

**Algebra Solution Key**  
**Chapter 12**  
**Rational Equations and Functions; Statistics**

**Lesson 12.1**  
**Inverse Variation Models**

1. Inverse variation is when the two variables change differently (when one goes up the other goes down, etc.)

2. In direct variation, the two variables change in the same way, while in inverse variation they change in opposite ways. Direct variation relationships have graphs that always go through the origin, while inverse variation relationships never cross the axes. The graph of a direct variation is a line, while the graph of an inverse variation creates a hyperbola. Additionally in an inverse variation  $x$  can never be zero and  $k$  must be a counting number, while these restrictions do not hold true for direct variation.

3. varies directly

4. neither

5. varies inversely

6. varies directly

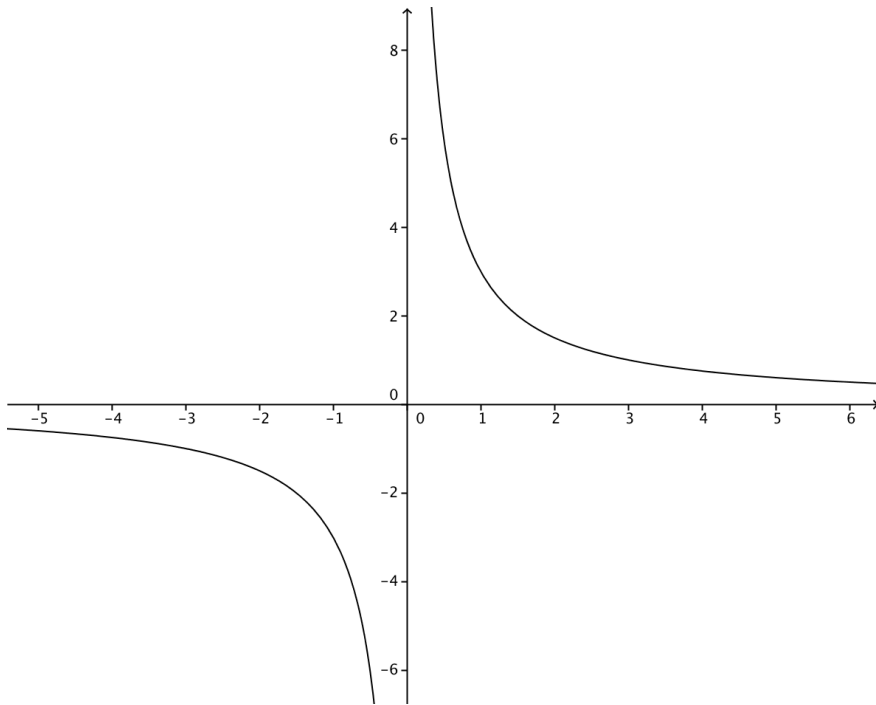
7. varies directly

8. neither

9. varies inversely

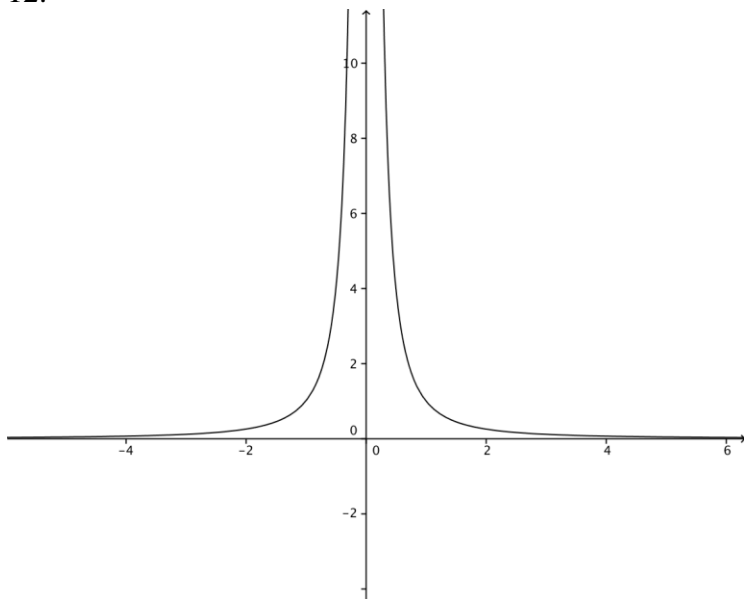
10. varies inversely

11.  $y = \frac{3}{x}$



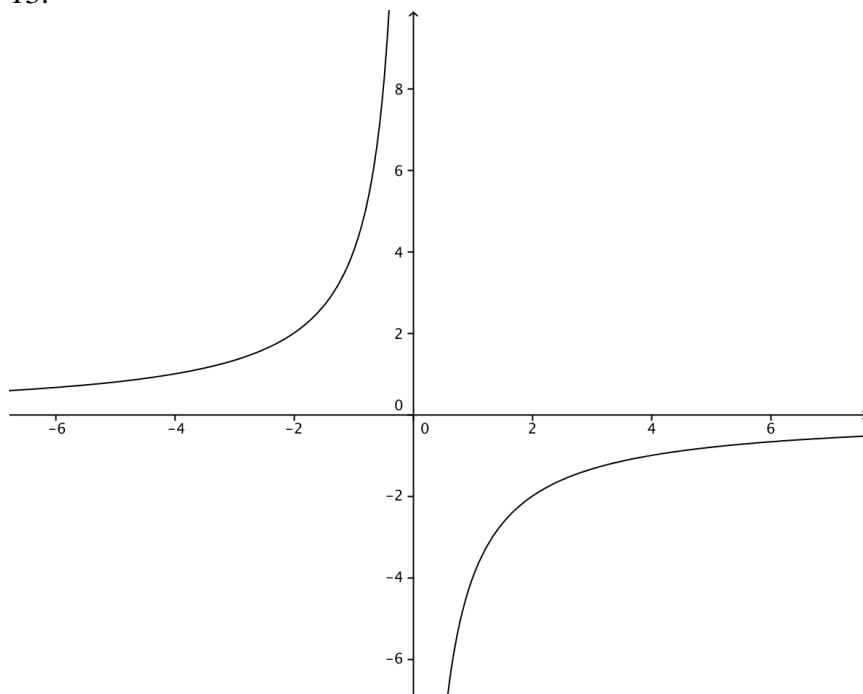
Domain :  $x \neq 0$ , Range:  $y \neq 0$

12.  $y = \frac{1}{x^2}$



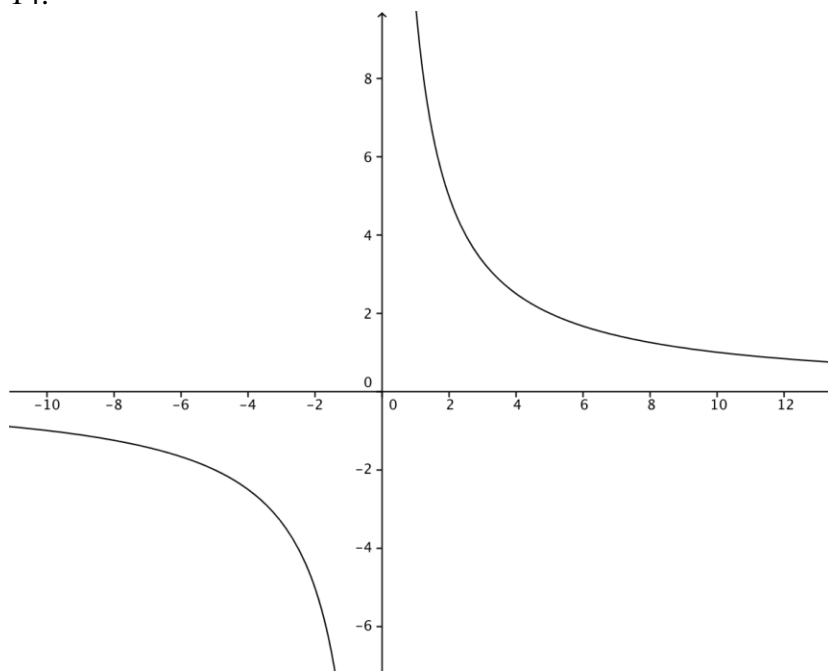
Domain :  $x \neq 0$ , Range:  $y \neq 0$

13.  $f(x) = -\frac{4}{x}$



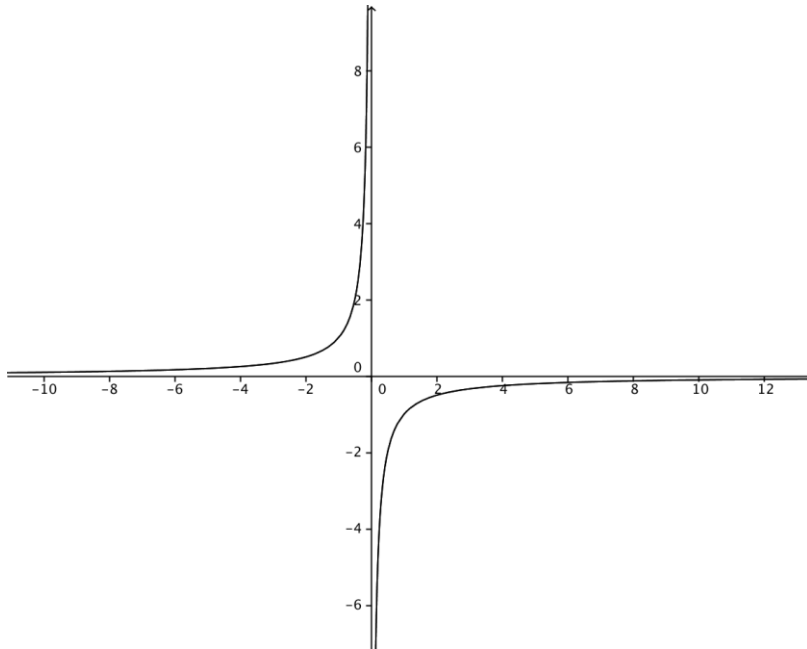
Domain :  $x \neq 0$ , Range:  $y \neq 0$

14.  $y = \frac{10}{x}$



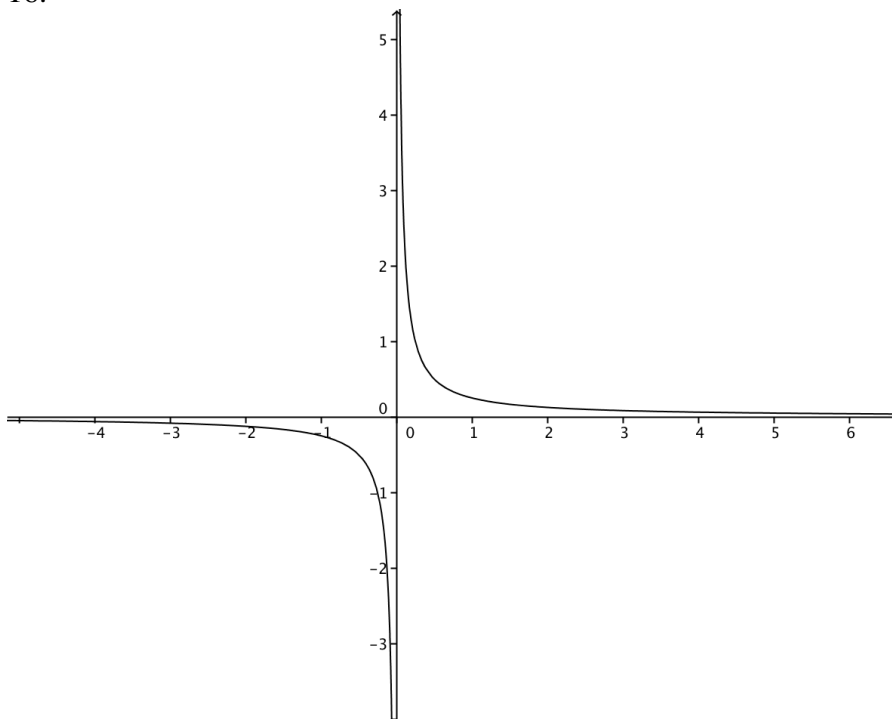
Domain :  $x \neq 0$ , Range:  $y \neq 0$

15.  $h(x) = -\frac{1}{x}$



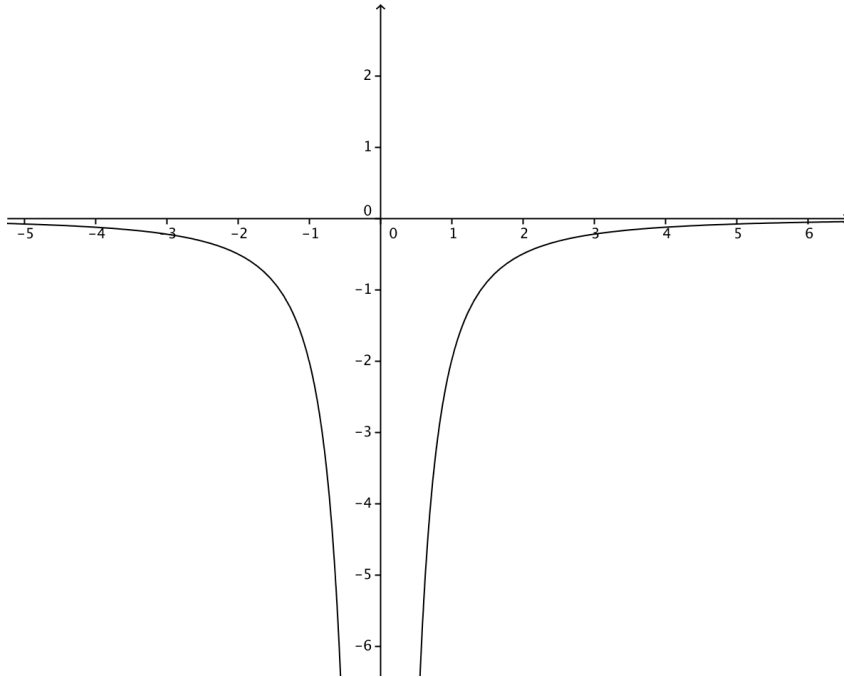
Domain :  $x \neq 0$ , Range:  $y \neq 0$

16.  $y = \frac{1}{4x}$



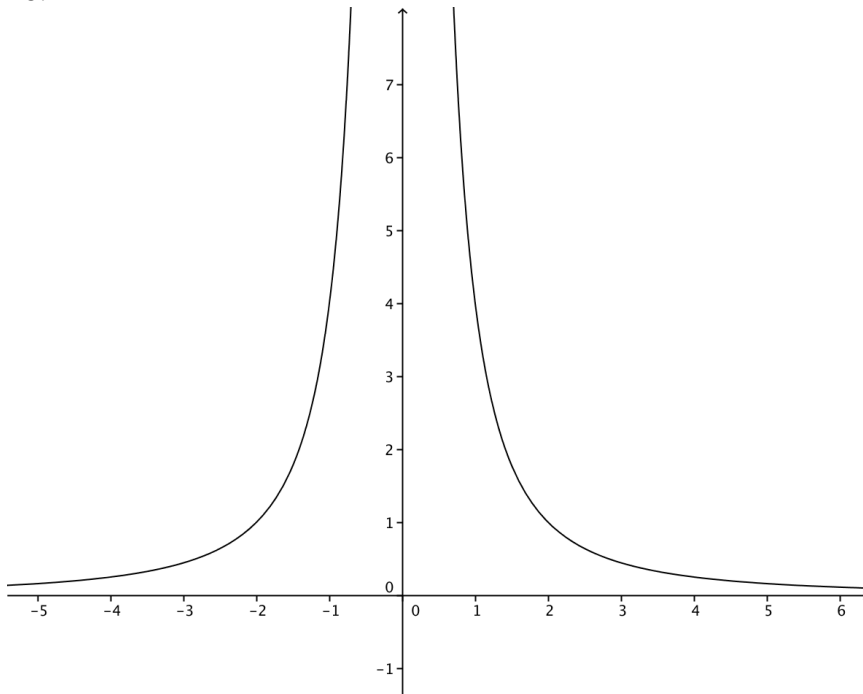
Domain :  $x \neq 0$ , Range:  $y \neq 0$

17.  $g(x) = -\frac{2}{x^2}$



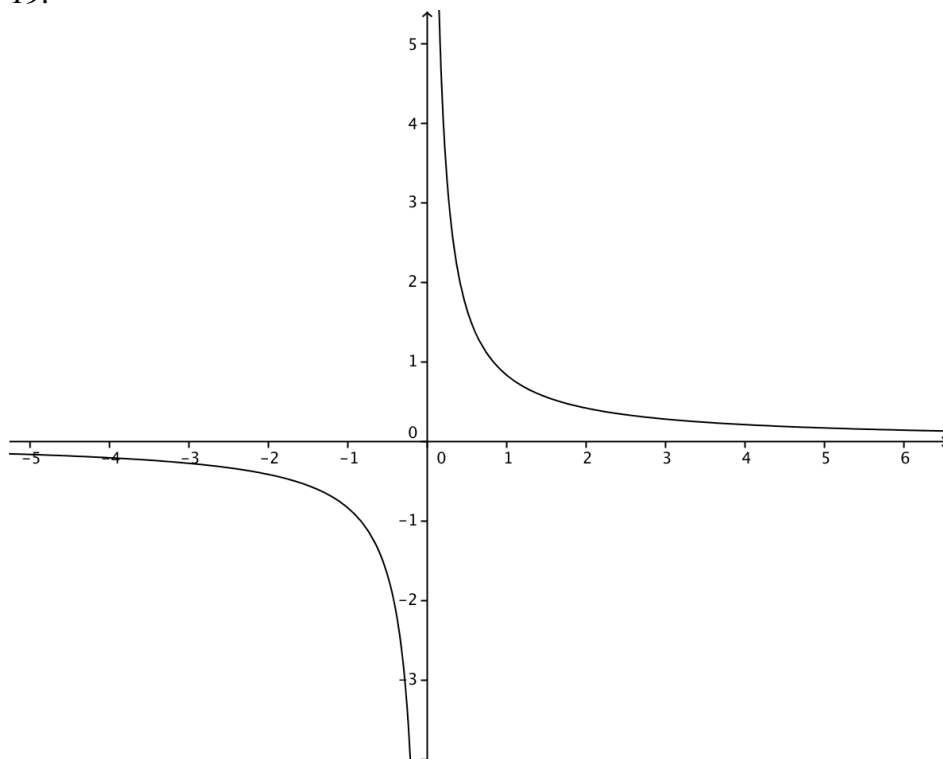
Domain :  $x \neq 0$ , Range:  $y \neq 0$

18.  $y = \frac{4}{x^2}$



Domain :  $x \neq 0$ , Range:  $y \neq 0$

19.  $y = \frac{5}{6x}$



Domain :  $x \neq 0$ , Range:  $y \neq 0$

20.

$$y = \frac{k}{x}$$

$$24 = \frac{k}{3}$$

$$k = 72$$

$$y = \frac{72}{-1.5} = -48$$

21.

$$d = \frac{k}{t^3}$$

$$-23.5 = \frac{k}{3^3}$$

$$-23.5 = \frac{k}{27}$$

$$k \approx -0.8704$$

$$d \approx \frac{-0.8704}{(0.25)^3} \approx \frac{-0.8704}{0.0156} \approx -55.7949$$

22.

$$z = \frac{k}{w}$$

$$81 = \frac{k}{9}$$

$$k = 729$$

$$24 = \frac{729}{w}$$

$$24w = 729$$

$$w = 30.375$$

23.

$$y = \frac{k}{x}$$

$$2 = \frac{k}{8}$$

$$k = 16$$

$$y = \frac{16}{12} = \frac{4}{3}$$

24.

$$a = \frac{k}{\sqrt{b}}$$

$$32 = \frac{k}{\sqrt{9}}$$

$$32 = \frac{k}{\pm 3}$$

$$k = \pm 96$$

$$6 = \frac{96}{\sqrt{b}}$$

$$6\sqrt{b} = 96$$

$$\sqrt{b} = 16$$

$$b = 16^2$$

$$b = 256$$

25.

$$w = \frac{k}{u^2}$$

$$4 = \frac{k}{2^2}$$

$$4 = \frac{k}{4}$$

$$k = 16$$

$$w = \frac{16}{8^2}$$

$$w = \frac{16}{64} = \frac{1}{4} = 0.25$$

26.  $d$  = distance from fulcrum,  $w$  = weight of object

$$d = \frac{k}{w}$$

$$6 = \frac{k}{40}$$

$$k = 240$$

$$d = \frac{240}{65} \approx 3.6923$$

Josh would have to sit approximately 3.6923 feet from the fulcrum to balance the seesaw.

27.  $l$  = intensity of light,  $d$  = distance between source and object

$$l = \frac{k}{d^2}$$

$$35 = \frac{k}{10^2}$$

$$35 = \frac{k}{100}$$

$$k = 3500$$

$$l = \frac{3500}{25^2} = \frac{3500}{625} = 5.6$$

The intensity would register 5.6 lux.

28.  $c$  = current in wire,  $r$  = resistance of wire

$$c = \frac{k}{r}$$

$$2.5 = \frac{k}{20}$$

$$k = 50$$

$$5 = \frac{50}{r}$$

$$5r = 50$$

$$r = 10$$

The resistance would be 10 ohms.

29.  $n$  = number of tiles,  $s$  = length of the side of a tile

$$n = \frac{k}{s^2}$$

$$112 = \frac{k}{6^2}$$

$$112 = \frac{k}{36}$$

$$k = 4032$$

$$n = \frac{4032}{8^2} = \frac{4032}{64} = 63$$

It would take 63 eight-inch tiles to cover the floor.

30.

$$16 \geq -3x + 5$$

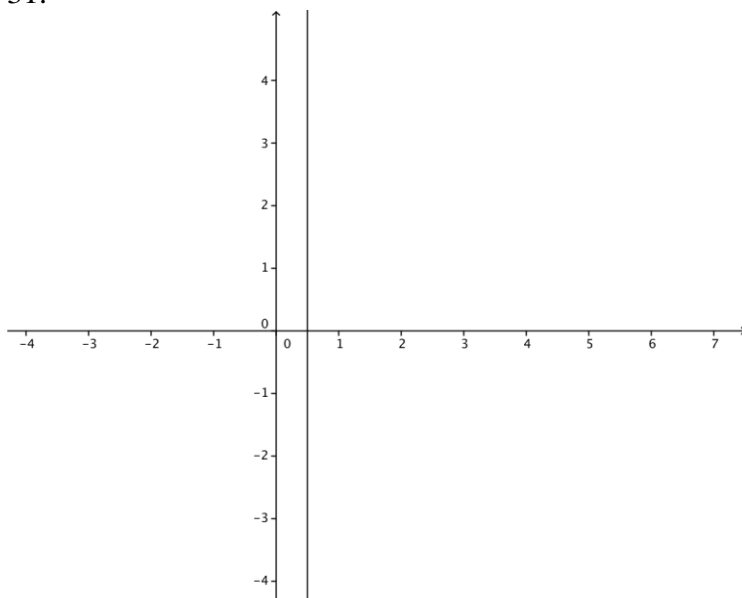
$$11 \geq -3x$$

$$-\frac{11}{3} \geq x$$

$$x \leq -3.6667$$



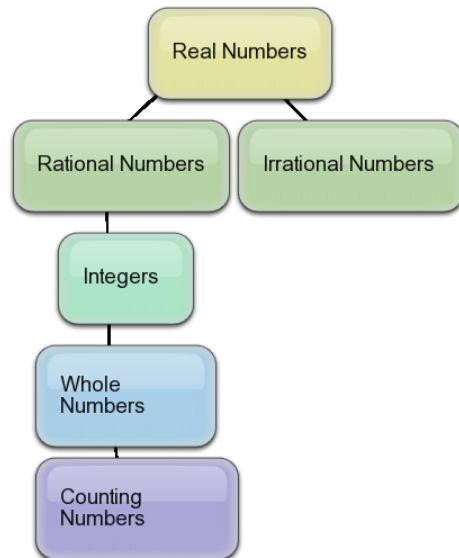
31.  $x = \frac{7}{14}$



32.  $\sqrt[3]{320} = \sqrt[3]{64} * \sqrt[3]{5} = 4\sqrt[3]{5}$

33. The Commutative Property of Multiplication states that  $a(b) = b(a)$ .

34.



Answers for examples will vary, any that fit the definitions are correct.

*Real Numbers:* all numbers of any size, that are not imaginary

*Rational Numbers:* any number that can be made by dividing one integer into another

*Irrational Numbers:* any number that cannot be written as a simple fraction, the decimals go on forever without repeating

*Integers:* are all numbers with no fractional or decimal piece, they include all positive and negative numbers, as well as zero

*Whole Numbers:* are numbers with no fractional or decimal part and zero, but they do not include negative numbers

*Counting Numbers:* All numbers greater than (not including) zero, that have no fractional or decimal part.

35.

$17.5\% \text{ of } 96$

$17.5\% * 96$

$.175 * 96$

$16.8$

## Lesson 12.2

### Graphs of Rational Functions

$$f(x) = \frac{g(x)}{h(x)}, h \neq 0$$

1. A rational function is a ratio of two polynomials (one divided by the other).
2. An asymptote is a straight line to which, as the distance from the origin gets larger, a curve get closer and closer but never intersects. The relationship between asymptotes and rational functions is that where an asymptote exists, the function is undefined.
3. This lesson describes vertical and horizontal asymptotes. The general equation for vertical asymptotes is

$$f(x) = \frac{g(x)}{h(x)}, h(x) = 0$$

(A vertical asymptote occurs when there is a zero in the denominator) The general equation for horizontal asymptotes is  $y=c$ , where  $c$  is the vertical shift.

4.

$$y = \frac{4}{x+2}$$

$$x+2 = 0$$

$$x = -2$$

$$y = c$$

$$y = 0$$

The vertical asymptote is  $x=-2$ .

The horizontal asymptote is  $y=0$ .

5.

$$f(x) = \frac{5}{2x-6} + 3$$

$$2x - 6 = 0$$

$$2x = 6$$

$$x = 3$$

$$y = c$$

$$y = 3$$

The vertical asymptote is  $x=3$ .

The horizontal asymptote is  $y=3$ .

6.

$$y = \frac{10}{x}$$

$$x = 0$$

$$y = 0$$

The vertical asymptote is  $x=0$ .

The horizontal asymptote is  $y=0$ .

7.

$$g(x) = \frac{4}{4x^2 + 1} - 2$$

$$4x^2 + 1 = 0$$

$$4x^2 = -1$$

$$x^2 = -\frac{1}{4}$$

$$x \neq \sqrt{-\frac{1}{4}}$$

$$y = c$$

$$y = -2$$

There is no vertical asymptote.

The horizontal asymptote is  $y=-2$ .

8.

$$h(x) = \frac{2}{x^2 - 9}$$

$$x^2 - 9 = 0$$

$$x^2 = 9$$

$$x = \pm 3$$

$$y = c$$

$$y = 0$$

The vertical asymptotes are  $x=3$  and  $x=-3$ .

The horizontal asymptote is  $y=0$ .

9.

$$y = \frac{1}{x^2 + 4x + 3} + \frac{1}{2}$$

$$x^2 + 4x + 3 = 0$$

$$(x + 3)(x + 1) = 0$$

$$x + 3 = 0$$

$$x = -3$$

$$x + 1 = 0$$

$$x = -1$$

$$y = c$$

$$y = \frac{1}{2}$$

The vertical asymptotes are  $x = -3$  and  $x = -1$ .

The horizontal asymptote is  $y = 1/2$ .

10.

$$y = \frac{3}{x^2 - 4} - 8$$

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

$$y = c$$

$$y = -8$$

The vertical asymptotes are  $x = 2$  and  $x = -2$ .

The horizontal asymptote is  $y = -8$ .

11.

$$f(x) = \frac{-3}{x^2 - 2x - 8}$$

$$x^2 - 2x - 8 = 0$$

$$(x - 4)(x + 2) = 0$$

$$x - 4 = 0$$

$$x = 4$$

$$x + 2 = 0$$

$$x = -2$$

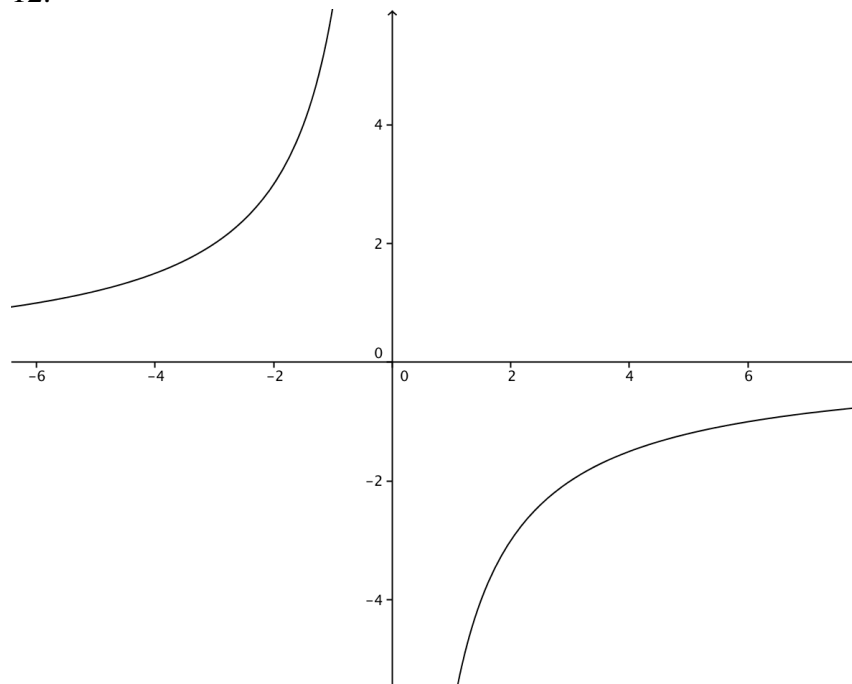
$$y = c$$

$$y = 0$$

The vertical asymptotes are  $x=4$  and  $x=-2$ .

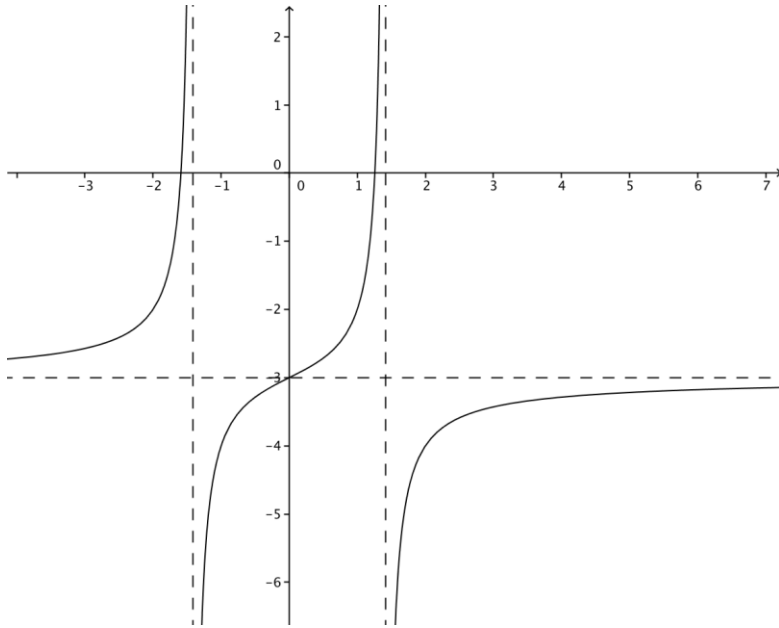
The horizontal asymptote is  $y=0$ .

12.  $y = -\frac{6}{x}$

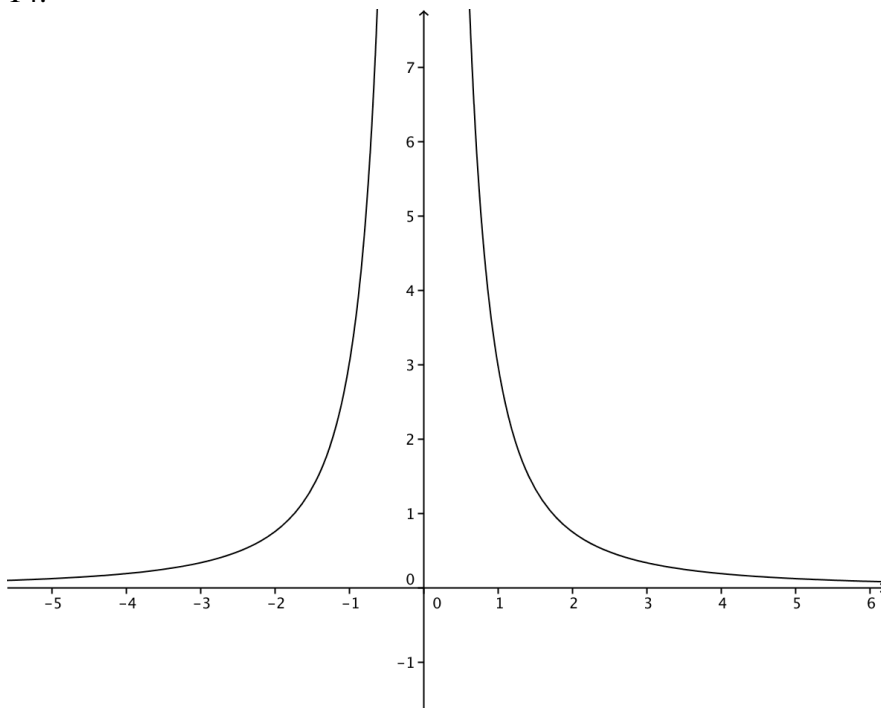


The axes are the horizontal and vertical asymptotes.

13.  $y = \frac{x}{2-x^2} - 3$

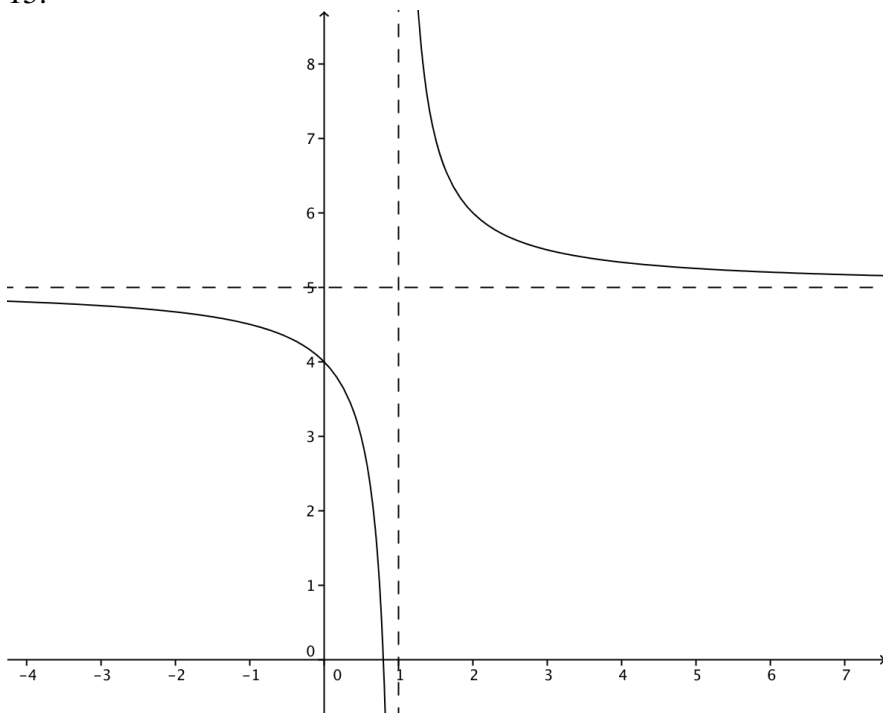


14.  $f(x) = \frac{3}{x^2}$

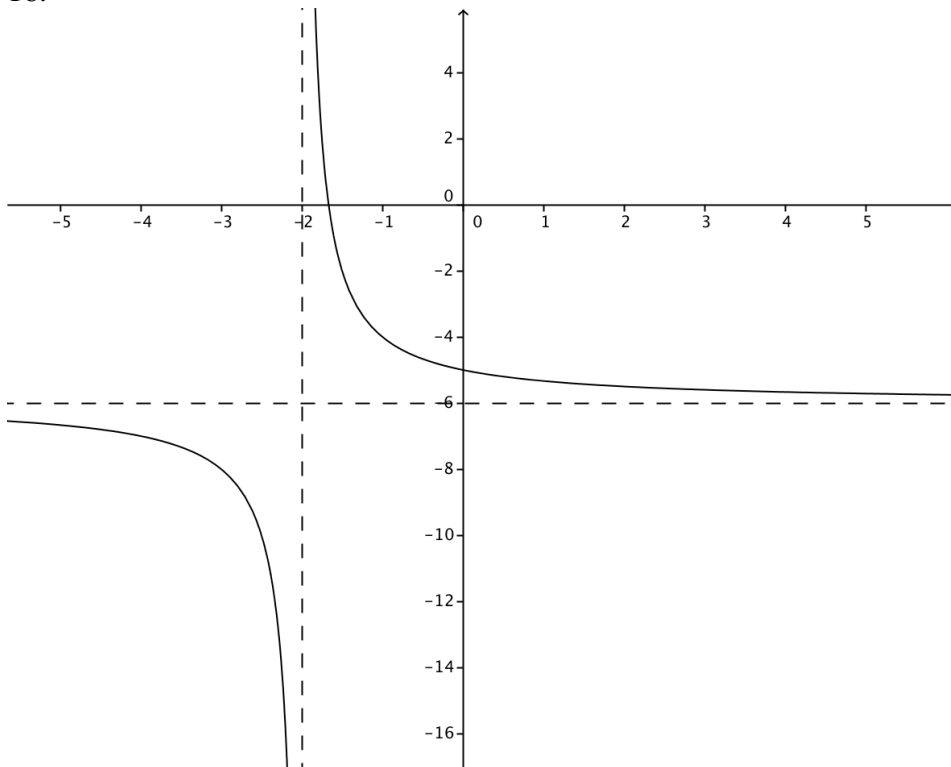


The axes are the horizontal and vertical asymptotes.

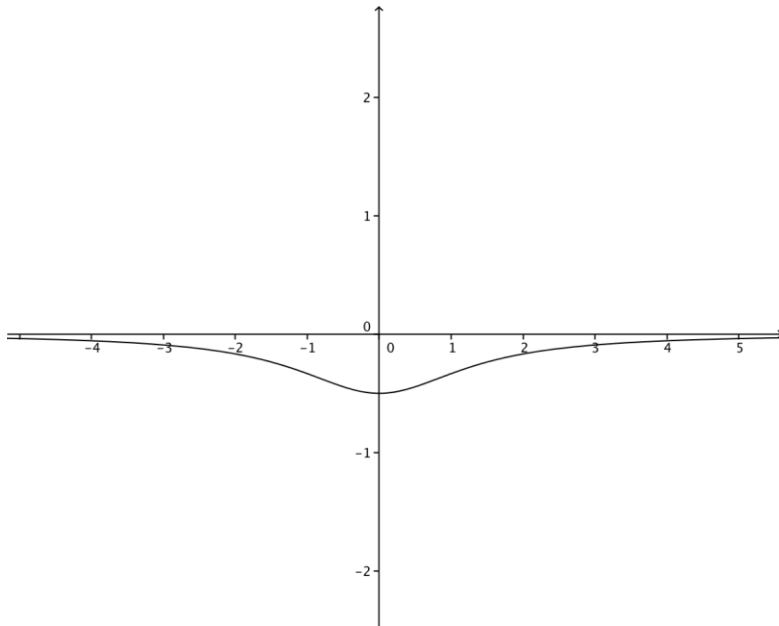
15.  $g(x) = \frac{1}{x-1} + 5$



16.  $y = \frac{2}{x+2} - 6$

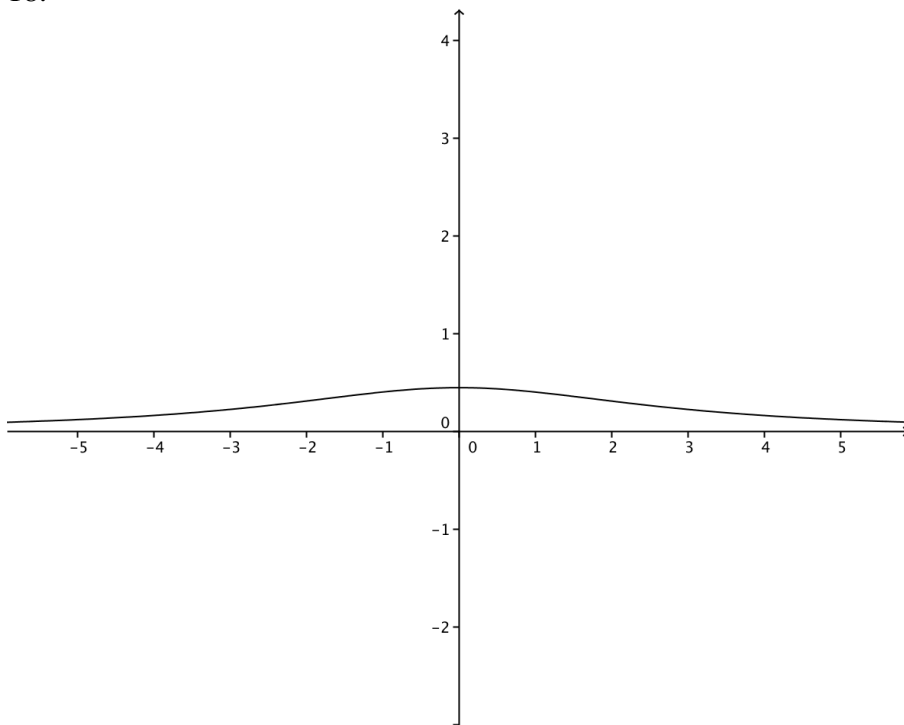


17.  $f(x) = \frac{-1}{x^2 + 2}$



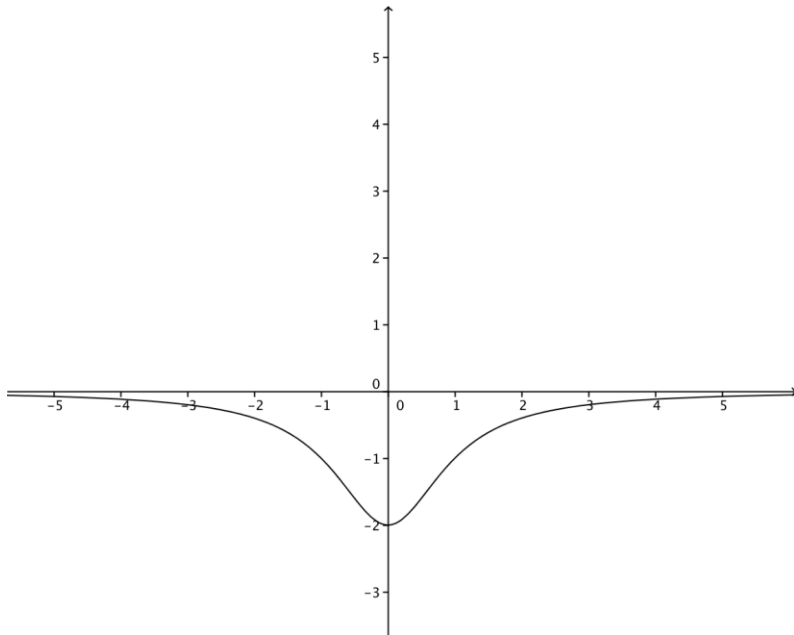
This function has no vertical asymptotes. The horizontal asymptote is the x-axis.

18.  $h(x) = \frac{4}{x^2 + 9}$



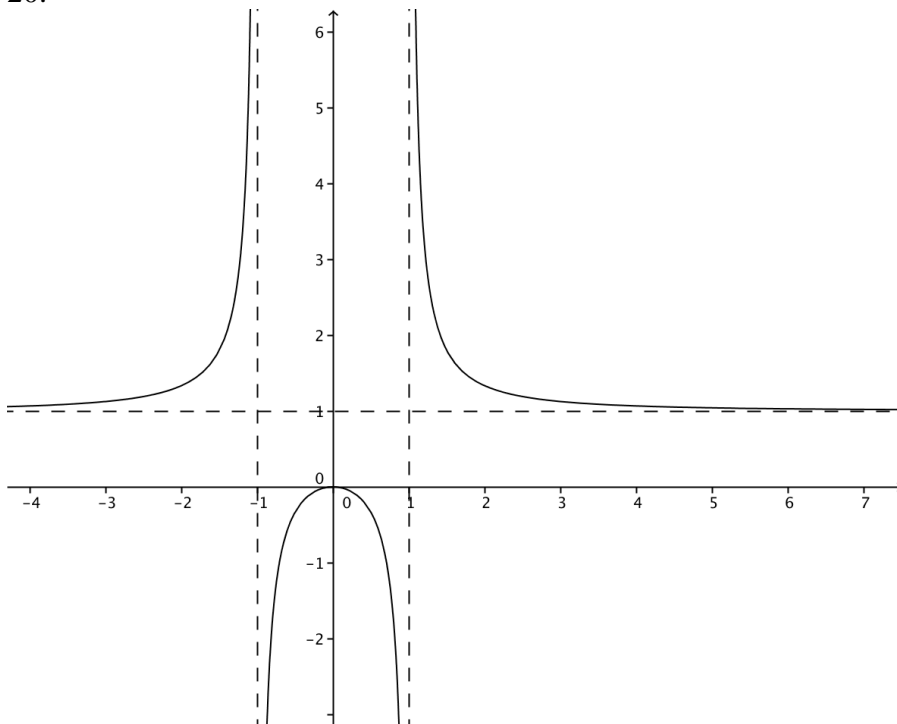
This function has no vertical asymptotes. The horizontal asymptote is the x-axis.

19.  $y = \frac{-2}{x^2 + 1}$

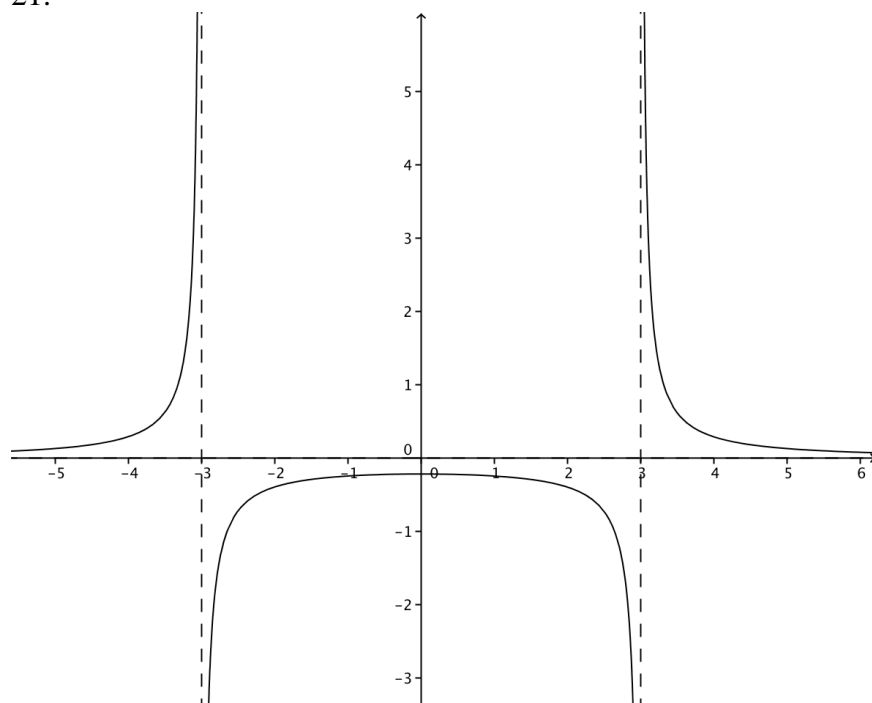


This function has no vertical asymptotes. The horizontal asymptote is the x-axis.

20.  $j(x) = \frac{1}{x^2 - 1} + 1$

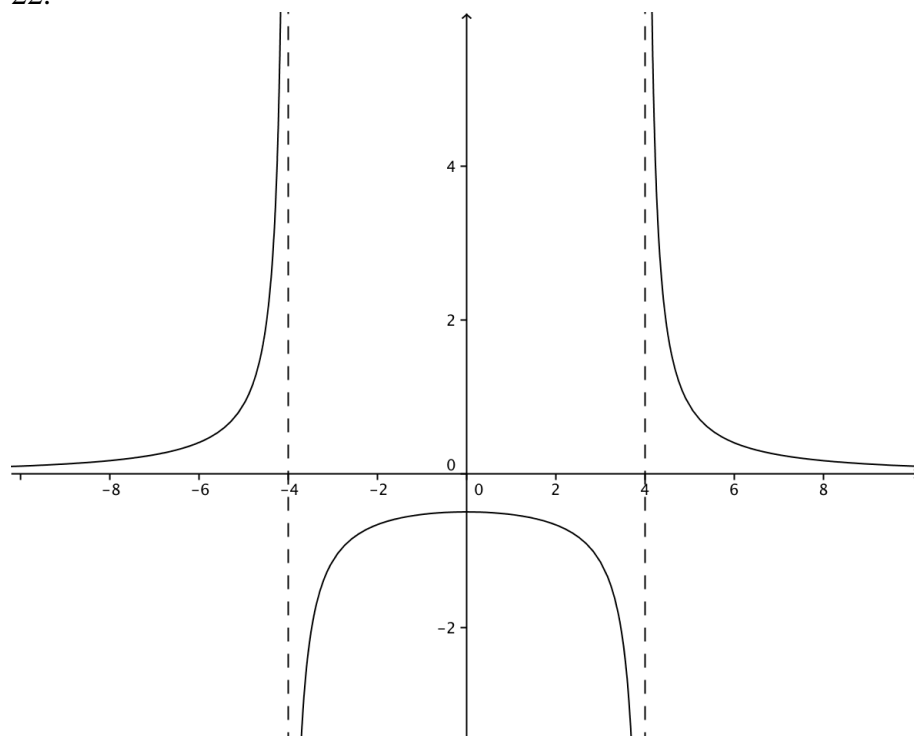


21.  $y = \frac{2}{x^2 - 9}$



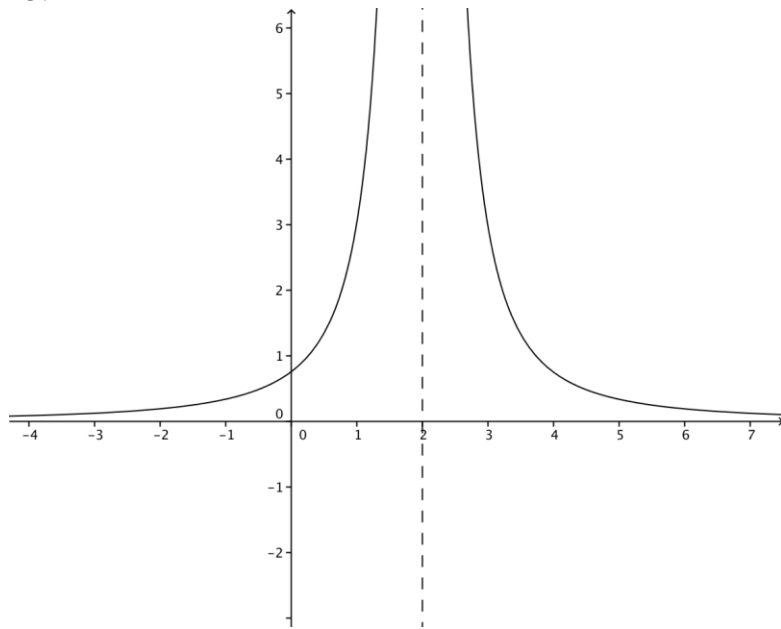
The horizontal asymptote is the  $x$ -axis.

22.  $f(x) = \frac{8}{x^2 - 16}$



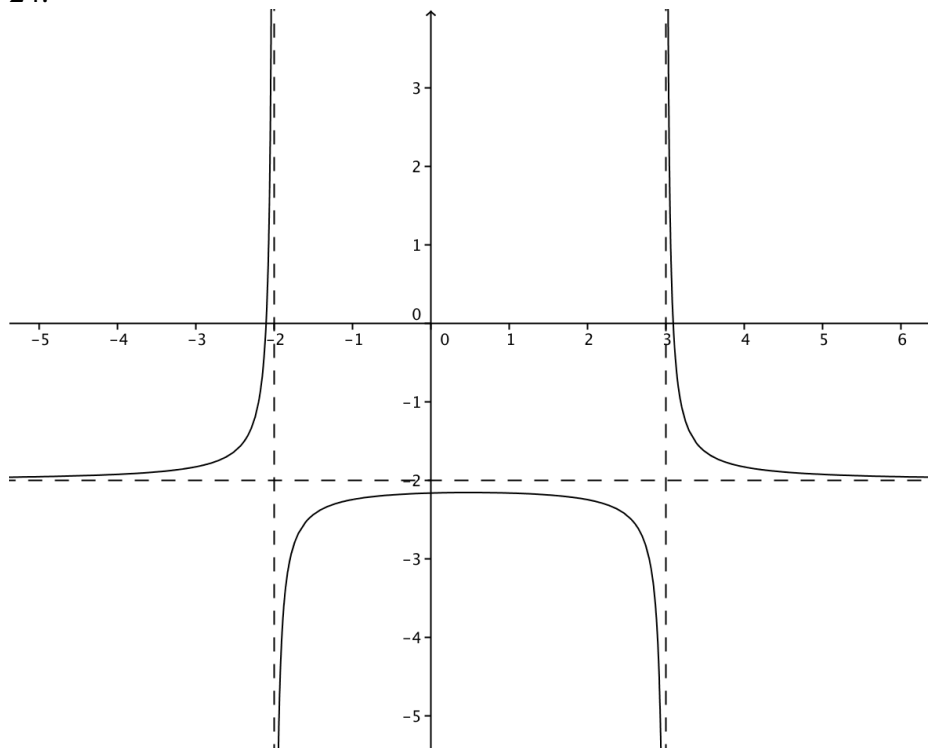
The horizontal asymptote is the  $x$ -axis.

23.  $g(x) = \frac{3}{x^2 - 4x + 4}$



The horizontal asymptote is the x-axis.

24.  $h(x) = \frac{1}{x^2 - x - 6}$



25.

$$I = \frac{V}{R}$$

$$1.5 = \frac{x}{12}$$

$$x = 18$$

The missing quantity is 18V.

26.

$$I = \frac{V}{R}$$

$$1.2 = \frac{27}{10+x}$$

$$1.2(10+x) = 27$$

$$12 + 1.2x = 27$$

$$1.2x = 15$$

$$x = 12.5$$

27.

$$I = \frac{V}{R}$$

$$x = \frac{15}{\frac{1}{10} + \frac{1}{15}}$$

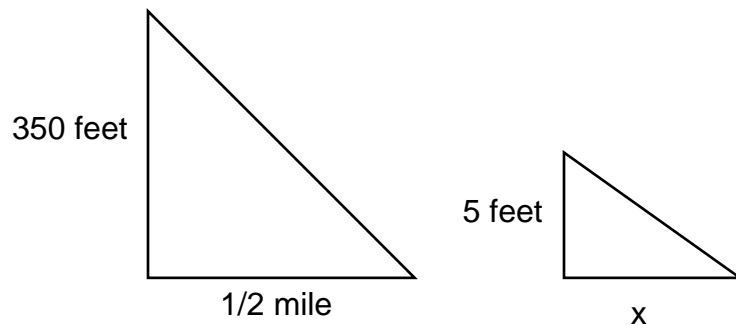
$$x = \frac{15}{\frac{1}{6}} = 15 \div \frac{1}{6} = 15 * \frac{6}{1} = 15 * 6$$

$$x = 90$$

The missing quantity is 90 A.

Mixed Review

28.



$$\frac{1}{2} \text{ mile} = 2640 \text{ feet}$$

$$\frac{350}{2640} = \frac{5}{x}$$

$$350x = 13200$$

$$x = 37.7143$$

The person's shadow is approximately 37.7143 feet.

29. The Cross Product Property states that you can solve a proportion by multiplying the means

and the extremes.  $\frac{a}{b} = \frac{c}{d} \rightarrow ad = cb$

30.

(1,1) & (-4,5)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{5 - 1}{-4 - 1} = \frac{4}{-5} = -0.8$$

31.

$$y = kx$$

$$12 = k(120)$$

$$\frac{12}{120} = k$$

$$k = .10$$

You receive \$0.10 per can.

32.

$$\frac{2}{8} = \frac{1}{4}$$

33. Any answer that includes a variable to the 6th power and 2 terms (with no negative exponents) is correct.

### Lesson 12.3

#### Division of Polynomials

1.

$$\frac{2x+4}{2} = \frac{2x}{2} + \frac{4}{2} = x+2$$

2.

$$\frac{x-4}{x} = \frac{x}{x} - \frac{4}{x} = 1 - \frac{4}{x}$$

3.

$$\frac{5x-35}{5x} = \frac{5x}{5x} - \frac{35}{5x} = 1 - \frac{7}{x}$$

4.

$$\frac{x^2+2x-5}{x} = \frac{x^2}{x} + \frac{2x}{x} - \frac{5}{x} = x+2 - \frac{5}{x}$$

5.

$$\frac{4x^2+12x-36}{-4x} = \frac{4x^2}{-4x} + \frac{12x}{-4x} - \frac{36}{-4x} = -x-3 + \frac{9}{x}$$

6.

$$\frac{2x^2+10x+7}{2x^2} = \frac{2x^2}{2x^2} + \frac{10x}{2x^2} + \frac{7}{2x^2} = 1 + \frac{5}{x} + \frac{7}{2x^2}$$

7.

$$\frac{x^3-x}{-2x^2} = \frac{x^3}{-2x^2} - \frac{x}{-2x^2} = -\frac{x}{2} + \frac{1}{2x}$$

8.

$$\frac{5x^4-9}{3x} = \frac{5x^4}{3x} - \frac{9}{3x} = \frac{5x^3}{3} - \frac{3}{x}$$

9.

$$\frac{x^3 - 12x^2 + 3x - 4}{12x^2} = \frac{x^3}{12x^2} - \frac{12x^2}{12x^2} + \frac{3x}{12x^2} - \frac{4}{12x^2} = \frac{x}{12} - 1 + \frac{1}{4x}$$

10.

$$\frac{3 - 6x + x^3}{-9x^3} = \frac{3}{-9x^3} - \frac{6x}{-9x^3} + \frac{x^3}{-9x^3} = -\frac{1}{3x^3} + \frac{2}{3x^2} - \frac{1}{9}$$

11.

$$\frac{x^2 + 3x + 6}{x + 1}$$

$$\begin{array}{r} x+2 \\ x+1 \overline{)x^2+3x+6} \\ \underline{-x^2-x} \phantom{6} \\ 0+2x+6 \\ \underline{0-2x-2} \\ 0+0+4 \end{array}$$

$$x + 2r4$$

12.

$$\frac{x^2 - 9x + 6}{x - 1}$$

$$\begin{array}{r} x-8 \\ x-1 \overline{)x^2-9x+6} \\ \underline{-x^2+x} \phantom{6} \\ 0-8x+6 \\ \underline{0+8x-8} \\ 0+0-2 \end{array}$$

$$x - 8r(-2)$$

13.

$$\frac{x^2 + 5x + 4}{x + 4}$$

$$\begin{array}{r} x+4 \overline{)x^2 + 5x + 4} \\ \underline{-x^2 - 4x} \phantom{4} \\ 0 + x + 4 \\ \underline{0 - x - 4} \\ 0 \end{array}$$

$x + 1$

14.

$$\frac{x^2 - 10x + 25}{x - 5}$$

$$\begin{array}{r} x-5 \overline{)x^2 - 10x + 25} \\ \underline{-x^2 + 5x} \phantom{25} \\ 0 - 5x + 25 \\ \underline{0 + 5x - 25} \\ 0 \end{array}$$

$x - 5$

15.

$$\frac{x^2 - 20x + 12}{x - 3}$$

$$\begin{array}{r} x-3 \overline{)x^2 - 20x + 12} \\ \underline{-x^2 + 3x} \phantom{12} \\ 0 - 17x + 12 \\ \underline{0 + 17x - 51} \\ 0 + 0 - 39 \end{array}$$

$x - 17r(-39)$

16.

$$\frac{3x^2 - x + 5}{x - 2}$$

$$\begin{array}{r} 3x + 5 \\ x - 2 \overline{) 3x^2 - x + 5} \\ \underline{-3x^2 + 6x} \phantom{+ 5} \\ 0 + 5x + 5 \\ \underline{0 - 5x + 10} \\ 0 + 0 + 15 \end{array}$$

$$3x + 5r15$$

17.

$$\frac{9x^2 + 2x - 8}{x + 4}$$

$$\begin{array}{r} 9x - 34 \\ x + 4 \overline{) 9x^2 + 2x - 8} \\ \underline{-9x^2 - 36x} \phantom{- 8} \\ 0 - 34x - 8 \\ \underline{0 + 34x + 136} \\ 0 + 0 + 128 \end{array}$$

$$9x - 34r128$$

18.

$$\frac{3x^2 - 8x - 3}{3x + 1}$$

$$\begin{array}{r} \phantom{3x+1} \overline{) 3x^2 - 8x - 3} \\ \underline{-3x^2 - x} \phantom{-3} \\ 0 - 9x - 3 \\ \underline{0 + 9x + 3} \\ 0 \end{array}$$

$$x - 3$$

19.

$$\frac{5x^2 + 2x - 9}{x - 1}$$

$$\begin{array}{r} \phantom{x-1} \overline{) 5x^2 + 2x - 9} \\ \underline{-5x^2 + 5x} \phantom{-9} \\ 0 + 7x - 9 \\ \underline{0 - 7x + 7} \\ -2 \end{array}$$

$$5x + 7r(-2)$$

20.

$$\frac{x^2 - 6x - 12}{x + 4}$$

$$\begin{array}{r} x + 4 \overline{) x^2 - 6x - 12} \\ \underline{-x^2 - 4x} \phantom{-12} \\ 0 - 10x - 12 \\ \underline{0 + 10x + 40} \\ 0 + 0 + 28 \end{array}$$

$$x - 10r(28)$$

21.

$$\frac{x^2 - 2x}{x + 3}$$

$$\begin{array}{r} x + 3 \overline{) x^2 - 2x} \\ \underline{-x^2 - 3x} \\ 0 - 5x \\ \underline{0 + 5x + 15} \\ 0 + 0 + 15 \end{array}$$

$$x - 5r(15)$$

22.

$$\frac{x^2 + 1}{x - 1}$$

$$\begin{array}{r} x+1 \\ x-1 \overline{)x^2 + 1} \\ \underline{-x^2 + x} \phantom{+ 1} \\ 0 + x + 1 \\ \underline{0 - x + 1} \\ 0 + 0 + 2 \end{array}$$

$$x + 1r2$$

Mixed Review

23.

$$y = \frac{k}{x}$$

$$200 = \frac{k}{16.75}$$

$$k = 3350$$

$$60 = \frac{3350}{x}$$

$$60x = 3350$$

$$x \approx 55.8333$$

The pressure is approximately 55.8333 psi.

24. No, it is not a polynomial because it has a negative exponent.

25.

$$-\frac{3}{4} * x = -1$$

$$x = \frac{4}{3}$$

26. 36 two person teams can be made by nine individuals.

27.

$$-4 = \frac{\sqrt{m-3}}{-2}$$

$$8 = \sqrt{m-3}$$

$$8^2 = (\sqrt{m-3})^2$$

$$64 = m - 3$$

$$m = 67$$

**Lesson 12.4**  
**Rational Expressions**

1.

$$\frac{4}{2x-8} = \frac{4}{2(x-4)} = \frac{2}{x-4}$$

2.

$$\frac{x^2+2x}{x} = \frac{x(x+2)}{x} = x+2$$

3.

$$\frac{9x+3}{12x+4} = \frac{3(3x+1)}{4(3x+1)} = \frac{3}{4}$$

4.

$$\frac{6x^2+2x}{4x} = \frac{2x(3x+1)}{4x} = \frac{3x+1}{2}$$

5.

$$\frac{x-2}{x^2-4x+4} = \frac{x-2}{(x-2)(x-2)} = \frac{1}{x-2}$$

6.

$$\frac{x^2-9}{5x+15} = \frac{(x+3)(x-3)}{5(x+3)} = \frac{x-3}{5}$$

7.

$$\frac{x^2+6x+8}{x^2+4x} = \frac{(x+4)(x+2)}{x(x+4)} = \frac{x+2}{x}$$

8.

$$\frac{2x^2+10x}{x^2+10x+25} = \frac{2x(x+5)}{(x+5)(x+5)} = \frac{2x}{x+5}$$

9.

$$\frac{x^2 + 6x + 5}{x^2 - x - 2} = \frac{(x+1)(x+5)}{(x+1)(x-2)} = \frac{x+5}{x-2}$$

10.

$$\frac{x^2 - 16}{x^2 + 2x - 8} = \frac{(x-4)(x+4)}{(x-4)(x+2)} = \frac{x+4}{x+2}$$

11.

$$\frac{3x^2 + 3x - 18}{2x^2 + 5x - 3} = \frac{3(x^2 + x - 6)}{(2x+3)(x-1)} = \frac{3(x+6)(x-1)}{(2x+3)(x-1)} = \frac{3(x+6)}{2x+3}$$

12.

$$\frac{x^3 + x^2 - 20x}{6x^2 + 6x - 120} = \frac{x(x^2 + x - 20)}{6(x^2 + x - 20)} = \frac{x}{6}$$

13.

$$\frac{2}{x}$$

$$x = 0$$

14.

$$\frac{4}{x+2}$$

$$x + 2 = 0$$

$$x = -2$$

15.

$$\frac{2x-1}{(x-1)^2}$$

$$x-1=0$$

$$x=1$$

16.

$$\frac{3x+1}{x^2-4}$$

$$x^2-4$$

$$(x-2)(x+2)$$

$$x-2=0$$

$$x=2$$

$$x+2=0$$

$$x=-2$$

$$x=2,-2$$

17.

$$\frac{x^2}{x^2+9}$$

There is no value for x that will make the denominator zero, therefore there are no excluded values.

18.

$$\frac{2x^2 + 3x - 1}{x^2 - 3x - 28}$$

$$x^2 - 3x - 28$$
$$(x - 7)(x + 4)$$

$$x - 7 = 0$$

$$x = 7$$

$$x + 4 = 0$$

$$x = -4$$

$$x = 7, -4$$

19.

$$\frac{5x^3 - 4}{x^2 + 3x}$$

$$x(x + 3)$$

$$x = 0$$

$$x + 3 = 0$$

$$x = -3$$

$$x = 0, -3$$

20.

$$\frac{9}{x^3 + 11x^2 + 30x}$$

$$x^3 + 11x^2 + 30x$$

$$x(x^2 + 11x + 30)$$

$$x(x+5)(x+6)$$

$$x = 0$$

$$x + 5 = 0$$

$$x = -5$$

$$x + 6 = 0$$

$$x = -6$$

$$x = 0, -5, -6$$

21.

$$\frac{4x-1}{x^2+3x-5}$$

$$x^2 + 3x - 5 = 0$$

$$x^2 + 3x = 5$$

$$x^2 + 3x + 2.25 = 5 + 2.25$$

$$(x+1.5)^2 = 7.25$$

$$\sqrt{(x+1.5)^2} = \sqrt{7.25}$$

$$x+1.5 = \sqrt{7.25}$$

$$x+1.5 \approx \pm 2.6926$$

$$x+1.5 \approx 2.6926$$

$$x \approx 1.1926$$

$$x+1.5 \approx -2.6926$$

$$x \approx -4.6926$$

$$x \approx 1.1926, -4.6926$$

22.

$$\frac{5x+11}{3x^2-2x-4}$$

$$3x^2 - 2x - 4 = 0$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-4)}}{2(3)}$$

$$y = \frac{2 \pm \sqrt{4 + 48}}{6}$$

$$y = \frac{2 \pm \sqrt{52}}{6}$$

$$y = \frac{2 + \sqrt{52}}{6} \approx \frac{2 + 7.2111}{6} \approx \frac{9.2111}{6} \approx 1.5352$$

$$y = \frac{2 - \sqrt{52}}{6} \approx \frac{2 - 7.2111}{6} \approx \frac{-5.2111}{6} \approx -0.8685$$

$$x = 1.5352, -0.8685$$

23.

$$\frac{x^2 - 1}{2x^3 + x + 3}$$

$$2x^3 + x + 3$$
$$(2x^2 - 2x + 3)(x + 1)$$

$$x + 1 = 0$$

$$x = -1$$

$$2x^2 - 2x + 3 = 0$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(3)}}{2(2)}$$

$$y = \frac{2 \pm \sqrt{4 - 24}}{4}$$

$$y = \frac{2 \pm \sqrt{-20}}{4}$$

*Undefined*

Excluded value:  $x = -1$

24.

$$\frac{12}{x^2 + 6x + 1}$$

$$x^2 + 6x + 1 = 0$$

$$x^2 + 6x = -1$$

$$x^2 + 6x + 9 = -1 + 9$$

$$(x + 3)^2 = 8$$

$$\sqrt{(x + 3)^2} = \sqrt{8}$$

$$x + 3 = \sqrt{8}$$

$$x + 3 \approx \pm 2.8284$$

$$x + 3 \approx 2.8284$$

$$x \approx -0.1716$$

$$x + 3 \approx -2.8284$$

$$x \approx -5.8284$$

$$x \approx -0.1716, -5.8284$$

25.

$$\frac{1}{R_c} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{10} = \frac{1}{25} + \frac{1}{R_2}$$

$$\frac{1}{R_2} = \frac{3}{50}$$

$$50 = 3R_2$$

$$R_2 \approx 16.6667\Omega$$

26.

$$F = \frac{G(m_1 m_2)}{(d^2)} \rightarrow y = \frac{k}{x}$$

$$20 = \frac{G(m_1 m_2)}{(d^2)} \rightarrow 20 = \frac{k}{x} \rightarrow k = 20x$$

$$y = \frac{G(m_1 m_2)}{2(d^2)} \rightarrow y = \frac{k}{2x} \rightarrow y = \frac{20x}{2x} \rightarrow y = 10$$

When the distance is doubled, the force of attraction is cut in half = 10 Newtons.

27.

$$F = \frac{G(m_1 m_2)}{(d^2)} \rightarrow y = \frac{k}{x}$$

$$36 = \frac{G(m_1 m_2)}{(d^2)} \rightarrow 36 = \frac{k}{x} \rightarrow k = 36x$$

$$y = \frac{G(2m_1 2m_2)}{2(d^2)} \rightarrow y = \frac{2k}{2x} \rightarrow y = \frac{2(36x)}{2x} \rightarrow y = \frac{72x}{2x} = 36$$

If both the distance and the mass are doubled, the force of attraction stays the same.

28.

$$SA = 4\pi r^2$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{SA}{V} = \frac{4\pi r^2}{\frac{4}{3}\pi r^3} = \frac{3}{r}$$

29.

$$\text{side} = x$$

$$A = lwh$$

$$A = x \cdot x \cdot x$$

$$A = x^3$$

$$\text{side} = 2x$$

$$A = lwh$$

$$A = 2x \cdot 2x \cdot 2x$$

$$A = 8x^3$$

$$\frac{x^3}{8x^3} = \frac{1}{8}$$

30.

$$V_1 = \frac{4}{3}\pi r^3$$

$$V_2 = \frac{4}{3}\pi(r-4)^3 = \frac{4}{3}\pi(r^3 - 12r^2 + 48r - 64)$$

$$\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi(r^3 - 12r^2 + 48r - 64)} = \frac{r^3}{r^3 - 12r^2 + 48r - 64} = \frac{1}{-12r^2 + 48r - 64}$$

### Mixed Review

31. "Four times p to the sixth power plus 7 times p to the third power minus 9"

32.

$$(4b^2 + b + 7b^3) + (5b^2 - 6b^4 + b^3)$$

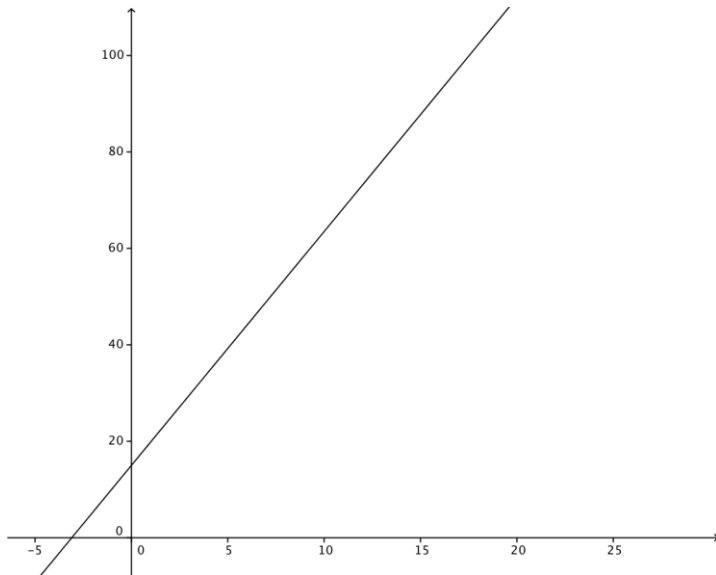
$$-6b^4 + (7b^3 + b^3) + (4b^2 + 5b^2) + b$$

$$-6b^4 + 8b^3 + 9b^2 + b$$

33. Zero Product Property: For any real numbers  $a$ ,  $(0) \times a = 0$ .

34. The zero product property cannot be used in this situation because none of the elements are zero.

35.  $E = 4.85h + 15$



36.

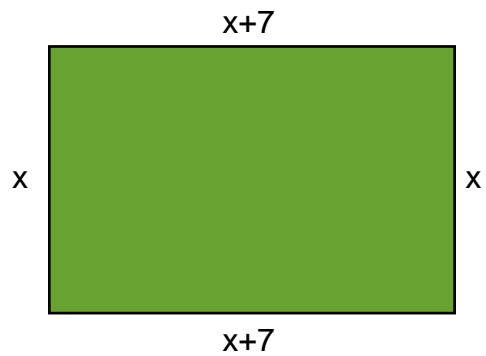
$$(-4x^2 + 8x - 1)(-7x^2 + 6x + 8)$$

$$28x^4 - 24x^3 - 32x^2 - 56x^3 + 48x^2 + 64x + 7x^2 - 6x - 8$$

$$28x^4 - 24x^3 - 56x^3 - 32x^2 + 48x^2 + 7x^2 + 64x - 6x - 8$$

$$28x^4 - 80x^3 + 23x^2 + 58x - 8$$

37.



$$P = w + w + l + l$$

$$P = 65$$

$$x + x + (x + 7) + (x + 7) = 65$$

$$x + x + x + 7 + x + 7 = 65$$

$$4x + 14 = 65$$

$$4x = 51$$

$$x = 12.75$$

$$A = l(w)$$

$$A = (12.75 + 7)(12.75)$$

$$A = 19.75(12.75)$$

$$A = 251.8125$$

The dimensions are 12.75 yards and 19.75 yards.

**Lesson 12.5**  
**Multiplication and Division of Rational Expressions**

1.

$$\frac{x^3}{2y^3} \cdot \frac{2y^2}{x} = \frac{2x^3y^2}{2xy^3} = \frac{x^2}{y}$$

2.

$$2xy \div \frac{2x^2}{y} = \frac{2xy}{1} \div \frac{2x^2}{y} = \frac{2xy}{1} \times \frac{y}{2x^2} = \frac{2xy^2}{2x^2} = \frac{y^2}{x}$$

3.

$$\frac{2x}{y^2} \cdot \frac{4y}{5x} = \frac{8xy}{5xy^2} = \frac{8}{5y}$$

4.

$$2xy \cdot \frac{2y^2}{x^3} = \frac{4xy^3}{x^3} = \frac{4y^3}{x^2}$$

5.

$$\frac{4y^2-1}{y^2-9} \cdot \frac{y-3}{2y-1} = \frac{(2y-1)(2y+1)}{(y-3)(y+3)} \cdot \frac{y-3}{2y-1} = \frac{2y+1}{y+3}$$

6.

$$\frac{6ab}{a^2} \cdot \frac{a^3b}{3b^2} = \frac{6a^4b^2}{3a^2b^2} = 2a^2$$

7.

$$\frac{x^2}{x-1} \div \frac{x}{x^2+x-2} = \frac{x^2}{x-1} \cdot \frac{x^2+x-2}{x} = \frac{x^2}{x-1} \cdot \frac{(x+2)(x-1)}{x} = \frac{x^2(x+2)}{x} = x(x+2) = x^2 + 2x$$

8.

$$\frac{33a^2}{-5} \cdot \frac{20}{11a^3} = \frac{660a^2}{-55a^3} = -\frac{12}{a}$$

9.

$$\frac{a^2 + 2ab + b^2}{ab^2 - a^2b} \div (a+b) = \frac{a^2 + 2ab + b^2}{ab^2 - a^2b} \times \frac{1}{a+b} = \frac{(a+b)(a+b)}{ab(b-a)} \times \frac{1}{a+b} = \frac{a+b}{ab(b-a)}$$

10.

$$\frac{2x^2 + 2x - 24}{x^2 + 3x} \cdot \frac{x^2 + x - 6}{x+4} = \frac{2(x+4)(x-3)}{x(x+3)} \cdot \frac{(x+3)(x-2)}{x+4} = \frac{2(x-3)(x-2)}{x(x+4)} = \frac{2x^2 - 10x + 12}{x^2 + 4x}$$

11.

$$\frac{3-x}{3x-5} \div \frac{x^2-9}{2x^2-8x-10} = \frac{3-x}{3x-5} \times \frac{2x^2-8x-10}{x^2-9} = \frac{3-x}{3x-5} \times \frac{2(x-5)(x+1)}{(x+3)(x-3)} = \frac{2(x-5)(x+1)(3-x)}{(x+3)(x-3)(3x-5)} = \frac{-2x^3 - 2x^2 - 34x - 30}{3x^3 - 5x^2 - 27x + 45}$$

12.

$$\frac{x^2 - 25}{x+3} \div (x-5) = \frac{(x+5)(x-5)}{x+3} \times \frac{1}{x-5} = \frac{x+5}{x+3}$$

13.

$$\frac{2x+1}{2x-1} \div \frac{4x^2-1}{1-2x} = \frac{2x+1}{2x-1} \times \frac{1-2x}{4x^2-1} = \frac{2x+1}{2x-1} \times \frac{1-2x}{(2x+1)(2x-1)} = \frac{1-2x}{(2x-1)(2x-1)} = \frac{1-2x}{4x^2-4x-1}$$

14.

$$\frac{x}{x-5} \cdot \frac{x^2-8x+15}{x^2-3x} = \frac{x}{x-5} \cdot \frac{(x-5)(x-3)}{x(x-3)} = 1$$

15.

$$\frac{3x^2 + 5x - 12}{x^2 - 9} \div \frac{3x - 4}{3x + 4} = \frac{(3x-4)(x+3)}{(x+3)(x-3)} \times \frac{3x+4}{3x-4} = \frac{3x+4}{x-3}$$

16.

$$\frac{5x^2 + 16x + 3}{36x^2 - 25} \cdot (6x^2 + 5x) = \frac{5x^2 + 16x + 3}{(6x-5)(6x+5)} \cdot \frac{x(6x+5)}{1} = \frac{x(5x^2 + 16x + 3)}{6x-5} = \frac{5x^3 + 16x^2 + 3x}{6x-5}$$

17.

$$\frac{x^2 + 7x + 10}{x^2 - 9} \cdot \frac{x^2 - 3x}{3x^2 + 4x - 4} = \frac{(x+5)(x+2)}{(x-3)(x+3)} \cdot \frac{x(x-3)}{(3x-2)(x+2)} = \frac{x(x+5)}{(x+3)(3x-2)} = \frac{x^2 + 5x}{3x^2 + 7x - 6}$$

18.

$$\frac{x^2 + x - 12}{x^2 + 4x + 4} \div \frac{x-3}{x+2} = \frac{(x+4)(x-3)}{(x+2)(x+2)} \times \frac{x+2}{x-3} = \frac{(x+4)}{(x+2)}$$

19.

$$\frac{x^4 - 16}{x^2 - 9} \div \frac{x^2 + 4}{x^2 + 6x + 9} = \frac{(x^2 + 4)(x^2 - 4)}{(x+3)(x-3)} \times \frac{(x+3)(x+3)}{x^2 + 4} = \frac{(x^2 - 4)(x+3)}{(x-3)} = \frac{x^3 + 3x^2 - 4x - 12}{x-3}$$

20.

$$\frac{x^2 + 8x + 16}{7x^2 + 9x + 2} \times \frac{7x + 2}{x^2 + 4x} = \frac{(x+4)(x+4)}{(7x+2)(x+1)} \times \frac{7x+2}{x(x+4)} = \frac{x+4}{x(x+1)} = \frac{x+4}{x^2 + x}$$

21.

$$\text{sugar} = s$$

$$\text{flour} = f$$

$$\frac{\frac{5}{2}f}{s} \times \frac{\frac{10}{3}}{\frac{10}{3}} = \frac{5f}{3\frac{1}{3}s}$$

22.

$$\frac{x}{15x} = \frac{1}{15}$$

23.

$$I = \frac{V}{R_c}$$

$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_c = R_{tot}$$

$$I = \frac{V}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}}$$

## Mixed Review

24.

$$t = \frac{k}{s}$$

$$3.6 = \frac{k}{65}$$

$$k = 234$$

$$t = \frac{234}{78}$$

$$t = 3$$

It would take 3 hours.

25.

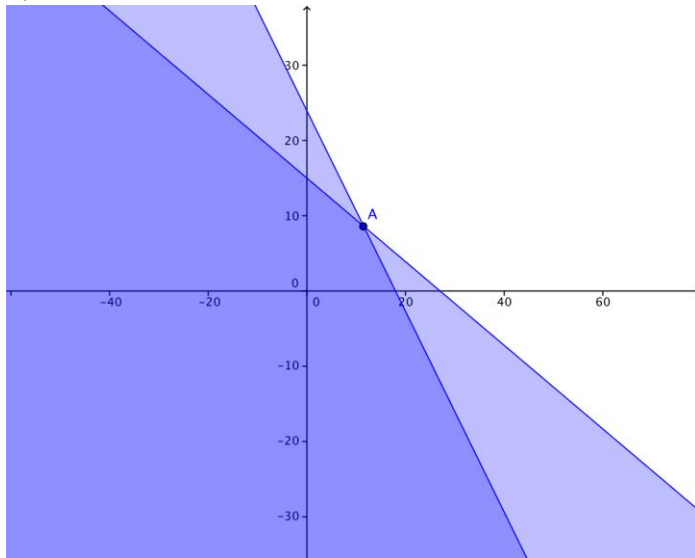
Let the number of arrangements of the the first type be  $x$  and that of the second type be  $y$ .

The total number of mums used is  $8x + 6y$

The total number of black-eyed susans is  $5x + 9y$

$$\begin{cases} 8x + 6y \leq 144 \\ 5x + 9y \leq 135 \end{cases}$$

$$\begin{cases} 8x + 6y \leq 144 \\ 5x + 9y \leq 135 \end{cases}$$



If  $x = 9$  and  $y = 10$  both the conditions are fulfilled.

$$8x + 6y \leq 144$$

$$8(9) + 6(10) \leq 144$$

$$72 + 60 \leq 144$$

$$\underline{132 \leq 144}$$

$$5x + 9y \leq 135$$

$$5(9) + 9(10) \leq 135$$

$$45 + 90 \leq 135$$

$$135 \leq 135$$

$$R = 49.99x + 38.95y$$

$$R = 49.99(9) + 38.95(10)$$

$$R = 449.91 + 389.50$$

The revenue is  $R = \$839.41$

$$R = 49.99x + 38.95y$$

$$R = 49.99(10) + 38.95(9)$$

$$R = 499.90 + 350.55$$

If  $x$  is made 10, only 9 of  $y$  can be made but the revenue is increased  $R = \$850.45$

To maximize revenue the nursery should sell 10 of the first arrangement and 9 of the second arrangement.

26.

$$-24 \geq |2r + 3|$$

No Solution (an absolute value cannot be negative, or less than a negative)

27.

$$5x + 9y = -36$$

$$9y = -5x - 36$$

$$y = -\frac{5}{9}x - 4$$

$$m = -\frac{5}{9}$$

Since parallel lines have the same slope, a line parallel to  $5x + 9y = -36$  has a slope of  $-5/9$ .

28.

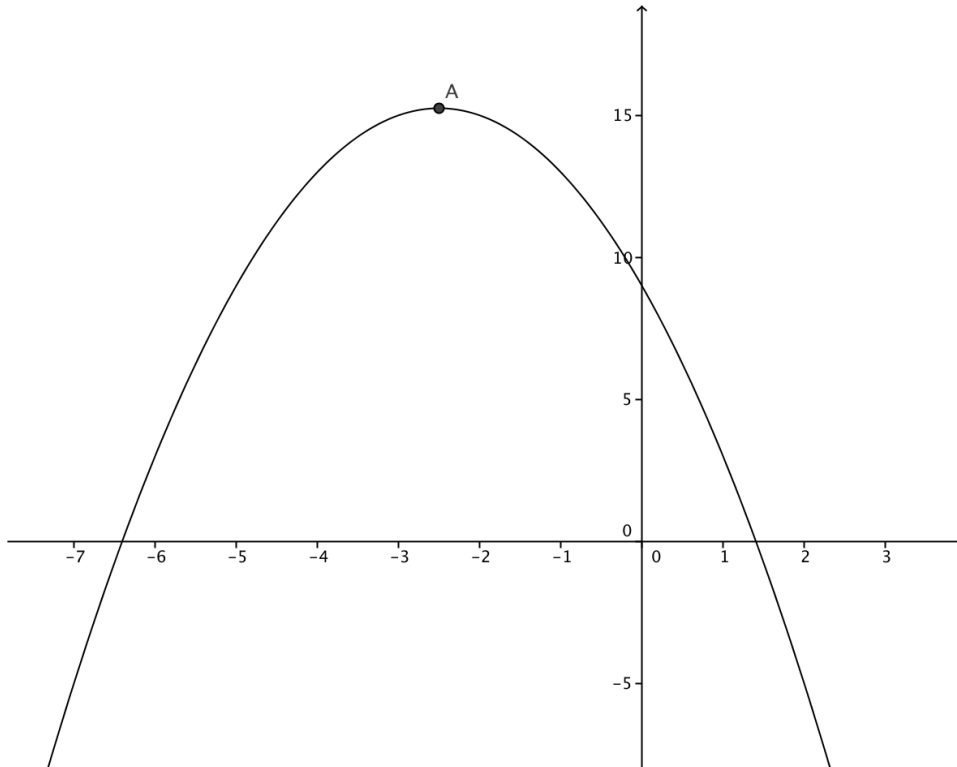
$$3 + 5d = -d - (3x - 3)$$

$$3 + 5d = -d - 3x + 3$$

$$6d = -3x$$

$$d = -0.5x$$

29.  $y - 9 = -x^2 - 5x$



Domain:  $x =$  all real numbers

Range:  $y \leq 15.25$

30.

$$y^2 - 16y + 3 = 4$$

$$y^2 - 16y = 1$$

$$y^2 - 16y + 64 = 1 + 64$$

$$(y - 8)^2 = 65$$

$$x = (y - 8)^2 - 65$$

$$y = a(x - h)^2 + k$$

$$(h, k) = (8, -65)$$

### Quick Quiz

1.

$$h = \frac{k}{t}$$

$$-16 = \frac{k}{-0.05153}$$

$$k = 0.82448$$

$$1.45 = \frac{0.82448}{t}$$

$$1.45t = 0.82448$$

$$t \approx 0.5686$$

2.  $f(x) = \frac{-5}{x^2 - 25}$

1.

$$x^2 - 25 = 0$$

$$(x + 5)(x - 5) = 0$$

$$x + 5 = 0$$

$$x = -5$$

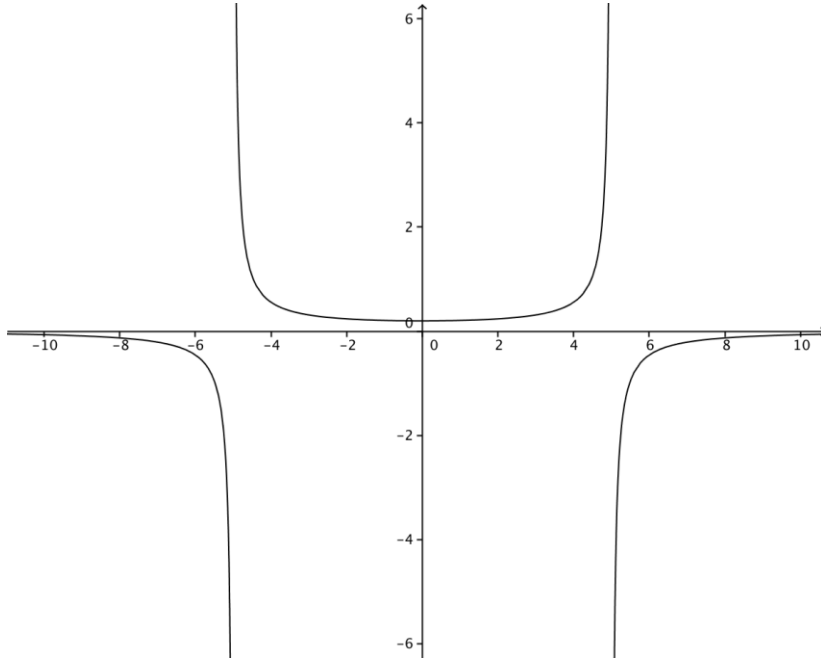
$$x - 5 = 0$$

$$x = 5$$

$$x = -5, 5$$

2.  
 $x=5$   
 $x=-5$

3.



4.  
Domain:  $x \neq 5, x \neq -5$   
Range:  $y \neq 0$

## Lesson 12.6

### Addition and Subtraction of Rational Expressions

1.

$$\frac{5}{24} - \frac{7}{24} = \frac{-2}{24} = \frac{-1}{12}$$

2.

$$\frac{10}{21} + \frac{9}{35} = \frac{50}{105} + \frac{27}{105} = \frac{77}{105} = \frac{11}{15}$$

3.

$$\frac{5}{2x+3} + \frac{2}{2x+3} = \frac{7}{2x+3}$$

4.

$$\frac{3x-1}{x+9} - \frac{4x+3}{x+9} = \frac{-x-4}{x+9} = \frac{-(x+4)}{x+9}$$

5.

$$\frac{4x+7}{2x^2} - \frac{3x-4}{2x^2} = \frac{x+11}{2x^2}$$

6.

$$\frac{x^2}{x+5} - \frac{25}{x+5} = \frac{x^2-25}{x+5} = \frac{(x+5)(x-5)}{x+5} = x-5$$

7.

$$\frac{2x}{x-4} + \frac{x}{4-x} = \frac{2x}{x-4} + \frac{x}{4-x} \cdot \frac{-1}{-1} = \frac{2x}{x-4} + \frac{-x}{x-4} = \frac{x}{x-4}$$

8.

$$\frac{10}{3x-1} - \frac{7}{1-3x} = \frac{10}{3x-1} - \frac{7}{1-3x} \cdot \frac{-1}{-1} = \frac{10}{3x-1} - \frac{-7}{3x-1} = \frac{17}{3x-1}$$

9.

$$\frac{5}{2x+3} - 3 = \frac{5}{2x+3} - 3 \cdot \frac{2x+3}{2x+3} = \frac{5}{2x+3} - \frac{3(2x+3)}{2x+3} = \frac{5}{2x+3} - \frac{6x+9}{2x+3} = \frac{-6x-4}{2x+3}$$

10.

$$\frac{5x+1}{x+4} + 2 = \frac{5x+1}{x+4} + 2 \cdot \frac{x+4}{x+4} = \frac{5x+1}{x+4} + \frac{2(x+4)}{x+4} = \frac{5x+1}{x+4} + \frac{2x+8}{x+4} = \frac{7x+9}{x+4}$$

11.

$$\frac{1}{x} + \frac{2}{3x} = \frac{3}{3} \cdot \frac{1}{x} + \frac{2}{3x} = \frac{3}{3x} + \frac{2}{3x} = \frac{5}{3x}$$

12.

$$\frac{4}{5x^2} - \frac{2}{7x^3} = \frac{4}{5x^2} \cdot \frac{7x}{7x} - \frac{2}{7x^3} \cdot \frac{5}{5} = \frac{28x}{35x^3} - \frac{10}{35x^3} = \frac{28x-10}{35x^3}$$

13.

$$\frac{4x}{x+1} - \frac{2}{2(x+1)} = \frac{4x}{x+1} \cdot \frac{2}{2} - \frac{2}{2(x+1)} = \frac{8x}{2(x+1)} - \frac{2}{2(x+1)} = \frac{8x-2}{2(x+1)} = \frac{2(4x-1)}{2(x+1)} = \frac{4x-1}{x+1}$$

14.

$$\frac{10}{x+5} + \frac{2}{x+2} = \frac{10}{x+5} \cdot \frac{x+2}{x+2} + \frac{2}{x+2} \cdot \frac{x+5}{x+5} = \frac{10(x+2)}{(x+5)(x+2)} + \frac{2(x+5)}{(x+2)(x+5)} = \frac{10x+20}{(x+5)(x+2)} + \frac{2x+10}{(x+2)(x+5)} = \frac{12x+30}{(x+2)(x+5)} = \frac{6(2x+5)}{(x+2)(x+5)}$$

15.

$$\begin{aligned} \frac{2x}{x-3} - \frac{3x}{x+4} &= \\ \frac{2x}{x-3} \cdot \frac{x+4}{x+4} - \frac{3x}{x+4} \cdot \frac{x-3}{x-3} &= \\ \frac{2x(x+4)}{(x-3)(x+4)} - \frac{3x(x-3)}{(x-3)(x+4)} &= \\ \frac{2x^2+8x}{(x-3)(x+4)} - \frac{3x^2-9x}{(x-3)(x+4)} &= \\ \frac{-x^2+17x}{(x-3)(x+4)} &= \end{aligned}$$

16.

$$\begin{aligned} & \frac{4x-3}{2x+1} + \frac{x+2}{x-9} \\ & \frac{4x-3}{2x+1} \cdot \frac{x-9}{x-9} + \frac{x+2}{x-9} \cdot \frac{2x+1}{2x+1} \\ & \frac{(4x-3)(x-9)}{(2x+1)(x-9)} + \frac{(x+2)(2x+1)}{(2x+1)(x-9)} \\ & \frac{4x^2 - 39x + 27}{(2x+1)(x-9)} + \frac{2x^2 + 5x + 2}{(2x+1)(x-9)} \\ & \frac{6x^2 - 34x + 29}{(2x+1)(x-9)} \end{aligned}$$

17.

$$\begin{aligned} & \frac{x^2}{x+4} - \frac{3x^2}{4x-1} \\ & \frac{x^2}{x+4} \cdot \frac{4x-1}{4x-1} - \frac{3x^2}{4x-1} \cdot \frac{x+4}{x+4} \\ & \frac{x^2(4x-1)}{(x+4)(4x-1)} - \frac{3x^2(x+4)}{(x+4)(4x-1)} \\ & \frac{4x^3 - x^2}{(x+4)(4x-1)} - \frac{3x^3 + 12x^2}{(x+4)(4x-1)} \\ & \frac{x^3 - 13x^2}{(x+4)(4x-1)} \end{aligned}$$

18.

$$\begin{aligned} & \frac{2}{5x+2} - \frac{x+1}{x^2} \\ & \frac{2}{5x+2} \cdot \frac{x^2}{x^2} - \frac{x+1}{x^2} \cdot \frac{5x+2}{5x+2} \\ & \frac{2x^2}{x^2(5x+2)} - \frac{(x+1)(5x+2)}{x^2(5x+2)} \\ & \frac{2x^2}{x^2(5x+2)} - \frac{5x^2 + 7x + 2}{x^2(5x+2)} \\ & \frac{-3x^2 - 7x - 2}{x^2(5x+2)} \end{aligned}$$

19.

$$\begin{aligned} & \frac{x+4}{2x} + \frac{2}{9x} \\ & \frac{x+4}{2x} \cdot \frac{9}{9} + \frac{2}{9x} \cdot \frac{2}{2} \\ & \frac{9(x+4)}{18x} + \frac{2(2)}{18x} \\ & \frac{9x+36}{18x} + \frac{4}{18x} \\ & \frac{9x+40}{18x} \end{aligned}$$

20.

$$\begin{aligned} & \frac{5x+3}{x^2+x} + \frac{2x+1}{x} \\ & \frac{5x+3}{x(x+1)} + \frac{2x+1}{x} \cdot \frac{x+1}{x+1} \\ & \frac{5x+3}{x(x+1)} + \frac{(2x+1)(x+1)}{x(x+1)} \\ & \frac{5x+3}{x(x+1)} + \frac{2x^2+3x+1}{x(x+1)} \\ & \frac{2x^2+8x+4}{x(x+1)} \end{aligned}$$

21.

$$\begin{aligned} & \frac{4}{(x+1)(x-1)} - \frac{5}{(x+1)(x+2)} \\ & \frac{4}{(x+1)(x-1)} \cdot \frac{x+2}{x+2} - \frac{5}{(x+1)(x+2)} \cdot \frac{x-1}{x-1} \\ & \frac{4(x+2)}{(x+1)(x-1)(x+2)} - \frac{5(x-1)}{(x+1)(x-1)(x+2)} \\ & \frac{4x+8}{(x+1)(x-1)(x+2)} - \frac{5x-5}{(x+1)(x-1)(x+2)} \\ & \frac{-x+13}{(x+1)(x-1)(x+2)} \end{aligned}$$

22.

$$\begin{aligned} & \frac{2x}{(x+2)(3x-4)} + \frac{7x}{(3x-4)^2} \\ & \frac{2x}{(x+2)(3x-4)} \cdot \frac{3x-4}{3x-4} + \frac{7x}{(3x-4)^2} \cdot \frac{x+2}{x+2} \\ & \frac{2x(3x-4)}{(x+2)(3x-4)^2} + \frac{7x(x+2)}{(x+2)(3x-4)^2} \\ & \frac{6x^2 - 8x}{(x+2)(3x-4)^2} + \frac{7x^2 + 14x}{(x+2)(3x-4)^2} \\ & \frac{13x^2 + 6x}{(x+2)(3x-4)^2} \end{aligned}$$

23.

$$\begin{aligned} & \frac{3x+5}{x(x-1)} - \frac{9x-1}{(x-1)^2} \\ & \frac{3x+5}{x(x-1)} \cdot \frac{x-1}{x-1} - \frac{9x-1}{(x-1)^2} \cdot \frac{x}{x} \\ & \frac{(3x+5)(x-1)}{x(x-1)^2} - \frac{x(9x-1)}{x(x-1)^2} \\ & \frac{3x^2 + 2x - 5}{x(x-1)^2} - \frac{9x^2 - x}{x(x-1)^2} \\ & \frac{-6x^2 + 3x - 5}{x(x-1)^2} \end{aligned}$$

24.

$$\begin{aligned} & \frac{1}{(x-2)(x-3)} + \frac{4}{(2x+5)(x-6)} \\ & \frac{1}{(x-2)(x-3)} \cdot \frac{(2x+5)(x-6)}{(2x+5)(x-6)} + \frac{4}{(2x+5)(x-6)} \cdot \frac{(x-2)(x-3)}{(x-2)(x-3)} \\ & \frac{1(2x+5)(x-6)}{(x-2)(x-3)(2x+5)(x-6)} + \frac{4(x-2)(x-3)}{(x-2)(x-3)(2x+5)(x-6)} \\ & \frac{2x^2 - 7x - 30}{(x-2)(x-3)(2x+5)(x-6)} + \frac{4x^2 - 20x + 24}{(x-2)(x-3)(2x+5)(x-6)} \\ & \frac{6x^2 - 27x - 6}{(x-2)(x-3)(2x+5)(x-6)} \end{aligned}$$

25.

$$\frac{3x-2}{x-2} + \frac{1}{x^2-4x+4}$$

$$\frac{3x-2}{x-2} + \frac{1}{(x-2)^2}$$

$$\frac{3x-2}{x-2} \cdot \frac{x-2}{x-2} + \frac{1}{(x-2)^2}$$

$$\frac{3x^2-8x+4}{(x-2)^2} + \frac{1}{(x-2)^2}$$

$$\frac{3x^2-8x+5}{(x-2)^2}$$

26.

$$\frac{-x^2}{x^2-7x+6} + x-4$$

$$\frac{-x^2}{(x-6)(x-1)} + \frac{(x-4)(x-6)(x-1)}{(x-6)(x-1)}$$

$$\frac{-x^2}{(x-6)(x-1)} + \frac{x^3-11x^2+34x-24}{(x-6)(x-1)}$$

$$\frac{x^3-12x^2+34x-24}{(x-6)(x-1)}$$

27.

$$\begin{aligned}
 & \frac{2x}{x^2+10x+25} - \frac{3x}{2x^2+7x-15} \\
 & \frac{2x}{(x+5)(x+5)} - \frac{3x}{(2x-3)(x+5)} \\
 & \frac{2x}{(x+5)(x+5)} \cdot \frac{2x-3}{2x-3} - \frac{3x}{(2x-3)(x+5)} \cdot \frac{x+5}{x+5} \\
 & \frac{2x(2x-3)}{(2x-3)(x+5)^2} - \frac{3x(x+5)}{(2x-3)(x+5)^2} \\
 & \frac{4x^2-6x}{(2x-3)(x+5)^2} - \frac{3x^2+15x}{(2x-3)(x+5)^2} \\
 & \frac{x^2-21x}{(2x-3)(x+5)^2}
 \end{aligned}$$

28.

$$\begin{aligned}
 & \frac{1}{x^2-9} + \frac{2}{x^2+5x+6} \\
 & \frac{1}{(x+3)(x-3)} + \frac{2}{(x+3)(x+2)} \\
 & \frac{1}{(x+3)(x-3)} \cdot \frac{x+2}{x+2} + \frac{2}{(x+3)(x+2)} \cdot \frac{x-3}{x-3} \\
 & \frac{x+2}{(x+3)(x-3)(x+2)} + \frac{2x-6}{(x+3)(x-3)(x+2)} \\
 & \frac{3x-4}{(x+3)(x-3)(x+2)}
 \end{aligned}$$

29.

$$\begin{aligned} & \frac{-x+4}{2x^2-x-15} + \frac{x}{4x^2+8x-5} \\ & \frac{-x+4}{(2x+5)(x-3)} + \frac{x}{(2x+5)(2x-1)} \\ & \frac{-x+4}{(2x+5)(x-3)} \cdot \frac{2x-1}{2x-1} + \frac{x}{(2x+5)(2x-1)} \cdot \frac{x-3}{x-3} \\ & \frac{-x+4(2x-1)}{(2x+5)(2x-1)(x-3)} + \frac{x(x-3)}{(2x+5)(2x-1)(x-3)} \\ & \frac{-2x^2+9x-4}{(2x+5)(2x-1)(x-3)} + \frac{x^2-3x}{(2x+5)(2x-1)(x-3)} \\ & \frac{-x^2+6x-4}{(2x+5)(2x-1)(x-3)} \end{aligned}$$

30.

$$\begin{aligned} & \frac{4}{9x^2-49} - \frac{1}{3x^2+5x-28} \\ & \frac{4}{(3x-7)(3x+7)} - \frac{1}{(3x-7)(x+4)} \\ & \frac{4}{(3x-7)(3x+7)} \cdot \frac{x+4}{x+4} - \frac{1}{(3x-7)(x+4)} \cdot \frac{3x+7}{3x+7} \\ & \frac{4x+16}{(3x-7)(3x+7)(x+4)} - \frac{3x+7}{(3x-7)(3x+7)(x+4)} \\ & \frac{x+9}{(3x-7)(3x+7)(x+4)} \end{aligned}$$

31.

$x$

$x - 5$

$$\frac{1}{x} + \frac{1}{x-5} = \frac{13}{36}$$

$$\frac{1}{x} \cdot \frac{x(x-5)}{x(x-5)} + \frac{1}{x-5} \cdot \frac{x(x-5)}{x(x-5)} = \frac{13}{36}$$

$$\frac{x-5}{x(x-5)} + \frac{x}{x(x-5)} = \frac{13}{36}$$

$$\frac{x-5+x}{x(x-5)} = \frac{13}{36}$$

$$\frac{2x-5}{x(x-5)} = \frac{13}{36}$$

$$36(2x-5) = 13 \cdot x(x-5)$$

$$72x - 180 = 13 \cdot x^2 - 5x$$

$$72x - 180 = 13x^2 - 65x$$

$$0 = 13x^2 - 65x - 72x + 180$$

$$0 = 13x^2 - 137x + 180$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-137) \pm \sqrt{(-137)^2 - 4(13)(180)}}{2(13)}$$

$$x = \frac{137 \pm \sqrt{18769 - 9360}}{26}$$

$$x = \frac{137 \pm \sqrt{9409}}{26}$$

$$x = \frac{137 + \sqrt{9409}}{26} = \frac{137 + 97}{26} = \frac{234}{26} = 9$$

$$x = \frac{137 - \sqrt{9409}}{26} = \frac{137 - 97}{26} = \frac{40}{26} \approx 1.5385$$

The numbers can be 9 and 4 or 1.5385 and -3.4615.

32.

$x$

$8x$

$$\frac{1}{x} - \frac{1}{8x} = \frac{21}{20}$$

$$\frac{1}{x} \cdot \frac{8}{8} - \frac{1}{8x} = \frac{21}{20}$$

$$\frac{8}{8x} - \frac{1}{8x} = \frac{21}{20}$$

$$\frac{7}{8x} = \frac{21}{20}$$

$$7(20) = 21(8x)$$

$$140 = 168x$$

$$x = \frac{140}{168} = \frac{5}{6}$$

The two numbers are  $5/6$  and  $20/3$ .

33.

The first pipe fills the tank in 4 hours.

In one hour it fills  $1/4$  of the tank.

The second pipe empties the tank in 8 hours.

In one hour it empties  $1/8$  of the tank.

For the first pipe: time X rate of work equals  $t * \frac{1}{4} = \frac{t}{4}$

For the second pipe: time X rate of work equals  $t * \frac{1}{8} = \frac{t}{8}$

$$\frac{t}{4} - \frac{t}{8} = 1$$

$$\frac{2t}{8} - \frac{t}{8} = 1$$

$$\frac{2t - t}{8} = 1$$

$$\frac{t}{8} = 1$$

Since one pipe is filling and one pipe is emptying, subtract:  $t = 8$

With both pipes open it will take 8 hours to fill the tank.

34.

Stefan could do the whole work in 6 hrs.

In one hour he can do  $\frac{1}{6}$  of the work.

Misha could do the whole work in 5 hrs.

In one hour he can do  $\frac{1}{5}$  of the work.

Stefan works for 2.5 hours: time X rate of work equals  $2.5 * \frac{1}{6} = \frac{2.5}{6} = \frac{25}{60}$

When he finished the amount of work left equals  $1 - \frac{25}{60} = \frac{60}{60} - \frac{25}{60} = \frac{35}{60}$

Now Misha has the remaining work divided by his rate of work  $\frac{35}{60} \div \frac{1}{5} = \frac{35}{60} * \frac{5}{1} = \frac{175}{60} = 2.9167$

It takes Misha 2 hours and 55 minutes to finish washing the cars.

35.

Amanda can clear the snow in 3 hours.

In one hour she can clear  $\frac{1}{3}$  of the driveway.

Chyna can clear the snow in 4 hours.

In one hour she can clear  $\frac{1}{4}$  of the driveway.

Amanda works for one hour: time X rate of work equals  $1 * \frac{1}{3} = \frac{1}{3}$

After that first hour the amount of work left equals  $1 - \frac{1}{3} = \frac{3}{3} - \frac{1}{3} = \frac{2}{3}$

Now the two girls work together  $\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$

And we divide the remaining work by the rate when they work together:

$$\frac{2}{3} \div \frac{7}{12} = \frac{2}{3} * \frac{12}{7} = \frac{24}{7} = 3.4286$$

Add this to the hour that Amanda worked alone. It takes approximately 2 hours and 8 minutes.

36.

The first machine can finish the work in 10 hours.

In one hour it can do  $\frac{1}{10}$  of the work.

The second machine can finish the work in 14 hours.

In one hour it can do  $\frac{1}{14}$  of the work.

For the first 4 hours the machines work together: time X rate of work equals

$$4 * \left( \frac{1}{10} + \frac{1}{14} \right) = 4 * \left( \frac{14}{140} + \frac{10}{140} \right) = 4 * \left( \frac{24}{140} \right) = \frac{96}{140}$$

When the second machine breaks the amount of work left equals:  $1 - \frac{96}{140} = \frac{140}{140} - \frac{96}{140} = \frac{44}{140}$

Now the first machine has the remaining work divided by its rate of work

$$\frac{44}{140} \div \frac{1}{10} = \frac{44}{140} \times \frac{10}{1} = \frac{440}{140} = 3.14$$

The fast machine works for approximately 3 hours and 8 minutes to finish the quota.

#### Mixed Review

37.

In the first situation the population is increasing mathematically by 500 people each year regardless of the size at the time, while in the second situation the population is increasing by 5% each year, meaning that each year it increases by 5% of the current population, increasing by a larger amount each year.

1.  
 $P = p + 500t$   
 $p = 10,000$   
 $t = 7$

2.  
 $P = p(1.05)^x$   
 $p = 10,000$   
 $x = 7$

Statement 2 becomes larger than statement one by the second year.

38.

$$\frac{16x^2y^7}{-2x^8y} \cdot \frac{1}{2}x^{-10} = \frac{16x^2y^7}{-2x^8y} \cdot \frac{1}{2x^{10}} = \frac{16x^2y^7}{-4x^{18}y} = -\frac{4y^6}{x^{16}}$$

39.

$$-12 = j^2 - 8j$$

$$j^2 - 8j + 12 = 0$$

$$(j - 6)(j - 2) = 0$$

$$j - 6 = 0$$

$$j = 6$$

$$j - 2 = 0$$

$$j = 2$$

$$j = 2, j = 6$$

Factoring is the most obvious method (since the equation factors so easily) but any method used correctly would be acceptable.

40.

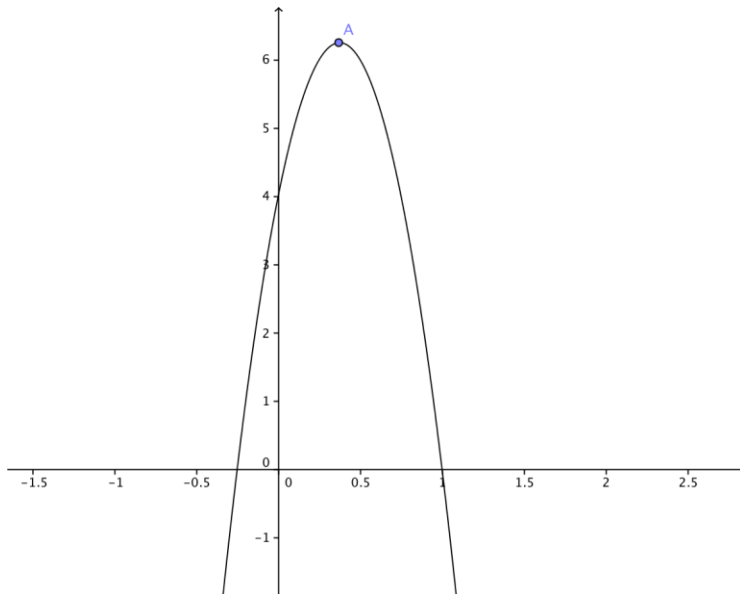
1.

$$h(t) = -\frac{1}{2}(g)t^2 + v_0t + h_0$$

$$h(t) = -\frac{1}{2}(32)t^2 + 12t + 4$$

$$h(t) = -16t^2 + 12t + 4$$

2.



41.

$$\frac{d}{s} = \frac{245}{5} = \frac{90}{x} =$$

$$245x = 450$$

$$x = 1.8367$$

42.

$$3.78 \div \frac{1}{2} = 7.56$$

$$7.56 \cdot 14 = 105.84$$

The true distance is 105.84 miles.

**Lesson 12.7**  
**Solution of Rational Equations**

1.

$$\frac{2x+1}{4} = \frac{x-3}{10}$$

$$10(2x+1) = 4(x-3)$$

$$20x+10 = 4x-12$$

$$16x = -22$$

$$x = -\frac{22}{16} = -\frac{11}{8}$$

2.

$$\frac{4x}{x+2} = \frac{5}{9}$$

$$4x(9) = 5(x+2)$$

$$36x = 5x+10$$

$$31x = 10$$

$$x = \frac{10}{31}$$

3.

$$\frac{5}{3x-4} = \frac{2}{x+1}$$

$$5(x+1) = 2(3x-4)$$

$$5x+5 = 6x-8$$

$$-x = -13$$

$$x = 13$$

4.

$$\frac{7x}{x-5} = \frac{x+3}{x}$$

$$7x(x) = (x+3)(x-5)$$

$$7x^2 = x^2 - 5x + 3x - 15$$

$$6x^2 - 2x - 15 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(6)(-15)}}{2(6)}$$

$$x = \frac{2 \pm \sqrt{4 + 360}}{12}$$

$$x = \frac{2 \pm \sqrt{364}}{12}$$

$$x = \frac{2 + \sqrt{364}}{12} \approx \frac{2 + 19.0788}{12} \approx \frac{21.0788}{12} \approx 1.7566$$

$$x = \frac{2 - \sqrt{364}}{12} \approx \frac{2 - 19.0788}{12} \approx \frac{-17.0788}{12} \approx -1.4232$$

5.

$$\frac{2}{x+3} - \frac{1}{x+4} = 0$$

$$\frac{2}{x+3} = \frac{1}{x+4}$$

$$2(x+4) = 1(x+3)$$

$$2x + 8 = x + 3$$

$$x = -5$$

6.

$$\frac{3x^2 + 2x - 1}{x^2 - 1} = -2$$

$$\frac{(3x-1)(x+1)}{(x+1)(x-1)} = -2$$

$$\frac{3x-1}{x-1} = -2$$

$$3x-1 = -2(x-1)$$

$$3x-1 = -2x+2$$

$$5x = 3$$

$$x = \frac{3}{5} = 0.6$$

7.

$$x + \frac{1}{x} = 2$$

$$x \cdot \frac{x}{x} + \frac{1}{x} = 2$$

$$\frac{x^2}{x} + \frac{1}{x} = 2$$

$$\frac{x^2 + 1}{x} = 2$$

$$x^2 + 1 = 2x$$

$$x^2 - 2x + 1 = 0$$

$$(x-1)(x-1) = 0$$

$$x-1 = 0$$

$$x = 1$$

8.

$$\frac{1}{x+1} - \frac{2}{x-1} = 3$$

$$\frac{1}{x+1} \cdot \frac{x-1}{x-1} - \frac{2}{x-1} \cdot \frac{x+1}{x+1} = 3$$

$$\frac{x-1}{(x-1)(x+1)} - \frac{2(x+1)}{(x-1)(x+1)} = 3$$

$$\frac{x-1-2x-2}{(x-1)(x+1)} = 3$$

$$\frac{-x-3}{(x-1)(x+1)} = 3$$

$$-x-3 = 3[(x-1)(x+1)]$$

$$-x-3 = 3(x^2-1)$$

$$-x-3 = 3x^2-3$$

$$0 = 3x^2+x$$

$$0 = x(3x+1)$$

$$x = 0$$

$$3x+1 = 0$$

$$3x = -1$$

$$x = -\frac{1}{3}$$

$$x = 0, x = -\frac{1}{3}$$

9.

$$\frac{1}{x} - \frac{x}{x-2} = 2$$

$$\frac{1}{x} \cdot \frac{x-2}{x-2} - \frac{x}{x-2} \cdot \frac{x}{x} = 2$$

$$\frac{x-2}{x(x-2)} - \frac{x^2}{x(x-2)} = 2$$

$$\frac{x-2-x^2}{x(x-2)} = 2$$

$$\frac{-x^2+x-2}{x(x-2)} = 2$$

$$-x^2+x-2 = 2[x(x-2)]$$

$$-x^2+x-2 = 2(x^2-2x)$$

$$-x^2+x-2 = 2x^2-4x$$

$$0 = 3x^2 - 5x + 2$$

$$0 = (3x-2)(x-1)$$

$$x-1 = 0$$

$$x = 1$$

$$3x-2 = 0$$

$$3x = 2$$

$$x = \frac{2}{3}$$

$$x = 1, x = \frac{2}{3}$$

10.

$$\frac{3}{2x-1} + \frac{2}{x+4} = 2$$

$$\frac{3}{2x-1} \cdot \frac{x+4}{x+4} + \frac{2}{x+4} \cdot \frac{2x-1}{2x-1} = 2$$

$$\frac{3(x+4)}{(2x-1)(x+4)} + \frac{2(2x-1)}{(2x-1)(x+4)} = 2$$

$$\frac{3x+12}{(2x-1)(x+4)} + \frac{4x-2}{(2x-1)(x+4)} = 2$$

$$\frac{3x+12+4x-2}{(2x-1)(x+4)} = 2$$

$$7x+10 = 2[(2x-1)(x+4)]$$

$$7x+10 = 2(2x^2 + 7x - 4)$$

$$7x+10 = 4x^2 + 14x - 8$$

$$0 = 4x^2 + 7x - 18$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{7^2 - 4(4)(-18)}}{2(4)}$$

$$x = \frac{-7 \pm \sqrt{49 + 288}}{8}$$

$$x = \frac{-7 \pm \sqrt{337}}{8}$$

$$x = \frac{-7 + \sqrt{337}}{8} \approx \frac{-7 + 18.3576}{8} \approx \frac{11.3576}{8} \approx 1.4197$$

$$x = \frac{-7 - \sqrt{337}}{8} \approx \frac{-7 - 18.3576}{8} \approx \frac{-25.3576}{8} \approx -3.1697$$

11.

$$\frac{2x}{x-1} - \frac{x}{3x+4} = 3$$

$$\frac{2x}{x-1} \cdot \frac{3x+4}{3x+4} - \frac{x}{3x+4} \cdot \frac{x-1}{x-1} = 3$$

$$\frac{2x(3x+4)}{(3x+4)(x-1)} - \frac{x(x-1)}{(3x+4)(x-1)} = 3$$

$$\frac{6x^2 + 8x}{(3x+4)(x-1)} - \frac{x^2 - x}{(3x+4)(x-1)} = 3$$

$$\frac{6x^2 + 8x - x^2 + x}{(3x+4)(x-1)} = 3$$

$$\frac{5x^2 + 9x}{(3x+4)(x-1)} = 3$$

$$5x^2 + 9x = 3[(3x+4)(x-1)]$$

$$5x^2 + 9x = 3(3x^2 + x - 4)$$

$$5x^2 + 9x = 9x^2 + 3x - 12$$

$$0 = 4x^2 - 6x - 12$$

$$0 = 2(2x^2 - 3x - 6)$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-6)}}{2(2)}$$

$$x = \frac{3 \pm \sqrt{9 + 48}}{4}$$

$$x = \frac{3 \pm \sqrt{57}}{4}$$

$$x = \frac{3 + \sqrt{57}}{4} \approx \frac{3 + 7.5498}{4} \approx \frac{10.5498}{4} \approx 2.6375$$

$$x = \frac{3 - \sqrt{57}}{4} \approx \frac{3 - 7.5498}{4} \approx \frac{-4.5498}{4} \approx -1.1375$$

12.

$$\frac{x+1}{x-1} + \frac{x-4}{x+4} = 3$$

$$\frac{x+1}{x-1} \cdot \frac{x+4}{x+4} + \frac{x-4}{x+4} \cdot \frac{x-1}{x-1} = 3$$

$$\frac{(x+1)(x+4)}{(x-1)(x+4)} + \frac{(x-4)(x-1)}{(x-1)(x+4)} = 3$$

$$\frac{x^2 + 5x + 4}{(x-1)(x+4)} + \frac{x^2 - 5x + 4}{(x-1)(x+4)} = 3$$

$$\frac{x^2 + 5x + 4 + x^2 - 5x + 4}{(x-1)(x+4)} = 3$$

$$\frac{2x^2 + 8}{(x-1)(x+4)} = 3$$

$$2x^2 + 8 = 3[(x-1)(x+4)]$$

$$2x^2 + 8 = 3(x^2 + 3x - 4)$$

$$2x^2 + 8 = 3x^2 + 9x - 12$$

$$0 = x^2 + 9x - 20$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-9 \pm \sqrt{9^2 - 4(1)(-20)}}{2(1)}$$

$$x = \frac{-9 \pm \sqrt{81 + 80}}{2}$$

$$x = \frac{-9 \pm \sqrt{161}}{2}$$

$$x = \frac{-9 + \sqrt{161}}{2} \approx \frac{-9 + 12.6886}{2} \approx \frac{3.6886}{2} \approx 1.8443$$

$$x = \frac{-9 - \sqrt{161}}{2} \approx \frac{-9 - 12.6886}{2} \approx \frac{-21.6886}{2} \approx -10.8443$$

13.

$$\frac{x}{x-2} + \frac{x}{x+3} = \frac{1}{x^2+x-6}$$

$$\frac{x}{x-2} + \frac{x}{x+3} = \frac{1}{(x-2)(x+3)}$$

$$(x-2)(x+3) \cdot \frac{x}{x-2} + (x-2)(x+3) \cdot \frac{x}{x+3} = (x-2)(x+3) \cdot \frac{1}{(x-2)(x+3)}$$

$$x(x+3) + x(x-2) = 1$$

$$x^2 + 3x + x^2 - 2x = 1$$

$$2x^2 + x - 1 = 0$$

$$(2x-1)(x+1) = 0$$

$$2x-1 = 0$$

$$2x = 1$$

$$x = \frac{1}{2}$$

$$x+1 = 0$$

$$x = -1$$

$$x = \frac{1}{2}, x = -1$$

14.

$$\frac{2}{x^2 + 4x + 3} = 2 + \frac{x-2}{x+3}$$

$$\frac{2}{(x+3)(x+1)} = 2 + \frac{x-2}{x+3}$$

$$(x+3)(x+1) \cdot \frac{2}{(x+3)(x+1)} = (x+3)(x+1) \cdot 2 + (x+3)(x+1) \cdot \frac{x-2}{x+3}$$

$$2 = 2[(x+3)(x+1)] + [(x+1)(x-2)]$$

$$2 = 2(x^2 + 4x + 3) + (x^2 - x - 2)$$

$$2 = 2x^2 + 8x + 6 + x^2 - x - 2$$

$$0 = 3x^2 + 7x + 2$$

$$0 = (3x+1)(x+2)$$

$$3x+1=0$$

$$3x=-1$$

$$x=-\frac{1}{3}$$

$$x+2=0$$

$$x=-2$$

$$x=-\frac{1}{3}, x=-2$$

15.

$$\frac{1}{x+5} - \frac{1}{x-5} = \frac{1-x}{x+5}$$

$$(x+5)(x-5) \cdot \frac{1}{x+5} - (x+5)(x-5) \cdot \frac{1}{x-5} = (x+5)(x-5) \cdot \frac{1-x}{x+5}$$

$$(x-5) - (x+5) = (x-5)(1-x)$$

$$x-5-x-5 = x-x^2-5+5x$$

$$0 = -x^2 + 4x - 5$$

$$0 = (x+5)(x-1)$$

$$x+5=0$$

$$x=-5$$

$$x-1=0$$

$$x=1$$

$$x=-5, x=1$$

16.

$$\frac{x}{x^2 - 36} + \frac{1}{x - 6} = \frac{1}{x + 6}$$

$$\frac{x}{(x - 6)(x + 6)} + \frac{1}{x - 6} = \frac{1}{x + 6}$$

$$(x - 6)(x + 6) \cdot \frac{x}{(x - 6)(x + 6)} + (x - 6)(x + 6) \cdot \frac{1}{x - 6} = (x - 6)(x + 6) \cdot \frac{1}{x + 6}$$

$$x + (x + 6) = x - 6$$

$$x + x + 6 = x - 6$$

$$2x + 6 = x - 6$$

$$x + 12 = 0$$

$$x = -12$$

17.

$$\frac{2x}{3x + 3} - \frac{1}{4x + 4} = \frac{2}{x + 1}$$

$$\frac{2x}{3(x + 1)} - \frac{1}{4(x + 1)} = \frac{2}{x + 1}$$

$$12(x + 1) \cdot \frac{2x}{3(x + 1)} - 12(x + 1) \cdot \frac{1}{4(x + 1)} = 12(x + 1) \cdot \frac{2}{x + 1}$$

$$4(2x) - 3 = 12(2)$$

$$8x - 3 = 24$$

$$8x = 27$$

$$x = 3.375$$

18.

$$\frac{-x}{x-2} + \frac{3x-1}{x+4} = \frac{1}{x^2+2x-8}$$

$$\frac{-x}{x-2} + \frac{3x-1}{x+4} = \frac{1}{(x-2)(x+4)}$$

$$(x-2)(x+4) \cdot \frac{-x}{x-2} + (x-2)(x+4) \cdot \frac{3x-1}{x+4} = (x-2)(x+4) \cdot \frac{1}{(x-2)(x+4)}$$

$$-x(x-4) + (3x-1)(x-2) = 1$$

$$(-x^2 + 4x) + (3x^2 - 7x + 2) = 1$$

$$-x^2 + 4x + 3x^2 - 7x + 2 = 1$$

$$2x^2 - 3x + 2 = 1$$

$$2x^2 - 3x + 1 = 0$$

$$(2x-1)(x-1) = 0$$

$$2x-1 = 0$$

$$2x = 1$$

$$x = \frac{1}{2}$$

$$x-1 = 0$$

$$x = 1$$

$$x = \frac{1}{2}, x = 1$$

19.

T = time it takes Maria to catch up to Juan

Juan's distance =  $3.5(T+2)$

Maria's distance =  $7T$

$$7T = 3.5(T+2)$$

$$7T = 3.5T + 7$$

$$3.5T = 7$$

$$T = 2$$

It takes Maria 2 hours to catch up so her distance will be  $7\text{mph}(2\text{hours}) = 14$  miles

At the same time Juan will have been walking for 4 hours so  $3.5\text{mph}(4\text{ hours}) = 14$  miles.

They will meet after 14 miles.

20.

Speed of the current =  $x$

Boat's speed in still water = 20 miles per hour

Upstream speed =  $x - 20$

Downstream speed =  $x + 20$

Upstream distance = 40 miles

Downstream distance = 60 miles

$$\frac{40}{x - 20} = \frac{60}{x + 20}$$

$$40(x + 20) = 60(x - 20)$$

$$40x + 800 = 60x - 1200$$

$$-20x = -2000$$

$$x = 100$$

The speed of the current is 100 mph.

21.

$T$  = the time it takes Paul's mother to catch up with Paul.

Paul's Distance =  $50(T + 2)$

Mom's distance =  $70T$

$$70T = 50(T + 2)$$

$$70T = 50T + 100$$

$$20T = 100$$

$$T = 5$$

22.

Airplane speed =  $x$

Wind speed = 30 miles per hour

Speed against the wind =  $x - 30$

Speed with the wind =  $x + 30$

Distance against the wind = 300 miles

Distance with the wind = 420 miles

$$\frac{300}{x - 30} = \frac{420}{x + 30}$$

$$300(x + 30) = 420(x - 30)$$

$$300x + 9000 = 420x - 12600$$

$$-120x = -21600$$

$$x = 180$$

The speed of the airplane with no wind is 180 mph.

23.

$x$

$x - 5$

$$\frac{420}{x-5} = \frac{420}{x} + 25$$

$$x(x-5) \cdot \frac{420}{x-5} = x(x-5) \cdot \frac{420}{x} + x(x-5) \cdot 25$$

$$420x = 420(x-5) + 25[x(x-5)]$$

$$420x = 420x - 2100 + 25(x^2 - 5x)$$

$$420x = 420x - 2100 + 25x^2 - 125x$$

$$0 = 25x^2 - 125x - 2100$$

$$0 = 25(x^2 - 5x - 84)$$

$$0 = 25(x-12)(x+7)$$

$$x-12=0$$

$$x=12$$

$$x+7=0$$

$$x=-7$$

The original group had 12 people in it.

24.

$x$

$x + 30$

$$\frac{2250}{x+30} = \frac{2250}{x} - 20$$

$$x(x+30) \cdot \frac{2250}{x+30} = x(x+30) \cdot \frac{2250}{x} - x(x+30) \cdot 20$$

$$2250x = 2250(x+30) - 20[x(x+30)]$$

$$2250x = 2250x + 67500 - 20(x^2 + 30x)$$

$$2250x = 2250x + 67500 - 20x^2 - 600x$$

$$0 = -20x^2 - 600x + 67500$$

$$0 = -20(x^2 + 30x - 3375)$$

$$0 = -20(x-45)(x+75)$$

$$x + 75 = 0$$

$$x = -75$$

$$x - 45 = 0$$

$$x = 45$$

The organization has 45 members.

### Mixed Review

25.

$$-2 \frac{9}{10} \div -\frac{15}{8} = -\frac{29}{10} \times -\frac{8}{15} = \frac{232}{150} = \frac{116}{75} = 1 \frac{41}{75}$$

26.

$$-1.5(-3\frac{4}{5} + g) = \frac{201}{20}$$

$$-1.5(-3.8 + g) = 10.05$$

$$5.7 - 1.5g = 10.05$$

$$-1.5g = 4.35$$

$$g = -2.9$$

27.

$$6x^2 + 3x + 4 = 0$$

$$b^2 - 4ac = 3^2 - 4(6)(4) = 9 - 96 = -87$$

There are no roots.

28.

$$\frac{6b}{2b+2} + 3$$

$$\frac{6b}{2(b+1)} + 3$$

$$\frac{3b}{(b+1)} + 3$$

$$\frac{3b}{(b+1)} + 3 \cdot \frac{b+1}{b+1}$$

$$\frac{3b}{b+1} + \frac{3b+3}{b+1} = \frac{3b+3b+3}{b+1} = \frac{6b+3}{b+1} = \frac{3(2b+1)}{b+1}$$

29.

$$\frac{8}{2x-4} - \frac{5x}{x-5}$$

$$\frac{8}{2x-4} \cdot \frac{x-5}{x-5} - \frac{5x}{x-5} \cdot \frac{2x-4}{2x-4}$$

$$\frac{8(x-5)}{(2x-4)(x-5)} - \frac{5x(2x-4)}{(2x-4)(x-5)}$$

$$\frac{8x-40}{(2x-4)(x-5)} - \frac{10x^2-20x}{(2x-4)(x-5)}$$

$$\frac{8x-40-10x^2+20x}{(2x-4)(x-5)}$$

$$\frac{-10x^2+28x-40}{(2x-4)(x-5)}$$

30.

$$(7x^2 + 16x - 10) \div (x + 3)$$

$$\begin{array}{r} 7x - 5 \\ x + 3 \overline{) 7x^2 + 16x - 10} \\ \underline{-7x^2 - 21x} \phantom{-10} \\ 0 - 5x - 10 \\ \underline{0 + 5x + 15} \\ 0 + 0 + 5 \end{array}$$

$$7x - 5(r5)$$

31.

$$(n - 1) * (3n + 2)(n - 4)$$

$$(n - 1) * (3n^2 - 10n - 8)$$

$$3n^3 - 10n^2 - 8n - 3n^2 - 10n - 8$$

$$3n^3 - 13n^2 - 18n - 8$$

## Lesson 12.8

### Surveys and Samples

1. The two most common types of sampling methods are random and stratified.

Random sampling is when everyone in the population has a equal chance of being chosen.

Stratified sampling is actively ensuring that people from different groups are in the sample.

-If you needed to survey a city about a new road project you would likely choose random sampling as it would be almost impossible to break an entire city into strata that would accurately represent the citizens of the city, all of whom would be affected by a road project.

2. A biased survey is when one or more subgroups of the population are either over represented or under represented in the sample. Bias can be eliminated by not cherry picking, by eliminating the non-response bias, and by not using a self selected sample. Also, it is important to carefully word questions so that they do not create bias.

3. Surveys are conducted by face to face interviews or self administered surveys.

Face to Face interviews have a high response rate, but are very time consuming. (any accurate answers from the list given in the lesson is acceptable)

Self administered surveys may get more honest results due to anonymity, but have lower response rates. (any accurate answers from the list given in the lesson is acceptable)

Surveys can also be conducted by phone. (other accurate answers would be acceptable)

4. Some keys to recognizing biased questions are finding polarizing language that assume emotion, finding a reference to an authority or apparent authority, finding phrasing that suggests the person asking the question already knows the answer, or finding ambiguous phrasing that often includes double negatives.

Answers to the second part will vary some may include: refusing to answer the question, pointing out the bias, etc

5. The most reliable sample would be choosing a stratified sample that includes equal numbers of boys and girls and equal numbers of students from each grade level and/or class.

6.

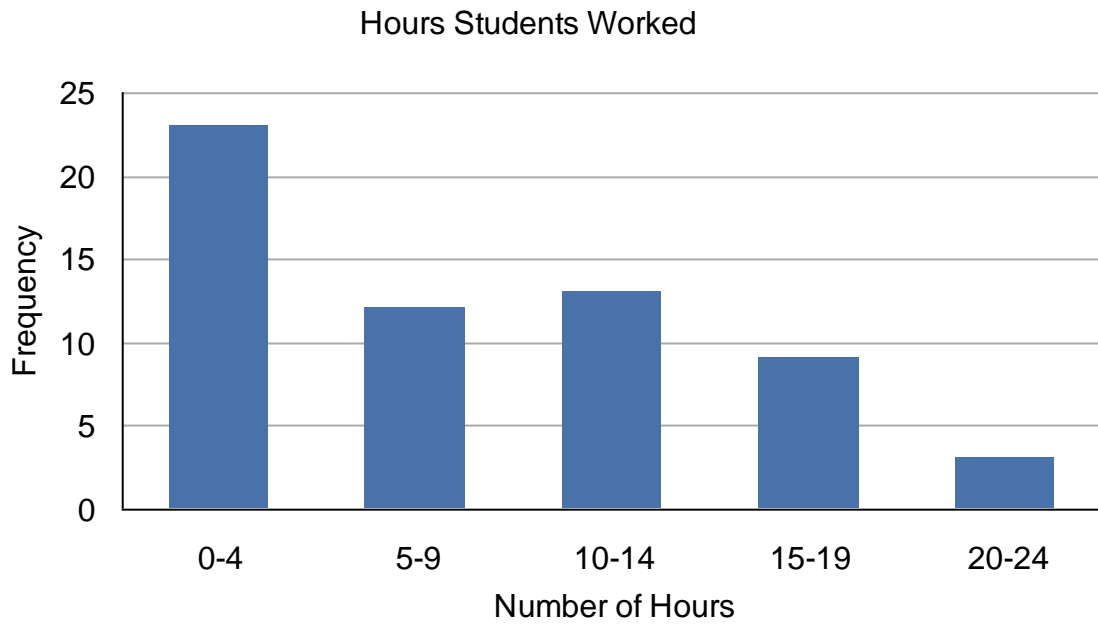
(a) not a good sample because you can't compare the choices of the wealthy with the choices of others.

(b) a biased sample because pedestrians will naturally want a crossing, your sample would also have to include the others affected by this like drivers and local businesses.

(c) this sample is pretty good, as long as it has a large enough sample that takes into account more than gender (age, occupation, geography, family status,etc) A random sample might be more effective



(c)



(d)

0,0,0,0,0,0,0,0,0,0,  
0,0,0,0,0,0,0,0,0,0,  
2,4,4,5,5,5,5,5,6,6,  
7,7,8,8,8,10,10,10,10,10,  
10,10,10,11,12,12,12,12,12,15,  
15,15,15,15,16,18,18,20,20,22

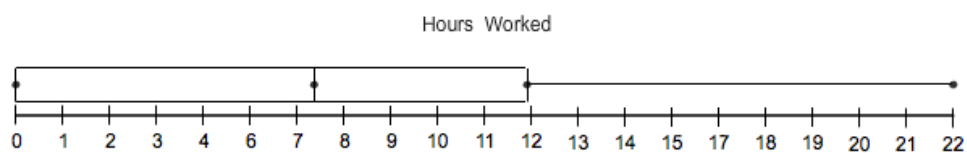
*Minimum* = 0

$Q_1$  = 0

*Median* = 6.5

$Q_3$  = 12

*Maximum* = 22



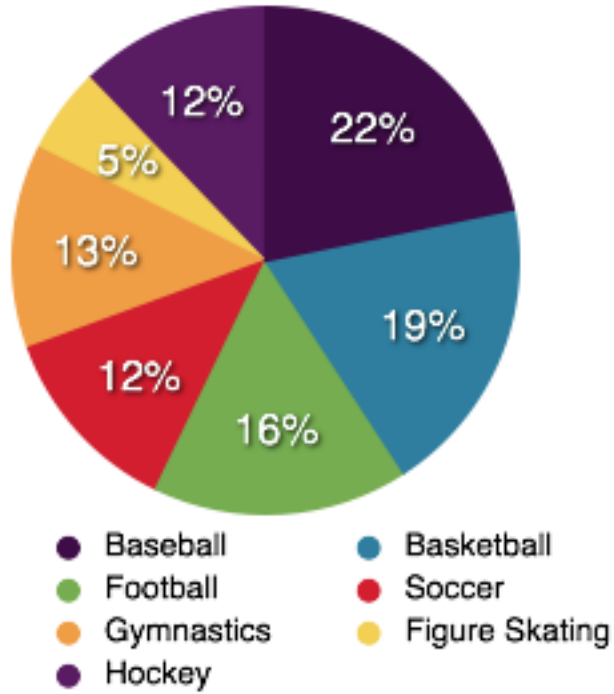
9. (Answers will vary, below are some possibilities)

- 1- The greatest number of students enjoy playing baseball.
- 2- The least number of students enjoy fencing.
- 3- A little more than half the students enjoy either baseball or soccer.
- 4- A little more than a quarter of the students enjoy either basketball or football.
- 5- A great majority of the students prefer team to individual sports.

10.

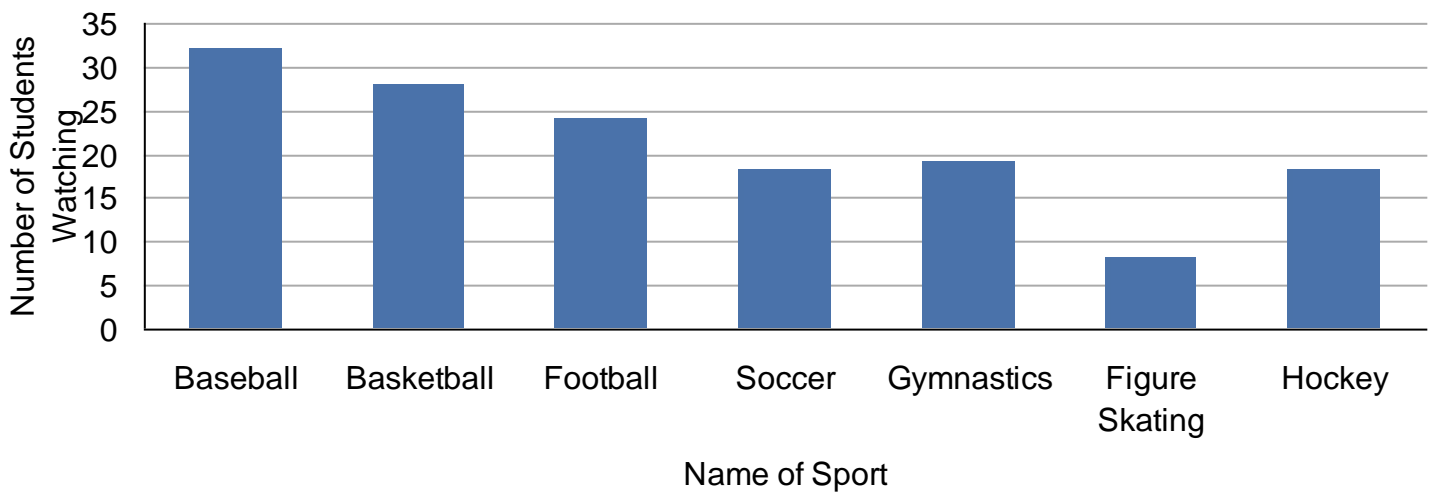
1.

Sports Students Watch on TV



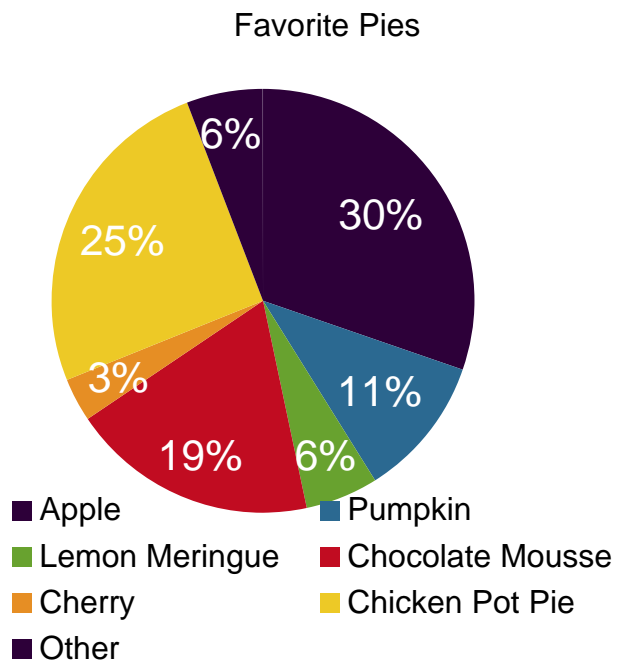
2.

Sports Students Watch on TV



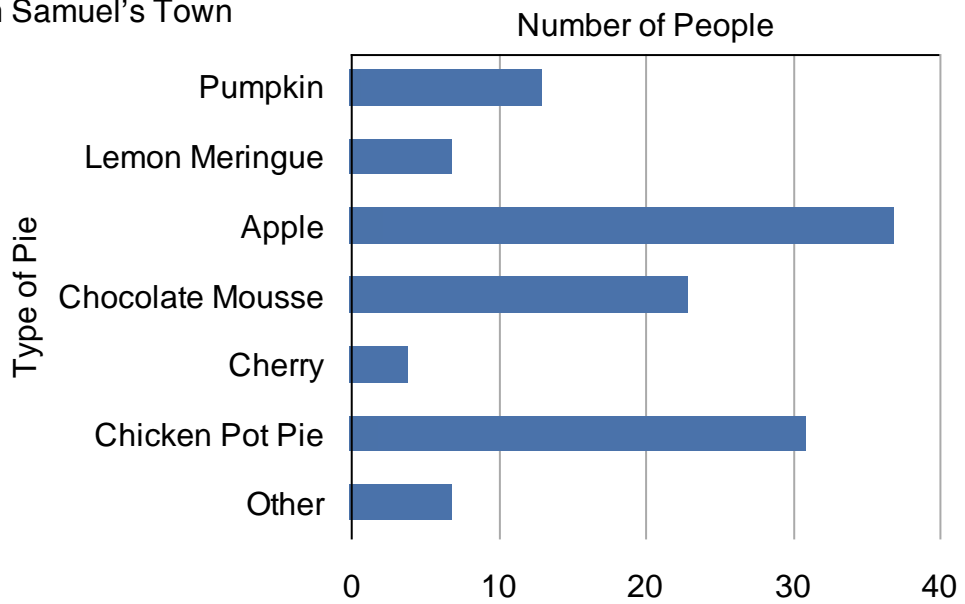
11.

1.



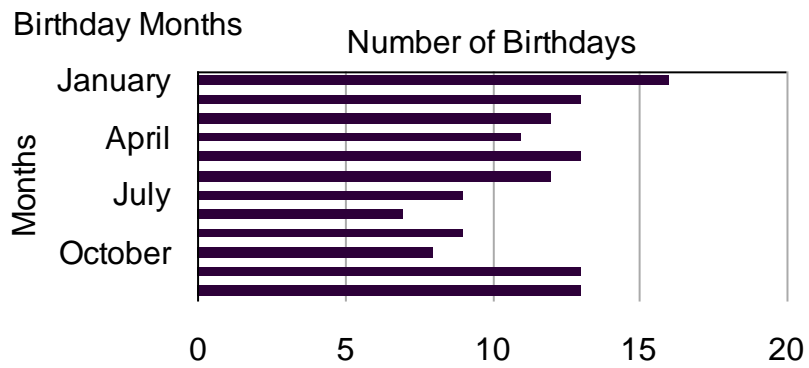
2.

Favorite Pies in Samuel's Town

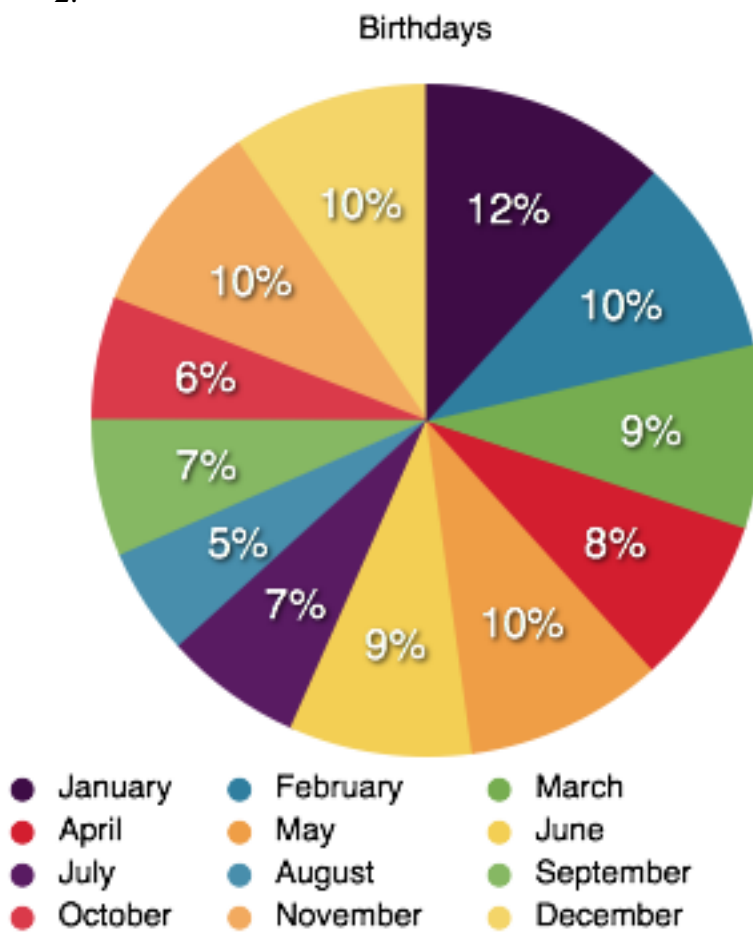


12.

1.



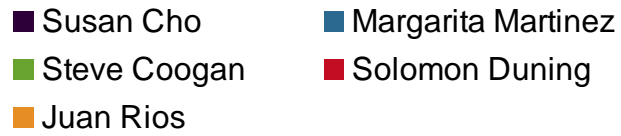
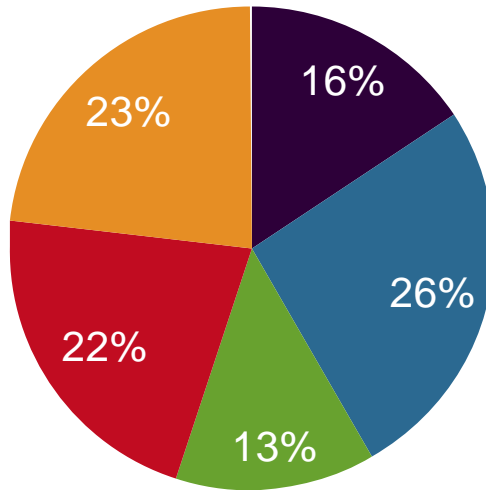
2.



13.

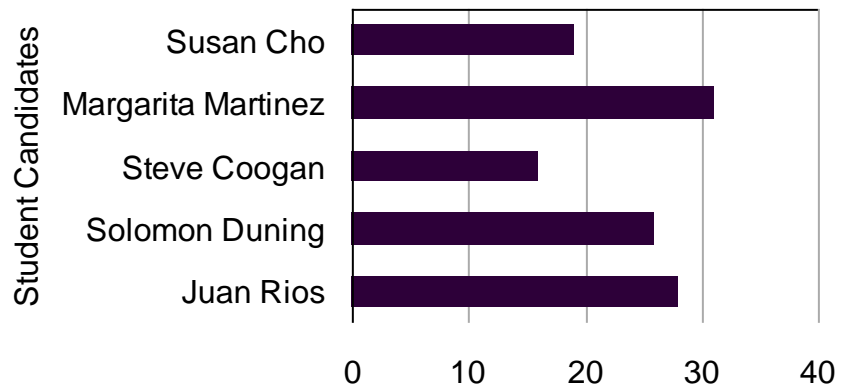
1.

Voting Poll



2.

Voting Poll Results Number of Voters



14.

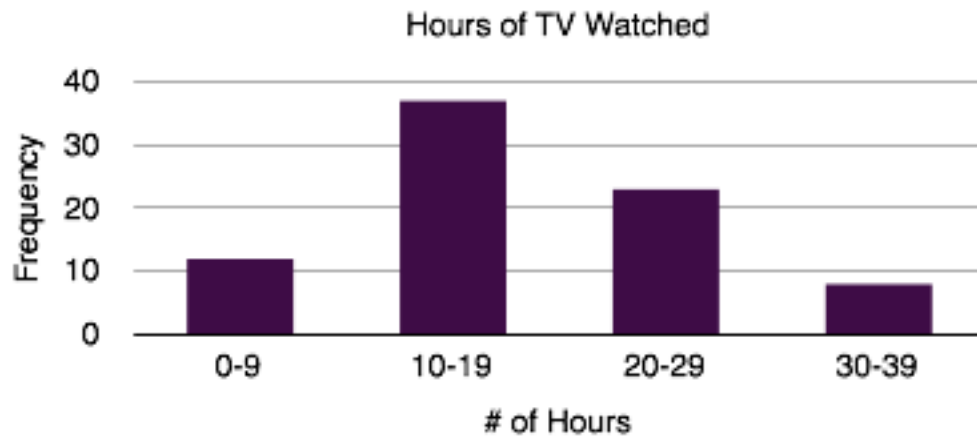
1.

Hours of TV Watched	
Stem	Leaf
0	0,0,5,6,6,6,8,8,8,8,9,9
1	0,0,0,0,0,0,0,0,0,0,0,2,2,2,2,2,2,2,2,2,2,4,5,5,5,5,5,5,6,6,6,8,8,8,8
2	0,0,0,0,0,0,0,2,2,2,2,4,4,4,4,4,5,5,5,6,8,8
3	0,0,2,2,2,2,4,8

2.

Hours of TV	Frequency
0-9	12
10-19	37
20-29	23
30-39	8

3.



4.  
 0,0,5,6,6,6,8,8,8,8,  
 9,9,10,10,10,10,10,10,10,  
 10,10,10,12,12,12,12,12,12,  
 12,12,12,12,14,15,15,15,15,  
 15,15,16,16,16,18,18,18,18,20,  
 20,20,20,20,20,20,22,22,22,22,  
 24,24,24,24,24,24,25,25,25,26,  
 28,28,30,30,32,32,32,32,34,38

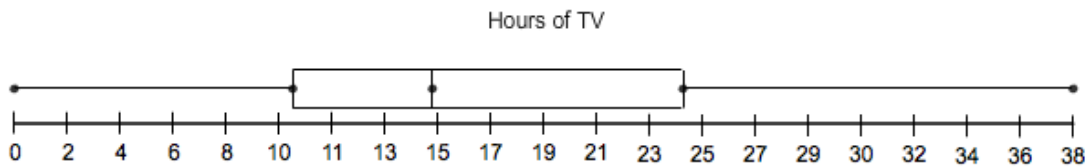
*Minimum* = 0

$Q_1 = 10$

*Median* = 15

$Q_3 = 23$

*Maximum* = 38



15. Answers will vary

16. Answers will vary

17. Answers will vary

18.

(a) Yes biased.

(b) Yes biased.

19. Answers will vary.

Mixed Review

20.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{1 - (-6)}{8 - 4} = \frac{1 + 6}{8 - 4} = \frac{7}{4} = 1\frac{3}{4}$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{7}{4}(x - 8)$$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -\frac{4}{7}(x - 0)$$

$$(a) \quad y = -\frac{4}{7}x$$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{7}{4}(x - 4)$$

$$y = \frac{7}{4}(x - 4)$$

$$(b) \quad y = \frac{7}{4}x - 7$$

21.  $\sqrt[3]{64} = 4$

It is a real number, a rational number, an integer, a whole number, and a counting number.

22.

First Trip:

Speed = time/ distance

distance = 22 miles

time = 45 minutes

Speed =  $45/22 = 2.0454$  mph

Second Trip

Time = speed X distance

distance = 22 miles

time = 70 minutes

speed = wind minus the ferry =  $w - 2.0454$

$70 = 22(w - 2.0454)$

$70 = 22w - 45$

$115 = 22w$

$w = 5.2272$

The ferry speed is approximately 2.0454 mph and the wind speed is approximately 5.2272 mph.

23.

$$\frac{6a}{a-1} = \frac{7}{a+7}$$

$$6a(a+7) = 7(a-1)$$

$$6a^2 + 42a = 7a - 7$$

$$6a^2 + 35a + 7 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-35 \pm \sqrt{35^2 - 4(6)(7)}}{2(6)}$$

$$x = \frac{-35 \pm \sqrt{1225 - 168}}{12}$$

$$x = \frac{-35 \pm \sqrt{1057}}{12}$$

$$x = \frac{-35 + \sqrt{1057}}{12} \approx \frac{-35 + 32.5115}{12} \approx \frac{-2.4885}{12} \approx -0.2074$$

$$x = \frac{-35 - \sqrt{1057}}{12} \approx \frac{-35 - 32.5115}{12} \approx \frac{-67.5115}{12} \approx -5.6259$$

24.

$$\frac{7}{18x^2} \cdot \frac{9x}{14} = \frac{1}{2x} \cdot \frac{1}{2} = \frac{1}{4x}$$

25.

$$\frac{2w^2 - 9w + 18}{w - 2}$$

$$\begin{array}{r} 2w - 5 \\ w - 2 \overline{) 2w^2 - 9w + 18} \\ \underline{-2w^2 + 4w} \phantom{+ 18} \\ 0 - 5w + 18 \\ \underline{0 + 5w - 10} \\ 0 + 0 + 8 \end{array}$$

$$2w - 5(r8)$$

26.

(a) It is a linear function. This situation can be written as  $h=16s$  which is a line.

$$h = 16s$$

$$16(4) = 64$$

(b)  $68.5 - 64 = 4.5$  The ballon started at 4.5 meters high.

**Lesson 12.9**  
**Chapter 12 Review**

1. Inverse variation is when the relationship between two variables is such that they change in opposite directions. When one variable goes up, the other goes down and when one variable goes down, the other goes up.
2. Asymptotes are the excluded values of an inverse variation equation. The graph of such an equation will approach the value of the asymptote but never touch or cross it.
3. A Hyperbola is the form the graph of an inverse function takes. It has branches in two quadrants and is contained by vertical and horizontal asymptotes.
4. Points of discontinuity (or excluded values) are the values for which the denominator of the rational function is zero. These create the asymptotes.
5. The Least Common Multiple is the smallest number that all of a set of numbers can divide equally into. It is used to create lowest common denominators.
6. Random sampling is when subjects for a survey are chosen at random with no consideration of their background or characteristics. In a random sample everyone in the population has an equal chance of being chosen.
7. Stratified sampling is when the surveyor divides the population by characteristic or strata and chooses participants by ensuring that each strata is equally represented.
8. Biased is when a survey has one or more subgroups that are either over represented or under represented.
9. Cherry Picking is when you chose your sample from a group that is more likely to answer the way you want. Instead of random or carefully stratified samples, you choose a sample from the group that best meets your needs and will likely provide the results you desire.
10. If  $k < 0$  then the branches of the hyperbola will be in quadrants II and IV.
11. inverse variation
12. direct variation
13. inverse variation
14. no relationship

15.

$$y = \frac{k}{x}$$

$$\frac{2}{15} = \frac{k}{5}$$

$$10 = 15k$$

$$k = \frac{10}{15} = \frac{2}{3}$$

$$y = \frac{\frac{2}{3}}{-\frac{1}{2}} = \frac{2}{3} \div -\frac{1}{2} = \frac{2}{3} * -\frac{2}{1} = -\frac{4}{3}$$

16.

$$y = \frac{k}{\sqrt{x}}$$

$$0.5625 = \frac{k}{\sqrt{16}}$$

$$0.5625 = \frac{k}{\pm 4}$$

$$k = \pm 2.25$$

$$y = \frac{\pm 2.25}{\frac{1}{8}} = \pm \frac{9}{4} \div \frac{1}{8} = \pm \frac{9}{4} * \frac{8}{1} = \pm 18$$

17.

Days to build the house = d

# of volunteers = v

constant of variation = k

$$d = \frac{k}{v}$$

$$8 = \frac{k}{20}$$

$$k = 160$$

$$d = \frac{160}{16} = 10$$

18.

distance from seesaw = d

weight in kg = w

k = constant of variation

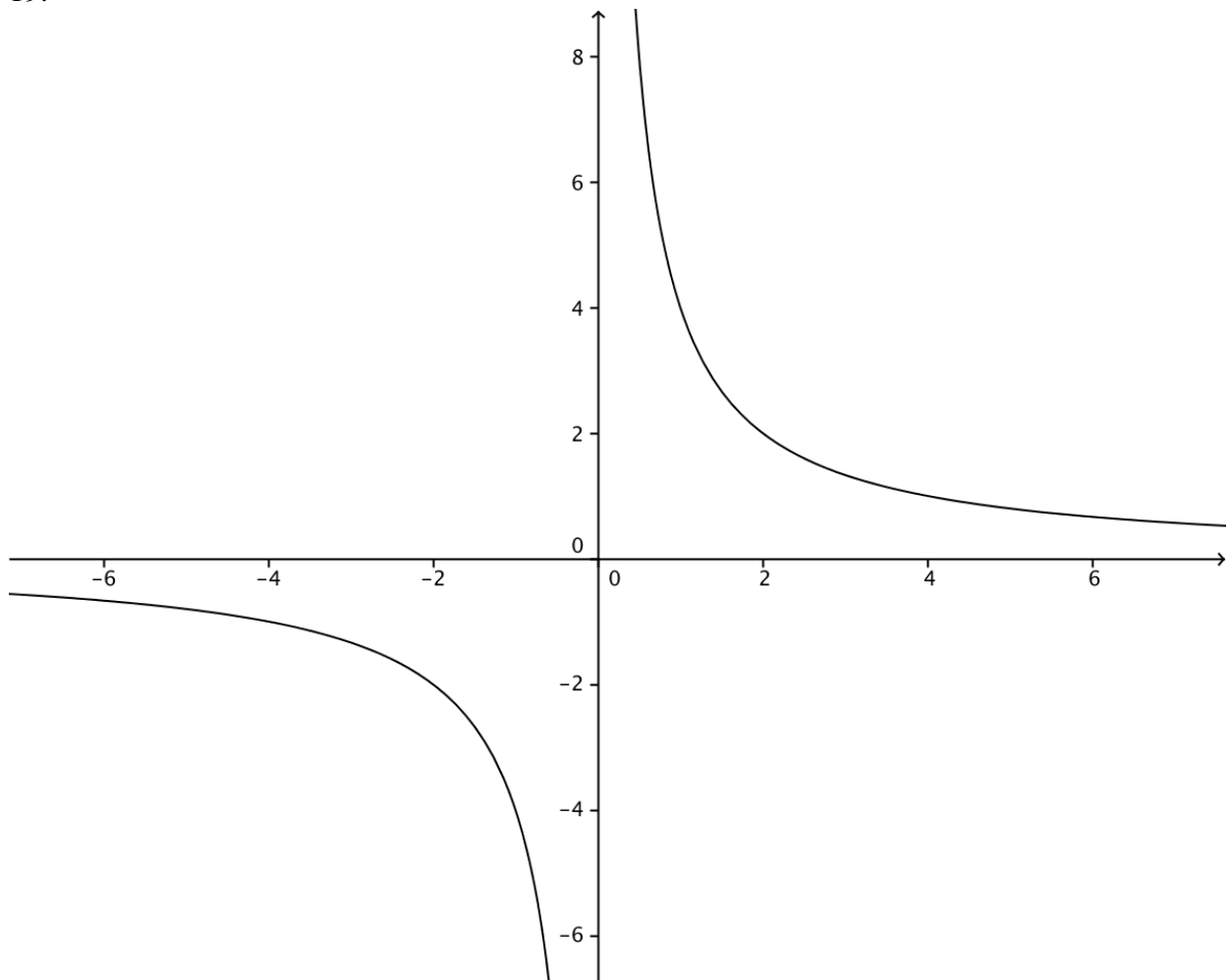
$$\text{Fulcrum} \rightarrow d = \frac{k}{w}$$

$$1.8 \text{ meters} = \frac{k}{20.43 \text{ kg}}$$

$$k = 36.774$$

$$d = \frac{36.774}{36.32 \text{ kg}} = 1.0125 \text{ meters}$$

19.  $y = \frac{4}{x}$



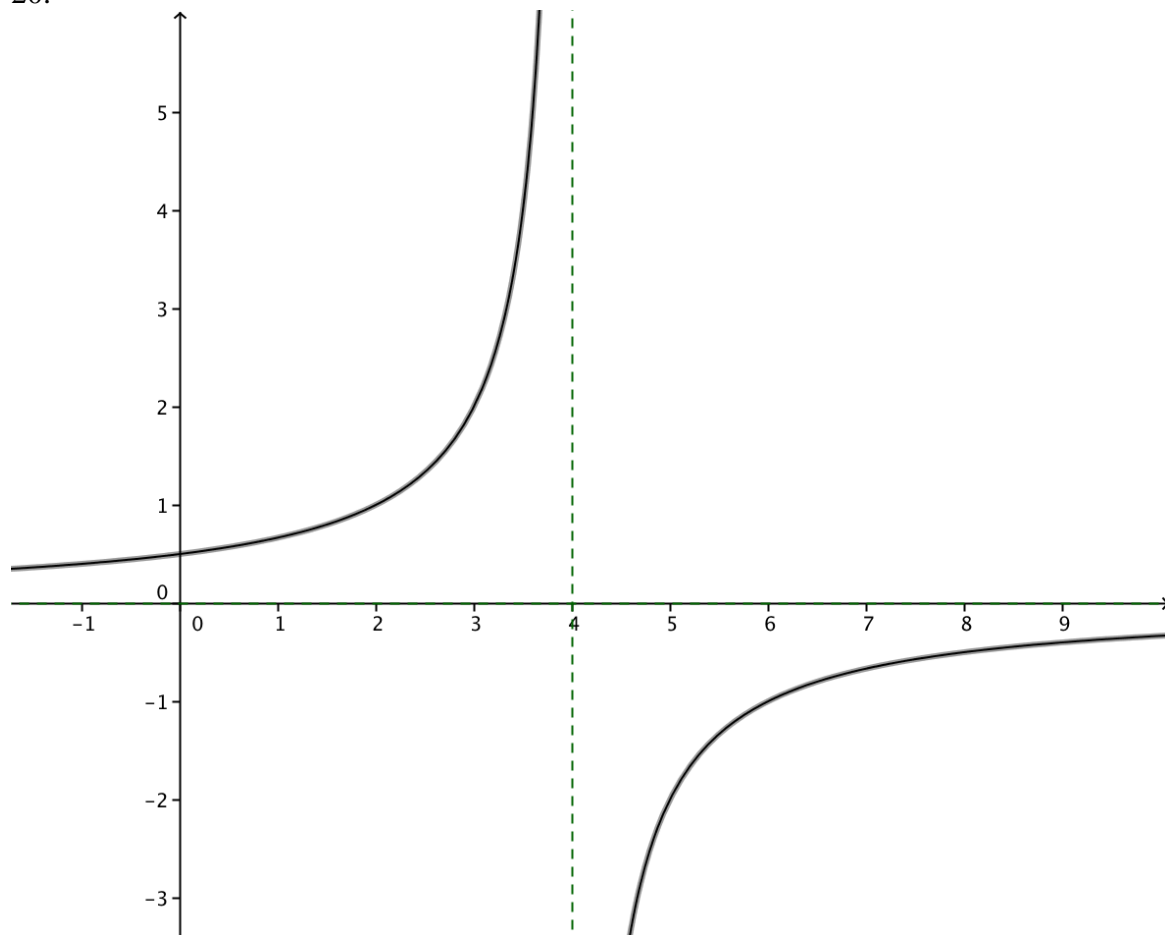
Domain:  $x \neq 0$

Range:  $y \neq 0$

Vertical asymptote:  $x=0$

Horizontal asymptote:  $y=0$

20.  $f(x) = \frac{2}{4-x}$



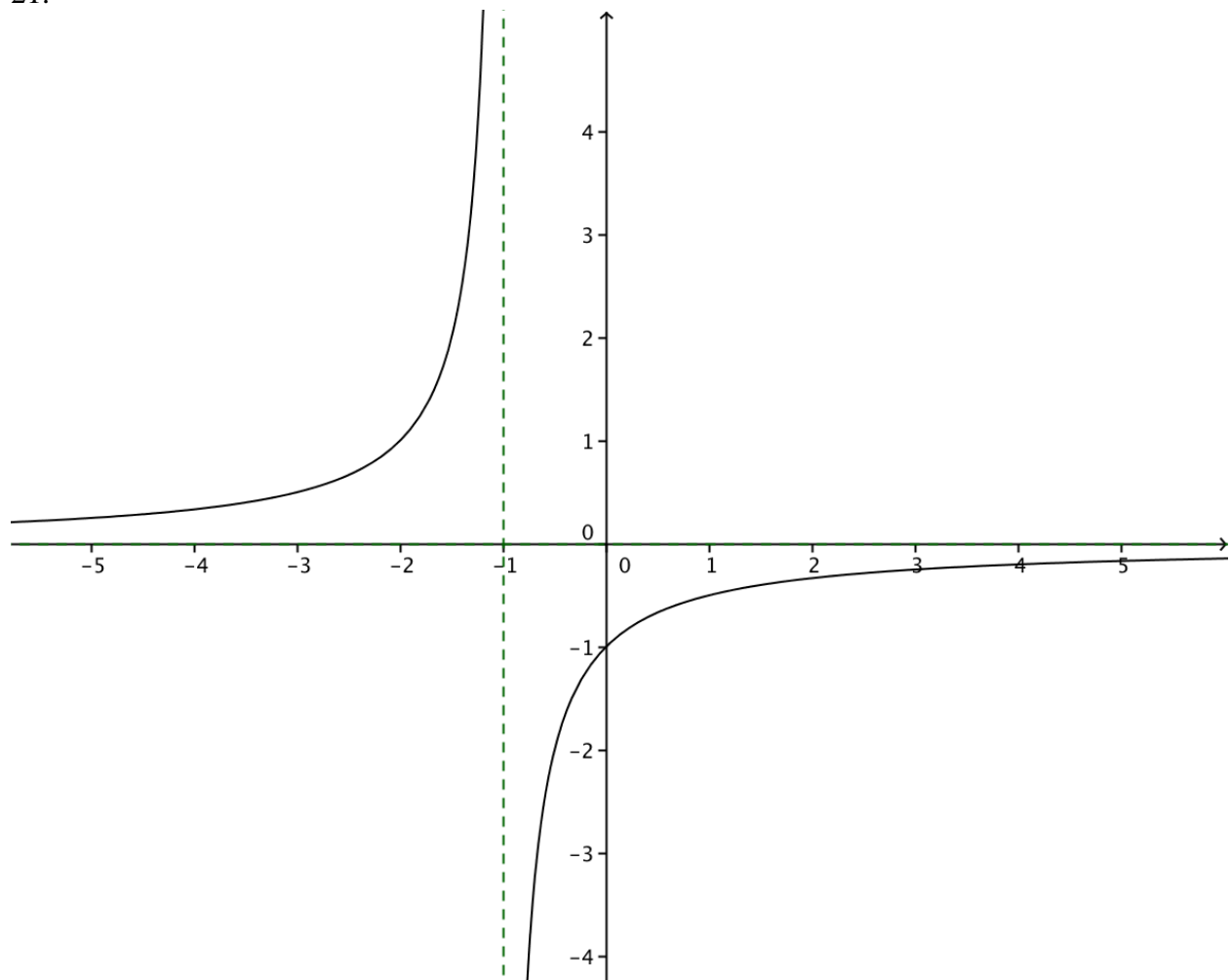
Domain:  $x \neq 4$

Range:  $y \neq 0$

Vertical Asymptote:  $x=4$

Horizontal Asymptote:  $y=0$

21.  $g(x) = \frac{-1}{x+1}$



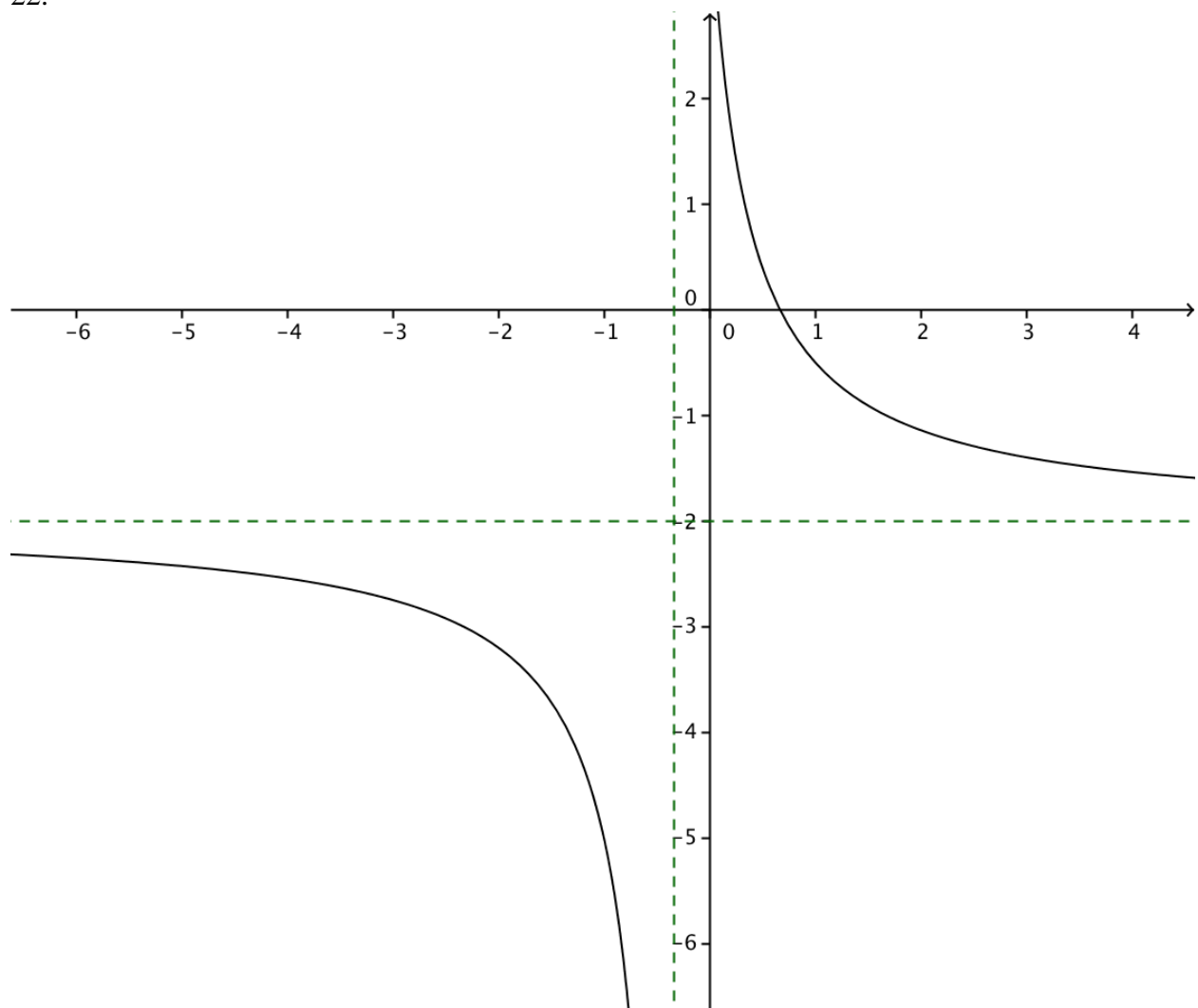
Domain:  $x \neq -1$

Range:  $y \neq 0$

Vertical asymptote:  $x = -1$

Horizontal asymptote:  $y = 0$

22.  $y = \frac{6}{3x+1} - 2$



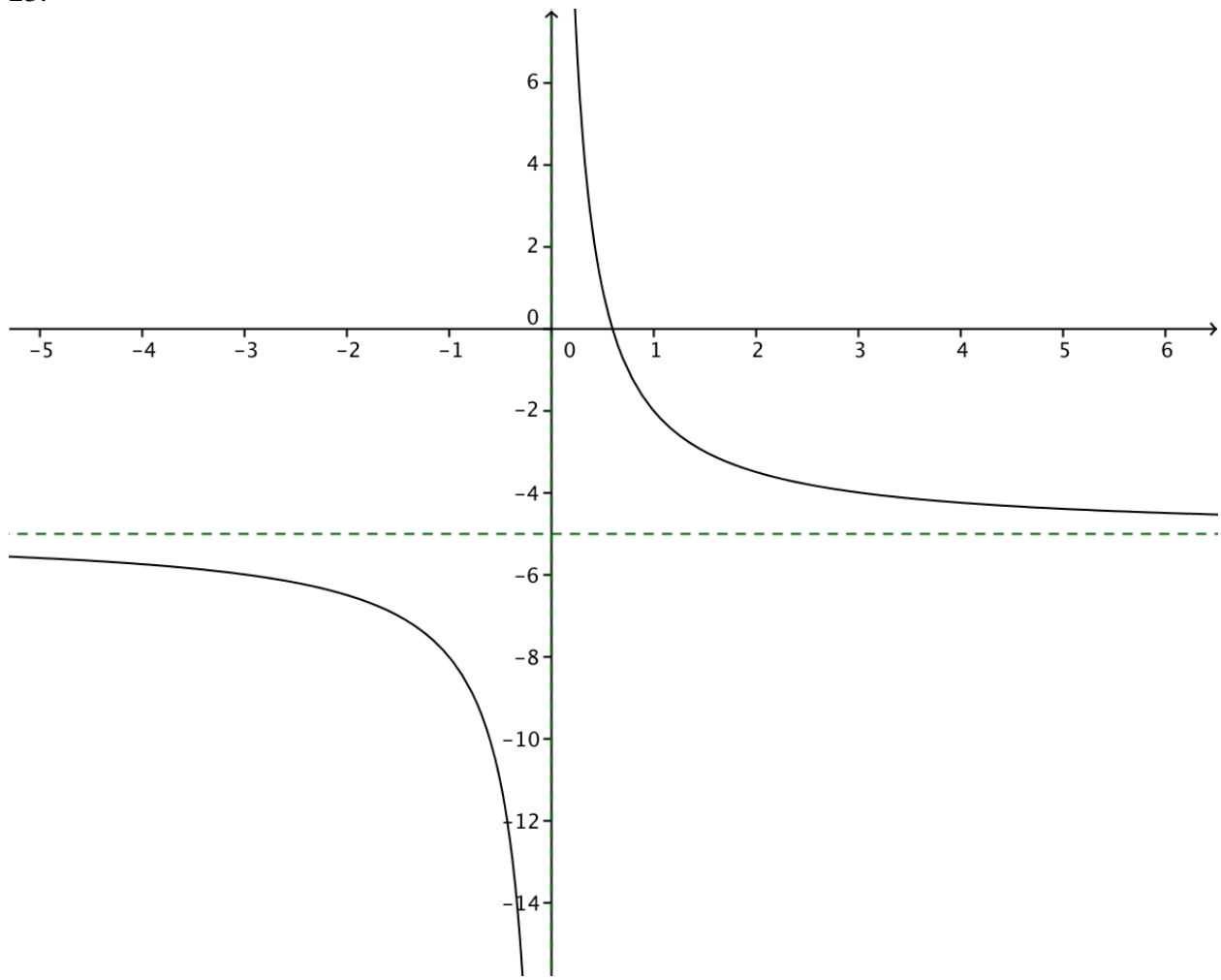
Domain:  $x \neq -0.33$

Range:  $y \neq -2$

Vertical asymptote:  $x = -0.33$

Horizontal Asymptote:  $y = -2$

23.  $f(x) = \frac{3}{x} - 5$



Domain:  $x \neq 0$

Range:  $y \neq -5$

Vertical Asymptote:  $x=0$

Horizontal Asymptote:  $y=-5$

24.

$$\frac{5a}{6} - \frac{5b}{4b}$$

$$\frac{5a(2b)}{12b} - \frac{5b(3)}{12b}$$

$$\frac{10ab}{12b} - \frac{15b}{12b} = \frac{10ab - 15b}{12b} = \frac{5b(2a - 3)}{12b} = \frac{5(2a - 3)}{12}$$

25.

$$\begin{aligned}\frac{4}{3m} + \frac{4m}{5} \\ \frac{4(5)}{15m} + \frac{4m(3m)}{15m} \\ \frac{20}{15m} + \frac{12m^2}{15m} = \frac{20+12m^2}{15m}\end{aligned}$$

26.

$$\begin{aligned}\frac{3x}{2xy} + \frac{4}{3} \\ \frac{3x(3)}{6xy} + \frac{4(2xy)}{6xy} \\ \frac{9x}{6xy} + \frac{8xy}{6xy} = \frac{9x+8xy}{6xy} = \frac{x(9+8y)}{6xy} = \frac{9+8y}{6y}\end{aligned}$$

27.

$$\begin{aligned}\frac{2}{5n-2} + \frac{2n}{2} \\ \frac{2(2)}{2(5n-2)} + \frac{2n(5n-2)}{2(5n-2)} \\ \frac{4}{2(5n-2)} + \frac{10n^2-4n}{2(5n-2)} \\ \frac{10n^2-4n+4}{2(5n-2)} = \frac{2(5n^2-2n+2)}{2(5n-2)} = \frac{(5n^2-2n+2)}{(5n-2)}\end{aligned}$$

28.

$$\frac{2x+1}{3x+9} - \frac{x+5}{3x+9} = \frac{2x+1-x-5}{3x-9} = \frac{x-4}{3x-9}$$

29.

$$\frac{5m+n}{30n^4} - \frac{4m+n}{30n^4} = \frac{5m+n-4m-n}{30n^4} = \frac{m}{30n^4}$$

30.

$$\frac{r-6}{4r^2-12r+8} - \frac{r+6}{4r^2-12r+8} = \frac{r-6-r-6}{4r^2-12r+8} = \frac{-12}{4(r^2-3r+2)} = \frac{-3}{r^2-3r+2}$$

31.

$$\frac{2}{16x^3y^2} + \frac{x-2y}{16x^3y^2} = \frac{2+x-2y}{16x^3y^2}$$

32.

$$\begin{aligned} & \frac{n-6}{n+2} + \frac{2n}{5} \\ & \frac{5(n-6)}{5(n-2)} + \frac{2n(n-2)}{5(n-2)} \\ & \frac{5n-30}{5(n-2)} + \frac{2n^2-4n}{5(n-2)} \\ & \frac{5n-30+2n^2-4n}{5(n-2)} = \frac{2n^2+n-30}{5(n-2)} \end{aligned}$$

33.

$$\begin{aligned} & \frac{8}{4} - \frac{x+5}{x+8} \\ & 2 - \frac{x+5}{x+8} \\ & \frac{2(x+8)}{x+8} - \frac{x+5}{x+8} \\ & \frac{2x+16-x-5}{x+8} = \frac{x+11}{x+8} \end{aligned}$$

34.

$$\begin{aligned} & \frac{3x}{2(x+1)} + \frac{6}{7x-6} \\ & \frac{3x(7x-6)}{(2x+2)(7x-6)} + \frac{6(2x+2)}{(2x+2)(7x-6)} \\ & \frac{21x^2-18x}{(2x+2)(7x-6)} + \frac{12x+12}{(2x+2)(7x-6)} \\ & \frac{21x^2-18x+12x+12}{(2x+2)(7x-6)} = \frac{21x^2-6x+12}{(2x+2)(7x-6)} = \frac{3(7x^2-2x+4)}{(2x+2)(7x-6)} \end{aligned}$$

35.

$$\frac{11}{8} \cdot \frac{20x^2}{2} = \frac{55x^2}{4}$$

36.

$$\frac{17r}{16} \cdot \frac{7r^4}{16} = \frac{119r^5}{256}$$

37.

$$\frac{15}{18} \cdot \frac{14}{17t} = \frac{35}{51t}$$

38.

$$\frac{2(b-11)}{14b} \cdot \frac{b+5}{(b+5)(b-11)} = \frac{1}{7b}$$

39.

$$\frac{17w^2}{w+4} \cdot \frac{18(w+4)}{17w^2(w-9)} = \frac{18}{w-9}$$

40.

$$\begin{aligned} & \frac{10s^3-30s^2}{30s^2-10s^3} \cdot \frac{s-3}{8} \\ & \frac{10s^2(s-3)}{10s^2(3-s)} \cdot \frac{s-3}{8} = \frac{s^2-6s+9}{24-8s} \end{aligned}$$

41.

$$\frac{1}{f-5} \div \frac{f+3}{f^2+6f+9}$$
$$\frac{1}{f-5} \div \frac{f+3}{(f+3)(f+3)} = \frac{1}{f-5} \div \frac{1}{f+3} = \frac{1}{f-5} * \frac{f+3}{1} = \frac{f+3}{f-5}$$

42.

$$\frac{(a+8)(a+3)}{4(a+3)} \div \frac{10a^2(a+10)}{4}$$
$$\frac{a+8}{4} * \frac{4}{10a^2(a+10)} = \frac{a+8}{10a^2(a+10)}$$

43.

$$\frac{1}{(h-10)(h+7)} \div \frac{(h-4)}{4h(h-10)} = \frac{1}{(h-10)(h+7)} * \frac{4h(h-10)}{(h-4)} = \frac{4h}{(h+7)(h-4)}$$

44.

$$\frac{2(5x-8)}{4x^2(8-5x)} \div \frac{6}{4x^2}$$
$$\frac{5x-8}{2x^2(8-5x)} \div \frac{3}{2x^2}$$
$$\frac{5x-8}{2x^2(8-5x)} * \frac{2x^2}{3} = \frac{5x-8}{3(8-5x)}$$

45.

$$\frac{2(q-7)}{40q(q+1)} \div \frac{1}{40q(q+1)}$$
$$\frac{2(q-7)}{40q(q+1)} * \frac{40q(q+1)}{1} = 2q-14$$

46.

$$\frac{3}{3x^2} = \frac{1}{x} + \frac{1}{3x^2}$$

$$3x^2 \cdot \frac{3}{3x^2} = 3x^2 \cdot \frac{1}{x} + 3x^2 \cdot \frac{1}{3x^2}$$

$$3 = 3x + 1$$

$$2 = 3x$$

$$x = \frac{2}{3}$$

47.

$$\frac{2}{5x^2} = -\frac{12}{x-3}$$

$$-12(5x^2) = 2(x-3)$$

$$-60x^2 = 2x - 6$$

$$0 = 60x^2 + 2x - 6$$

$$0 = 2(30x^2 + x - 3)$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(30)(-3)}}{2(30)}$$

$$x = \frac{-1 \pm \sqrt{1 + 360}}{60}$$

$$x = \frac{-1 \pm \sqrt{361}}{60}$$

$$x = \frac{-1 + \sqrt{361}}{60} = \frac{-1 + 19}{60} = \frac{18}{60} = 0.3$$

$$x = \frac{-1 - \sqrt{361}}{60} = \frac{-1 - 19}{60} = \frac{-20}{60} = -0.3333$$

48.

$$\frac{7x}{x-6} = \frac{3}{4x+16}$$

$$7x(4x+16) = 3(x-6)$$

$$28x^2 + 112x = 3x - 18$$

$$28x^2 + 109x + 18 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-109 \pm \sqrt{109^2 - 4(28)(18)}}{2(28)}$$

$$x = \frac{-109 \pm \sqrt{11881 - 2016}}{56}$$

$$x = \frac{-109 \pm \sqrt{9865}}{56}$$

$$x = \frac{-109 + \sqrt{9865}}{56} \approx \frac{-109 + 99.3227}{56} \approx \frac{-9.6773}{56} \approx -0.1728$$

$$x = \frac{-109 - \sqrt{9865}}{56} \approx \frac{-109 - 99.3227}{56} \approx \frac{-208.3227}{56} \approx -3.72$$

49.

$$\frac{4}{c-2} = \frac{3}{c+4}$$

$$4(c+4) = 3(c-2)$$

$$4c + 16 = 3c - 6$$

$$c = -22$$

50.

$$\frac{d-4}{4d^2} = \frac{1}{4d^2} + \frac{2}{4d}$$

$$4d^2 \cdot \frac{d-4}{4d^2} = 4d^2 \cdot \frac{1}{4d^2} + 4d^2 \cdot \frac{2}{4d}$$

$$d-4 = 1+2d$$

$$-5 = d$$

$$d = -5$$

51.

$$\frac{1}{2} = \frac{2z-12}{z} - \frac{z+1}{4z}$$

$$\frac{1}{2} = \frac{4}{4} \cdot \frac{2z-12}{z} - \frac{z+1}{4z}$$

$$\frac{1}{2} = \frac{4(2z-12)}{4z} - \frac{z+1}{4z}$$

$$\frac{1}{2} = \frac{8z-48}{4z} - \frac{z+1}{4z}$$

$$\frac{1}{2} = \frac{8z-48-z-1}{4z}$$

$$\frac{1}{2} = \frac{7z-49}{4z}$$

$$4z = 2(7z-49)$$

$$4z = 14z - 98$$

$$98 = 10z$$

$$z = 9.8$$

52.

$$\frac{1}{n} = \frac{1}{n^2} + \frac{6}{n}$$

$$n^2 \cdot \frac{1}{n} = n^2 \cdot \frac{1}{n^2} + n^2 \cdot \frac{6}{n}$$

$$n = 1 + 6n$$

$$-1 = 5n$$

$$n = -\frac{1}{5} = -0.2$$

53.

$$\frac{1}{2a} = \frac{1}{2a^2} + \frac{1}{a}$$

$$2a^2 \cdot \frac{1}{2a} = 2a^2 \cdot \frac{1}{2a^2} + 2a^2 \cdot \frac{1}{a}$$

$$a = 1 + 2a$$

$$-a = 1$$

$$a = -1$$

54.

$$\frac{k+4}{k^2} = \frac{5k-30}{3k^2} + \frac{1}{3k^2}$$
$$3k^2 \cdot \frac{k+4}{k^2} = 3k^2 \cdot \frac{5k-30}{3k^2} + 3k^2 \cdot \frac{1}{3k^2}$$
$$3(k+4) = (5k-30)+1$$
$$3k+12 = 5k-29$$
$$12 = 2k-29$$
$$41 = 2k$$
$$k = 20.5$$

55.

Part of task completed = rate of work X time spent on task

Painter	Rate of Work (per hour)	Time Worked	Part of Task
Jayden	1/7	t	t/7
Andie	1/5	t	t/5

Part of task Jayden + Part of task Andie = 1 complete task

$$\frac{t}{7} + \frac{t}{5} = 1$$
$$\frac{5t}{35} + \frac{7t}{35} = 1$$
$$\frac{12t}{35} = 1$$
$$12t = 35$$
$$t = \frac{35}{12} \approx 2.917$$

It takes approximately 2.917 hours to paint a room if Jayden and Andie work together.

56.

Part of task completed = rate of work X time spent on task

Mower	Rate of Work (per hour)	Time Worked	Part of Task
Kiefer	1/4.5	t	t/4.5
Brad	1/2	t	t/2

Part of task Kiefer + Part of task Brad = 1 complete task

$$\frac{t}{4.5} + \frac{t}{2} = 1$$

$$\frac{2t}{9} + \frac{4.5t}{9} = 1$$

$$\frac{6.5t}{9} = 1$$

$$6.5t = 9$$

$$t \approx 1.3846$$

It takes approximately 1.3846 hours to mow the lawn if Kiefer and Brad work together.

57.

Melissa can mop the floor in 1.75 hours or 105 minutes.

Brad + Melissa can mop the floor in 50 minutes.

Part of task completed = rate of work X time spent on task

Mower	Rate of Work (per hour)	Time Worked	Part of Task
Melissa	1/105	50	50/105
Brad	1/x	50	50/x

Part of task Melissa + Part of task Brad = 1 complete task

$$\frac{50}{105} + \frac{50}{x} = 1$$

$$\frac{50x}{105x} + \frac{50(105)}{105x} = 1$$

$$\frac{50x + 5250}{105x} = 1$$

$$50x + 5250 = 105x$$

$$5250 = 55x$$

$$x = 95.4545$$

It would take Brad approximately 95.4545 minutes or 1.59 hours to mop the floor himself.

58.

Frankie can frame a room in 15 hours.

Frankie + Ricky can frame a room in 8 hours.

Part of task completed = rate of work X time spent on task

Framer	Rate of Work (per hour)	Time Worked	Part of Task
Frankie	1/15	8	8/15
Ricky	1/x	8	8/x

Part of task Frankie + Part of task Ricky = 1 complete task

$$\frac{8}{15} + \frac{8}{x} = 1$$

$$\frac{8x}{15x} + \frac{8(15)}{15x} = 1$$

$$\frac{8x + 120}{15x} = 1$$

$$8x + 120 = 15x$$

$$120 = 7x$$

$$x = 17.1429$$

It would take Ricky approximately 17.1429 hours to frame a room by himself.

59.

$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{16} = \frac{1}{50} + \frac{1}{R_2}$$

$$\frac{1}{16} - \frac{1}{50} = \frac{1}{R_2}$$

$$\frac{50}{800} - \frac{16}{800} = \frac{1}{R_2}$$

$$\frac{34}{800} = \frac{1}{R_2}$$

$$34R_2 = 800$$

$$R_2 \approx 23.5294\Omega$$

60.

$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_t} = \frac{1}{6} + \frac{1}{9}$$

$$\frac{1}{R_t} = \frac{3}{18} + \frac{2}{18}$$

$$\frac{1}{R_t} = \frac{5}{18}$$

$$18 = 5R_t$$

$$R_t = 3.6\Omega$$

61.

$$R_t = R_1 + R_2$$

$$300 = 200 + R_2$$

$$100 = R_2$$

$$R_2 = 100\Omega$$

62.

$$R_t = R_1 + R_2$$

$$R_t = 11 + 25$$

$$R_t = 36\Omega$$

63.

$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

64. The bias occurs because when you only ask people already in the chip aisle, you know they already buy chips. If you want a truly unbiased sample you have to include people who don't always or normally buy chips.

65.

1. Determine the goal of your survey, What question do you want to answer?
2. Identify the sample population. Who will you interview?
3. Choose an interviewing method, face-to-face interview, phone interview or self-administered paper survey or internet survey.
4. Conduct the interview and collect the information.
5. Analyze the results by making graphs and drawing conclusions.

66. Answers will vary. (Should follow the steps from #65 and have pertinent questions.)

67. A stratified sample is a sample where the person conducting the survey divides the population into groups by a certain characteristic. These groups are called strata. The surveyor then chooses a sample in which each strata has the same number of people in the sample. Examples will vary.

**Lesson 12.10**  
**Chapter 12 Test**

1. False: a horizontal asymptote DOES have the equation  $y=c$ , but it represents the vertical shift of the rational function NOT where the denominator of the rational function is equal to zero.

2. The potential bias is that by wearing SADD t-shirts they tell the respondents what they are against and likely bias the results. The respondents will be more likely to tell them what they want to hear. By not wearing the t-shirts and not telling the members of the sample what group they are from they will get more accurate results. Or they could use an anonymous self directed survey.

3. The four ways questions can be biased are:

-polarizing language that assume emotion

-a reference to an authority or apparent authority

-phrasing that suggests the person asking the question already knows the answer

-ambiguous phrasing that often includes double negatives

4. Box and whisker plots are very useful in showing data comparing two categories.

5. 
$$f(x) = -\frac{4}{x}$$

Domain:  $x \neq 0$

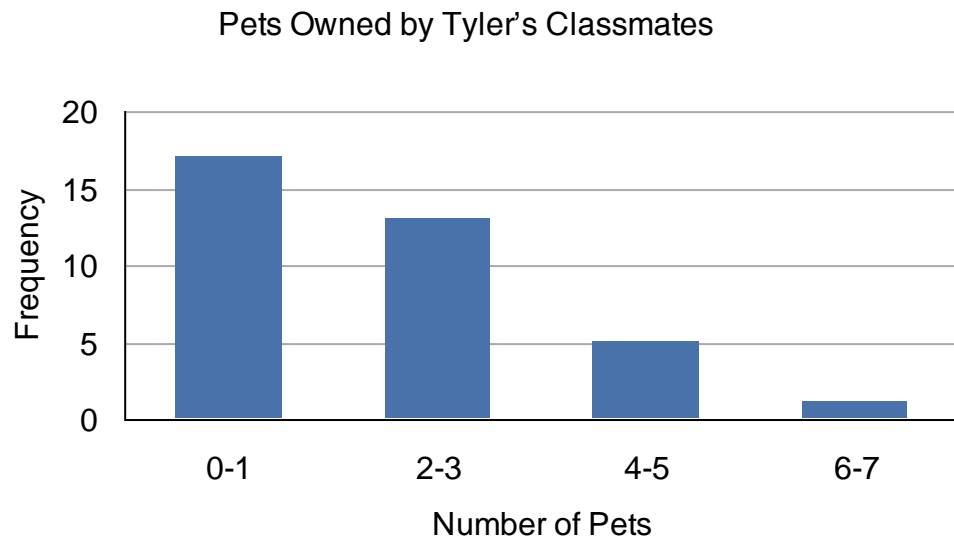
Range:  $y \neq 0$

Vertical Asymptote:  $x=0$

Horizontal Asymptote:  $y=0$

Branches are in quadrants II and IV

6.



$$h = \frac{k}{r}$$

$$-2.25 = \frac{k}{0.125}$$

$$k = -0.28125$$

$$h = \frac{-0.28125}{12}$$

$$h = -0.0234375$$

7. Two types of visual displays that could be used with a frequency distribution are a histogram and a pie chart.

8.

1.

Number of Pets	Tally	Frequency
0-1	IIII-IIII-IIII-II	17
2-3	IIII-IIII-III	13
4-5	IIII-	5
6-7	I	1

2.

3.

0,0,0,0,0,0,0,1,1,1,

1,1,1,1,1,1,2,2,2,

2,2,2,2,3,3,3,3,3,3,

4,4,4,4,5,7

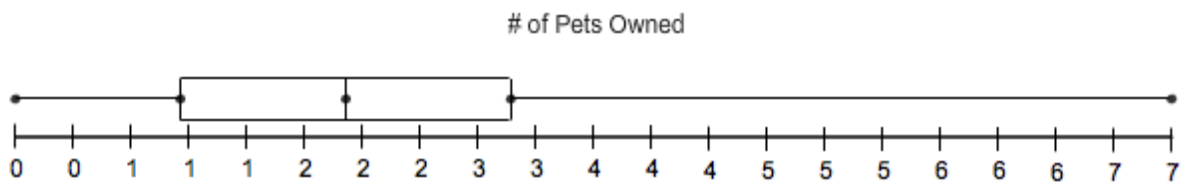
*Minimum* = 0

$Q_1$  = 1

*Median* = 2

$Q_3$  = 3

*Maximum* = 7



4.

5. Answers will vary.

9.

$$f(x) = -\frac{9}{x^2 - 16} + 4$$

Excluded Values:  $x=4$ ,  $x=-4$

Domain:  $x \neq 4$ ,  $x \neq -4$

Range:  $y \neq 4$

Vertical Asymptotes:  $x=4$ ,  $x=-4$

Horizontal Asymptotes:  $y=4$

10.

$$\frac{4}{21r^4} + \frac{4r+5t}{21r^4} = \frac{4+4r+5t}{21r^4}$$

11.

$$\frac{a-v}{12a^3} - \frac{a+5v}{12a^3} = \frac{a-v-a-5v}{12a^3} = \frac{-6v}{12a^3}$$

12.

$$\begin{aligned} & \frac{8}{g+8} + \frac{g-3}{g-5} \\ & \frac{8}{g+8} \cdot \frac{g-5}{g-5} + \frac{g-3}{g-5} \cdot \frac{g+8}{g+8} \\ & \frac{8(g-5)}{(g+8)(g-5)} + \frac{(g-3)(g+8)}{(g+8)(g-5)} \\ & \frac{8g-40}{(g+8)(g-5)} + \frac{g^2+5g-24}{(g+8)(g-5)} \\ & \frac{8g-40+g^2+5g-24}{(g+8)(g-5)} \\ & \frac{g^2+13g-64}{(g+8)(g-5)} \end{aligned}$$

13.

$$\begin{aligned} & \frac{4t}{5t-8} + \frac{24}{12} \\ & \frac{4t}{5t-8} + 2 \\ & \frac{4t}{5t-8} + \frac{5t-8}{5t-8} \cdot 2 \\ & \frac{4t}{5t-8} + \frac{10t-16}{5t-8} \\ & \frac{4t+10t-16}{5t-8} \\ & \frac{14t-16}{5t-8} \end{aligned}$$

14.

$$\frac{4}{5} \cdot \frac{80}{48m} = \frac{1}{1} \cdot \frac{16}{12m} = \frac{16}{12m} = \frac{4}{3m}$$

15.

$$\frac{1}{d-8} \div \frac{d+7}{2d+14} = \frac{1}{d-8} \div \frac{d+7}{2(d+7)} = \frac{1}{d-8} \div \frac{1}{2} = \frac{1}{d-8} * \frac{2}{1} = \frac{2}{d-8}$$

16.

$$\frac{1}{u-3} \div \frac{u-4}{2u-6} = \frac{1}{u-3} \div \frac{u-4}{2(u-3)} = \frac{1}{u-3} * \frac{2(u-3)}{u-4} = \frac{1}{1} * \frac{2}{u-4} = \frac{2}{u-4}$$

17.

$$\begin{aligned} & \frac{7w}{w-7} = \frac{7w}{w+5} \\ & 7w(w+5) = 7w(w-7) \\ & 7w^2 + 35w = 7w^2 - 49w \\ & 84w = 0 \\ & w = 0 \end{aligned}$$

18.

$$\frac{p-6}{3p^2-6p} = \frac{7}{3}$$

$$3(p-6) = 7(3p^2-6p)$$

$$3p-18 = 21p^2-42p$$

$$0 = 21p^2-45p+18$$

$$0 = 3(7p^2-15p+6)$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-15) \pm \sqrt{(-15)^2-4(7)(6)}}{2(7)}$$

$$x = \frac{15 \pm \sqrt{225-168}}{14}$$

$$x = \frac{15 \pm \sqrt{57}}{14}$$

$$x = \frac{15 + \sqrt{57}}{14} \approx 1.6107$$

$$x = \frac{15 - \sqrt{57}}{14} \approx 0.5322$$

19.

$$\frac{2}{x^2} = \frac{1}{2x^2} - \frac{x+1}{2x^2}$$

$$2x^2 \cdot \frac{2}{x^2} = 2x^2 \cdot \frac{1}{2x^2} - 2x^2 \cdot \frac{x+1}{2x^2}$$

$$2 \cdot 2 = 1 - x - 1$$

$$4 = -x$$

$$x = -4$$

20.

$$\frac{1}{2} - \frac{1}{4r} = \frac{3}{4}$$

$$4r \cdot \frac{1}{2} - 4r \cdot \frac{1}{4r} = 4r \cdot \frac{3}{4}$$

$$2r(1) - 1 = 3r$$

$$2r - 1 = 3r$$

$$-1 = r$$

$$r = -1$$

21.

$$\frac{y-5}{3y^2} = -\frac{1}{3y} + \frac{1}{y^2}$$

$$3y^2 \cdot \frac{y-5}{3y^2} = -\frac{1}{3y} \cdot 3y^2 + \frac{1}{y^2} \cdot 3y^2$$

$$y-5 = -y+3$$

$$2y = 8$$

$$y = 4$$

22.

Part of task completed = rate X time

Tiler	Rate of work (per hour)	Total time	Part of Task
Ashton	1/2 hrs = 1/120min	25min	25/120
Matt	1/x	25min	25/x

Part of task Ashton + Part of task Matt = 1 completed task

$$\frac{25}{120} + \frac{25}{x} = 1$$

$$\frac{25x}{120x} + \frac{25(120)}{120x} = 1$$

$$\frac{25x + 3000}{120x} = 1$$

$$25x + 3000 = 120x$$

$$3000 = 95x$$

$$x \approx 32$$

It would take Matt approximately 32 minutes to tile the floor.

23.

Part of task completed= rate X time

Painter	Rate of work (per hour)	Total time	Part of Task
Bethany	1/12	t	t/12
Melissa	1/5	t	t/5

Part of task Bethany+ Part of task Melissa= 1 completed task

$$\frac{t}{12} + \frac{t}{5} = 1$$

$$\frac{5t}{60} + \frac{12t}{60} = 1$$

$$\frac{17t}{60} = 1$$

$$17t = 60$$

$$t \approx 3.5294$$

It would take approximately 3.5 hours to paint the deck if the two girls work together.

24.

$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{115} = \frac{1}{R_1} + \frac{1}{75}$$

$$\frac{1}{115} - \frac{1}{75} = \frac{1}{R_1}$$

$$\frac{15}{1725} - \frac{23}{1725} = \frac{1}{R_1}$$

$$\frac{-8}{1725} = \frac{1}{R_1}$$

$$-8R_1 = 1725$$

$$R_1 = -215.625\Omega$$

25.

$$R_t = R_1 + R_2$$

$$21 = 13 + R_2$$

$$8 = R_2$$

$$R_2 = 8\Omega$$