narrowed by increasing the sample size from 250 to a larger number.

2. a) 90% confidence interval: 0.564 to 0.656; 95% confidence interval: 0.538 – 0.682; 99% confidence interval: 0.555 to 0.665.

   b) The confidence interval got wider as the confidence level increased. To increase the probability of enclosing the population, you should choose a wider confidence interval.

   c) Yes, with a population proportion of 0.58, all three confidence intervals would enclose it.

3. For large samples ($n \geq 30$), the population standard deviation (σ) will be fairly well approximated by the sample standard deviation, s. increasing the size of the n will increase the chance of capturing the unknown population mean.

4. $3.56 \pm 0.049 = (3.511, 3.609)$

5. $(9.7, 27.7)$

6. $(3244.06, 3475.94)$; Yes, because confidence intervals are based on normal distribution curves.

7. $(636.84, 663.16)$

8. $(13.86, 14.14)$

9. (1) The width of the confidence interval is wider for a greater standard deviation.

   (2) The width of the confidence interval is wider for a smaller sample size, n.

   (3) The width of the confidence interval is wider for a greater confidence interval.

10. a) $(63.71, 64.69)$

    b) $(121.52, 124.48)$

    c) $(122.68, 124.32)$

11. a) 0.25

 b) $(26.86, 28.14)$ At a 99% confidence interval, the mean foot length for men falls between 26.86 cm and 28.14 cm.

12. a) ± 0.0817

 b) ± 0.0408

 c) ± 0.0356

 d) ± 0.0356

13. a) 0.70

 b) $0.70 \pm 0.0519 \; or \; 0.648 < p < 0.752$. Therefore we are 95% confident that the interval from 0.648 to 0.753 contains the population proportion. If we took 100 samples of size n, then 95 out of the 100 confidence intervals would capture the population proportion, p.

14. With a margin of error of 3%, polling results reported in the survey will be $\pm 3\%$. So if 65% of the sample surveyed said yes, the results can be reported as $65\% \pm 3\%$ or a yes can be obtained for 62% to 68% of those surveyed.

15. a) $0.63 \pm 0.0192 \; or \; 0.611 < p < 0.649$

 b) Therefore with a margin of error of 2%, polling results reported in the survey will be $\pm 2\%$. So of the 63% of the sample that said it was a good idea, we are 95% confident that the interval from 0.611 to 0.649 contains the population proportion.

16. a) $0.7 \pm 0.0397 \; or \; 0.660 < p < 0.740$

 b) Therefore with a margin of error of 5%, there is a 0.95 probability that the method used to produce the interval from 65% to 75% results in a confidence interval that encloses the population mean (the number of people who believe the doctor's advice is accurate and do not need a second opinion).

17. a) A success would be those who are in the interval that covers the population mean.

 b) 95%

 c) 95 out of 100 confidence intervals would capture the population proportion.

18. a) Confidence interval will be wider as the level of confidence is increasing.

    b) Confidence interval will be narrower as the sample size is increasing.

    c) Confidence interval will be not increase or decrease as a sample size cannot be the same size and increased by 10.

19. This question should say problem 13.

$0.70 \pm 0.0682 \ or \ 0.632 < p < 0.768$ The interval is wider because the level of confidence increased.

20. a) The confidence interval $\pm$ the margin of error would tell you the range of respondents that answered positively to the question of how they got along with their parents. If the margin of error was 5%, then the confidence interval would have to be 90% (therefore a range of 85% - 90%). A confidence interval of 95% or 99% would not have a margin of error as 5%. These confidence intervals would give unlikely or impossible ranges.

    b) $0.54 \pm 0.0366 \ or \ 0.503 < p < 0.577$