6.1 Basic Trigonometric Identities

Answers

1. $\cot\theta = \frac{adj}{opp} = \frac{\left(\frac{adj}{hyp}\right)}{\left(\frac{opp}{hyp}\right)} = \frac{\cos\theta}{\sin\theta}$

2. Start with the graph of $\cos\theta$. This is the same as the graph of $\cos(-\theta)$. Then, $\cos\left(-\left(\theta - \frac{\pi}{2}\right)\right)$ shifts horizontally to the right $\frac{\pi}{2}$, creating the graph of $\sin\theta$.

3. $\sec\theta = \frac{hyp}{adj} = \frac{1}{\frac{adj}{hyp}} = \frac{1}{\cos\theta}$

4. $\tan\theta\cot\theta = \frac{sin\theta}{\cos\theta} \cdot \frac{cos\theta}{\sin\theta} = 1$

5. $\sin\theta \cdot \csc\theta = \sin\theta \cdot \frac{1}{\sin\theta} = 1$

6. $\sin\theta \cdot \sec\theta = \sin\theta \cdot \frac{1}{\cos\theta} = \tan\theta$

7. $\cos\theta \cdot \csc\theta = \cos\theta \cdot \frac{1}{\sin\theta} = \cot\theta$

8. -0.81

9. 0.5

10. 4

11. $-\frac{1}{.7} = -1.43$

12. If a function is even then its graph is symmetric with respect to the y-axis. If a function is odd, then is has 180° rotation symmetry about the origin.

13. $\frac{\tan x \cdot \sec x}{\csc x} \cdot \cot x = \frac{\tan x \sin x \cos x}{\cos x \sin x} = \tan x$

14. $\frac{\sin^2 x \cdot \sec x}{\tan x} \cdot \csc x = \frac{\sin x \sin x \cos x}{\sin x \sin x \cos x} = 1$

15. $\cos x \cdot \tan x = \cos x \cdot \frac{\sin x}{\cos x} = \sin x$

6.2 Pythagorean Identities

Answers

1. $(1-\cos^2 x)(1+\cot^2 x) = \sin^2 x \cdot \csc^2 x = \sin^2 x \cdot \frac{1}{\sin^2 x} = 1$

2. $\cos x(1-\sin^2 x) = \cos x(\cos^2 x) = \cos^3 x$

3. $\sin^2 x = (1-\cos^2 x) = (1-\cos x)(1+\cos x)$

4. $\frac{\sin^2 + \cos^2 x}{\csc x} = \frac{1}{\csc x} = \sin x$

5. $\sin^4 x - \cos^4 x = (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) = (\sin^2 x - \cos^2 x) \cdot 1 = \sin^2 x - \cos^2 x$

6. $(\sin^2 x - \sin^4 x)(\cos x) = \sin^2 x(1-\sin^2 x)(\cos x) = \sin^2 x(\cos^2 x)(\cos x) = \sin^2 x(\cos^3 x)$

7. $\sec^3 x$

8. $\sin^2 x$

9. $1 - \sin x$

10. $\sin x$

11. $\sin^2 x$

12. $\sec^4 x$

13. $\sec x$

14. $\tan^2 x$

15. $\cos x$

6.3 Sum and Difference Identities

Answers

1. $\frac{\sqrt{3}-1}{2\sqrt{2}}$

2. $-\frac{\sqrt{3}-1}{2\sqrt{2}}$

3. $-\frac{1+\sqrt{3}}{2\sqrt{2}}$

4. $\frac{1+\sqrt{3}}{2\sqrt{2}}$

5. $-\sqrt{2}(1+\sqrt{3})$

6. $2+\sqrt{3}$

7. $\sin(\alpha+\beta)=\sin\big(\alpha-(-\beta)\big)=\sin\alpha\cos(-\beta)-\cos(\alpha)\sin(-\beta)=\sin\alpha\cos\beta-\cos(\alpha)\,(-1)(\sin\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

8. $\tan(\alpha+\beta)=\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}=\frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{cos\alpha cos\beta-\sin\alpha\sin\beta}=\frac{\frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta}+\frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\frac{cos\alpha cos\beta}{\cos\alpha\cos\beta}+\frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

9. $\tan(\alpha-\beta)=\tan\big(\alpha+(-\beta)\big)=\frac{\tan\alpha+\tan(-\beta)}{1-\tan\alpha\tan(-\beta)}=\frac{\tan\alpha-\tan(\beta)}{1+\tan\alpha\tan(\beta)}$

10. $\frac{1}{2}$

11. $\frac{1}{2}$

12. $\frac{\sqrt{3}}{2}$

13. $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta=2\cos\alpha\cos\beta$

14. $\tan\left(x+\frac{\pi}{4}\right)=\frac{\tan x+\tan\frac{\pi}{4}}{1-\tan x\tan\frac{\pi}{4}}=\frac{\tan x+1}{1-\tan x}=\frac{1+\tan x}{1-\tan x}$

15. $\sin(x+\pi)=\sin x\cos\pi+\cos x\sin\pi=\sin x\,(-1)+\cos x\,(0)=-\sin x$

6.4 Double, Half, and Power Reducing Identities

Answers

1. $\sin 2x = \sin(x+x) = \sin x \cos x + \cos x \sin x = 2 \sin x \cos x$

2. $\cos 2x = \cos(x+x) = \cos \mathrm{x} \cos \mathrm{x} - \sin \mathrm{x} \sin \mathrm{x} = \cos^2 x - \sin^2 x$

3. $\tan 2x = \tan(x+x) = \frac{\tan x + \tan x}{1 - tanx \tan x} = \frac{2 \tan x}{1 - \tan^2 x}$

4. $\frac{1+\cos 2x}{2} = \frac{1+\cos^2 x - \sin^2 x}{2} = \frac{1+\cos^2 x - (1-\cos^2 x)}{2} = \frac{2\cos^2 x}{2} = \cos^2 x$

5. $\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{1-\cos 2x}{2}}{\frac{1+\cos 2x}{2}} = \frac{1-\cos 2x}{1+\cos 2x}$

6. $\sin \frac{x}{2} = \pm\sqrt{\sin^2 \frac{x}{2}} = \pm\sqrt{\frac{1-\cos x}{2}}$

7. $\cos \frac{x}{2} = \pm\sqrt{\cos^2 \frac{x}{2}} = \pm\sqrt{\frac{1+\cos x}{2}}$

8. $\tan \frac{x}{2} = \pm\sqrt{\tan^2 \frac{x}{2}} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$

9. $\csc 2x = \frac{1}{\sin 2x} = \frac{1}{2 \sin x \cos x} = \frac{1}{2} \csc x \sec x$

10. $\cot 2x = \frac{\cos 2x}{\sin 2x} = \frac{\cos^2 x - \sin^2 x}{2 \sin x \cos x} = \frac{\frac{\cos^2 x - \sin^2 x}{\sin^2 x}}{\frac{2 \sin x \cos x}{\sin^2 x}} = \frac{\cot^2 x - 1}{2 \cot x}$

11. $2 - \sqrt{3}$

12. $\frac{1-\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}}$

13. $\pm\sqrt{\frac{2}{1+\frac{\sqrt{2}}{2}}}$

14. $\tan \frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}} \cdot \sqrt{\frac{1-\cos x}{1-\cos x}} = \frac{1-\cos x}{\sqrt{1-\cos^2 x}} = \frac{1-\cos x}{\sin x}$

15. $\tan \frac{x}{2} = \frac{1-\cos x}{\sin x} = \frac{1-\cos x}{\sin x} \cdot \frac{1+\cos x}{1+\cos x} = \frac{1-\cos^2 x}{\sin x(1+\cos x)} = \frac{\sin^2 x}{\sin x(1+\cos x)} = \frac{\sin x}{(1+\cos x)}$

6.5 Trigonometric Equations

Answers

1. $x = 0$

2. $x = 0, \frac{\pi}{2}, \frac{3\pi}{2}$

3. $x = 1.786, 4.497$

4. No solution

5. $x = 0.916, 1.98, \ 4.058, 5.12$

6. No solution

7. Identity

8. $x = 120°$ or $240°$

9. $x = 180°$

10. $x = 3\pi$

11. $x = \frac{13\pi}{6}, \frac{17\pi}{6}$

12. $x = 2\pi, 3\pi$

13. $x = \frac{5\pi}{2}$

14. $x = \frac{7\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}$

15. Identity