Trigonometric Ratios

Right Triangle Trigonometry

Review Queue

1. yes, independent 2. 0.2 3. a. $4\sqrt{15}$ b. $37\sqrt{3}$ c. $60\sqrt{2}$

Pythagorean Theorem and its Converse

1. $\sqrt{194}$ 2. $\sqrt{63}$ 3. $5\sqrt{3}$ 4. $c=10$
5. $4\sqrt{10}$ 6. $6\sqrt{5}$ 7. Yes 8. No
9. No 10. Yes 11. Not a triangle 12. No
13. a. $\frac{1}{2}(b+a)(a+b)=\frac{1}{2}(a^2+2ab+b^2)$

b. $2\left(\frac{1}{2}ab\right)+\frac{1}{2}c^2=\frac{1}{2}(2ab+c^2)$

$\frac{1}{2}(a^2+2ab+b^2)=\frac{1}{2}(2ab+c^2)$

c. $a^2+2ab+b^2=2ab+c^2$

$a^2+b^2=c^2$

Sine, Cosine and Tangent

1. 0.5736 2. 3.0777 3. 0.6691
4. $\sin N=\frac{3}{5},\cos N=\frac{4}{5},\tan N=\frac{3}{4}$; $\sin M=\frac{4}{5},\cos M=\frac{3}{5},\tan M=\frac{4}{3}$
5. $x\approx 5.14$, $y\approx 6.13$ 6. $x\approx 11.03$, $y\approx 4.66$ 7. $x\approx 8.66$, $y\approx 10$
8. $b\approx 17.60$, $c\approx 12.87$ 9. $c\approx 23.18$, $a\approx 6.21$ 10. $a\approx 15.01$, $b\approx 16.56$
11. 17.3 ft 12. 12.7 km

Inverse Trig Functions and Solving Right Triangles

1. $44°$ 2. $60°$ 3. $81°$
4. $x\approx 45°$, $y\approx 45°$ 5. $x\approx 71°$, $y\approx 19°$ 6. $x\approx 27°$, $y\approx 63°$
7. $x\approx 45°$, $y\approx 45°$ 8. $x\approx 50°$, $y\approx 40°$ 9. $x\approx 33°$, $y\approx 57°$

10. $m\angle B \approx 41°$, $m\angle A \approx 39°$, $b \approx 39.7$

11. $m\angle B \approx 56°$, $m\angle A \approx 24°$, $c \approx 10.8$

12. $m\angle B \approx 40°$, $m\angle A \approx 50°$, $a \approx 10.7$

Application Problems

1. 11.3 in
2. 477 m
3. 35 m
4. 97 ft
5. $39°$
6. 88 ft
7. $31°$
8. 97 ft
9. 29 ft
10. 13 miles
11. The hypotenuse is always the longest side. Therefore, the ratios, $\frac{O}{H}$ and $\frac{A}{H}$ are both less than 1.

The Unit Circle

Review Queue

1. a. $69.6°$ b. $72.7°$ c. $44.4°$
2. $AC = 33.5$, $m\angle A = 41.8°$, $m\angle B = 48.2°$
3. 19.3 ft

Introduction to Angles of Rotations, Coterminal Angles and Reference Angles

1. $-458°, 262°$
2. $115°$, $-245°$
3. $-570°$, $150°$
4. $-313°$, $407°$
5. $-302°$, $58°$
6. QII, $78°$
7. QIV, $40°$
8. QIII, $47°$
9. QIII, $80°$
10. QIV, $56°$
11. All the angles between $0°$ and $90°$ are acute angles between the terminal side of the angle and the x-axis.

Introduction to the Unit Circle and Radian Measure

1. $\frac{3\pi}{4}$
2. $\frac{4\pi}{3}$
3. $-\frac{11\pi}{6}$
4. $\frac{5\pi}{2}$
5. $-\frac{7\pi}{4}$
6. $420°$
7. $-390°$
8. $810°$
9. $-135°$
10. $150°$
11. coterminal angles: $\frac{2\pi}{3}, -\frac{4\pi}{3}$; reference angle: $\frac{\pi}{3}$ QII
12. coterminal angles: $\frac{3\pi}{4}, -\frac{5\pi}{4}$; reference angle: $\frac{\pi}{4}$ QII

13. coterminal angles: $\frac{11\pi}{6}, -\frac{13\pi}{6}$; reference angle: $\frac{\pi}{6}$ QIV

14. coterminal angles: $\frac{10\pi}{3}, -\frac{2\pi}{3}$; reference angle: $\frac{\pi}{3}$ QIII

Trigonometric Ratios on the Unit Circle

1.	$\frac{\sqrt{2}}{2}$	2.	0	3.	$-\sqrt{3}$	4.	$\frac{1}{2}$	5.	$-\frac{1}{2}$
6.	0	7.	$\frac{\sqrt{2}}{2}$	8.	$-\frac{\sqrt{2}}{2}$	9.	$\frac{\sqrt{3}}{3}$	10.	$-\frac{\sqrt{2}}{2}$
11.	0	12.	1	13.	$-\frac{\sqrt{3}}{2}$	14.	Undefined	15.	$\frac{1}{2}$

Reciprocal Trigonometric Functions

1.	1.0038	2.	-0.1405	3.	-1.2361	4.	-0.4663
5.	-1.1099	6.	-1.5080	7.	-1.9626	8.	-1.7013
9.	$-\frac{2\sqrt{3}}{3}$	10.	1	11.	1	12.	2
13.	-2	14.	Undefined	15.	$-\frac{\sqrt{3}}{3}$	16.	$-\sqrt{2}$

Inverse Trigonometric Functions

1.	$102.6°, 257.4°$	2.	$84.7°, 275.3°$	3.	$92.8°, 272.8°$	4.	$61.5°, 118.5°$
5.	$188.3°, 351.7°$	6.	$50.3°, 230.3°$	7.	3.80, 5.62	8.	1.43, 4.85
9.	2.80, 5.94	10.	1.68, 4.82	11.	0.78, 3.92	12.	0.08, 3.06
13.	$0, \pi$	14.	$\frac{3\pi}{4}, \frac{5\pi}{4}$	15.	$\frac{3\pi}{4}, \frac{7\pi}{4}$	16.	$\frac{\pi}{6}, \frac{11\pi}{6}$
17.	$\frac{\pi}{6}, \frac{5\pi}{6}$	18.	$0, \pi$	19.	$\frac{2\pi}{3}, \frac{4\pi}{3}$	20.	$\frac{\pi}{4}, \frac{3\pi}{4}$
21.	$\frac{\pi}{6}, \frac{7\pi}{6}$						

Introduction to Polar Coordinates

Review Queue

1. -2
2. undefined
3. -1
4. $\frac{\pi}{6}, \frac{11\pi}{6}$
5. $\frac{5\pi}{4}, \frac{7\pi}{4}$
6. $\frac{\pi}{3}, \frac{4\pi}{3}$

Trigonometric Ratios of Points on the Terminal Side of an Angle

1. $(34, 298°)$
2. $(5\sqrt{2}, 45°)$
3. $(13, 4.39)$
4. $(41, 1.79)$
5. $(10, 127°)$
6. $(15, 270°)$
7. $(2\sqrt{41}, 321°)$
8. $(8, 30°)$

$\sin 127° = \frac{4}{5}$
$\cos 127° = -\frac{3}{5}$
$\tan 127° = -\frac{4}{3}$
$\csc 127° = \frac{5}{4}$
$\sec 127° = -\frac{5}{3}$
$\cot 127° = -\frac{3}{4}$

$\sin 270° = -1$
$\cos 270° = 0$
$\tan 270° = undef$
$\csc 270° - 1$
$\sec 270° = undef$
$\cot 270° = 0$

$\sin 321° = -\frac{4\sqrt{41}}{41}$
$\cos 321° = \frac{5\sqrt{41}}{41}$
$\tan 321° = -\frac{4}{5}$
$\csc 321° = -\frac{\sqrt{41}}{4}$
$\sec 321° = \frac{\sqrt{41}}{5}$
$\cot 321° = -\frac{5}{4}$

$\sin 30° = \frac{1}{2}$
$\cos 30° = \frac{\sqrt{3}}{2}$
$\tan 30° = \frac{\sqrt{3}}{3}$
$\csc 30° = 2$
$\sec 30° = \frac{2\sqrt{3}}{3}$
$\cot 30° = \sqrt{3}$

9. $(9, \pi)$
10. $\left(13\sqrt{2}, \frac{7\pi}{4}\right)$
11. $\left(\sqrt{13}, 0.98\right)$
12. $\left(14, \frac{4\pi}{3}\right)$

$\sin \pi = 0$
$\cos \pi = -1$
$\tan \pi = 0$
$\csc \pi = undef$
$\sec \pi = -1$
$\cot \pi = undef$

$\sin \frac{7\pi}{4} = -\frac{\sqrt{2}}{2}$
$\cos \frac{7\pi}{4} = \frac{\sqrt{2}}{2}$
$\tan \frac{7\pi}{4} = -11$
$\csc \frac{7\pi}{4} = -\sqrt{2}$
$\sec \frac{7\pi}{4} = \sqrt{2}$
$\cot \frac{7\pi}{4} = -1$

$\sin 0.98 = \frac{3\sqrt{13}}{13}$ $\sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$
$\cos 0.98 = \frac{2\sqrt{13}}{13}$ $\cos \frac{4\pi}{3} = -\frac{1}{2}$
$\tan 0.98 = \frac{3}{2}$ $\tan \frac{4\pi}{3} = \sqrt{3}$
$\csc 0.98 = \frac{\sqrt{13}}{3}$ $\csc \frac{4\pi}{3} = -\frac{2\sqrt{3}}{3}$
$\sec 0.98 = \frac{\sqrt{13}}{2}$ $\sec \frac{4\pi}{3} = -2$
$\cot 0.98 = \frac{2}{3}$ $\cot \frac{4\pi}{3} = \frac{\sqrt{3}}{3}$

Using r and θ to find a Point in the Coordinate Plane

1. (10.24, 8.00)
2. (-16.07, 19.15)
3. (16.42, -4.40)
4. (-1.53, 1.29)
5. (2.16, 6.66)
6. (-8.88, 1.45)
7. (2.75, 1.20)
8. (9.01, -4.34)
9. $\left(\frac{5}{2},\frac{5\sqrt{3}}{2}\right)$
10. $\left(3\sqrt{2},-3\sqrt{2}\right)$
11. $\left(-6\sqrt{3},6\right)$
12. $\left(-7,0\right)$
13. $\left(0,-11\right)$
14. $\left(-7,-7\sqrt{3}\right)$
15. $\left(-\frac{27\sqrt{2}}{2},\frac{27\sqrt{2}}{2}\right)$
16. $\left(-20\sqrt{3},-20\right)$

Law of Sines

Review Queue

1. $-\frac{\sqrt{2}}{2}$
2. $-\frac{2\sqrt{3}}{3}$
3. $-\frac{2\sqrt{3}}{3}$
4. undefined

Law of Sines with AAS and ASA

1. $m\angle A=56°, a\approx 8.7, b\approx 10.4$
2. $m\angle C=30°, a\approx 9.4, b\approx 6.4$
3. $m\angle A=65°, c\approx 5.6, a\approx 13.6$
4. $m\angle A=106°, a\approx 73.8, c\approx 59.7$
5. $m\angle B=83°, c\approx 37.6, b\approx 41.2$
6. $m\angle C=33°, b\approx 16.3, a\approx 15.2$
7. $m\angle B=55°, c\approx 7.7, b\approx 9.7$
8. $m\angle A=95°, b\approx 24.3, c\approx 11.9$
9. $m\angle C=102°, a\approx 7.0, c\approx 11.7$
10. $m\angle C=25°, a\approx 87.2, b\approx 53.2$
11. 79 feet
12. 123.5 meters

The Ambiguous Case – SSA

1. 2 triangles
2. 2 triangles
3. 1 triangle
4. No triangle
5. 2 triangles
6. one triangle, $m\angle B\approx 39.4°$, $m\angle C\approx 75.6°$ and $c\approx 10.7$
7. two triangles, $m\angle B\approx 61°$, $m\angle C\approx 78°$ and $c\approx 13.4$ or $m\angle B\approx 119°$, $m\angle C\approx 20°$ and $c\approx 4.7$
8. two triangles, $m\angle B\approx 59.6°$, $m\angle C\approx 87.4°$ and $c\approx 22$ or $m\angle B\approx 120.4°$, $m\angle C\approx 26.6°$ and $c\approx 9.9$
9. one triangle, $m\angle B\approx 41°$, $m\angle A\approx 87°$ and $a\approx 76$
10. no triangle
11. two triangles, $m\angle B\approx 78.1°$, $m\angle C\approx 67.9°$ and $c\approx 33.1$ or $m\angle B\approx 101.9°$, $m\angle C\approx 44.1°$ and $c\approx 24.9$

Area of a Triangle

1. 371 u^2 2. 681 u^2 3. 35 u^2 4. 135 u^2 5. 152 u^2
6. 94 u^2 7. 463 u^2 8. 312 u^2 9. 1945 u^2
10. The two possible measures are $35°$ and $145°$ because the sine of an angle and its supplement are equal.
11. 191.5 ft^2 12. $97.43

Law of Cosines

Review Queue

1. 46.7 2. 61.2 3. $20.5°$ or $83.5°$

Law of Cosines with SAS (to find the third side)

1. 18.0 2. 23.0 3. 24.9 4. 47.2 5. 15.4
6. 30.9 7. 92.1 8. 15.5 9. 20.1 10. 31.9
11. $c^2 = a^2 + b^2 - 2ab\cos 90°$, since $\cos 90° = 0$, we can substitute to get $c^2 = a^2 + b^2 - 2ab(0)$, or $c^2 = a^2 + b^2$.
12. 30.4

Law of Cosines with SSS (to find an angle)

1. $38°$ 2. $138°$ 3. $65°$ 4. $56°$ 5. $50°$
6. $123°$ 7. $47°$ 8. $88°$ 9. $119°$ 10. $26°$
11. $88°$ 12. $49°$

Heron's Formula for the Area of a Triangle and Problem Solving with Trigonometry

1. 0.51 mi 2. 550 ft 3. 3.9 and 7.2 4. $94°$ 5. 8575 m^2
6. 88 in^2 7. 1.63 mi; 0.64 mi 8. 73 m^2 9. 87 ft
10. 185 ft; 181 ft