Sequences and Series

General Sequences

Review Queue Answers

1. Domain: all reals; Range: all reals
2. Domain: all reals; Range: $y \geq -2$
3. Domain: all reals; Range: $y \geq -\frac{1}{8}$

Finding the Next Term in a Sequence

1. 39, 45, 51
2. -324, 972, -2916
3. 35, 31, 27
4. 0.01, 0.001, 0.0001
5. 16, 32, 64
6. $\frac{-1}{6}, \frac{-3}{7}, \frac{-5}{8}$
7. 9; 36
8. $\frac{4}{5}; \frac{6}{7}$
9. 5; 20
10. 8; 216
11. 6; 45
12. 15; 35

Describing the Pattern and Writing a Recursive Rule for a Sequence

1. Each term is multiplied by -2 to get the next term: $a_n = -2a_{n-1}$
2. Each term is increased by 6 to get the next term: $a_n = a_{n-1} + 6$
3. Subtract 5 from each term to find the next term: $a_n = a_{n-1} - 5$
4. Multiply each term by 4 to find the next term: $a_n = 4a_{n-1}$
5. Each term is 9 more than the previous term: $a_n = a_{n-1} + 9$
6. Each term is 25 less than the previous term: $a_n = a_{n-1} - 25$
7. Each term is two thirds the previous term: $a_n = \frac{2}{3}a_{n-1}$
8. Each term is multiplied by ¾ to get the next term: $a_n = \frac{3}{4}a_{n-1}$
9. First add 4, then 5, then 6, etc (add one more each time): $a_n = a_{n-1} + (n+2)$
10. Each term is the sum of the two previous terms: $a_n = a_{n-1} + a_{n-2}$
11. Add the term number to the previous term to get each term: $a_n = a_{n-1} + n$
12. Add twice the previous term number to the previous term to get the term: $a_n = a_{n-1} + 2(n-1)$

Using and Writing Nth Term Rules for Sequences

1. 9, 11, 13, 15, 17; 27
2. -6, -11, -16; -251
3. 1, 3, 7; 1023
4. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}; \frac{1}{256}$
5. 1, 3, 6, 10; 210
6. 1, 5, 9, 13, 17

7.	$\frac{9}{2}, 4, \frac{7}{2}, 3, \frac{5}{2}$	8.	$\frac{5}{3}, \frac{13}{9}, \frac{35}{27}, \frac{97}{81}, \frac{275}{243}$	9.	0, 4, 12, 24, 40
10.	1, 5, 14, 30, 55	11.	$2n+1$	12.	$3^n - 2$
13.	$n(n+5)$	14.	$-n+7$	15.	$\frac{n(n+3)}{2}$

Series and Summation Notation

1. 2 + 4 + 6 + 8 + 10 = 30
2. 8 + 9 +10 + 11 = 38
3. 70 + 88 + 108 + 130 + 154 + 180 = 730
4. 3 + 6 + 10 + 15 + 21 = 55
5. 4 + 5 + 7 +11 + 19 + 35 = 81
6. 55.5
7. 0
8. 31
9. 654
10. 2525
11. a. 5 + 7 + 9 + 11 + 13 = 45 b. 3(5) + (2 + 4 + 6 + 8 + 10) = 45
    Both sums are the same. The constant, 3, is added to each of the five terms in part a but is just added "five times" at once in part b.
12. a. 1 + 3 + 6 + 10 + 15 = 35 b. ½ (2 + 6 + 12 + 20 + 30) = 35
    This time the ½ is distributed within the sum in part a but is kept outside and multiplied by the sum in part b.

Arithmetic Sequences and Series

Review Queue Answers

1. add two to each term to get the next term
2. add 3, then add 4, then add 5 (add one more than the last time)
3. 120

Arithmetic Sequences and Finding the Nth Term Given the Common Difference and a Term

1.	arithmetic; $a_n = n+1$	2.	not arithmetic	3.	arithmetic; $a_n = -5n+10$
4.	not arithmetic	5.	arithmetic; $a_n = 3n-3$	6.	arithmetic; $-n+14$
7.	$a_n = -8n+25$	8.	$a_n = \frac{1}{2}n - \frac{21}{2}$	9.	$a_n = -2n+30$
10.	$a_n = 3n-18$	11.	$a_n = -11n+95$	12.	$a_n = 7n-17$

Finding the Nth Term Given Two Terms

1.	$a_n = -3n+4$	2.	$a_n = 2n+1$	3.	$a_n = \frac{1}{3}n - 7$
4.	$a_n = \frac{5}{2}n + 4$	5.	$a_n = -5n+3$	6.	$a_n = 6n+13$
7.	$a_n = -2n+6$	8.	$a_n = 7n+2$	9.	$a_n = -n-1$
10.	$a_n = 8n-15$	11.	$a_n = -\frac{4}{5}n + 6$	12.	$a_n = \frac{3}{4}n + 2$

Finding the Sum of a Finite Arithmetic Series

1.	1356	2.	-84	3.	2499	4.	13	5.	875
6.	861	7.	240	8.	9900	9.	91	10.	-1860
11.	361	12.	180	13.	1207	14.	-483	15.	63
16.	630	17.	1378						

Geometric Sequences and Series

Review Queue Answers

1. $\frac{1}{2}n-3$
2. 345
3. -375

Geometric Sequences and Finding the Nth Term Given the Common Ratio and the First Term

1. arithmetic
2. geometric
3. neither
4. geometric
5. arithmetic
6. neither
7. $a_n=32\left(\frac{3}{2}\right)^{n-1}$; $\{32,48,72,108,162\}$
8. $a_n=-81\left(-\frac{1}{3}\right)^{n-1}$; $\{-81,27,-9,3,-1\}$
9. $a_n=7(2)^{n-1}$; $\{7,14,28,56,112\}$
10. $a_n=\frac{8}{125}\left(-\frac{5}{2}\right)^{n-1}$; $\left\{\frac{8}{125},-\frac{4}{25},\frac{2}{5},-1,\frac{5}{2}\right\}$
11. $a_n=162\left(\frac{2}{3}\right)^{n-1}$
12. $a_n=-625\left(\frac{3}{5}\right)^{n-1}$
13. $a_n=\frac{9}{4}\left(-\frac{2}{3}\right)^{n-1}$
14. $a_n=3(5)^{n-1}$
15. $a_n=5(2)^{n-1}$
16. $a_n=\frac{1}{2}(-4)^{n-1}$
17. $81,445
18. $29,647

Finding the Nth Term Given the Common Ratio and any Term or Two Terms

1. $a_n=54\left(\frac{2}{3}\right)^{n-1}$
2. $a_n=160\left(-\frac{3}{4}\right)^{n-1}$
3. $a_n=\frac{125}{72}\left(\frac{6}{5}\right)^{n-1}$
4. $a_n=320\left(-\frac{1}{2}\right)^{n-1}$
5. $a_n=\frac{11}{8}(2)^{n-1}$
6. $a_n=24\left(\frac{3}{2}\right)^{n-1}$
7. $a_n=48\left(\frac{1}{4}\right)^{n-1}$
8. $a_n=\frac{343}{216}\left(\frac{6}{7}\right)^{n-1}$
9. $a_n=2(3)^{n-1}$
10. $a_n=128\left(\frac{3}{2}\right)^{n-1}$
11. $a_n=\frac{8}{27}\left(\frac{3}{2}\right)^{n-1}$
12. $a_n=\frac{1}{12}(-2)^{n-1}$

13. $34,000

14. $93,000

Finding the Sum of a Finite Geometric Series

1. $\frac{844}{9}$
2. 99
3. 5
4. $\frac{1031}{50}$
5. $-\frac{15}{8}$
6. $\frac{5461}{128}$
7. $\frac{11529}{2000}$
8. $\frac{43}{2}$
9. $a_n = 3(-2)^{n-1}$
10. $a_n = 216\left(\frac{5}{6}\right)^{n-1}$
11. $a_n = 2(-3)^{n-1}$
12. $a_n = 96\left(-\frac{1}{2}\right)^{n-1}$
13. $9799
14. $1500

Infinite Series

Review Queue Answers

1. 7.96875
2. 238
3. 162

Partial Sums

1. $S_1 = 10$; $S_2 = 19$; $S_3 = 27.1$; $S_4 = 34.39$; $S_5 = 40.95$; $S_{50} = 99.485$; $S_{100} = 99.997$; $S_{500} = 100$
This infinite series converges to a sum of 100.
2. $S_1 = 8$; $S_2 = 16.24$; $S_3 = 24.727$; $S_4 = 33.469$; $S_5 = 42.473$; $S_{50} = 902.375$; $S_{100} = 4858.302$;
This infinite series will continue to grow without bound.
3. $S_1 = 0.5$; $S_2 = 1.5$; $S_3 = 3$; $S_4 = 5$; $S_5 = 7.5$; $S_{50} = 637.5$; $S_{100} = 2525$;
This infinite series will continue to grow without bound.
4. $S_1 = 10$; $S_2 = 15$; $S_3 = 18.333$; $S_4 = 20.833$; $S_5 = 22.833$; $S_{50} = 44.992$; $S_{100} = 51.874$;
This infinite series will continue to grow without bound.
5. $S_1 = 0.5$; $S_2 = 0.875$; $S_3 = 1.156$; $S_4 = 1.367$; $S_5 = 1.525$; $S_{50} = 2$; $S_{100} = 2$;
This infinite series converges to a sum of 2.
6. $S_1 = 1$; $S_2 = 1.25$; $S_3 = 1.361$; $S_4 = 1.424$; $S_5 = 1.464$; $S_{50} = 1.625$; $S_{100} = 1.635$; $S_{500} = 1.643$; $S_{750} = 1.644$; This infinite series appears to approach a sum of approximately 1.644.
7. $S_1 = 6$; $S_2 = 6.6$; $S_3 = 6.66$; $S_4 = 6.666$; $S_5 = 6.667$; $S_{50} = 6.667$;
This infinite series converges to a sum of approximately 6.667.
8. $S_1 = 5.01$; $S_2 = 10.03$; $S_3 = 15.06$; $S_4 = 20.1$; $S_5 = 25.15$; $S_{50} = 262.75$;
This infinite series will continue to grow without bound.
9. $S_1 = 2$; $S_2 = 3.75$; $S_3 = 5.281$; $S_4 = 6.621$; $S_5 = 7.793$; $S_{50} = 15.980$; $S_{100} = 16$;
This infinite series converges to a sum of 16.
10. The series in problems 3 and 8 are both arithmetic and do not have a finite sum. As n gets very large, the absolute value of each term in an arithmetic sequence is greater than that of the previous term. Therefore, the sum of the n terms will continue to grow faster and faster and not converge.
11. The series in problems 1, 2, 5, 7 and 9 are all geometric. Of these, problem 2 is the only one that does not converge. In problem 2 the value of the common ratio is greater than 1 which causes the values of the terms in the geometric sequence to grow without bound and thus the sum grows without bound.

In the convergent series, the common ratios are between 0 and 1 and thus the values of the terms in the sequences decrease in size and therefore the sums approach a finite value.

Finding the Sum of an Infinite Geometric Series

1. 15
2. no sum, $\left|-\frac{4}{3}\right| \geq 1$
3. $\frac{3}{2}$
4. no sum, $|1.1| \geq 1$
5. 10
6. $\frac{7}{8}$
7. 2
8. no sum, $|1.05| \geq 1$
9. 4
10. 180
11. no sum, $\left|\frac{3}{2}\right| \geq 1$
12. 125