Chapter 3 – Introduction to Discrete Random Variables *Answer Key*

3.1 Discrete Random Variables

Answers

1. Answer:

TABLE 3.1:

Probability Statement	$P(X)$
a. The probability of this event will never occur.	e $P(X) = 1.0$
b. The probability of this event is highly likely.	d $P(X) = 0.33$
c. The probability of this event is very likely.	c $P(X) = 0.67$
d. The probability of this event is somewhat likely.	a $P(X) = 0.00$
e. The probability of this event is certain.	b $P(X) = 0.95$

2. Answer

TABLE 3.2:

Probability Statement	$P(X)$
a. I bought a ticket for the State Lottery. The probability of a successful event (winning) is likely to be:	d $P(X) = 0.80$
b. I have a bag of equal numbers of red and green jelly beans. The probability of reaching into the bag and picking out a red jelly bean is likely to be:	b $P(X) = 0.50$
c. My dad teaches math and my mom chemistry. The probability that I will be expected to study science or math is likely to be:	c $P(X) = 0.67$
d. Our class has the highest test scores in the State Math Exams. The probability that I have scored a great mark is likely to be:	e $P(X) = 1.0$
e. The Chicago baseball team has won every game in the season. The probability that the team will make it to the play offs is likely to be:	a $P(X) = 0.01$

3. Answers will vary, depending on who is doing the problem and where he or she lives.

4. Answers will vary, depending on who is doing the problem and where he or she lives.

5. The amount of lemonade in a pitcher cannot be represented by a discrete random variable, because the amount can take on any value within a certain range. For example, it could be 1 liter, 1.1 liters, 1.11 liters, 1.111 liters, and so on.

6. The number of musicians in an orchestra can be represented by a discrete random

variable, because the number will always be an integer.

7. The weights of the alligators in a swamp cannot be represented by a discrete random variable, because the weights can take on any values within a certain range. For example, a weight could be 800 pounds, 800.8 pounds, 800.88 pounds, 800.888 pounds, and so on.

8. The number of tickets sold to a movie can be represented by a discrete random variable, because the number will always be an integer.

9. Answers will vary. One possible answer is the number of trees in a forest, because the number will always be an integer.

10. Answers will vary. One possible answer is the temperature in an oven, because the temperature could take on any value within a certain range.

3.2 Probability Distribution

Answers

1. No, it does not.

2. No, it does not.

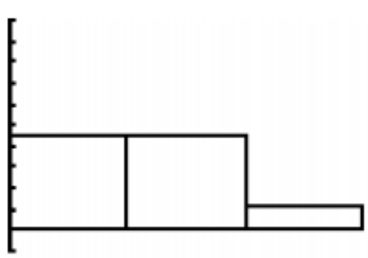
3. Yes, it does.

4. The probability that the spinner doesn't land on green on either of the spins can be calculated as follows:

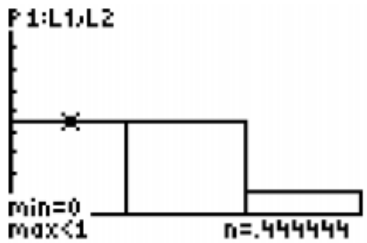
5. The probability that the spinner lands on green on 1 of the spins can be calculated as follows:

6. The probability that the spinner lands on green on both of the spins is as follows:

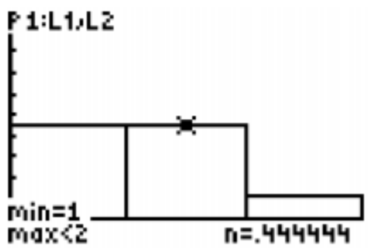
7. Answer:



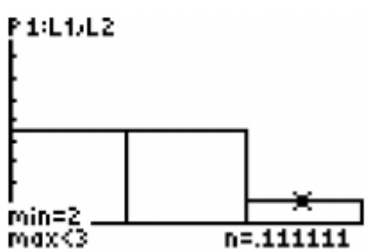
8. Answer:



9. Answer:



10. Answer:



3.3 Binomial Distributions and Probability

Answers

1. The probability of scoring below 80% can be calculated as follows: 1−0.20 = 0.80

2. The probability of not getting a job after university can be calculated as follows: 1−0.85 = 0.15

3. The probabilities are as follows:

   a.

   b.

$$P(X=1) = {}_{10}C_1 \times (0.167)^1 \times (0.833)^{(10-1)}$$
$$P(X=1) = 10 \times 0.167 \times 0.193$$
$$P(X=1) = 0.322$$

4. The probabilities are as follows:

   a.

   b.

$$P(X=1) = {}_{15}C_1 \times (0.167)^1 \times (0.833)^{(15-1)}$$
$$P(X=1) = 15 \times 0.167 \times 0.0774$$
$$P(X=1) = 0.194$$

5. The probabilities are as follows:

   a.

   b.

$$P(X=7) = {}_{15}C_7 \times (0.167)^7 \times (0.833)^{(15-7)}$$
$$P(X=7) = 6{,}435 \times (3.62 \times 10^{-6}) \times 0.231$$
$$P(X=7) = 0.00538$$

6.

+	1	2	3	4	5	6
1	1,1	2,1	3,1	4,1	5,1	6,1
2	1,2	2,2	3,2	4,2	5,2	6,2
3	1,3	2,3	3,3	4,3	5,3	6,3
4	1,4	2,4	3,4	4,4	5,4	6,4
5	1,5	2,5	3,5	4,5	5,5	6,5
6	1,6	2,6	3,6	4,6	5,6	6,6

a.

b.

$$P(X=5) = {}_{10}C_5 \times (0.306)^5 \times (0.694)^{(10-5)}$$
$$P(X=5) = 252 \times 0.00268 \times 0.161$$
$$P(X=5) = 0.109$$

7.

+	1	2	3	4	5	6
1	1,1	2,1	3,1	4,1	5,1	6,1
2	1,2	2,2	3,2	4,2	5,2	6,2
3	1,3	2,3	3,3	4,3	5,3	6,3
4	1,4	2,4	3,4	4,4	5,4	6,4
5	1,5	2,5	3,5	4,5	5,5	6,5
6	1,6	2,6	3,6	4,6	5,6	6,6

a.
b.

$$P(X=8) = {}_{15}C_8 \times (0.306)^8 \times (0.694)^{(15-8)}$$
$$P(X=8) = 6{,}435 \times (7.69 \times 10^{-5}) \times 0.0775$$
$$P(X=8) = 0.384$$

8. The probabilities are as follows:

   a.

   b.

$$P(X=11) = {}_{25}C_{11} \times (0.5)^{11} \times (0.5)^{(25-11)}$$
$$P(X=11) = 4{,}457{,}400 \times (4.88 \times 10^{-4}) \times (6.10 \times 10^{-5})$$
$$P(X=11) = 0.133$$

9. The probabilities are as follows:

   a.

   b.

$$P(X=20) = {}_{30}C_{20} \times (0.5)^{20} \times (0.5)^{(30-20)}$$
$$P(X=20) = 30{,}045{,}015 \times (9.54 \times 10^{-7}) \times (9.77 \times 10^{-4})$$
$$P(X=20) = 0.028$$

10. When tossing 2 coins, there are 4 possibilities: HH, HT, TH, and TT. Therefore, the probabilities are as follows:

    a.

    b.

$$P(X=25) = {}_{30}C_{25} \times (0.75)^{25} \times (0.25)^{(30-25)}$$

$$P(X = 25) = 142{,}506 \times (7.53 \times 10^{-4}) \times (9.77 \times 10^{-4})$$
$$P(X = 25) = 0.105$$

3.4 Multinomial Distributions

Answers

1. The probability can be calculated as follows:

$$P = \frac{9!}{2!2!2!3!} \times (0.25^2 \times 0.25^2 \times 0.25^2 \times 0.25^3)$$
$$P = 7{,}560 \times 0.0625 \times 0.0625 \times 0.0625 \times 0.015625$$
$$P = 0.029$$

2. The probability can be calculated as follows:

$$P = \frac{9!}{3!1!1!4!} \times (0.25^3 \times 0.25^1 \times 0.25^1 \times 0.25^4)$$
$$P = 2{,}520 \times 0.015625 \times 0.25 \times 0.25 \times 0.00390625$$
$$P = 0.0096$$

3. The probability can be calculated as follows:

$$P = \frac{8!}{1!2!3!2!} \times (0.35^1 \times 0.40^2 \times 0.10^3 \times 0.15^2)$$
$$P = 1{,}680 \times 0.35 \times 0.16 \times 0.001 \times 0.0225$$
$$P = 0.002$$

4. The probability can be calculated as follows:

$$P = \frac{8!}{2!1!2!3!} \times (0.35^2 \times 0.40^1 \times 0.10^2 \times 0.15^3)$$
$$P = 1{,}680 \times 0.1225 \times 0.40 \times 0.01 \times 0.003375$$
$$P = 0.003$$

5. The probability can be calculated as follows:

$$P = \frac{10!}{1!2!2!2!3!} \times (0.2^1 \times 0.2^2 \times 0.2^2 \times 0.2^2 \times 0.2^3)$$
$$P = 75{,}600 \times 0.2 \times 0.04 \times 0.04 \times 0.04 \times 0.008$$
$$P = 0.008$$

6. The probability can be calculated as follows:

$$P = \frac{10!}{2!2!2!2!2!} \times (0.2^2 \times 0.2^2 \times 0.2^2 \times 0.2^2 \times 0.2^2)$$
$$P = 113{,}400 \times 0.04 \times 0.04 \times 0.04 \times 0.04 \times 0.04$$
$$P = 0.012$$

7. The probability can be calculated as follows:

$$P = \frac{7!}{2!2!1!2!} \times (0.23^2 \times 0.19^2 \times 0.13^1 \times 0.45^2)$$
$$P = 630 \times 0.0529 \times 0.0361 \times 0.13 \times 0.2025$$

$P = 0.032$

8. The probability can be calculated as follows:

$$P = \frac{7!}{2!1!1!3!} \times (0.23^2 \times 0.19^1 \times 0.13^1 \times 0.45^3)$$
$$P = 420 \times 0.0529 \times 0.19 \times 0.13 \times 0.091125$$
$$P = 0.050$$

9. The probability can be calculated as follows:

$$P = \frac{14!}{3!2!1!4!2!2!} \times ((\frac{1}{6})^3 \times (\frac{1}{6})^2 \times (\frac{1}{6})^1 \times (\frac{1}{6})^4 \times (\frac{1}{6})^2 \times (\frac{1}{6})^2)$$
$$P = 75,675,600 \times 0.00463 \times 0.02778 \times 0.16667 \times 0.00077 \times 0.02778 \times 0.02778$$
$$P = 0.00097$$

10. The probability can be calculated as follows:

$$P = \frac{14!}{2!3!3!2!2!2!} \times ((\frac{1}{6})^2 \times (\frac{1}{6})^3 \times (\frac{1}{6})^3 \times (\frac{1}{6})^2 \times (\frac{1}{6})^2 \times (\frac{1}{6})^2)$$
$$P = 151,351,200 \times 0.02778 \times 0.00463 \times 0.00463 \times 0.02778 \times 0.02778 \times 0.02778$$
$$P = 0.00193$$

3.5 Theoretical and Experimental Spinners

Answers

1. Answers will vary, but they should be somewhat similar to the following definitions: Theoretical Probability is the calculated probability of an event. Experimental Probability is the actual probability of an event found through experimentation.

2. Answers will vary, but they should be somewhat similar to the following:
   a. The difference between theoretical and experimental probability is that theoretical probability is calculated and experimental probability is actual. In theoretical probability, you are analyzing equally likely outcomes.
   b. In general, the more data that is collected, the closer the experimental probability gets to the theoretical probability.
   c. Yes, spinning 1 spinner 100 times is the same as spinning 100 spinners 1 time, because each spin of a spinner (whether it's the same spinner or different spinners) is independent of any other spin.

3. The experimental probability of landing on blue can be calculated as follows:

4. The experimental probability of landing on purple can be calculated as follows:

5. The experimental probability of landing on green can be calculated as follows:

6. The experimental probability of landing on red can be calculated as follows:

7. The experimental probability of landing on blue can be calculated as follows:

8. The experimental probability of landing on purple can be calculated as follows:

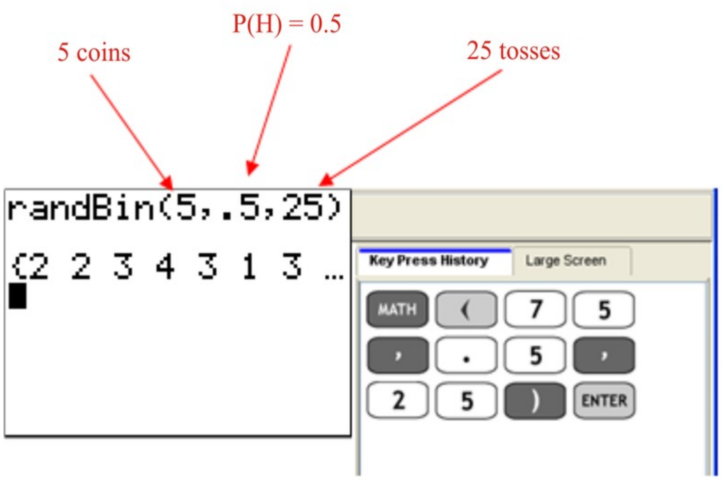
9. The experimental probability of landing on green can be calculated as follows:

10. . The experimental probability of landing on red can be calculated as follows:

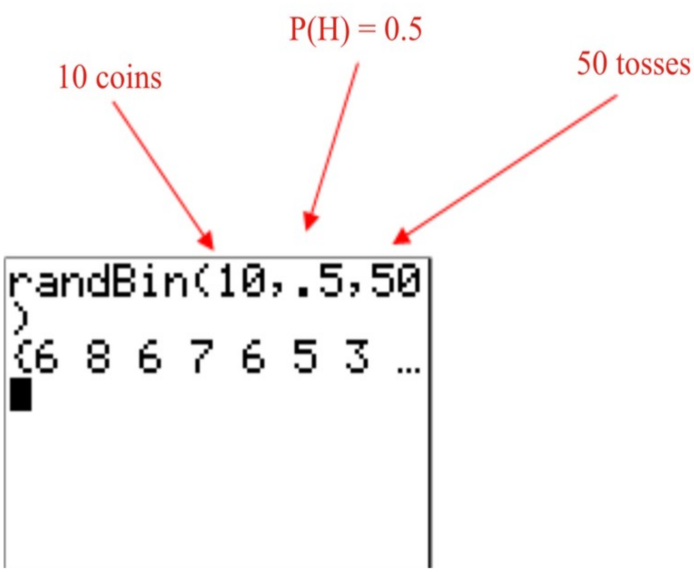
3.6 Theoretical and Experimental Coin Tosses

Answers

1. The probability will vary, depending on what the calculator returns.

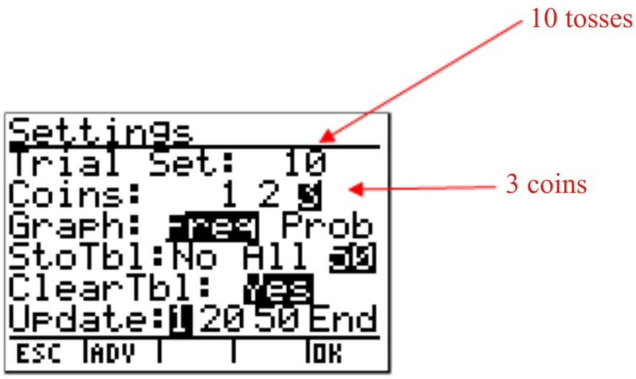


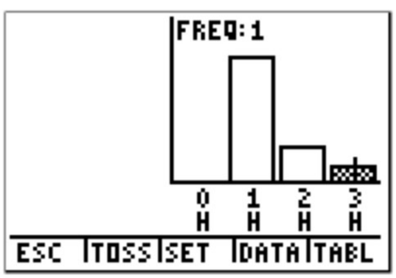
2. The probability will vary, depending on what the calculator returns.



3. The probability can be calculated as follows:

4. The probability will vary, depending on what the calculator returns.





5. The probability can be calculated as follows:

6. The probability can be calculated as follows:

7. The experimental probability of getting 0 heads can be calculated as follows:

8. The experimental probability of getting 1 head can be calculated as follows:

9. The experimental probability of getting 2 heads can be calculated as follows:

10. The experimental probability of getting 3 heads can be calculated as follows: