2.1 Inductive Reasoning from Patterns

Answers

1. 4^{th} figure: 9 dots, 10^{th} figure: 21dots

2. 4^{th} figure: 20 dots, 10^{th} figure: 110 dots

3. 4^{th} figure: 13 dots, 10^{th} figure: 37 dots

4. a)

b) There are two more points in each star than its figure number.

5. a) 10 triangles

b) 48

6. 20, 23, 26

7. -19, -24, -29

8. 64, 128, 256

9. 12, 1, -10

10. ***Possible Answers*** -16, 0, 32 or -12, 0, -16

11. $\frac{6}{7}, \frac{7}{8}, \frac{8}{9}$

12. $\frac{12}{23}, \frac{14}{27}, \frac{16}{31}$

13. 21, -25, 29

14. 38, 57; the amount that is added is increasing by two with each term. From the first to the second term 3 was added, then 5, then 7, then 9, etc.

15. 48, 67; the amount that is added is increasing by two with each term. From the first to the second term 5 was added, then 7, then 9, then 11, etc.

16. 216, 343; the term number cubed; $1^3, 2^3, 3^3, 4^3, \ldots, n^3$.

17. 8, 13; add the previous two terms together to get the next term. This particular sequence is called a *Fibonacci* sequence.

    1 + **1** = **2**, 1 + 2 = **3**, 2 + 3 = **5**, 3 + 5 = **8**,

2.2 Deductive Reasoning

Answers

1. I am a smart person. Law of Detachment

2. No conclusion, the fact that Ann is driving today has nothing to do with the previous statement.

3. ΔABC is equilateral. Law of Detachment

4. If North wins, then East loses. Law of Syllogism.

5. If $z > 5$, then $y > 7$. Law of Syllogism.

6. I am not cold. Law of Contrapositive.

7. No conclusion. You cannot say "I don't need an umbrella," because that would be completing the inverse of the first statement.

8. If a shape is a circle, then we don't need to study it. Law of Syllogism.

9. You don't text while driving. Law of Contrapositive.

10. It is sunny outside. Law of Detachment.

11. You are not wearing sunglasses. Law of Contrapositive; you can think of "cloudy" as "not sunny."

12. Switch the first statement to be "If my mom asks me to, then I will clean my room." The conclusion would then be: My mom didn't ask me to. Law of Contrapositive.

13. This is a logical argument, but it doesn't make sense because we know that circles exist.

$p \rightarrow q$
$q \rightarrow r$
$r \rightarrow s$
$s \rightarrow t$
$\therefore p \rightarrow t$

14. Law of Detachment.

$p \rightarrow q$
p
$\therefore q$

15. Law of Contrapositive.

$p \rightarrow q$
$\sim q$
$\therefore \sim p$

2.3 If-Then Statements

Answers

1. Hypothesis: 5 divides evenly into *x*.

   Conclusion: *x* ends in 0 or 5.

2. Hypothesis: A triangle has three congruent sides.

   Conclusion: It is an equilateral triangle.

3. **Here, the "if" is in the middle of the statement, making the hypothesis the second half.**

   Hypothesis: Three points lie in the same plane.

   Conclusion: The three points are coplanar.

4. Hypothesis: $x = 3$.

   Conclusion: $x^2 = 9$.

5. Hypothesis: You take yoga.

   Conclusion: You are relaxed.

6. Hypothesis: You are a baseball player.

   Conclusion: You wear a hat.

7. Hypothesis: I am 16 years old.

   Conclusion: I will learn how to drive.

8. Hypothesis: You do your homework.

   Conclusion: You can watch TV.

9. Hypothesis: The lines are parallel.

   Conclusion: Alternate interior angles are congruent.

10. Hypothesis: You are a kid.

    Conclusion: You like ice cream.

2.4 Converse, Inverse, and Contrapositive

Answers

1. Not necessarily, A, B, and C need to be collinear in order for B to be a midpoint.

2. If B is the midpoint of $\overline{AC}$, then AB = 5 and BC = 5. This could be true, but we don't know the length of AC. AB = BC, but we cannot say they are 5 without knowing the length of AC.

3. If $AB \neq 5$ and $BC \neq 5$, then B is not the midpoint of $\overline{AC}$. Again, this could be true, but we don't know AC. Also, A, B and C might not be collinear.

4. If $AB \neq 5$ and $BC \neq 5$, then B is not the midpoint of $\overline{AC}$. It is the same as #3.

5. If an angle is less than 90°, then it is acute. *True*.

   Biconditional: An angle is acute if and only if it is less than 90°.

6. If you are sun burnt, then you are at the beach. False, you could be anywhere there is sun (amusement park, baseball game, on a boat, etc).

7. If $x + 3 > 7$, then $x > 4$. *True*.

   Biconditional: $x + 3 > 7$ if and only if $x > 4$.

8. If a U.S. citizen can vote, then he or she is 18 or more years old.

   If a U.S. citizen is 18 or more years old, then he or she can vote.

9. If a whole number is prime, then its factors are 1 and itself.

   If a whole number's factors are only 1 and itself, then it is prime.

10. If $2x = 18$, then $x = 9$.

    If $x = 9$, then $2x = 18$.

2.5 Conjectures and Counterexamples

Answers

1. *n* = 1 would be a counterexample because $1^2 \not> 1$. Recall that a whole number is 0, 1, 2, 3, … n, so 0 could also be a counterexample.

2. Counterexamples include: 21, 51, 81, 121, and 151

3. $\frac{4}{3}$ is one counterexample. Any positive improper fraction (where the numerator is greater than the denominator) could be a counterexample.

4. A triangle is a counterexample.

5. A girl that doesn't like ice cream would be a counterexample.

6. Not everyone takes choir in high school.

7. Obtuse angles do not have complementary angles.

8. 13, 14, 15 year-olds are teenagers and cannot drive yet.

9. Any negative integer could be a counterexample.

10. Equations can have any real number as a solution.

2.6 Properties of Equality and Congruence

Answers

1. $3x + 11 = -16$

$3x = -27$	Subtraction PoE (subtract 11 from both sides)
$x = -9$	Division PoE (divide both sides by 3)

2. $7x - 3 = 3x - 35$

$4x - 3 = -35$	Subtraction PoE
$4x = -32$	Addition PoE
$x = -8$	Division PoE

3. $\frac{2}{3}g + 1 = 19$

$\frac{2}{3}g = 18$	Subtraction PoE
$g = 27$	Multiplication PoE

4. $\frac{1}{2}MN = 5$

$MN = 10$	Multiplication PoE

5. $5m\angle ABC = 540°$

$m\angle ABC = 108°$	Division PoE

6. $10b - 2(b + 3) = 5b$

$10b - 2b + 6 = 5b$	Distributive Property
$8b + 6 = 5b$	Combine like terms
$6 = -3b$	Subtraction PoE
$-2 = b$	Division PoE
$b = -2$	Symmetric PoE (this step is not necessary, but helpful to see how the Symmetric property works)

7. $\frac{1}{4}y + \frac{5}{6} = \frac{1}{3}$

$3y + 10 = 4$	Multiplication PoE (multiplied everything by 12)
$3y = -6$	Subtraction PoE
$y = -2$	Division PoE

Students could have also found a common denominator and would have ended up with the same answer.

8. $\frac{1}{4}AB + \frac{1}{3}AB = 12 + \frac{1}{2}AB$

$3AB + 4AB = 144 + 6AB$	Multiplication PoE (multiplied everything by 12)
$7AB = 144 + 6AB$	Combine like terms
$AB = 144$	Subtraction PoE

9. $y + z = x + y$

10. $CD = 5$

11. $y - 7 = z + 4$

2.7 Two-Column Proofs

Answers

1.

Statement	*Reason*
1. $\angle ABC \cong \angle DEF$, $\angle GHI \cong \angle JKL$	Given
2. $m\angle ABC = m\angle DEF$ $m\angle GHI = m\angle JKL$	$\cong \angle$s have = measures
3. $m\angle ABC + m\angle GHI = m\angle DEF + m\angle GHI$	Addition PoE
4. $m\angle ABC + m\angle GHI = m\angle DEF + m\angle JKL$	Substitution

2.

Statement	*Reason*
1. M is the midpoint of $\overline{AN}$, N is the midpoint $\overline{MB}$	Given
2. $AM = MN$, $MN = NB$	Definition of a midpoint
3. $AM = NB$	Transitive

3.

Statement	*Reason*
1. $\overline{AC} \perp \overline{BD}$, $\angle 1 \cong \angle 4$	Given
2. $m\angle 1 = m\angle 4$	$\cong$ angles have = measures
3. $\angle ACB$ and $\angle ACD$ are right angles	$\perp$ lines create right angles
4. $m\angle ACB = 90°$ $m\angle ACD = 90°$	Definition of right angles
5. $m\angle 1 + m\angle 2 = m\angle ACB$ $m\angle 3 + m\angle 4 = m\angle ACD$	Angle Addition Postulate
6. $m\angle 1 + m\angle 2 = 90°$ $m\angle 3 + m\angle 4 = 90°$	Substitution
7. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	Substitution
8. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 1$	Substitution
9. $m\angle 2 = m\angle 3$	Subtraction PoE
10. $\angle 2 \cong \angle 3$	$\cong$ angles have = measures

4.

Statement	***Reason***
1. $\angle MLN \cong \angle OLP$	Given
2. $m\angle MLN = m\angle OLP$	$\cong$ angles have = measures
3. $m\angle MLO = m\angle MLN + m\angle NLO$ $m\angle NLP = m\angle NLO + m\angle OLP$	Angle Addition Postulate
4. $m\angle NLP = m\angle NLO + m\angle MLN$	Substitution
5. $m\angle MLO = m\angle NLP$	Substitution
6. $\angle NLP \cong \angle MLO$	$\cong$ angles have = measures

5.

Statement	***Reason***
1. $\overline{AE} \perp \overline{EC}$, $\overline{BE} \perp \overline{ED}$	Given
2. $\angle BED$ is a right angle $\angle AEC$ is a right angle	$\perp$ lines create right angles
3. $m\angle BED = 90°$ $m\angle AEC = 90°$	Definition of a right angle
4. $m\angle BED = m\angle 2 + m\angle 3$ $m\angle AEC = m\angle 1 + m\angle 3$	Angle Addition Postulate
5. $90° = m\angle 2 + m\angle 3$ $90° = m\angle 1 + m\angle 3$	Substitution
6. $m\angle 2 + m\angle 3 = m\angle 1 + m\angle 3$	Substitution
7. $m\angle 2 = m\angle 1$	Subtraction PoE
8. $\angle 2 \cong \angle 1$	$\cong$ angles have = measures

6.

Statement	***Reason***
1. $\angle L$ is supplementary to $\angle M$ $\angle P$ is supplementary to $\angle O$ $\angle L \cong \angle O$	Given
2. $m\angle L = m\angle O$	$\cong$ angles have = measures
3. $m\angle L + m\angle M = 180°$ $m\angle P + m\angle O = 180°$	Definition of supplementary angles
4. $m\angle L + m\angle M = m\angle P + m\angle O$	Substitution
5. $m\angle L + m\angle M = m\angle P + m\angle L$	Substitution
6. $m\angle M = m\angle P$	Subtraction PoE
7. $\angle M \cong \angle P$	$\cong$ angles have = measures

7.

Statement	***Reason***
1. $\angle 1 \cong \angle 4$	Given
2. $m\angle 1 = m\angle 4$	$\cong$ angles have = measures
3. $\angle 1$ and $\angle 2$ are a linear pair $\angle 3$ and $\angle 4$ are a linear pair	Given (by looking at the picture) could also be Definition of a Linear Pair
4. $\angle 1$ and $\angle 2$ are supplementary $\angle 3$ and $\angle 4$ are supplementary	Linear Pair Postulate
5. $m\angle 1 + m\angle 2 = 180°$ $m\angle 3 + m\angle 4 = 180°$	Definition of supplementary angles
6. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	Substitution
7. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 1$	Substitution
8. $m\angle 2 = m\angle 3$	Subtraction PoE
9. $\angle 2 \cong \angle 3$	$\cong$ angles have = measures

8.

Statement	***Reason***
1. $\angle C$ and $\angle F$ are right angles	Given
2. $m\angle C = 90°$, $m\angle F = 90°$	Definition of a right angle
3. $90° + 90° = 180°$	Addition of real numbers
4. $m\angle C + m\angle F = 180°$	Substitution

9.

Statement	***Reason***
1. $l \perp m$	Given
2. $\angle 1$ and $\angle 2$ are right angles	$\perp$ lines create right angles.
3. $\angle 1 \cong \angle 2$	Right Angles Theorem

10.

Statement	***Reason***
1. $m\angle 1 = 90°$	Given
2. $\angle 1$ and $\angle 2$ are a linear pair	Definition of a linear pair
3. $\angle 1$ and $\angle 2$ are supplementary	Linear Pair Postulate
4. $m\angle 1 + m\angle 2 = 180°$	Definition of supplementary angles
5. $90° + m\angle 2 = 180°$	Substitution
6. $m\angle 2 = 90°$	Subtraction PoE

11.

Statement	***Reason***
1. $l \perp m$	Given
2. $\angle 1$ and $\angle 2$ make a right angle	$\perp$ lines create right angles
3. $m\angle 1 + m\angle 2 = 90°$	Definition of a right angle OR Substitution PoE
4. $\angle 1$ and $\angle 2$ are complementary	Definition of complementary angles

12.

Statement	***Reason***
1. $l \perp m$, $\angle 2 \cong \angle 6$	Given
2. $m\angle 2 = m\angle 6$	$\cong$ angles have = measures
3. $\angle 5 \cong \angle 2$	Vertical Angles Theorem
4. $m\angle 5 = m\angle 2$	$\cong$ angles have = measures
5. $m\angle 5 = m\angle 6$	Transitive